

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO4"];
```

```
<< FreeLie.m;
```

```
<< AwCalculus.m;
```

```
$SeriesShowDegree = 4;
```

FreeLie` implements / extends

```
{*, +, **, $SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Λ, σ, ħ, ↦, ↪}.
```

FreeLie` is in the public domain. Dror Bar-Natan is committed to

support it within reason until July 15, 2022. This is version 150806.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm,

hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed

to support it within reason until July 15, 2022. This is version 150806.

```
x = LW@"x"; y = LW@"y";
```

```
{α, β, γ} = LS /@ {x + b[x, y], y - b[x, b[x, y]], x + y - 2 b[x, y]}
```

```
{LS[ $\overline{x}$ ,  $\overline{xy}$ , 0, 0, ...], LS[ $\overline{y}$ , 0,  $-\overline{x\overline{xy}}$ , 0, ...], LS[ $\overline{x + y}$ ,  $-2\overline{xy}$ , 0, 0, ...]}
```

```
{λ = <x → α, y → β>, γ // Dλ}
```

```
{<LW[x] → LS[ $\overline{x}$ ,  $\overline{xy}$ , 0, 0, ...], LW[y] → LS[ $\overline{y}$ , 0,  $-\overline{x\overline{xy}}$ , 0, ...]>,
LS[0, 0,  $\overline{x\overline{xy}}$ ,  $-\overline{x\overline{xy}y}$ , ...]}
```

```
λ1 = λ; λ2 = <x → β, y → γ>; tb[λ1, λ2]
```

```
{LW[x] → LS[0, 0,  $\overline{x\overline{xy}}$ ,  $-\overline{x\overline{xy}y}$ , ...], LW[y] → LS[0, 0,  $\overline{x\overline{xy}}$ ,  $-\overline{x\overline{xy}y}$ , ...]}
```

```
j[λ]
```

```
CWS[ $\overline{x + y}$ ,  $-\overline{xy}$ ,  $-\overline{xy}$ ,  $-\overline{xyxy} + \overline{xyxy}$ , ...]
```

```
div[λ]
```

```
CWS[ $\overline{x + y}$ ,  $-\overline{xy}$ ,  $-\overline{xy}$ , 0, ...]
```

```
divy[λy]
```

```
CWS[ $\overline{y}$ , 0,  $-\overline{xy}$ , 0, ...]
```

```
divy[x + y]
```

```
CW[y]
```

```

τ = FreeLie`Private`τ
FreeLie`Private`τ

τ[y, y]
AW[]

tr[AW["y"]]
CW[y]

tr["y"]
y

lhs = j[BCHtb[λ1, λ2]]; rhs = j[λ1] + eDλ1[j[λ2]];
{lhs, (lhs ≡ rhs)@{8}}
{CWS[x̄ + 2 ȳ̄, -3 xȳ̄, 0, -9 xxyȳ̄ + 9 xyxȳ, ...], BS[9 True, ...]}

```