

Pensieve header: A ν^{-1} -like computation, except with the middle strand punctured, as suggested by Zsuzsi.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\WKO3"];
<< "../WKO4/FreeLie.m";
<< "../WKO4/AwCalculus.m";
$SeriesShowDegree = 4;
```

FreeLie` implements / extends

{*, +, **, \$SeriesShowDegree, <>, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords, AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw, CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE, Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization, Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support, t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ℓ, Δ, σ, ħ, ↦, ↠}.

FreeLie` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA, hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Δ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed to support it within reason until July 15, 2022. This is version 150909.

```
ϕs[2, 1] = ϕs[3, 1] = ϕs[3, 2] = 0; ϕs[3, 1, 2] = 1/24; ϕ0 = DKS[3, ϕs];
SeriesSolve[ϕ0, (ϕ0σ[3,2,1] ≡ -ϕ0) ∧ (ϕ0 ** ϕ0σ[1,2,3,4] ** ϕ0σ[2,3,4] ≡ ϕ0σ[12,3,4] ** ϕ0σ[1,2,34])]];
ϕ0@{6}
```

SeriesSolve: In degree 3 arbitrarily setting {ϕs[3, 1, 1, 2] → 0}.

SeriesSolve: In degree 5 arbitrarily setting {ϕs[3, 1, 1, 1, 1, 2] → 0}.

$$DKS\left[0, \frac{1}{24} t_{13} t_{23}, 0, -\frac{7 t_{13} t_{23} t_{23} t_{23}}{5760} + \frac{7 t_{13} t_{13} t_{23} t_{23}}{5760} - \frac{t_{13} t_{13} t_{13} t_{23}}{1440}, \right. \\ \left. 0, \frac{31 t_{13} t_{23} t_{23} t_{23} t_{23}}{967680} - \frac{157 t_{13} t_{13} t_{23} t_{23} t_{13} t_{23}}{1935360} - \frac{31 t_{13} t_{23} t_{13} t_{23} t_{23} t_{23}}{387072} - \right. \\ \left. \frac{31 t_{13} t_{13} t_{23} t_{23} t_{23} t_{23}}{483840} + \frac{11 t_{13} t_{13} t_{13} t_{23} t_{13} t_{23}}{290304} + \frac{31 t_{13} t_{13} t_{23} t_{13} t_{23} t_{23}}{725760} + \right. \\ \left. \frac{83 t_{13} t_{13} t_{13} t_{23} t_{23} t_{23}}{967680} - \frac{13 t_{13} t_{13} t_{13} t_{13} t_{23} t_{23}}{241920} + \frac{t_{13} t_{13} t_{13} t_{13} t_{13} t_{23}}{60480}, \dots \right]$$

```
DK2Es[s___][s_] := El[s // aMap[s], CWS[0]] // r;
DK2Es[1, 2, 3][a_0]
```

$$\begin{aligned}
 \text{Es} \left[\left(1 \rightarrow \text{LS} \left[0, \frac{23}{24}, 0, -\frac{1123}{1440} + \frac{71223}{5760} + \frac{1233}{5760} - \frac{72223}{5760} + \right. \right. \right. \\
 \left. \left. \frac{72233}{5760} + \frac{1}{480} \frac{1213}{1213} - \frac{1323}{1920} + \frac{1}{640} \frac{1232}{1232} - \frac{1322}{1152} - \frac{1332}{1152} - \frac{2333}{1440}, \dots \right], \right. \\
 2 \rightarrow \text{LS} \left[0, -\frac{13}{24}, 0, \frac{1113}{1440} - \frac{1123}{1152} + \frac{71223}{1920} - \frac{1}{480} \frac{1132}{1132} - \frac{1133}{5760} + \frac{1233}{1152} + \right. \\
 \left. \frac{71213}{5760} + \frac{191323}{5760} + \frac{71232}{1920} + \frac{71322}{5760} + \frac{71332}{5760} + \frac{1333}{1440}, \dots \right], \\
 3 \rightarrow \text{LS} \left[0, \frac{12}{24}, 0, -\frac{1112}{1440} + \frac{1123}{5760} + \frac{71223}{5760} + \frac{71122}{5760} - \frac{1132}{1440} - \frac{1233}{1440} + \frac{1213}{5760} + \right. \\
 \left. \frac{1323}{1440} - \frac{1232}{1152} - \frac{71222}{5760} - \frac{71322}{5760} - \frac{1332}{1440}, \dots \right], \text{CWS}[0, 0, 0, 0, \dots] \Big]
 \end{aligned}$$

```
vinv = a_0 // DK2Es[1, 2, 3] // dS[2] // dm[3, 2, 2] // dm[2, 1, x]
```

$$\text{Es} \left[\langle x \rightarrow \text{LS}[0, 0, 0, 0, \dots] \rangle, \text{CWS} \left[0, \frac{xx}{24}, 0, -\frac{xxxx}{2880}, \dots \right] \right]$$

```
vinv@{6}
```

$$\text{Es} \left[\langle x \rightarrow \text{LS}[0, 0, 0, 0, 0, 0, \dots] \rangle, \text{CWS} \left[0, \frac{xx}{24}, 0, -\frac{xxxx}{2880}, 0, \frac{xxxxxx}{181440}, \dots \right] \right]$$

```
zv = a_0 // DK2Es[1, 2, 3] // dS[2] // t[2] // dm[3, 2, 2] // dm[2, 1, x];
zv@{6}
```

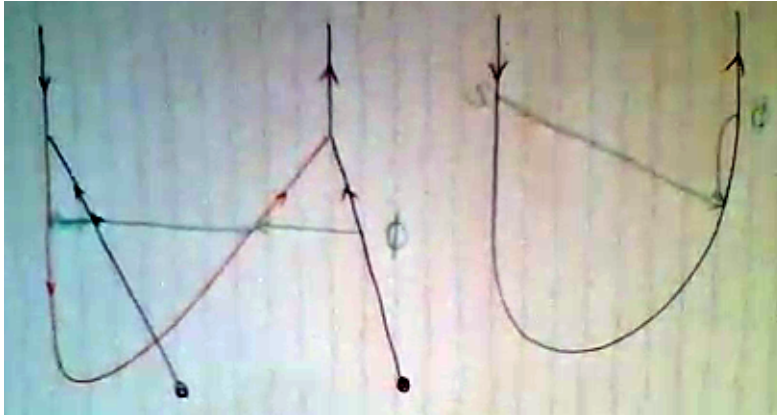
$$\text{Es} \left[\langle x \rightarrow \text{LS}[0, 0, 0, 0, 0, 0, \dots] \rangle, \text{CWS} \left[0, \frac{xx}{24}, 0, -\frac{7xxxx}{2880}, 0, \frac{31xxxxxx}{181440}, \dots \right] \right]$$

```
(zv ** vinv ** zv ** vinv)@{6}
```

$$\text{Es} \left[\langle x \rightarrow \text{LS}[0, 0, 0, 0, 0, 0, \dots] \rangle, \text{CWS} \left[0, \frac{xx}{6}, 0, -\frac{xxxx}{180}, 0, \frac{xxxxxx}{2835}, \dots \right] \right]$$

```
(vinv // d[x, x, y] // dm[x, y, x])@{6}
```

$$\text{Es} \left[\langle x \rightarrow \text{LS}[0, 0, 0, 0, 0, 0, \dots] \rangle, \text{CWS} \left[0, \frac{xx}{6}, 0, -\frac{xxxx}{180}, 0, \frac{xxxxxx}{2835}, \dots \right] \right]$$



```
zV1 =  $\emptyset$  // DK2Es[1, 2, 3] // dS[1] // t $\eta$ [2] // dm[1, 2, 1] // dm[1, 3, x];
zV1@{6}
```

```
Es[⟨x → LS[0, 0, 0, 0, 0, 0, ...]⟩, CWS[0, - $\frac{xx}{24}$ , 0,  $\frac{xxxx}{2880}$ , 0, - $\frac{xxxxxx}{181440}$ , ...]]
```

```
(zV1 ** vinv)@{6}
```

```
Es[⟨x → LS[0, 0, 0, 0, 0, 0, ...]⟩, CWS[0, 0, 0, 0, 0, 0, ...]]
```