

A^v in Degree 2

Pensieve Header: We analyze A^v in degree 2.

```

SetDirectory["C:/drorbn/AcademicPensieve/Projects/WKO"];
<< Dimensions.m

ShowMatrix[mat_] := MatrixForm[Prepend[mat,
  diags /. Diag[ar[i_, j_], ar[k_, l_]] -> Column[{
    1000 i + 100 j + 10 k + 1
  }]
]]

Print[Diagrams[2 ar]]

{Diag[ar[1, 2], ar[3, 4]], Diag[ar[1, 2], ar[4, 3]], Diag[ar[2, 1], ar[3, 4]],
  Diag[ar[2, 1], ar[4, 3]], Diag[ar[1, 3], ar[2, 4]], Diag[ar[1, 3], ar[4, 2]],
  Diag[ar[2, 4], ar[3, 1]], Diag[ar[3, 1], ar[4, 2]], Diag[ar[1, 4], ar[2, 3]],
  Diag[ar[1, 4], ar[3, 2]], Diag[ar[2, 3], ar[4, 1]], Diag[ar[3, 2], ar[4, 1]]}

diags = {
  Diag[ar[1, 3], ar[4, 2]], Diag[ar[2, 4], ar[3, 1]],
  Diag[ar[1, 3], ar[2, 4]], Diag[ar[3, 1], ar[4, 2]],
  Diag[ar[1, 4], ar[3, 2]], Diag[ar[1, 4], ar[2, 3]],
  Diag[ar[2, 3], ar[4, 1]], Diag[ar[3, 2], ar[4, 1]],
  Diag[ar[1, 2], ar[3, 4]], Diag[ar[2, 1], ar[3, 4]],
  Diag[ar[1, 2], ar[4, 3]], Diag[ar[2, 1], ar[4, 3]]
}

{Diag[ar[1, 3], ar[4, 2]], Diag[ar[2, 4], ar[3, 1]], Diag[ar[1, 3], ar[2, 4]],
  Diag[ar[3, 1], ar[4, 2]], Diag[ar[1, 4], ar[3, 2]], Diag[ar[1, 4], ar[2, 3]],
  Diag[ar[2, 3], ar[4, 1]], Diag[ar[3, 2], ar[4, 1]], Diag[ar[1, 2], ar[3, 4]],
  Diag[ar[2, 1], ar[3, 4]], Diag[ar[1, 2], ar[4, 3]], Diag[ar[2, 1], ar[4, 3]]}

Diagrams[R6T]

{Diag[R6T[1, 2, 3]], Diag[R6T[1, 3, 2]], Diag[R6T[2, 1, 3]],
  Diag[R6T[2, 3, 1]], Diag[R6T[3, 1, 2]], Diag[R6T[3, 2, 1]]}

rels = R /@ Diagrams[R6T]

{Diag[ar[1, 2], ar[3, 4]] + Diag[ar[1, 3], ar[2, 4]] - 2 Diag[ar[1, 4], ar[2, 3]],
  Diag[ar[1, 2], ar[4, 3]] - Diag[ar[1, 3], ar[2, 4]] +
  Diag[ar[1, 4], ar[2, 3]] - Diag[ar[1, 4], ar[3, 2]], -Diag[ar[1, 3], ar[2, 4]] +
  Diag[ar[1, 4], ar[2, 3]] - Diag[ar[1, 4], ar[3, 2]] + Diag[ar[2, 1], ar[3, 4]],
  -Diag[ar[2, 1], ar[3, 4]] + Diag[ar[2, 3], ar[4, 1]] + Diag[ar[3, 1], ar[4, 2]] -
  Diag[ar[3, 2], ar[4, 1]], -Diag[ar[1, 2], ar[4, 3]] +
  Diag[ar[2, 3], ar[4, 1]] + Diag[ar[3, 1], ar[4, 2]] - Diag[ar[3, 2], ar[4, 1]],
  -Diag[ar[2, 1], ar[4, 3]] - Diag[ar[3, 1], ar[4, 2]] + 2 Diag[ar[3, 2], ar[4, 1]]}

```

```

mat = SparseArray[
  Join @@ Table[
    rel = rels[[i]];
    {i, Position[diags, #][[1, 1]]} → Coefficient[rel, #] & /@
    Cases[{rel}, diag_Diag, Infinity],
    {i, Length[rels]}
  ],
  {Length[rels], Length[diags]}
];
mat // ShowMatrix

```

$$\begin{pmatrix} 1342 & 2431 & 1324 & 3142 & 1432 & 1423 & 2341 & 3241 & 1234 & 2134 & 1243 & 2143 \\ 0 & 0 & 1 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
ShowMatrix[RowReduce[mat]]
```

$$\begin{pmatrix} 1342 & 2431 & 1324 & 3142 & 1432 & 1423 & 2341 & 3241 & 1234 & 2134 & 1243 & 2143 \\ 0 & 0 & 1 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The following can only be executed after loading code from "AcademicPensieve/Projects/Arrow_Diagrams_and_gl(N)/Arrow_Diagrams_and_gl(N).nb":

```

Import[
  "http://katlas.math.toronto.edu/drorbn/AcademicPensieve/Projects/Arrow_Diagrams_and
    _gl(N)/Arrow_Diagrams_and_gl(N).m"
]
(# → Wgl[#][n, k]) & /@ diags

```

$$\left\{ \begin{aligned} & \text{Diag}[\text{ar}[1, 3], \text{ar}[4, 2]] \rightarrow \frac{1}{4}, \text{Diag}[\text{ar}[2, 4], \text{ar}[3, 1]] \rightarrow \frac{1}{4}, \text{Diag}[\text{ar}[1, 3], \text{ar}[2, 4]] \rightarrow \frac{1}{4}, \\ & \text{Diag}[\text{ar}[3, 1], \text{ar}[4, 2]] \rightarrow \frac{1}{4}, \text{Diag}[\text{ar}[1, 4], \text{ar}[3, 2]] \rightarrow -\frac{1}{4} + \frac{k}{2} - \frac{k^2}{2} + \frac{n^2}{2}, \\ & \text{Diag}[\text{ar}[1, 4], \text{ar}[2, 3]] \rightarrow -\frac{1}{4} - \frac{k}{2} + \frac{k^2}{2} + \frac{n}{2} - kn + \frac{n^2}{2}, \\ & \text{Diag}[\text{ar}[2, 3], \text{ar}[4, 1]] \rightarrow -\frac{1}{4} + \frac{k}{2} - \frac{k^2}{2} - \frac{n}{2} + kn, \text{Diag}[\text{ar}[3, 2], \text{ar}[4, 1]] \rightarrow \frac{1}{4} - \frac{k}{2} + \frac{k^2}{2}, \\ & \text{Diag}[\text{ar}[1, 2], \text{ar}[3, 4]] \rightarrow \frac{1}{4} - k + k^2 + n - 2kn + n^2, \\ & \text{Diag}[\text{ar}[2, 1], \text{ar}[3, 4]] \rightarrow -\frac{1}{4} + k - k^2 - \frac{n}{2} + kn, \\ & \text{Diag}[\text{ar}[1, 2], \text{ar}[4, 3]] \rightarrow -\frac{1}{4} + k - k^2 - \frac{n}{2} + kn, \text{Diag}[\text{ar}[2, 1], \text{ar}[4, 3]] \rightarrow \frac{1}{4} - k + k^2 \end{aligned} \right\}$$