Review of the paper
"An unexpected cyclic symmetry of $I\mathfrak{u}_n$"
by D. Bar-Natan and R. van der Veen
submitted to *Transformation Groups*

Let $\mathfrak{u}_n$ be the Lie algebra of upper triangular $n \times n$ matrices. Consider the solvable Lie algebra $I\mathfrak{u}_n = \mathfrak{u}_n \ltimes \mathfrak{u}_n^*$, where $\mathfrak{u}_n^*$ is an Abelian ideal on which $\mathfrak{u}_n$ acts via the coadjoint representation; it is a special case of the Drinfeld double construction. The aim of this note is to describe the Lie algebra structure of $I\mathfrak{u}_n$ of order n which mixes $\mathfrak{u}_n$ with $\mathfrak{u}_n^*$ and does not come from an automorphism of $\mathfrak{u}_n$ or from an inner automorphism of $I\mathfrak{u}_n$. The authors motivate their interest in the Lie algebra $I\mathfrak{u}_n$ and its automorphisms by claiming that the algebras of this kind are of importance in knot theory and in representation theory. (Such kind of applications is not discussed in the note, however.) A similar automorphism is constructed for a certain deformation of the Lie algebra $I\mathfrak{u}_n$.

The construction of the above-mentioned automorphism is handmade, just by representing $I\mathfrak{u}_n$ by generators and relations. No conceptual explanation for existence of such an automorphism is given. The result by itself is a kind of peculiar example which does not lead to further developments (at least not in this note). For some unexplained reason, the authors also add the script of a computer program for testing the main theorem (which looks strange, because a general theoretical proof exists).

A conceptual explanation for the existence of the above-mentioned automorphism of $I\mathfrak{u}_n$ (or its deformed version) and in fact a wide extension of this result to a complete description of the automorphism group of $I\mathfrak{b}$, where $\mathfrak{b}$ is a Borel subalgebra in any simple Lie algebra, was obtained in a subsequent preprint of M. Bulois and N. Ressayre `arXiv:2002.03395`, which completely overrides this note. Paying tribute to the priority of the note under review in raising the problem of describing automorphisms of this kind of Drinfeld doubles, I still don't think that this is a sufficient reason for recommending this note for publication in TG; in my opinion, it does not match the high level of the journal.