

Pensieve Header: The separation powers of various invariants.

Loading the invariants

(Alt) In[]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
```

(Alt) In[]:=

```
Do[
  NumberOfKnots[n, Alternating] =
    {0, 0, 1, 1, 2, 3, 7, 18, 41, 123, 367, 1288, 4878, 19536, 85263}[[n]];
  NumberOfKnots[n, NonAlternating] =
    {0, 0, 0, 0, 0, 0, 0, 3, 8, 42, 185, 888, 5110, 27436, 168030}[[n]],
  {n, 15}
];
NumberOfKnots[n_Integer] :=
  NumberOfKnots[n, Alternating] + NumberOfKnots[n, NonAlternating];
NumberOfKnots[{n_}] := Sum[NumberOfKnots[n1], {n1, n}];
{NumberOfKnots[15, Alternating],
  NumberOfKnots[15, NonAlternating], NumberOfKnots[15], NumberOfKnots[{15}]}
```

(Alt) Out[]:=

```
{85263, 168030, 253293, 313230}
```

The data files for H, Kh, and Θ are available at <https://drorbn.net/AcademicPensieve/Projects/HigherRank/Data>.

(Alt) In[]:=

```
Get["../..//Projects/HigherRank/Data/HOMFLYPT3-15.m"] /. (K_ -> V_) -> (H[K] = Expand@V);
H[Knot[3, 1]]
```

(Alt) Out[]:=

$$2a^2 - a^4 + a^2z^2$$

(Alt) In[]:=

```
Get["../..//Projects/HigherRank/Data/Kh3-15.m"] /. (K_ -> V_) -> (Kh[K] = Expand@V);
Kh[Knot[3, 1]]
```

(Alt) Out[]:=

$$\frac{1}{q^3} + \frac{1}{q} + \frac{1}{q^9 t^3} + \frac{1}{q^5 t^2}$$

(Alt) In[]:=

```
J[K_] := J[K] = Kh[K] /. t -> -1;
J[Knot[3, 1]]
```

(Alt) Out[]:=

$$-\frac{1}{q^9} + \frac{1}{q^5} + \frac{1}{q^3} + \frac{1}{q}$$

(Alt) In[]:=

```
Get["../.. /Projects/HigherRank/Data/theta3-15.m"] /. (K_ -> V_) := (Theta[K] = Expand@V);
Theta[Knot[3, 1]]
```

(Alt) Out[]:=

$$\left\{ -1 + \frac{1}{T} + T, -\frac{1}{T^2} - T^2 - \frac{1}{T^2} - \frac{1}{T^2 T^2} + \frac{1}{T^2 T^2} + \frac{1}{T^2 T^2} + \frac{T^2}{T^2} + \frac{T^2}{T^2} + T^2 T^2 - T^2^2 + T^2 T^2^2 - T^2^2 T^2^2 \right\}$$

(Alt) In[]:=

```
Delta[K_] := Delta[K] = Theta[K][[1]];
rho1[K_] := rho1[K] = -Theta[K][[2]] /. {T1 -> T, T2 -> 1};
Factor@{Delta[Knot[3, 1]], rho1[Knot[3, 1]]}
```

(Alt) Out[]:=

$$\left\{ \frac{1 - T + T^2}{T}, \frac{(-1 + T)^2 (1 + T^2)}{T^2} \right\}$$

The data files for ρ_2 are available at <https://drorbn.net/AcademicPensieve/Talks/KnotTheoryCongress-2502/Rho2Data>.

(Alt) In[]:=

```
(Join@@(Get["../.. /Talks/KnotTheoryCongress-2502/Rho2Data/" <> #] & /@
{"Rho2_3-10.m", "Rho2_11.m", "Rho2_12.m", "Rho2_13.m",
"Rho2_14.m", "Rho2_15.m"})) /. (K_ -> V_) := (rho2[K] = Expand@V);
rho2[Knot[3, 1]]
```

(Alt) Out[]:=

$$2 - 4 z^2 + 3 z^4 + 4 z^6 + z^8$$

The data file for Vol is available at <https://drorbn.net/AcademicPensieve/Projects/Theta/Vols/>.

(Alt) In[]:=

```
Get["../.. /Projects/Theta/Vols/Vols.m"] /. (K_ -> v_) := (Vol[K] = v);
Vol[Knot[4, 1]]
Vol[Knot[4, 1]] // Precision
```

(Alt) Out[]:=

2.02988321281930725004240510854904057188337861506059958403497821

(Alt) Out[]:=

63.3075

(Alt) In[]:=

```
LowVolKnots = Select[AllKnots[{3, 15}], Vol[#] < Vol[Knot[4, 1]] &]
```

(Alt) Out[]:=

AllKnots[]

(Alt) In[]:=

```
Vol /@ LowVolKnots
```

(Alt) Out[]:=

AllKnots[]

(Alt) In[]:=

```
Get["../Talks/ICERM-2305/TLSigData.m"] /. (K_ -> v_) -> (σ[K] = v);
σ[Knot[3, 1]]
σ[Knot[14, NonAlternating, 27436]]
```

(Alt) Out[]:=

$$-2 \theta \left[-\frac{\sqrt{3}}{2} + u \right] + 2 \theta \left[\frac{\sqrt{3}}{2} + u \right]$$

(Alt) Out[]:=

$$4 \theta \left[-\frac{\sqrt{3}}{2} + u \right] - 4 \theta \left[\frac{\sqrt{3}}{2} + u \right] - 2 \theta \left[u - \sqrt[6]{-0.992...} \right] - \\ 2 \theta \left[u - \sqrt[6]{-0.782...} \right] + 2 \theta \left[u - \sqrt[6]{0.782...} \right] + 2 \theta \left[u - \sqrt[6]{0.992...} \right]$$

Stats

(Alt) In[]:=

```
Knots[n_Integer] := If[n ≤ 10,
  Table[Knot[n, k], {k, NumberOfKnots[n]}],
  Join[
    Table[Knot[n, Alternating, k], {k, NumberOfKnots[n, Alternating]}],
    Table[Knot[n, NonAlternating, k], {k, NumberOfKnots[n, NonAlternating]}]
  ];
Knots[{n_}] := Join@@Table[Knots[n1], {n1, n}];
Knots[{11}] // Short
```

(Alt) Out[]//Short=

```
{Knot[3, 1], Knot[4, 1], Knot[5, 1], Knot[5, 2], <<794>>, Knot[11, NonAlternating, 183],
Knot[11, NonAlternating, 184], Knot[11, NonAlternating, 185]}
```

(Alt) In[]:=

```
Stats[{n_}, ls_List] :=
  NumberOfKnots[{n}] - CountDistinct@Table[ℓ[K], {K, Knots[{n]}}, {ℓ, ls}];
Stats[ls_List] := Monitor[
  Table[def = Stats[{n}, ls], {n, 10, 15}],
  {n - 1, def}
];
```

(Alt) In[]:=

```
Stats[{Δ}]
```

(Alt) Out[]:=

```
{38, 250, 1204, 7326, 39741, 236326}
```

(Alt) In[]:=

```
Stats[{σ}]
```

(Alt) Out[]:=

```
{108, 356, 1525, 7736, 40101, 230592}
```

```

(Alt) In[ ]:=
  Stats[{Δ, σ}]

(Alt) Out[ ]=
  {25, 167, 907, 5927, 34132, 210536}

(Alt) In[ ]:=
  Stats[{J}]

(Alt) Out[ ]=
  {7, 70, 482, 3434, 21250, 138591}

(Alt) In[ ]:=
  Stats[{σ, J}]

(Alt) Out[ ]=
  {6, 43, 302, 2235, 13527, 87430}

(Alt) In[ ]:=
  Stats[{Kh}]

(Alt) Out[ ]=
  {6, 65, 452, 3226, 19754, 127261}

(Alt) In[ ]:=
  Stats[{σ, Kh}]

(Alt) Out[ ]=
  {6, 43, 300, 2196, 13164, 83817}

(Alt) In[ ]:=
  Stats[{H}]

(Alt) Out[ ]=
  {2, 31, 222, 1839, 11251, 73892}

(Alt) In[ ]:=
  Stats[{σ, H}]

(Alt) Out[ ]=
  {1, 30, 216, 1806, 11097, 73214}

(Alt) In[ ]:=
  Stats[{H, Kh}]

(Alt) Out[ ]=
  {1, 30, 214, 1771, 10788, 70245}

(Alt) In[ ]:=
  Stats[{σ, H, Kh}]

(Alt) Out[ ]=
  {1, 30, 214, 1771, 10785, 70227}

(Alt) In[ ]:=
  Table[d → Stats[{If[Vol@# < 1, 0, N[Vol@#, d]] &}], {d, 0, 70}] // Column

(Alt) Out[ ]=
  0 → {247, 799, 2975, 12963, 59935, 313228}
  1 → {236, 787, 2962, 12949, 59921, 313212}
  2 → {182, 725, 2892, 12873, 59838, 313122}
  3 → {64, 456, 2538, 12460, 59360, 312585}
  4 → {13, 79, 851, 8652, 54096, 306184}
  5 → {7, 35, 222, 3100, 34925, 274804}
  6 → {7, 27, 126, 1306, 12452, 157229}

```

$7 \rightarrow \{6, 25, 113, 1036, 6774, 54\,336\}$
 $8 \rightarrow \{6, 25, 113, 1015, 6411, 45\,048\}$
 $9 \rightarrow \{6, 25, 113, 1012, 6358, 43\,767\}$
 $10 \rightarrow \{6, 25, 113, 1012, 6356, 43\,625\}$
 $11 \rightarrow \{6, 25, 113, 1012, 6354, 43\,612\}$
 $12 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $13 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $14 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $15 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $16 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $17 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $18 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $19 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $20 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $21 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $22 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $23 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $24 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $25 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $26 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $27 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $28 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $29 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $30 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $31 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $32 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $33 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $34 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $35 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $36 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $37 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $38 \rightarrow \{6, 25, 113, 1012, 6354, 43\,610\}$
 $39 \rightarrow \{6, 25, 113, 1012, 6353, 43\,609\}$
 $40 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $41 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $42 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $43 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $44 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $45 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $46 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $47 \rightarrow \{6, 25, 113, 1012, 6353, 43\,608\}$
 $48 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $49 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $50 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $51 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $52 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $53 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $54 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $55 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $56 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $57 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $58 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$
 $59 \rightarrow \{6, 25, 113, 1012, 6353, 43\,607\}$

$60 \rightarrow \{6, 25, 113, 1012, 6353, 43605\}$
 $61 \rightarrow \{6, 25, 113, 1012, 6353, 43604\}$
 $62 \rightarrow \{6, 25, 113, 1011, 6348, 43561\}$
 $63 \rightarrow \{6, 22, 108, 990, 6216, 42723\}$
 $64 \rightarrow \{5, 20, 86, 841, 5294, 36701\}$
 $65 \rightarrow \{5, 19, 74, 690, 4394, 29931\}$
 $66 \rightarrow \{5, 19, 74, 690, 4394, 29931\}$
 $67 \rightarrow \{5, 19, 74, 690, 4394, 29931\}$
 $68 \rightarrow \{5, 19, 74, 690, 4394, 29931\}$
 $69 \rightarrow \{5, 19, 74, 690, 4394, 29931\}$
 $70 \rightarrow \{5, 19, 74, 690, 4394, 29931\}$

(Alt) In[]:=

```
Clear[TVol]
TVol[K_] := TVol[K] = If[Vol@K < 1, 0, N[Vol@K, 58]]
```

(Alt) In[]:=

```
Stats[{TVol}]
```

(Alt) Out[]=

```
{6, 25, 113, 1012, 6353, 43607}
```

(Alt) In[]:=

```
Stats[{σ, TVol}]
```

(Alt) Out[]=

```
{0, 14, 89, 928, 6016, 41868}
```

(Alt) In[]:=

```
Stats[{J, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5921, 41444}
```

(Alt) In[]:=

```
Stats[{σ, J, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5919, 41439}
```

(Alt) In[]:=

```
Stats[{Kh, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5918, 41435}
```

(Alt) In[]:=

```
Stats[{σ, Kh, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5918, 41435}
```

(Alt) In[]:=

```
Stats[{H, TVol}]
```

(Alt) Out[]=

```
{0, 14, 84, 911, 5920, 41443}
```

```

(Alt) In[ ]:=
  Stats[{σ, H, TVol}]
(Alt) Out[ ]=
  {0, 14, 84, 911, 5918, 41438}

(Alt) In[ ]:=
  Stats[{Kh, H, TVol}]
(Alt) Out[ ]=
  {0, 14, 84, 911, 5917, 41434}

(Alt) In[ ]:=
  Stats[{σ, Kh, H, TVol}]
(Alt) Out[ ]=
  {0, 14, 84, 911, 5917, 41434}

(Alt) In[ ]:=
  Stats[{Δ, ρ1}]
(Alt) Out[ ]=
  {0, 14, 95, 959, 6253, 42914}

(Alt) In[ ]:=
  Stats[{Δ, ρ1, ρ2}]
(Alt) Out[ ]=
  {0, 14, 84, 911, 5926, 41469}

(Alt) In[ ]:=
  Stats[{ρ1, ρ2, Kh, H, TVol}]
(Alt) Out[ ]=
  {0, 14, 84, 911, 5916, 41432}

(Alt) In[ ]:=
  Stats[{σ, ρ1, ρ2, Kh, H, TVol}]
(Alt) Out[ ]=
  {0, 14, 84, 911, 5916, 41432}

```

```

(Alt) In[ ]:=
  Stats[{Θ}]
(Alt) Out[ ]=
  {0, 3, 19, 194, 1118, 6758}

(Alt) In[ ]:=
  6758 / 41432 // N
(Alt) Out[ ]=
  0.163111

(Alt) In[ ]:=
  Stats[{σ, Θ}]
(Alt) Out[ ]=
  {0, 3, 19, 194, 1118, 6758}

(Alt) In[ ]:=
  Stats[{Θ, Kh}]
(Alt) Out[ ]=
  {0, 3, 18, 185, 1062, 6555}

```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ , H}]
(Alt) Out[ ]=
  {0, 3, 18, 185, 1064, 6563}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ , H, Kh}]
(Alt) Out[ ]=
  {0, 3, 18, 185, 1062, 6555}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ }]
(Alt) Out[ ]=
  {0, 3, 10, 169, 982, 6341}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ , TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 973, 6308}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 973, 6308}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , Kh}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 981, 6337}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , H}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 982, 6341}
```

```
(Alt) In[ ]:=
  Stats[{ $\sigma$ ,  $\Theta$ ,  $\rho_2$ }]
(Alt) Out[ ]=
  {0, 3, 10, 169, 982, 6341}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , H, Kh}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 981, 6337}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 973, 6308}
```

```
(Alt) In[ ]:=
  Stats[{ $\Theta$ ,  $\rho_2$ , H, Kh, TVol}]
(Alt) Out[ ]=
  {0, 3, 10, 169, 972, 6304}
```



```
(Alt) In[ ]:=  
  Stats[{ $\sigma$ ,  $\theta$ ,  $\rho_2$ , H, Kh, TVol}]
```

```
(Alt) Out[ ]=  
  {0, 3, 10, 169, 972, 6304}
```