Pensieve Header: The separation powers of various invariants.

Loading the invariants

(Alt) In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
```

(Alt) In[ ]:=
```
Do[
  NumberOfKnots[n, Alternating] =
   {0, 0, 1, 1, 2, 3, 7, 18, 41, 123, 367, 1288, 4878, 19536, 85263}[[n]];
  NumberOfKnots[n, NonAlternating] =
   {0, 0, 0, 0, 0, 0, 0, 3, 8, 42, 185, 888, 5110, 27436, 168030}[[n]],
  {n, 15}
 ];
NumberOfKnots[n_Integer] :=
  NumberOfKnots[n, Alternating] + NumberOfKnots[n, NonAlternating];
```

(Alt) Out[ ]=
```
{85263, 168030, 253293, 313230}
```

The data files for H, Kh, and Θ are available at https://drorbn.net/AcademicPensieve/Projects/HigherRank/Data

(Alt) In[ ]:=
```
Get["../../Projects/HigherRank/Data/HOMFLYPT3-15.m"] /. (K_ → V_) :> (H[K] = Expand@V);
```

(Alt) Out[ ]=
$$2\,a^2 - a^4 + a^2\,z^2$$

(Alt) In[ ]:=
```
Get["../../Projects/HigherRank/Data/Kh3-15.m"] /. (K_ → V_) :> (Kh[K] = Expand@V);
```

(Alt) Out[ ]=
$$\frac{1}{3} + \frac{1}{\quad} + \frac{1}{9\,,3} + \frac{1}{5\,,2}$$

(Alt) In[ ]:=
```
J[K_] := J[K] = Kh[K] /. t → -1;
```

(Alt) Out[ ]=
$$-\frac{1}{9} + \frac{1}{5} + \frac{1}{3} + \frac{1}{\quad}$$

(Alt) In[ ]:=
```
Get["../../Projects/HigherRank/Data/theta3-15.m"] /. (K_ → V_) :> (Θ[K] = Expand@V);
```

(Alt) Out[ ]=
$$\left\{-1 + \frac{1}{\quad} + T,\ -\frac{1}{\quad} - T1^2 - \frac{1}{\quad} - \frac{1}{\quad\quad} + \frac{1}{\quad} + \frac{1}{\quad} + \frac{T1}{\quad} + \frac{T2}{\quad} + T1^2\,T2 - T2^2 + T1\,T2^2 - T1^2\,T2^2\right\}$$

(Alt) In[ ]:=
```
Δ[K_] := Δ[K] = Θ[K][[1]];
ρ1[K_] := ρ1[K] = -Θ[K][[2]] /. {T1 → T, T2 → 1};
```

(Alt) Out[ ]=
$$\left\{\frac{1 - T + T^2}{\quad},\ \frac{(-1+T)^2\,\left(1+T^2\right)}{\quad}\right\}$$

The data files for ρ_2 are available at https://drorbn.net/AcademicPensieve/Talks/KnotTheory-

(Alt) In[ ]:=
```
(Join @@ (Get["../../Talks/KnotTheoryCongress-2502/Rho2Data/" <> #] & /@
      {"Rho2_3-10.m", "Rho2_11.m", "Rho2_12.m", "Rho2_13.m",
       "Rho2_14.m", "Rho2_15.m"})) /. (K_ → V_) :> (ρ2[K] = Expand@V);
```
(Alt) Out[ ]=
$$2 - 4\,z^2 + 3\,z^4 + 4\,z^6 + z^8$$

The data file for Vol is available at https://drorbn.net/AcademicPensieve/Projects/Theta/Vols/.

(Alt) In[ ]:=
```
Get["../../Projects/Theta/Vols/Vols.m"] /. (K_ → v_) :> (Vol[K] = v);
Vol[Knot[4, 1]]
```
(Alt) Out[ ]=
2.029883212819307250042405108549040571883378615060599584034978 21
(Alt) Out[ ]=
63.3075

(Alt) In[ ]:=
```
LowVolKnots = Select[AllKnots[{3, 15}], Vol[#] < Vol[Knot[4, 1]] &]
```
(Alt) Out[ ]=
AllKnots[ ]

(Alt) In[ ]:=
```
Vol /@ LowVolKnots
```
(Alt) Out[ ]=
AllKnots[ ]

(Alt) In[ ]:=
```
Get["../../Talks/ICERM-2305/TLSigData.m"] /. (K_ → v_) :> (σ[K] = v);
σ[Knot[3, 1]]
```
(Alt) Out[ ]=
$$-2\,\theta\left[-\frac{\sqrt{3}}{} + u\right] + 2\,\theta\left[\frac{\sqrt{3}}{} + u\right]$$
(Alt) Out[ ]=
$$4\,\theta\left[-\frac{\sqrt{3}}{2} + u\right] - 4\,\theta\left[\frac{\sqrt{3}}{2} + u\right] - 2\,\theta\left[u - \sqrt[6]{-0.992\ldots}\right] -$$
$$2\,\theta\left[u - \sqrt[6]{-0.782\ldots}\right] + 2\,\theta\left[u - \sqrt[6]{0.782\ldots}\right] + 2\,\theta\left[u - \sqrt[6]{0.992\ldots}\right]$$

Stats

(Alt) In[ ]:=
```
Knots[n_Integer] := If[n ≤ 10,
    Table[Knot[n, k], {k, NumberOfKnots[n]}],
    Join[
     Table[Knot[n, Alternating, k], {k, NumberOfKnots[n, Alternating]}],
     Table[Knot[n, NonAlternating, k], {k, NumberOfKnots[n, NonAlternating]}]
    ]
   ];
Knots[{n_}] := Join @@ Table[Knots[n1], {n1, n}];
Knots[{11}] // Short
```
(Alt) Out[ ]//Short=
{Knot[3, 1], Knot[4, 1], <<797>>,
 Knot[11, NonAlternating, 184], Knot[11, NonAlternating, 185]}

(Alt) In[•]:=

```
Stats[{n_}, ∠s_List] := Module[{Ks = Knots[{n}]},
   Count[Tally@Table[ι[K], {K, Ks}, {ι, ∠s}], {_, 1}]
                          Length@Ks
  ] * 100.
Stats[∠s_List] := Monitor[
   Table[def = Stats[{n}, ∠s], {n, 10, 15}],
   {n - 1, def}
  ];
```

(Alt) In[•]:=

```
Stats[{Δ}]
```

(Alt) Out[•]=

```
{70.2811, 47.8152, 38.4615, 23.5712, 16.7459, 11.2093}
```

(Alt) In[•]:=

```
Stats[{σ}]
```

(Alt) Out[•]=

```
{49.7992, 46.1923, 36.4461, 26.5561, 19.3753, 14.1363}
```

(Alt) In[•]:=

```
Stats[{Δ, σ}]
```

(Alt) Out[•]=

```
{80.3213, 63.6704, 50.5207, 33.9607, 24.1554, 16.8857}
```

(Alt) In[•]:=

```
Stats[{J}]
```

(Alt) Out[•]=

```
{94.3775, 83.1461, 70.7088, 55.511, 44.2148, 34.7537}
```

(Alt) In[•]:=

```
Stats[{σ, J}]
```

(Alt) Out[•]=

```
{95.1807, 89.5131, 81.3571, 69.6953, 61.7315, 54.6819}
```

(Alt) In[•]:=

```
Stats[{Kh}]
```

(Alt) Out[•]=

```
{95.1807, 84.3945, 72.5227, 57.9329, 47.4331, 38.7562}
```

(Alt) In[•]:=

```
Stats[{σ, Kh}]
```

(Alt) Out[•]=

```
{95.1807, 89.5131, 81.4914, 70.2121, 62.6775, 56.2829}
```

(Alt) In[•]:=

```
Stats[{H}]
```

(Alt) Out[•]=

```
{98.3936, 92.3845, 85.6903, 74.0069, 66.6216, 59.8241}
```

(Alt) In[•]:=
```
Stats[{σ, H}]
```
(Alt) Out[•]=
```
{99.1968, 92.6342, 86.0934, 74.4312, 66.997, 60.1038}
```

(Alt) In[•]:=
```
Stats[{H, Kh}]
```
(Alt) Out[•]=
```
{99.1968, 92.6342, 86.2277, 74.9248, 67.8396, 61.5129}
```

(Alt) In[•]:=
```
Stats[{σ, H, Kh}]
```
(Alt) Out[•]=
```
{99.1968, 92.6342, 86.2277, 74.9248, 67.8479, 61.5225}
```

(Alt) In[•]:=
```
Table[d → Stats[{If[Vol@# < 1, 0, N[Vol@#, d]] &}], {d, 0, 70}] // Column
```
(Alt) Out[•]=
```
0 → {0., 0., 0., 0., 0., 0.}
1 → {0.401606, 0.124844, 0.0335909, 0.00771307, 0.00166842, 0.000638508}
2 → {6.82731, 1.49813, 0.201545, 0.0462784, 0.00834209, 0.00191553}
3 → {55.0201, 16.2297, 2.98959, 0.570767, 0.118458, 0.022667}
4 → {91.5663, 81.7728, 50.5878, 11.8241, 1.89199, 0.302653}
5 → {95.9839, 91.8851, 85.8918, 58.01, 17.4033, 2.60767}
6 → {95.9839, 93.8826, 91.9046, 81.0953, 63.1747, 24.3795}
7 → {96.7871, 94.382, 92.7444, 84.9826, 79.1982, 69.3114}
8 → {96.7871, 94.382, 92.7444, 85.2603, 80.276, 74.2678}
9 → {96.7871, 94.382, 92.7444, 85.3066, 80.4244, 74.9618}
10 → {96.7871, 94.382, 92.7444, 85.3066, 80.4311, 75.0394}
11 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0461}
12 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
13 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
14 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
15 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
16 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
17 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
18 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
19 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
20 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
21 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
22 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
23 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
24 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
25 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
26 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
27 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
28 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
29 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
30 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
31 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
32 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
33 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
34 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
35 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
```

```
36 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
37 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
38 → {96.7871, 94.382, 92.7444, 85.3066, 80.4361, 75.0471}
39 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0474}
40 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
41 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
42 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
43 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
44 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
45 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
46 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
47 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.048}
48 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
49 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
50 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
51 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
52 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
53 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
54 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
55 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
56 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
57 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
58 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
59 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
60 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.05}
61 → {96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0503}
62 → {96.7871, 94.382, 92.7444, 85.322, 80.4545, 75.0749}
63 → {96.7871, 95.1311, 93.0803, 85.6305, 80.8499, 75.5371}
64 → {97.5904, 95.6305, 94.5247, 87.7594, 83.6128, 78.8596}
65 → {97.5904, 95.8801, 95.3309, 89.9344, 86.2989, 82.5866}
66 → {97.5904, 95.8801, 95.3309, 89.9344, 86.2989, 82.5866}
67 → {97.5904, 95.8801, 95.3309, 89.9344, 86.2989, 82.5866}
68 → {97.5904, 95.8801, 95.3309, 89.9344, 86.2989, 82.5866}
69 → {97.5904, 95.8801, 95.3309, 89.9344, 86.2989, 82.5866}
70 → {97.5904, 95.8801, 95.3309, 89.9344, 86.2989, 82.5866}
```

(Alt) In[ ]:=

```
Clear[TVol]
TVol[K_] := TVol[K] = If[Vol@K < 1, 0, N[Vol@K, 58]]
```

(Alt) In[ ]:=

```
Stats[{TVol}]
```

(Alt) Out[ ]=

```
{96.7871, 94.382, 92.7444, 85.3066, 80.4395, 75.0487}
```

(Alt) In[ ]:=

```
Stats[{σ, TVol}]
```

(Alt) Out[ ]=

```
{100., 96.5044, 94.1888, 86.3941, 81.3638, 75.9049}
```

(Alt) In[•]:=
```
Stats[{J, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.5973, 76.1048}
```

(Alt) In[•]:=
```
Stats[{σ, J, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.604, 76.1073}
```

(Alt) In[•]:=
```
Stats[{Kh, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.6074, 76.1099}
```

(Alt) In[•]:=
```
Stats[{σ, Kh, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.6074, 76.1099}
```

(Alt) In[•]:=
```
Stats[{H, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.599, 76.1051}
```

(Alt) In[•]:=
```
Stats[{σ, H, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.6057, 76.1077}
```

(Alt) In[•]:=
```
Stats[{Kh, H, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.609, 76.1102}
```

(Alt) In[•]:=
```
Stats[{σ, Kh, H, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.609, 76.1102}
```

(Alt) In[•]:=
```
Stats[{Δ, ρ1}]
```
(Alt) Out[•]=
```
{100., 96.5044, 93.7185, 85.9236, 80.6447, 75.3398}
```

(Alt) In[•]:=
```
Stats[{Δ, ρ1, ρ2}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.5757, 76.0869}
```

(Alt) In[•]:=
```
Stats[{ρ1, ρ2, Kh, H, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.609, 76.1105}
```

(Alt) In[•]:=
```
Stats[{σ, ρ1, ρ2, Kh, H, TVol}]
```
(Alt) Out[•]=
```
{100., 96.5044, 94.4239, 86.5793, 81.609, 76.1105}
```

(Alt) In[•]:=
```
Stats[{Θ}]
```
(Alt) Out[•]=
```
{100., 99.2509, 98.7571, 97.0382, 96.3979, 95.8899}
```

(Alt) In[•]:=
```
Stats[{σ, Θ}]
```
(Alt) Out[•]=
```
{100., 99.2509, 98.7571, 97.0382, 96.3979, 95.8899}
```

(Alt) In[•]:=
```
Stats[{Θ, Kh}]
```
(Alt) Out[•]=
```
{100., 99.2509, 98.8243, 97.1693, 96.543, 95.98}
```

(Alt) In[•]:=
```
Stats[{Θ, H}]
```
(Alt) Out[•]=
```
{100., 99.2509, 98.8243, 97.1693, 96.5364, 95.9758}
```

(Alt) In[•]:=
```
Stats[{Θ, H, Kh}]
```
(Alt) Out[•]=
```
{100., 99.2509, 98.8243, 97.1693, 96.543, 95.98}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.7733, 96.0904}
```

(Alt) In[•]:=
```
Stats[{Θ, TVol}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.8033, 96.1115}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2, TVol}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.8033, 96.1115}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2, Kh}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.7766, 96.093}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2, H}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.7733, 96.0904}
```

(Alt) In[•]:=
```
Stats[{σ, Θ, ρ2}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.7733, 96.0904}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2, H, Kh}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.7766, 96.093}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2, TVol}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.8033, 96.1115}
```

(Alt) In[•]:=
```
Stats[{Θ, ρ2, H, Kh, TVol}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.8066, 96.114}
```

(Alt) In[•]:=
```
Stats[{σ, Θ, ρ2, H, Kh, TVol}]
```
(Alt) Out[•]=
```
{100., 99.2509, 99.3282, 97.393, 96.8066, 96.114}
```