Pensieve header: Playing games with magnitudes.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
```

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```
In[•]:= Once[
  << KnotTheory`;
  << "../../Talks/MonteVerita-2604/Rot.m";
  << "../../Talks/MonteVerita-2604/PolyPlot.m"
]
```

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```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

```
Loading Rot.m from http://drorbn.net/v25/ap to compute rotation numbers.
```

pdf

```
Loading PolyPlot.m from http://drorbn.net/mv26/ap to plot 2-variable polynomials.
```

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```
In[•]:= CF[ℰ_] := Expand@Collect[ℰ, g__, F] /. F → Factor;
```

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```
In[•]:= T3 = T1 T2;
```

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```
In[•]:= F1[{s_, i_, j_}] := CF[
  s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
     g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^2s g2ji - T2^s g2jj + T2^s g3jj) +
        (T3^s - 1) g3ji (1 - T2^s g1ii + g2ij + (T2^s - 2) g2jj - (T1^s - 1) (T2^s + 1) g1ji)) / (T2^s - 1))]
```

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```
In[•]:= F2[{s0_, i0_, j0_}, {s1_, i1_, j1_}] := CF[
  s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
```

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```
In[•]:= F3[φ_, k_] = φ g3kk - φ/2;
```

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In[•]:=
```
Θ[K_] := Θ[K] = Module[{X, φ, n, A, Δ, G, ev, Θ, k, k1, k2},
  (* 1 *) {X, φ} = Rot[K]; n = Length[X]; A = IdentityMatrix[2 n + 1];
  (* 2 *) Cases[X, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += {{-T^s, T^s - 1}, {0, -1}})];
  (* 3 *) Δ = T^((-Total[φ]-Total[X[[All,1]]])/2) Det[A];
  (* 4 *) G = Inverse[A];
  (* 5 *) ev[ℰ_] := Factor[ℰ /. g_{ν_,α_,β_} :> (G[[α, β]] /. T → T_ν)];
  (* 6 *) Θ = ev[Sum[F1[X[[k]]], {k, n}]];
  (* 7 *) Θ += ev[Sum[F2[X[[k1]], X[[k2]]], {k1, n}, {k2, n}]];
  (* 8 *) Θ += ev[Sum[F3[φ[[k]], k], {k, Length@φ}]];
  (* 9 *) Factor@{Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) Θ}
 ];
```

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In[•]:=
```
Expand[Θ[Knot[3, 1]]]
```

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KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
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$$\left\{-1+\frac{1}{T}+T,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2\,T_2^2}+\frac{1}{T_1\,T_2^2}+\frac{1}{T_1^2\,T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2\,T_2-T_2^2+T_1\,T_2^2-T_1^2\,T_2^2\right\}$$

In[•]:=
```
DunfieldKnots = ReadList["../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;
DK[n_] := DunfieldKnots[[n - 2]];
DKString[n_] := StringDrop[ToString[1000 + n], 1]
```

In[•]:=
```
ΘDK[n_] := Get["../HigherRank/DunfieldKnots/D" <> DKString[n] <> ".m"][[2]]
```

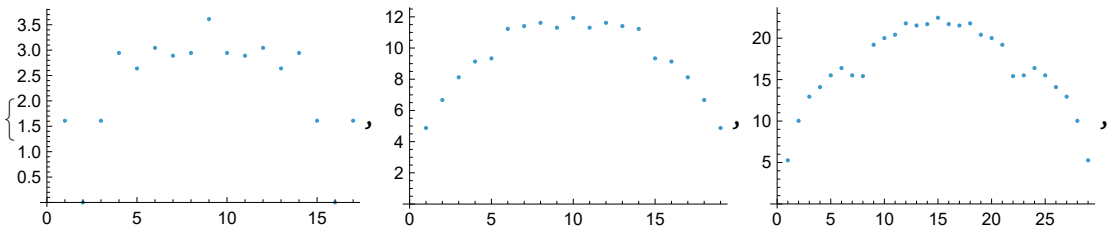
In[•]:=
```
ΘDK[3]
```

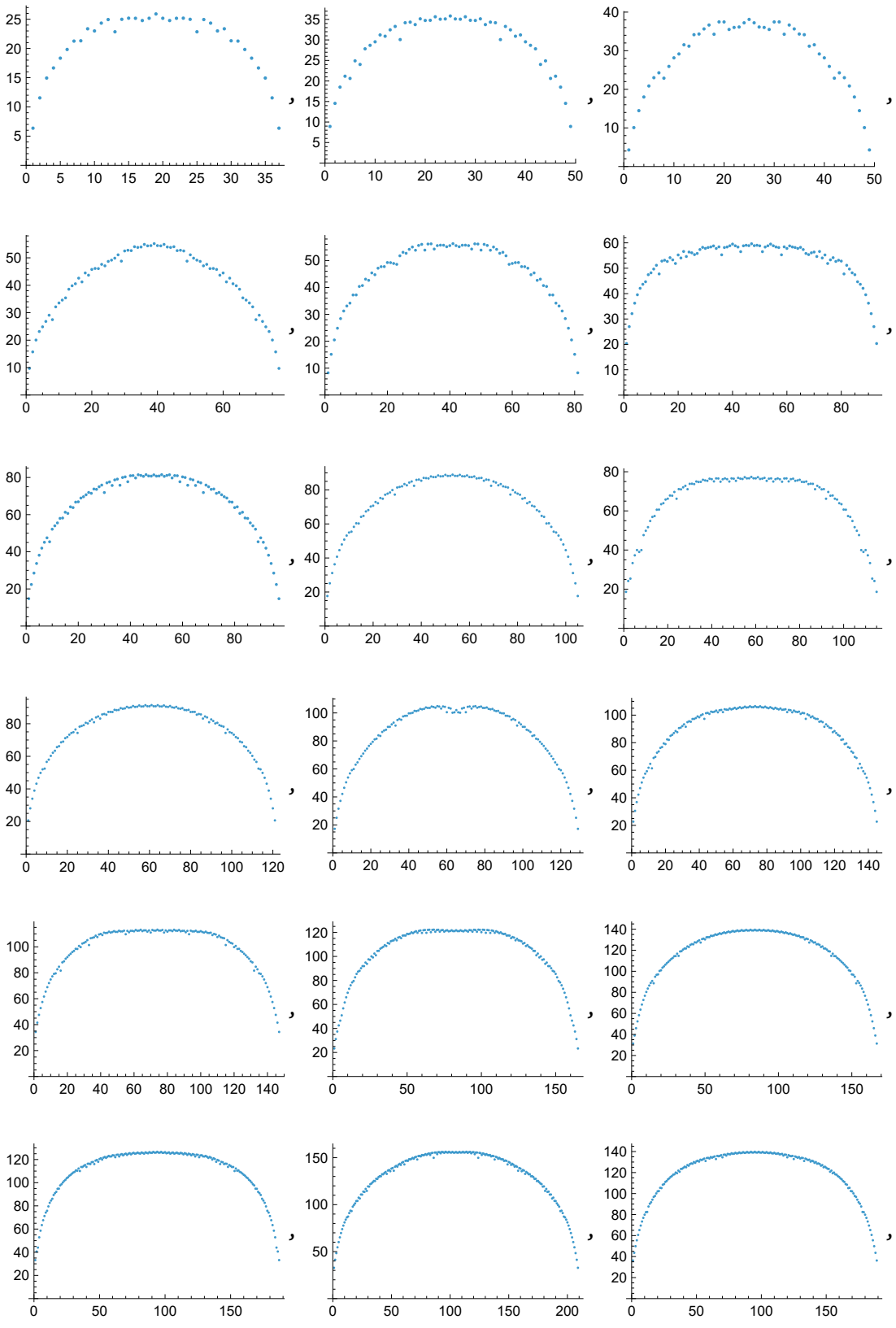
Out[•]=

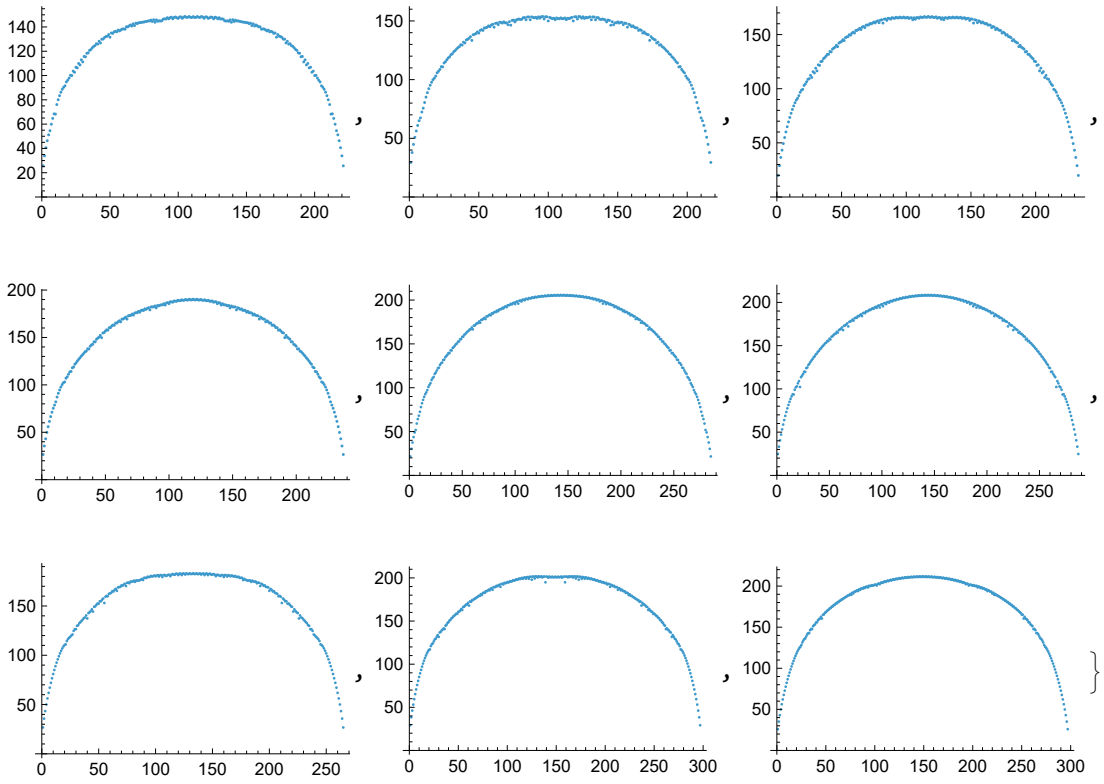
$$\left\{\frac{1-T+T^2}{T},\ \frac{1-T1+T1^2-T2-T1^3\,T2+T2^2+T1^4\,T2^2-T1\,T2^3-T1^4\,T2^3+T1^2\,T2^4-T1^3\,T2^4+T1^4\,T2^4}{T1^2\,T2^2}\right\}$$

In[•]:=
```
Table[
 sec = Coefficient[Expand@ΘDK[n][[2]], T2, 0] /. T1 → T;
 me = Exponent[sec, T, Min];
 ListPlot[Log[1 + Abs@CoefficientList[sec / T^me, T]]],
 {n, 10, 300, 10}
]
```

Out[•]=

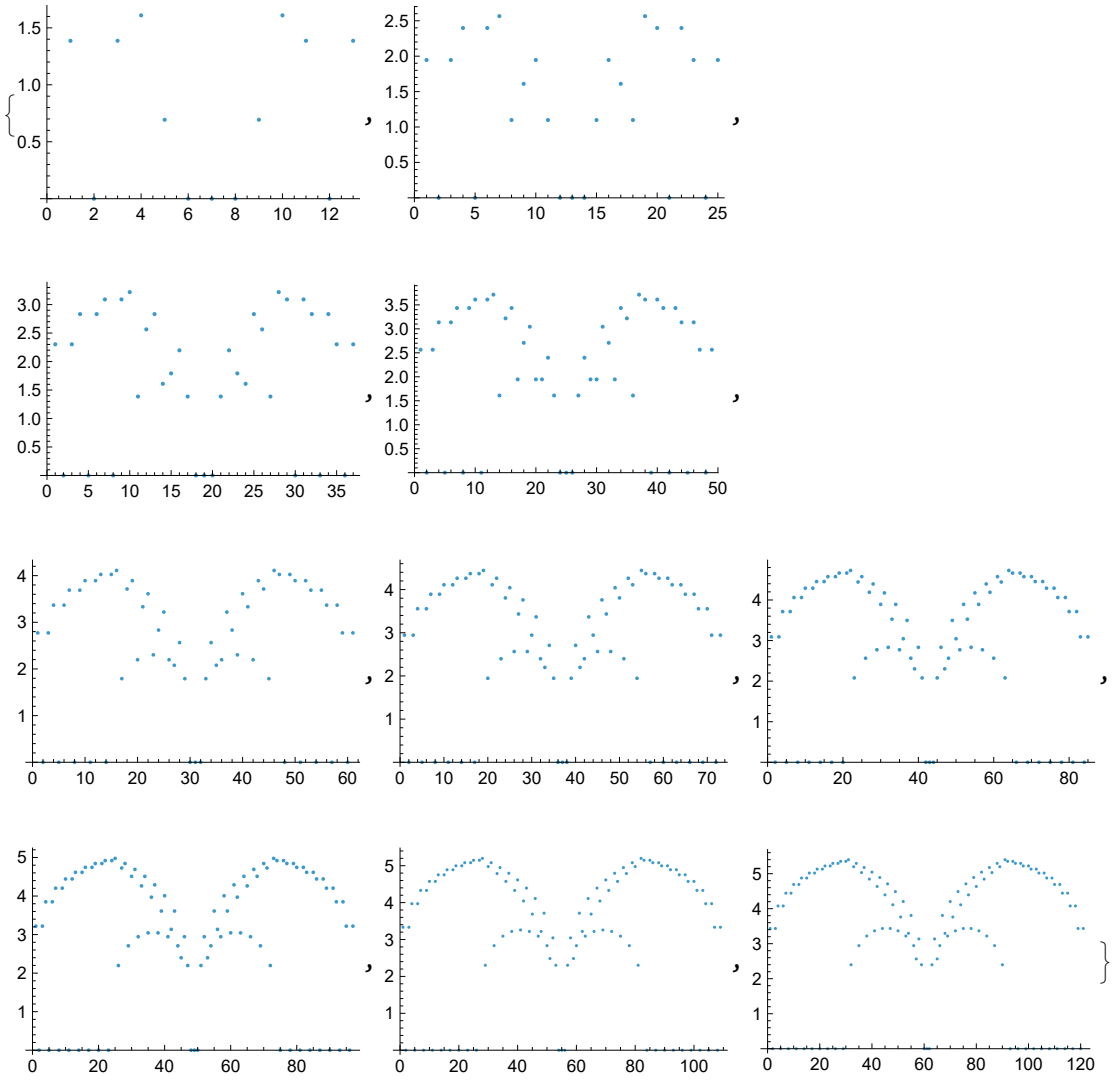






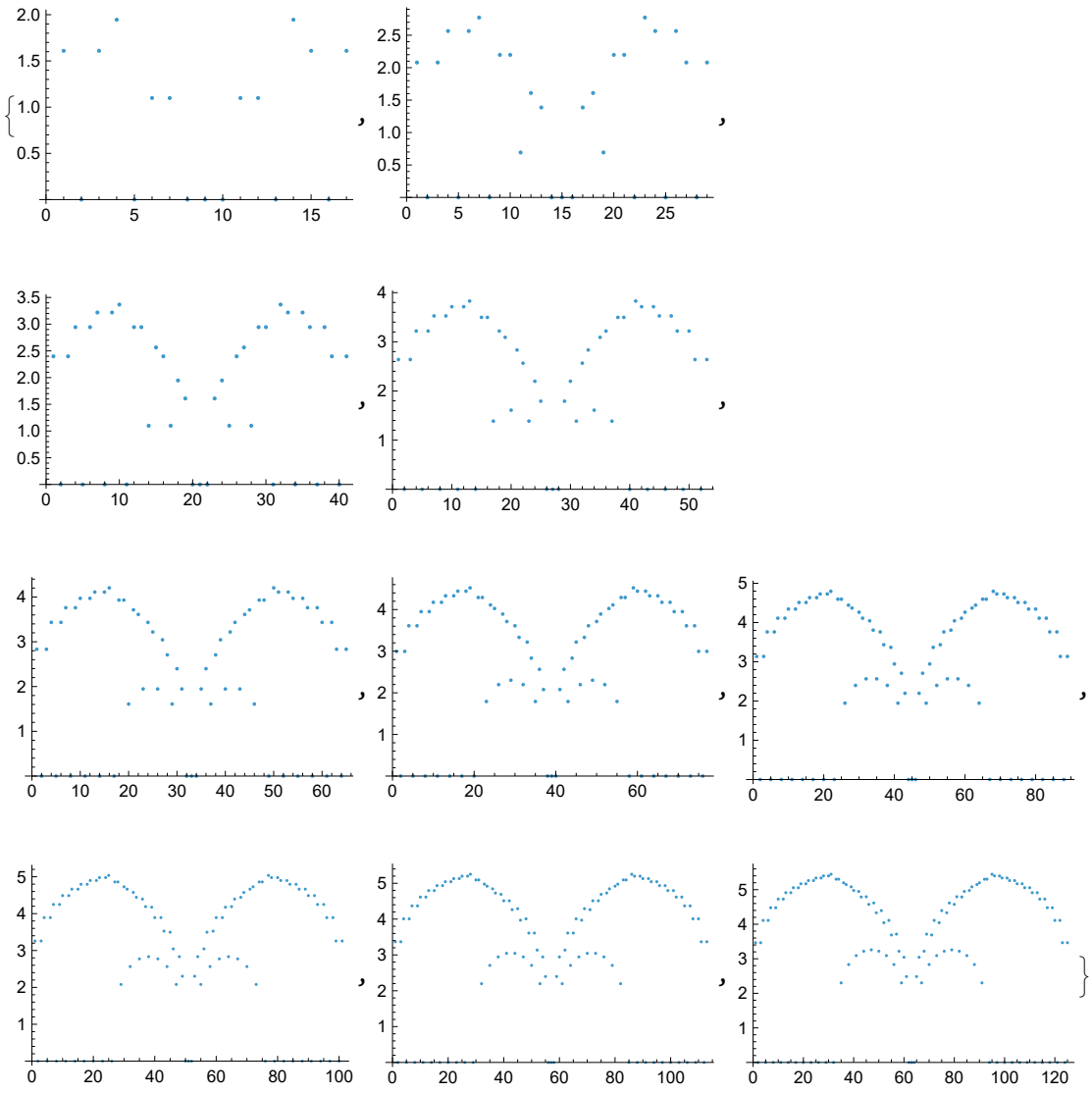
In[•]:=
```
Table[
 sec = Coefficient[Expand@⊖[TorusKnot[3 n + 1, 3]][[2]], T_2, 0] /. T_1 → T;
 me = Exponent[sec, T, Min];
 ListPlot[Log[1 + Abs@CoefficientList[sec / T^me, T]]],
 {n, 1, 10}
]
```

Out[•]=



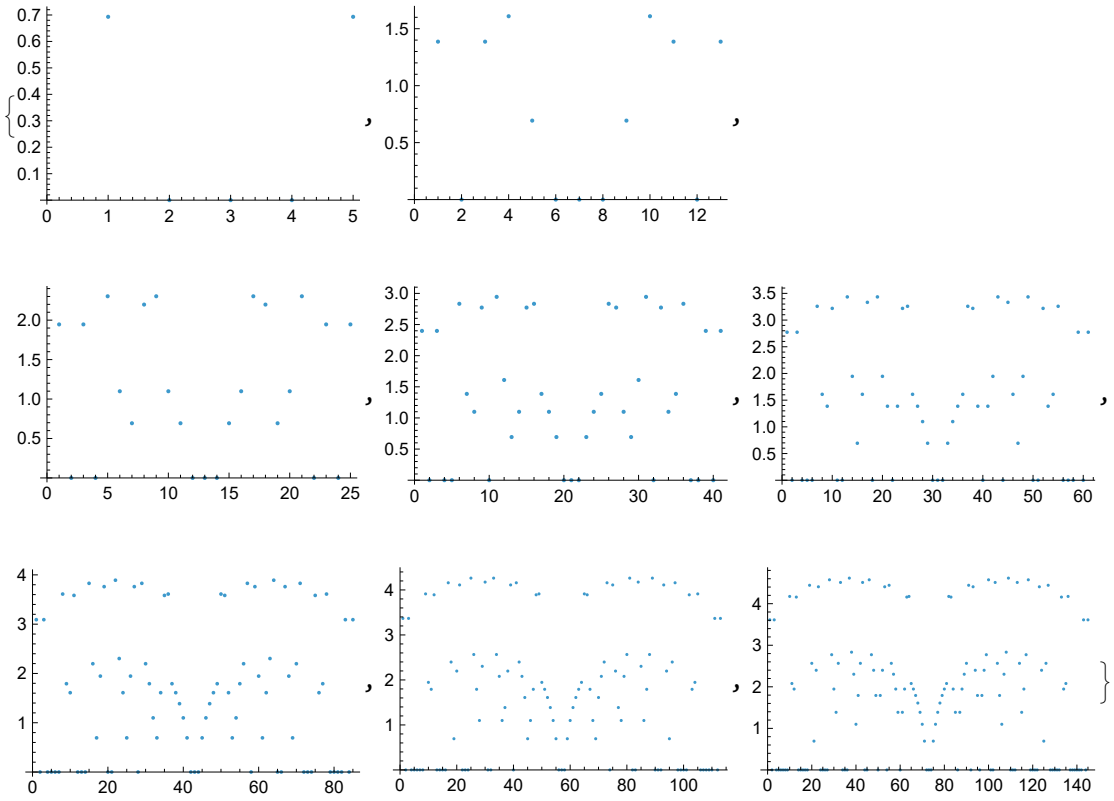
In[•]:=
```
Table[
 sec = Coefficient[Expand@θ[TorusKnot[3 n + 2, 3]][[2]], T_2, 0] /. T_1 → T;
 me = Exponent[sec, T, Min];
 ListPlot[Log[1 + Abs@CoefficientList[sec / T^me, T]]],
 {n, 1, 10}
]
```

Out[•]=



In[•]:=
```
Table[
 sec = Coefficient[Expand@θ[TorusKnot[n, n - 1]][[2]], T_2, 0] /. T_1 → T;
 me = Exponent[sec, T, Min];
 ListPlot[Log[1 + Abs@CoefficientList[sec / T^me, T]]],
 {n, 3, 10}
]
```

Out[•]=



In[•]:=
```
Table[
 sec = Coefficient[Expand@θ[TorusKnot[n, n - 1]][[2]], T2, 0] /. T1 → T;
 me = Exponent[sec, T, Min];
 ListPlot[Log[1 + Abs@CoefficientList[sec / T^me, T]]],
 {n, 3, 12}
]
```

Out[•]=