```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\Theta"];
```

pdf

```
In[•]:= << KnotTheory`
```

pdf

```
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at http://katlas.org/wiki/KnotTheory.
```

pdf

```
In[•]:= (* Rot suppressed *)
```

pdf

```
In[•]:= (* PolyPlot suppressed *)
```

```
In[•]:= PD[GST48] = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
    X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
    X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
    X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
    X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
    X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
    X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
    X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
    X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
    X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
    X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
    X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];
```

```
In[•]:= DunfieldKnots = ReadList["../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;
DK[n_] := DunfieldKnots[[n - 2]];
```

Current Version

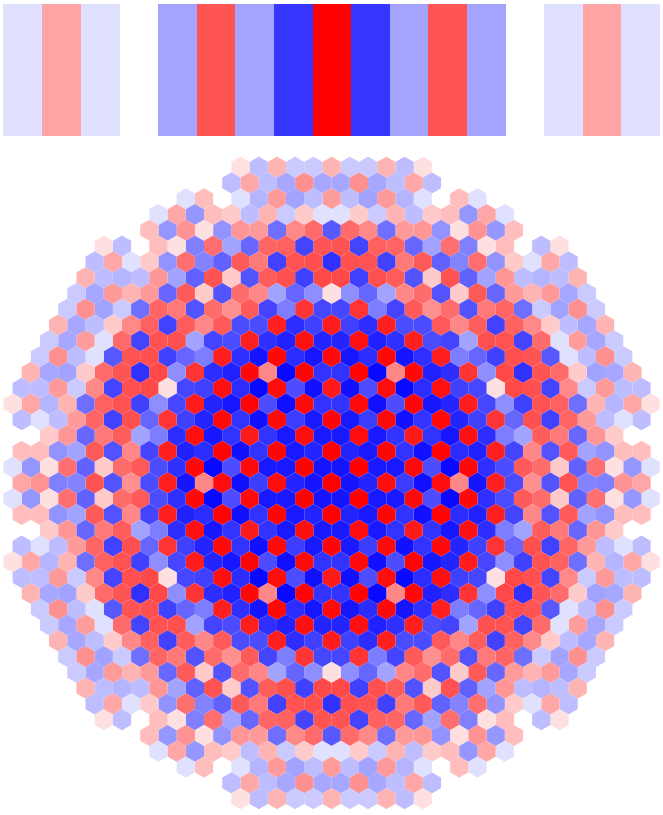
```
In[•]:= T3 = T1 T2;
        R11[{s_, i_, j_}] =
          s (1 / 2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
              g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^2 s g2ji - T2^s g2jj + T2^s g3jj) +
                  (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji + (T2^s - 2) g2jj + g2ij)) / (T2^s - 1));
        R12[{s0_, i0_, j0_}, {s1_, i1_, j1_}] =
          s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0));
        Γ1[φ_, k_] = -φ / 2 + φ g3kk;
        Θ[K_] := Module[{Cs, φ, n, A, Δ, G, ev, θ, t = AbsoluteTime[]},
          {Cs, φ} = Rot[K]; n = Length[Cs];
          A = IdentityMatrix[2 n + 1];
          Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
          Δ = T^((-Total[φ] - Total[Cs[[All, 1]]]) / 2) Det[A];
          G = Inverse[A];
          Print["Done with commons at ", AbsoluteTime[] - t];
          ev[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν)];
          θ = ev[Sum[Sum[R12[Cs[[k1]], Cs[[k2]]], {k2, 1, n}], {k1, 1, n}]];
          θ += ev[Sum[R11[Cs[[k]]], {k, 1, n}]];
          θ += ev[Sum[Γ1[φ[[k]], k], {k, 1, 2 n}]];
          Print["Done with θ at ", AbsoluteTime[] - t];
          Factor@{Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) θ}
          ];

In[•]:= AbsoluteTiming@Short[th = Θ[GST48]]
        PolyPlot[th]
```

```
Done with commons at 2.2435727

Done with Θ at 17.6794061
```

Out[•]=

$$\left\{17.7758,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\ \left(-1+\text{<<7>>}+T^8\right)}{T^8},\ \frac{\text{<<1764>>}+T_1^{35}\,T_2^{40}}{T_1^{20}\,T_2^{20}}\right\}\right\}$$

Out[•]=



In[•]:=
```
AbsoluteTiming[th = Θ[TorusKnot[13, 5]]]
PolyPlot[th]
```

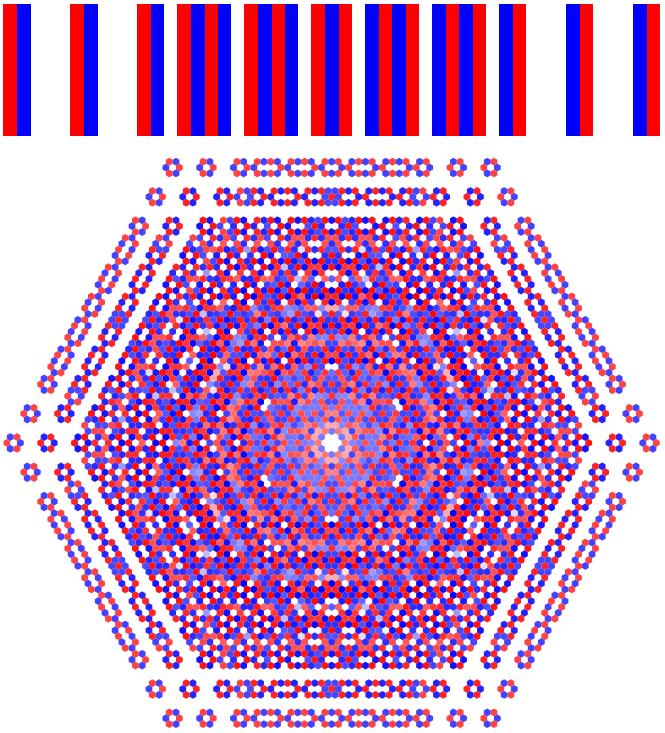
```
Done with commons at 3.3399959

Done with Θ at 79.1477238
```

Out[•]=

$$\left\{79.433,\ \left\{\frac{1-T+T^5-T^6+T^{10}-T^{11}+T^{13}-T^{14}+T^{15}-T^{16}+T^{18}-T^{19}+T^{20}-T^{21}+T^{23}-T^{24}+T^{25}-T^{27}+T^{28}-T^{29}+T^{30}-T^{32}+T^{33}-T^{34}+T^{35}-T^{37}+T^{38}-T^{42}+T^{43}-T^{47}+T^{48}}{T^{24}},\ \frac{1}{T_1^{48}\,T_2^{48}}\right.\right.$$
$$\left(24 - 24\,T_1 + 24\,T_1^5 - 24\,T_1^6 + 24\,T_1^{10} - 24\,T_1^{11} + 24\,T_1^{13} - 24\,T_1^{14} + 24\,T_1^{15} - 24\,T_1^{16} + 24\,T_1^{18} - 24\,T_1^{19} + 24\,T_1^{20} - 24\,T_1^{21} + 24\,T_1^{23} - \right.$$
$$24\,T_1^{24} + \langle\!\langle 7325 \rangle\!\rangle + 24\,T_1^{71}\,T_2^{96} - 24\,T_1^{72}\,T_2^{96} + 24\,T_1^{73}\,T_2^{96} - 24\,T_1^{75}\,T_2^{96} + 24\,T_1^{76}\,T_2^{96} - 24\,T_1^{77}\,T_2^{96} + 24\,T_1^{78}\,T_2^{96} - 24\,T_1^{80}\,T_2^{96} +$$
$$\left.\left.24\,T_1^{81}\,T_2^{96} - 24\,T_1^{82}\,T_2^{96} + 24\,T_1^{83}\,T_2^{96} - 24\,T_1^{85}\,T_2^{96} + 24\,T_1^{86}\,T_2^{96} - 24\,T_1^{90}\,T_2^{96} + 24\,T_1^{91}\,T_2^{96} - 24\,T_1^{95}\,T_2^{96} + 24\,T_1^{96}\,T_2^{96}\right)\right\}\right\}$$

Full expression not available (original memory size: 1.8 MB)

Out[•]=



In[•]:=

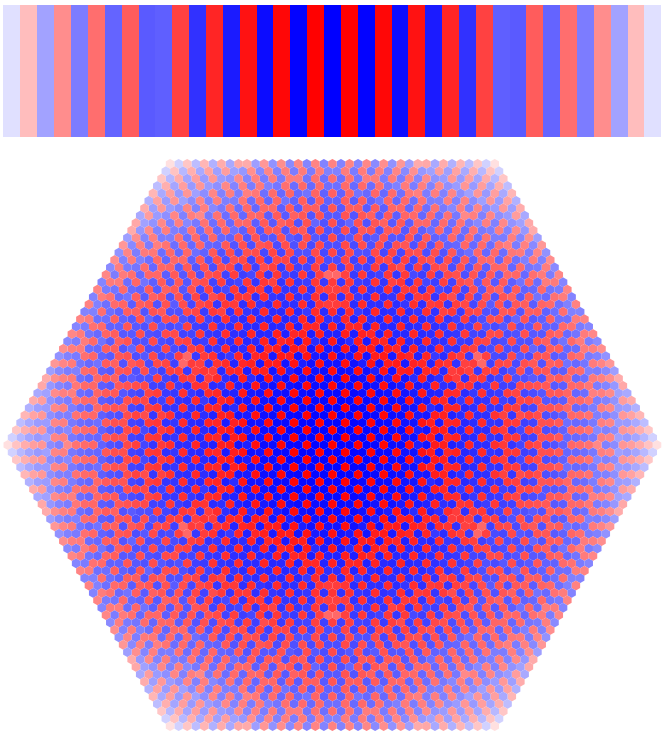
```
AbsoluteTiming@Short[th = Θ[DK[70]]]
PolyPlot[th]
```

```
Done with commons at 13.8219616

Done with Θ at 659.7819274
```

Out[•]=

$$\left\{660.376,\ \left\{-\frac{16-332\,T+3178\,T^2-19\,008\,T^3+\text{<<47>>}+3178\,T^{36}-332\,T^{37}+16\,T^{38}}{T^{19}},\right.\right.$$
$$\left.\left.\frac{16\,384-339\,968\,T_1+\text{<<6789>>}+16\,384\,T_1^{76}\,T_2^{76}}{T_1^{38}\,T_2^{38}}\right\}\right\}$$

Out[•]=



In[•]:=
```
AbsoluteTiming@Short[th = Θ[DK[150]]]
PolyPlot[th]
```

```
Done with commons at 216.3740451
```

Out[•]=

```
$Aborted
```

Out[•]=

```
$Aborted
```

Version with Δv and Gv

```
T3 = T1 T2;
R11[{s_, i_, j_}] =
  s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
      g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^2 s g2ji - T2^s g2jj + T2^s g3jj) +
          (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji + (T2^s - 2) g2jj + g2ij)) / (T2^s - 1));
R12[{s0_, i0_, j0_}, {s1_, i1_, j1_}] =
  s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0));
Γ1[φ_, k_] = -φ/2 + φ g3kk;
Θ[K_] := Module[{Cs, φ, n, A, Δ, Δv, G, Gv, ev, Θ},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
   Δ = T^(-Total[φ]-Total[Cs[[All,1]]])/2 Det[A]; Δv = Table[Δ /. T → Tv, {v, 3}];
   G = Inverse[A]; Gv = Table[G /. T → Tv, {v, 3}];
   ev[ℰ_] := Factor[ℰ /. gv_,α_,β_ :> Gv[[v, α, β]]];
   Θ = ev[∑_{k1=1}^n ∑_{k2=1}^n R12[Cs[[k1]], Cs[[k2]]]];
   Θ += ev[∑_{k=1}^n R11[Cs[[k]]]];
   Θ += ev[∑_{k=1}^{2 n} Γ1[φ[[k]], k]];
   Factor@{Δ, (Times @@ Δv) Θ}
  ];
```

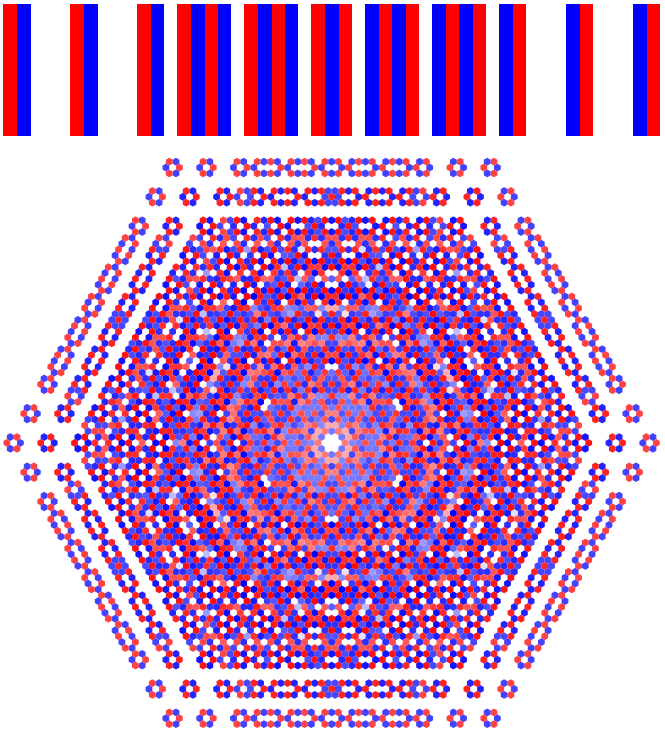
In[•]:=
```
AbsoluteTiming[th = Θ[TorusKnot[13, 5]]]
PolyPlot[th]
```

Out[•]=

$$\left\{85.0704,\ \left\{\frac{1-T+T^5-T^6+T^{10}-T^{11}+T^{13}-T^{14}+T^{15}-T^{16}+T^{18}-T^{19}+T^{20}-T^{21}+T^{23}-T^{24}+T^{25}-T^{27}+T^{28}-T^{29}+T^{30}-T^{32}+T^{33}-T^{34}+T^{35}-T^{37}+T^{38}-T^{42}+T^{43}-T^{47}+T^{48}}{T^{24}},\ \frac{1}{T_1^{48}\,T_2^{48}}\right.\right.$$
$$\left(24 - 24\,T_1 + 24\,T_1^5 - 24\,T_1^6 + 24\,T_1^{10} - 24\,T_1^{11} + 24\,T_1^{13} - 24\,T_1^{14} + 24\,T_1^{15} - 24\,T_1^{16} + 24\,T_1^{18} - 24\,T_1^{19} + 24\,T_1^{20} - 24\,T_1^{21} + 24\,T_1^{23} -\right.$$
$$24\,T_1^{24} + \langle\!\langle 7325 \rangle\!\rangle + 24\,T_1^{71}\,T_2^{96} - 24\,T_1^{72}\,T_2^{96} + 24\,T_1^{73}\,T_2^{96} - 24\,T_1^{75}\,T_2^{96} + 24\,T_1^{76}\,T_2^{96} - 24\,T_1^{77}\,T_2^{96} + 24\,T_1^{78}\,T_2^{96} - 24\,T_1^{80}\,T_2^{96} +$$
$$\left.\left.24\,T_1^{81}\,T_2^{96} - 24\,T_1^{82}\,T_2^{96} + 24\,T_1^{83}\,T_2^{96} - 24\,T_1^{85}\,T_2^{96} + 24\,T_1^{86}\,T_2^{96} - 24\,T_1^{90}\,T_2^{96} + 24\,T_1^{91}\,T_2^{96} - 24\,T_1^{95}\,T_2^{96} + 24\,T_1^{96}\,T_2^{96}\right)\right\}\right\}$$

Full expression not available (original memory size: 1.8 MB)

Out[•]=



Version from Talks\Toronto-241030\Theta.nb

In[•]:=
```
T3 = T1 T2;
CF[ℰ_] := Module[{vs = Union@Cases[ℰ, g__, ∞], ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :↦ Factor[c] (Times @@ vs^ps)]];
R11[{s_, i_, j_}] =
  CF[s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
      g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^(2 s) g2ji - T2^s g2jj + T2^s g3jj) +
        (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji + (T2^s - 2) g2jj + g2ij)) / (T2^s - 1))];
R12[{s0_, i0_, j0_}, {s1_, i1_, j1_}] := CF[
  s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))]
Γ1[φ_, k_] = -φ/2 + φ g3kk;
Θ[K_] := Module[{Cs, φ, n, A, Δ, G, ev, θ, t = AbsoluteTime[]},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :↦ (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
   Δ = T^(-Total[φ] - Total[Cs[[All, 1]]])/2 Det[A];
   G = Inverse[A];
   Print["Done with commons at ", AbsoluteTime[] - t];
   ev[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :↦ (G[[α, β]] /. T → T_ν)];
   θ = ev[∑_{k1=1}^n ∑_{k2=1}^n R12[Cs[[k1]], Cs[[k2]]]];
   θ += ev[∑_{k=1}^n R11[Cs[[k]]]];
   θ += ev[∑_{k=1}^{2 n} Γ1[φ[[k]], k]];
   Print["Done with θ at ", AbsoluteTime[] - t];
   Factor@{Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) θ}];
```

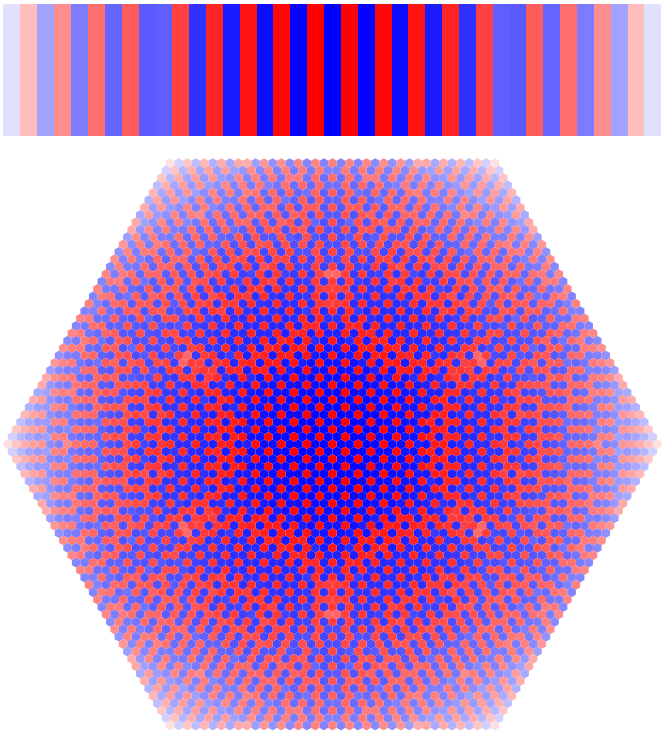
In[•]:=
```
AbsoluteTiming@Short[th = Θ[DK[70]]]
PolyPlot[th]
```

```
Done with commons at 14.4189511

Done with Θ at 125.2023122
```

Out[•]=

$$\left\{125.793,\ \left\{-\frac{16-332\,T+\text{<<53>>}+16\,T^{38}}{T^{19}},\ \frac{\text{<<6792>>}+16\,384\,T_1^{76}\,T_2^{76}}{T_1^{38}\,T_2^{38}}\right\}\right\}$$

Out[•]=



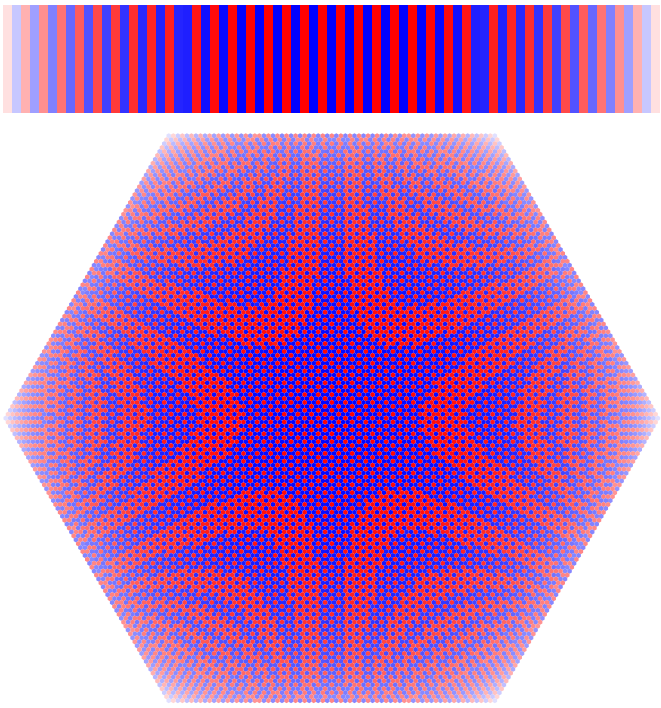
In[•]:=
```
AbsoluteTiming@Short[th = Θ[DK[150]]]
PolyPlot[th]
```

```
Done with commons at 210.0849949

Done with Θ at 1281.7855698
```

Out[•]=

$$\left\{1287.27,\ \left\{\frac{\left(1-3\,T+T^2\right)^2\ (\ll 1\gg)^2\ \ll 1\gg\ (\ll 6\gg+2\ \ll 1\gg)\ \left(\ll 83\gg+640\,T^{56}\right)}{T^{36}},\right.\right.$$
$$\left.\left.-\frac{(\ll 1\gg)^2\ \ll 5\gg\ (\ll 1\gg)}{T_1^{72}\,T_2^{72}}\right\}\right\}$$

Out[•]=



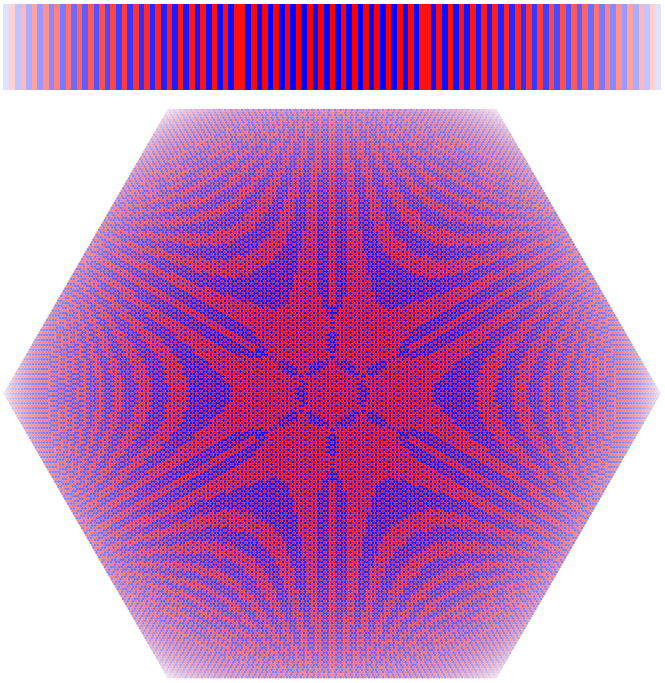
In[•]:=
```
AbsoluteTiming@Short[th = Θ[DK[240]]]
PolyPlot[th]
```

```
Done with commons at 1467.0389535

Done with Θ at 22 301.8891706
```

Out[•]=

$$\left\{22\,316.7,\ \left\{-\frac{384-35\,088\,T+\text{<<172>>}+384\,T^{116}}{T^{58}},\ -\frac{\text{<<1>>}}{T_1^{116}\,T_2^{\text{<<3>>}}}\right\}\right\}$$

Out[•]=



Version from Projects\HigherRank\DunfieldKnots\theta.nb

pdf

In[•]:=

```
T3 = T1 T2;
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | π | g)__, ∞] ⋃ {x, p, ϵ}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]];
R1[1, i_, j_] = CF[
   1/2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
    T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
       T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj)/(T1 - 1)];
R1[-1, i_, j_] = CF[
   -1/2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
    T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
    (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
       g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj)/(T1 - 1)];
Θ[{1, i0_, j0_}, {1, i1_, j1_}] =
```

```
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
   T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
Θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
  (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
   (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
Θ[{-1, i0_, j0_}, {1, i1_, j1_}] = CF[
  T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 -
     T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 - g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1)];
Θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = CF[
  (1 - T3^-1) (-T1^-1 g1,j1,i0 g2,i1,i0 g3,j0,i1 +
     g1,j1,j0 g2,i1,i0 g3,j0,i1 + T1^-1 g1,j1,i0 g2,j1,i0 g3,j0,i1 - g1,j1,j0 g2,j1,i0 g3,j0,i1)];
Γ1[φ_, k_] = -φ / 2 + φ g3,k,k;
Θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z, t = AbsoluteTime[]},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A];
  Print["Done with commons at ", AbsoluteTime[] - t];
  gEval[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T -> T_ν)];
  z = gEval[Sum[Sum[Θ[Cs[[k1]], Cs[[k2]]], {k2, n}], {k1, n}]];
  z += gEval[Sum[R1 @@ Cs[[k]], {k, n}]];
  z += gEval[Sum[Γ1[φ[[k]], k], {k, 2 n}]];
  Print["Done with z at ", AbsoluteTime[] - t];
  {Δ, (Δ /. T -> T1) (Δ /. T -> T2) (Δ /. T -> T3) z} // Factor];
```

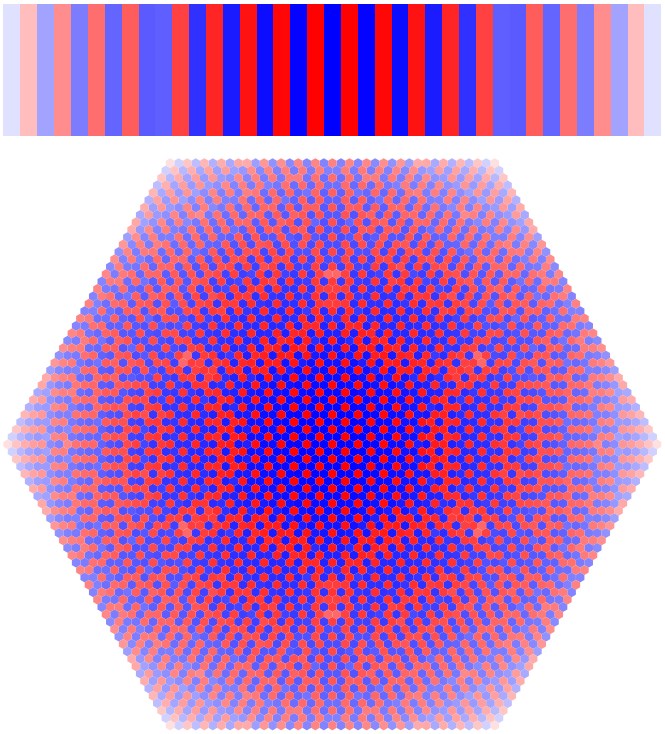
In[•]:=
```
AbsoluteTiming@Short[th = Θ[DK[70]]]
PolyPlot[th]
```

```
Done with commons at 11.1891489

Done with z at 110.1266331
```

Out[•]=

$$\left\{110.568,\ \left\{-\frac{16-332\,T+\text{<<50>>}+3178\,T^{36}-332\,T^{37}+16\,T^{38}}{T^{19}},\ \frac{\text{<<6792>>}+16\,384\,T_1^{\text{<<2>>}}\,T_2^{76}}{T_1^{38}\,T_2^{38}}\right\}\right\}$$

Out[•]=



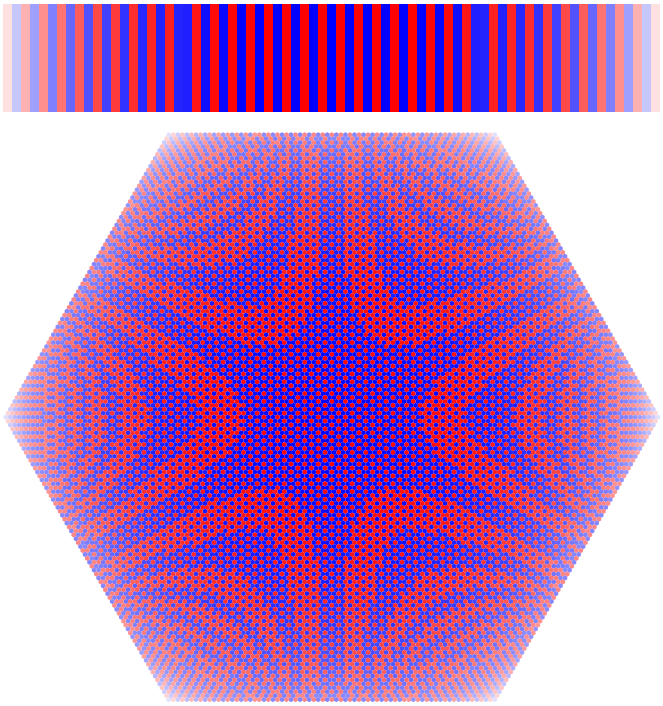
In[•]:=
```
AbsoluteTiming@Short[th = Θ[DK[150]]]
PolyPlot[th]
```

```
Done with commons at 210.0128410

Done with z at 1267.9159827
```

Out[•]=

$$\left\{1272.99,\ \left\{\frac{\left(1-3\,T+T^2\right)^2\ \text{<<3>>}\ \left(640-24\,256\,T+\text{<<80>>}+640\,T^{56}\right)}{T^{36}},\ -\frac{(1-\text{<<1>>}+\text{<<1>>})^2\ \text{<<5>>}\ (\text{<<1>>})}{T_1^{72}\,T_2^{72}}\right\}\right\}$$

Out[•]=



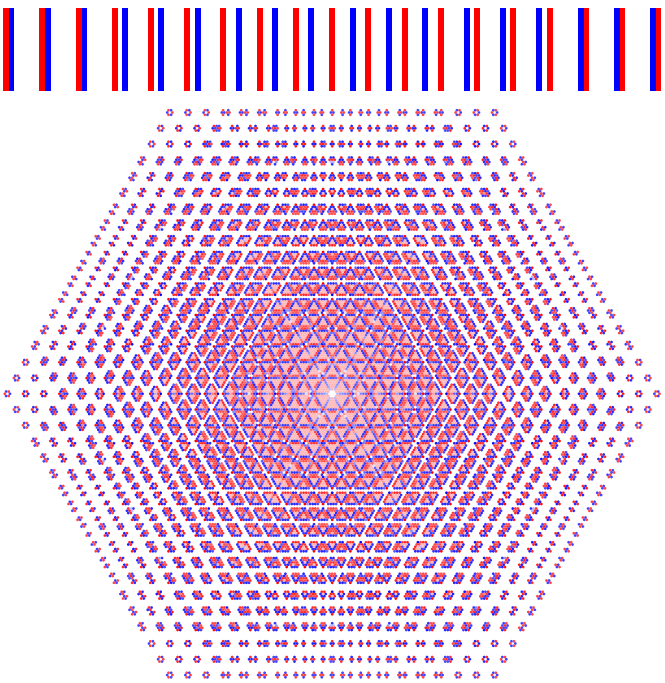
In[•]:=
```
AbsoluteTiming@Short[th = Θ[TorusKnot[22, 7]]]
PolyPlot[th]
```

```
Done with commons at 71.3250148

Done with z at 623.7988225
```

Out[•]=

$$\left\{625.729,\ \left\{\frac{\left(1-T+T^2-T^3+T^4-T^5+T^6\right)\ \left(\text{<<45>>}+T^{60}\right)\ \left(1+\text{<<43>>}+T^{59}+T^{60}\right)}{T^{63}},\right.\right.$$
$$\left.\left.\frac{\text{<<30\,647>>}+63\,T_1^{\text{<<3>>}}\,T_2^{\text{<<3>>}}}{T_1^{126}\,T_2^{126}}\right\}\right\}$$

Out[•]=



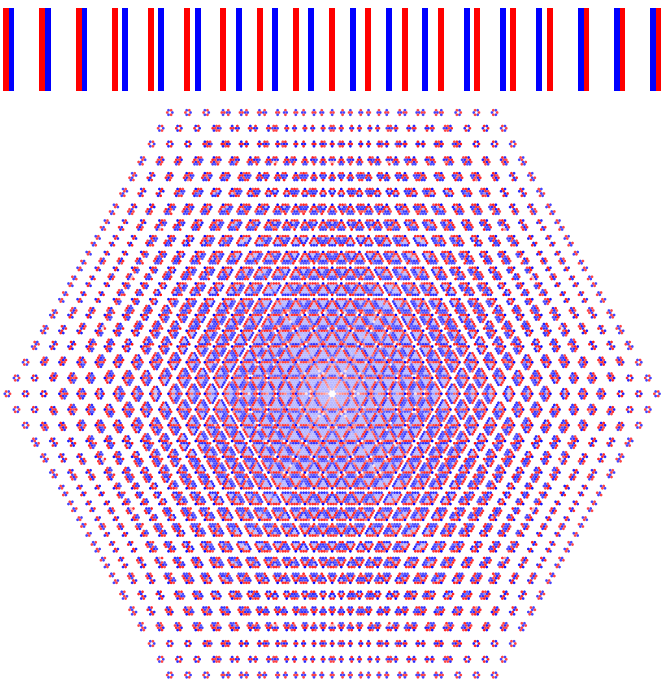
In[•]:=
```
AbsoluteTiming@Short[th = Θ[Mirror@TorusKnot[22, 7]]]
PolyPlot[th]
```

```
Done with commons at 104.0091145

Done with z at 1108.1460322
```

Out[•]=

$$\left\{1110.13,\ \left\{\frac{\left(1-T+T^2-T^3+T^4-T^5+T^6\right)\ \left(\text{<<45>>}+T^{60}\right)\ \left(1+\text{<<43>>}+T^{59}+T^{60}\right)}{T^{63}},\right.\right.$$
$$\left.\left.-\frac{\text{<<30\,647>>}+63\,T_1^{\text{<<3>>}}\,T_2^{\text{<<3>>}}}{T_1^{126}\,T_2^{126}}\right\}\right\}$$

Out[•]=



Eval with a Dispatch Table

```
T3 = T1 T2;
R11[{s_, i_, j_}] =
 s (1/2 - g3ii + T2^s g1ii g2ji - g1ii g2jj - (T2^s - 1) g2ji g3ii + 2 g2jj g3ii - (1 - T3^s) g2ji g3ji -
    g2ii g3jj - T2^s g2ji g3jj + g1ii g3jj + ((T1^s - 1) g1ji (T2^(2 s) g2ji - T2^s g2jj + T2^s g3jj) +
       (T3^s - 1) g3ji (1 - T2^s g1ii - (T1^s - 1) (T2^s + 1) g1ji + (T2^s - 2) g2jj + g2ij)) / (T2^s - 1))
R12[{s0_, i0_, j0_}, {s1_, i1_, j1_}] :=
 s1 (T1^s0 - 1) (T2^s1 - 1)^-1 (T3^s1 - 1) g1,j1,i0 g3,j0,i1 ((T2^s0 g2,i1,i0 - g2,i1,j0) - (T2^s0 g2,j1,i0 - g2,j1,j0))
Γ1[φ_, k_] = -φ/2 + φ g3kk
Θ[K_] :=
  Module[{Cs, φ, n, A, s, i, j, k, Δ, G, Δν, gναβ, gEval, z, t = AbsoluteTime[], out},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   A = IdentityMatrix[2 n + 1];
   Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
   Δ = Factor[T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A]];
   Δν = Product[Δ /. T → Tν, {ν, 3}];
   Print["After Δ: ", AbsoluteTime[] - t];
   G = Factor@Inverse[A];
   Print["After G: ", AbsoluteTime[] - t];
   gναβ =
    Dispatch@Flatten@Table[gν,α,β → (G[[α, β]] /. T → Tν), {ν, 3}, {α, 2 n + 1}, {β, 2 n + 1}];
   gEval[ℰ_] := Expand[Δν (ℰ /. gναβ)];
   Print["After gEval: ", AbsoluteTime[] - t];
   z = Sum[gEval@R12[Cs[[k1]], Cs[[k2]]], {k1, 1, n}, {k2, 1, n}];
   z += Sum[gEval@R11[Cs[[k]]], {k, 1, n}]; z += Sum[gEval@Γ1[φ[[k]], k], {k, 1, 2 n}];
   Print["After z: ", AbsoluteTime[] - t];
   out = {Δ, z} // Factor;
   Print["After out: ", AbsoluteTime[] - t];
   out
   ];
```

In[•]:=
```
Expand[Θ[Knot[3, 1]]]
```

```
After Δ: 0.0019521
After G: 0.0029286
After gEval: 0.0039052
After z: 0.0132342
After out: 0.0712492
```

Out[•]=
$$\left\{-1+\frac{1}{T}+T,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2\,T_2^2}+\frac{1}{T_1\,T_2^2}+\frac{1}{T_1^2\,T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2\,T_2-T_2^2+T_1\,T_2^2-T_1^2\,T_2^2\right\}$$

In[•]:=
```
Expand[θ[Knot[10, 165]]]
```

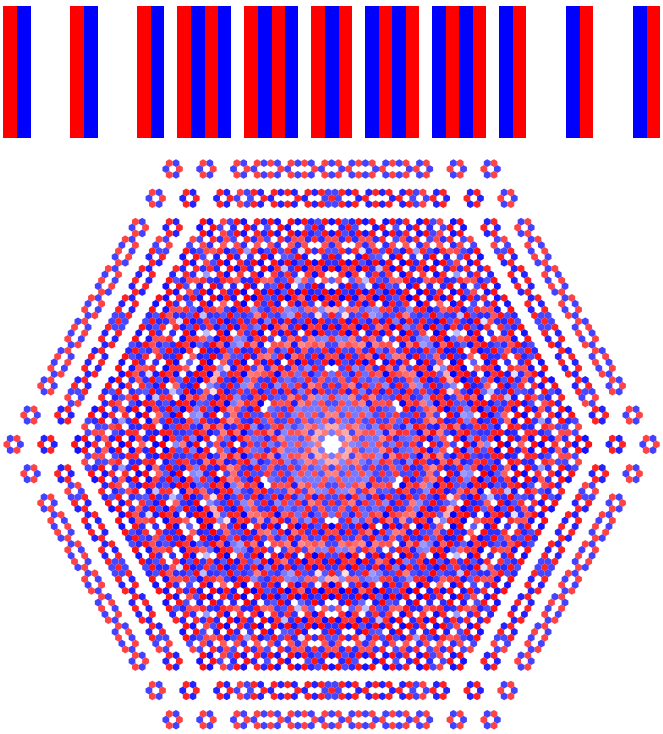
Out[•]=

$$\left\{-15-\frac{2}{T^2}+\frac{10}{T}+10\,T-2\,T^2,\right.$$
$$-1404-\frac{9}{T_1^4}-\frac{178}{T_1^3}+\frac{607}{T_1^2}+\frac{624}{T_1}+624\,T_1+607\,T_1^2-178\,T_1^3-9\,T_1^4-\frac{9}{T_2^4}-\frac{9}{T_1^4\,T_2^4}+\frac{44}{T_1^3\,T_2^4}-$$
$$\frac{65}{T_1^2\,T_2^4}+\frac{44}{T_1\,T_2^4}-\frac{178}{T_2^3}+\frac{44}{T_1^4\,T_2^3}-\frac{178}{T_1^3\,T_2^3}+\frac{104}{T_1^2\,T_2^3}+\frac{104}{T_1\,T_2^3}+\frac{44\,T_1}{T_2^3}+\frac{607}{T_2^2}-\frac{65}{T_1^4\,T_2^2}+\frac{104}{T_1^3\,T_2^2}+\frac{607}{T_1^2\,T_2^2}-$$
$$\frac{1041}{T_1\,T_2^2}+\frac{104\,T_1}{T_2^2}-\frac{65\,T_1^2}{T_2^2}+\frac{624}{T_2}+\frac{44}{T_1^4\,T_2}+\frac{104}{T_1^3\,T_2}-\frac{1041}{T_1^2\,T_2}+\frac{624}{T_1\,T_2}-\frac{1041\,T_1}{T_2}+\frac{104\,T_1^2}{T_2}+\frac{44\,T_1^3}{T_2}+$$
$$624\,T_2+\frac{44\,T_2}{T_1^3}+\frac{104\,T_2}{T_1^2}-\frac{1041\,T_2}{T_1}+624\,T_1\,T_2-1041\,T_1^2\,T_2+104\,T_1^3\,T_2+44\,T_1^4\,T_2+$$
$$607\,T_2^2-\frac{65\,T_2^2}{T_1^2}+\frac{104\,T_2^2}{T_1}-1041\,T_1\,T_2^2+607\,T_1^2\,T_2^2+104\,T_1^3\,T_2^2-65\,T_1^4\,T_2^2-178\,T_2^3+\frac{44\,T_2^3}{T_1}+$$
$$\left.104\,T_1\,T_2^3+104\,T_1^2\,T_2^3-178\,T_1^3\,T_2^3+44\,T_1^4\,T_2^3-9\,T_2^4+44\,T_1\,T_2^4-65\,T_1^2\,T_2^4+44\,T_1^3\,T_2^4-9\,T_1^4\,T_2^4\right\}$$

In[•]:=
```
PolyPlot[θ[TorusKnot[13, 5]]]
```

```
After △: 0.2141511

After G: 17.9732460

After gEval: 20.1062978

After z: 20.1514453

After out: 93.0097957
```

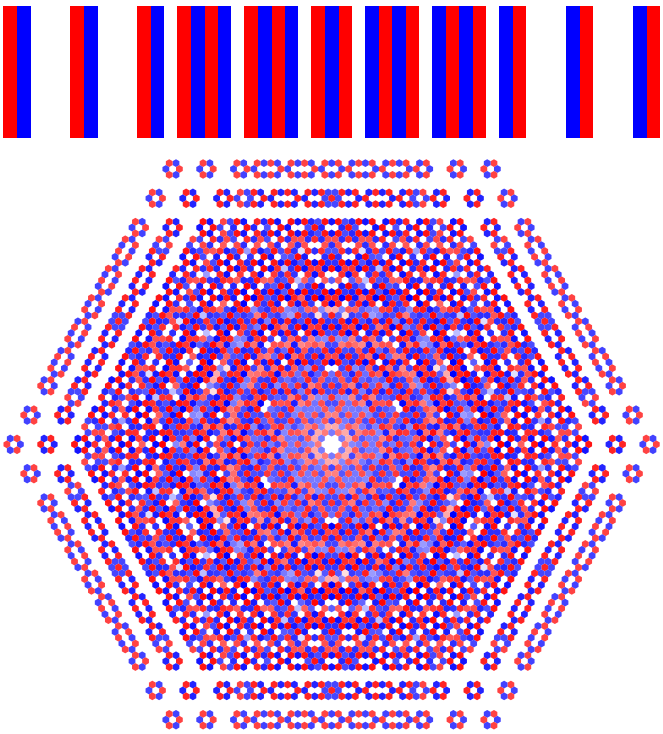
Out[•]=



In[•]:=
```
PolyPlot[∂[TorusKnot[13, 5]]]
```

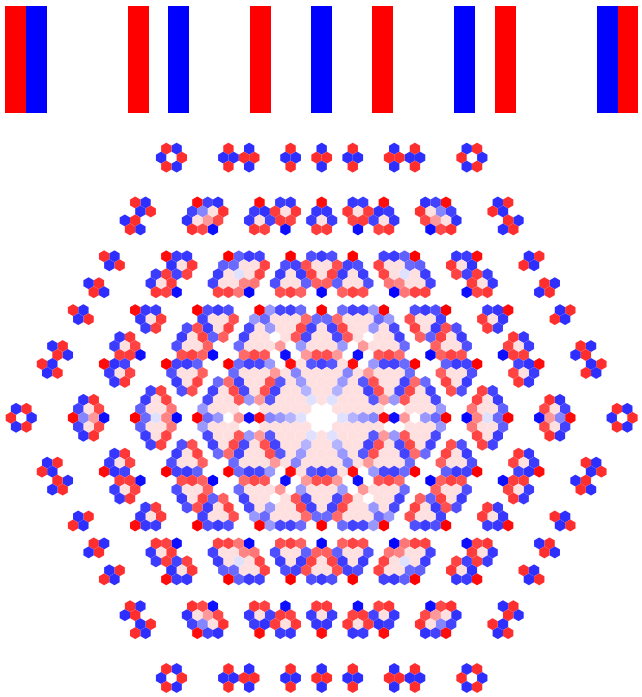
```
After △: 0.2155422

After G: 18.1539619

After gEval: 20.3798731

After z: 7366.6256760

After out: 8397.7290368
```

Out[•]=



In[•]:=
```
PolyPlot[Θ[TorusKnot[7, 6]]]
```

Out[•]=



In[•]:=
```
AbsoluteTiming[th = θ[TorusKnot[22, 7]]]
PolyPlot[th]
```

Out[•]=

$$\Big\{4740.09,\ \Big\{\frac{1}{T^{63}}\left(1-T+T^2-T^3+T^4-T^5+T^6\right)\left(1-T+T^7-T^8+T^{11}-T^{12}+T^{14}-T^{15}+T^{18}-T^{19}+T^{21}-T^{23}+T^{25}-T^{26}+T^{28}-T^{30}+T^{32}-T^{34}+T^{35}-T^{37}+T^{39}-T^{41}+T^{42}-T^{45}+T^{46}-T^{48}+T^{49}-T^{52}+T^{53}-T^{59}+T^{60}\right)\left(1+T-T^7-T^8-T^{11}-T^{12}+T^{14}+T^{15}+T^{18}+T^{19}-T^{21}+T^{23}-T^{25}-T^{26}+T^{28}-T^{30}+T^{32}-T^{34}-T^{35}+T^{37}-T^{39}+T^{41}+T^{42}+T^{45}+T^{46}-T^{48}-T^{49}-T^{52}-T^{53}+T^{59}+T^{60}\right),$$
$$\frac{1}{T_1^{126}\,T_2^{126}}\left(63-63\,T_1+63\,T_1^7-63\,T_1^8+63\,T_1^{14}-63\,T_1^{15}+63\,T_1^{21}-63\,T_1^{23}+63\,T_1^{28}-63\,T_1^{30}+63\,T_1^{35}-63\,T_1^{37}+\cdots 30\,611\cdots+63\,T_1^{210}\,T_2^{252}-63\,T_1^{215}\,T_2^{252}+63\,T_1^{217}\,T_2^{252}-63\,T_1^{222}\,T_2^{252}+63\,T_1^{224}\,T_2^{252}-63\,T_1^{229}\,T_2^{252}+63\,T_1^{231}\,T_2^{252}-63\,T_1^{237}\,T_2^{252}+63\,T_1^{238}\,T_2^{252}-63\,T_1^{244}\,T_2^{252}+63\,T_1^{245}\,T_2^{252}-63\,T_1^{251}\,T_2^{252}+63\,T_1^{252}\,T_2^{252}\right)\Big\}\Big\}$$

Full expression not available (original memory size: 8.1 MB)

Out[•]=