Pensieve header: A solvable Lie algebra which is not a semi-direct product of an Abelian and a nilpotent.

In[∘]:=
```
Clear[e, A, B, X, Y, Z, Comm];

e[i_, j_] := SparseArray[{{i, j} → 1}, {7, 7}] // Normal;

A = e[1, 2] + e[4, 4];
B = e[2, 3] + e[6, 6];

X = e[4, 5];
Y = e[6, 7];
Z = e[1, 3];

Comm[u_, v_] := u.v - v.u;

{Comm[A, B] == Z, Comm[A, X] == X, Comm[B, Y] == Y, Comm[A, Y] == 0 Y, Comm[B, X] == 0 X,
 Comm[A, Z] == 0 Z, Comm[B, Z] == 0 Z, Comm[X, Y] == 0 X, Comm[X, Z] == 0 Z, Comm[Y, Z] == 0 Z}
```

Out[∘]=
```
{True, True, True, True, True, True, True, True, True, True}
```

Convex basis: azbxy.

In[∘]:=
```
AllExps =
  ((p ↦ (p → Dot @@ (MatrixExp /@ (p /. {a → α A, b → β B, x → ξ X, y → η Y, z → ζ Z})))) /@
    Permutations[{a, b, x, y, z}]);
```

In[ ]:=
```
Union[AllExps, SameTest → ((#1〚2〛 === #2〚2〛) &)] /. (p_ → m_) :> (p → MatrixForm[m])
```

Out[ ]=

$$\left\{\{a, b, x, y, z\} \to \begin{pmatrix} 1 & \alpha & \alpha\beta+\zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & e^{\alpha}\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & e^{\beta}\eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},\right.$$

$$\{a, x, y, b, z\} \to \begin{pmatrix} 1 & \alpha & \alpha\beta+\zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & e^{\alpha}\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & \eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\{b, a, x, y, z\} \to \begin{pmatrix} 1 & \alpha & \zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & e^{\alpha}\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & e^{\beta}\eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \{b, x, a, y, z\} \to \begin{pmatrix} 1 & \alpha & \zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & e^{\beta}\eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\{x, a, b, y, z\} \to \begin{pmatrix} 1 & \alpha & \alpha\beta+\zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & e^{\beta}\eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \{x, a, y, b, z\} \to \begin{pmatrix} 1 & \alpha & \alpha\beta+\zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & \eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\left.\{x, y, b, a, z\} \to \begin{pmatrix} 1 & \alpha & \zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & \xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & \eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \{y, b, a, x, z\} \to \begin{pmatrix} 1 & \alpha & \zeta & 0 & 0 & 0 & 0 \\ 0 & 1 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\alpha} & e^{\alpha}\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta} & \eta \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\right\}$$