

Pensieve header: An isolated attempt to compute the multiplication tensor of QU.

The implementation of QU is modified from pensieve://Projects/SL2Portfolio/SL2PortfolioProgram.nb.

Initialization / Utilities

```
In[ ]:= $k = 1; $U = QU; $E := {$k};
$trim := {e^{k-} /; k > $k -> 0};
SetAttributes[{SS, SST}, HoldAll];
q_h = e^{y e^h};
(* Upper to lower and lower to upper: *)
U2l = {B_{i-}^{p-} -> e^{-p h y} b_i, B_{-}^{p-} -> e^{-p h y} b, T_{i-}^{p-} -> e^{p h} t_i, T_{-}^{p-} -> e^{p h} t, A_{i-}^{p-} -> e^{p y} a_i, A_{-}^{p-} -> e^{p y} a};
l2U = {e_{-}^{c-} b_{i-} + d_{-} -> B_{i-}^{-c/(h y)} e^d, e_{-}^{c-} b + d_{-} -> B^{-c/(h y)} e^d,
e_{-}^{c-} t_{i-} + d_{-} -> T_{i-}^{c/h} e^d, e_{-}^{c-} t + d_{-} -> T^{c/h} e^d,
e_{-}^{c-} a_{i-} + d_{-} -> A_{i-}^{c/y} e^d, e_{-}^{c-} a + d_{-} -> A^{c/y} e^d,
e_{-}^{c-} -> e^{Expand@c}};
SS[_e_, op_] := Collect[Normal@Series[_e_, {e, 0, $k}], e, op];
SS[_e_] := SS[_e_, Together];
SST[_e_, op___] := SS[_e_ /. U2l, op];
Simp[_e_, op_] := Collect[_e_, _CU | _QU, op];
Simp[_e_] := Simp[_e_, SS[#, Expand] &];
SimpT[_e_] := Collect[_e_, _CU | _QU, SST[#, Expand] &];
Kd /: Kd_{i,j} := If[i == j, 1, 0];
c_Integer_{k_Integer} := c + 0[e]^{k+1};
```

```
In[ ]:= CF[_e_] := ExpandDenominator@
ExpandNumerator@Together[Expand[_e_] /. e^x- e^y- -> e^{x+y} /. e^x- -> e^{CF[x]}];
```

```
In[ ]:= Unprotect[SeriesData];
SeriesData /: CF[sd_SeriesData] := MapAt[CF, sd, 3];
SeriesData /: Expand[sd_SeriesData] := MapAt[Expand, sd, 3];
SeriesData /: Simplify[sd_SeriesData] := MapAt[Simplify, sd, 3];
SeriesData /: Together[sd_SeriesData] := MapAt[Together, sd, 3];
SeriesData /: Collect[sd_SeriesData, specs___] := MapAt[Collect[#, specs] &, sd, 3];
Protect[SeriesData];
```

Self-Pair (SP):

```
In[ ]:= SP_{ }[_P_] := P; SP_{_e- -> x-, ps___}[_P_] := Expand[P // SP_{ps}] /. f_{-} . \xi^{d-} -> \partial_{\{x,d\}} f
```

DeclareAlgebra

```
In[*]:= Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z_] := (x ⋈ y) ⋈ z;
0 ⋈ _ = _ ⋈ 0 = 0;
(x_Plus) ⋈ y_ := (# ⋈ y) & /@ x; x_ ⋈ (y_Plus) := (x ⋈ #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ⋈ y - y ⋈ x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

```
In[*]:= DeclareAlgebra[U_Symbol, opts__Rule] := Module[{gp, sr, g, cp, M, pow, k = 0,
  gs = Generators /. {opts},
  cs = Centrals /. {opts} /. Centrals -> {}},
  (#U = U@#) & /@ gs;
  gp = Alternatives @@ gs; gp = gp | gp_ (* gens *)
  sr = Flatten@Table[{g -> ++k, gi_ -> {i, k}}, {g, gs}]; (* sorting -> *)
  cp = Alternatives @@ cs; (* cents *)
  SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
  CE[ε_] := Collect[ε, _U, Expand] /. e^x_ -> e^Expand[x] /. $trim;
  Ui[ε_] := ε /. {t : cp -> ti, u_U -> (ti &) /@ u};
  Ui[NCM[]] = pow[ε_, 0] = U@{} = 1U = U[];
  B[U@(x_)i_, U@(y_)j_] := Ui@B[U@x, U@y];
  B[U@(x_)i_, U@(y_)j_] /; i != j := 0;
  B[U@y_, U@x_] := CE[-B[U@x, U@y]];
  x_ ⋈ (c_. 1U) := CE[c x]; (c_. 1U) ⋈ x_ := CE[c x];
  (a_. U[xx_, x_]) ⋈ (b_. U[y_, yy_]) := If[OrderedQ[{x, y} /. sr],
    CE@M[a b /. $trim, U[xx, x, y, yy]],
    U@xx ⋈ CE@M[a b /. $trim, U@y ⋈ U@x + B[U@x, U@y, $E]] ⋈ U@yy];
  U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ⋈ U@{r}];
  U@{c_. * l : gp, r___} := CE[c U[l] ⋈ U@{r}];
  U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
  U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
  U@{l_, r___} := U@{Expand[l], r};
  U[ε_NonCommutativeMultiply] := U /@ ε;
  Ou[specs___, poly_] := Module[{sp, null, vs, us},
    sp = Replace[{specs}, l_List -> lnull, {1}];
    vs = Join@@ (First /@ sp);
    us = Join@@ (sp /. l_s_ -> (l /. x_i_ -> xs));
    CE[Total[
      CoefficientRules[poly, vs] /. (p_ -> c_) -> c U@ (us^p)
    ] /. x_nnull -> x];
  Ou[specs___, E[l_, Q_, P_]] := Ou[specs, SS@Normal[P e^(L+Q)]];
  pow[ε_, n_] := pow[ε, n - 1] ⋈ ε;
  Su[ε_, ss__Rule] := CE@Total[
```

```

CoefficientRules[ $\mathcal{E}$ , First /@ {ss}] /.
  (p_ -> c_) := c NCM@@MapThread[pow, {Last /@ {ss}, p}]]];
 $\sigma_{rs\_}$ [c_. * u_U] := (c /. (t : cp)j_ -> tj/.{rs}) U[List@@(u /. v_j_ -> vj/.{rs})];
mj_ -> k_[c_. * u_U] := CE[(c /. (t : cp)j_ -> tk) DeleteCases[u, _j|k] ]  $\otimes$ 
  U@@Cases[u, w_j_ -> wk]  $\otimes$  U@@Cases[u, _k];
U /: c_. * u_U * v_U := CE[c u  $\otimes$  v];
Si_[c_. * u_U] := CE[(c /. Si[U, Centrals]) DeleteCases[u, _i] ]  $\otimes$ 
  Ui[NCM@@Reverse@Cases[u, x_i -> S@U@x]];
 $\Delta_{i_ -> j_ , k_}$ [c_. * u_U] := CE[(c /.  $\Delta_{i_ -> j_ , k}$ [U, Centrals]) DeleteCases[u, _i] ]  $\otimes$ 
  (NCM@@Cases[u, x_i ->  $\sigma_{1 -> j_ , 2 -> k}$ @ $\Delta$ @U@x] /. NCM[] -> U[])]];

```

Meta-Operations

```

In[ ]:=
 $\sigma_{rs\_}$ [ $\mathcal{E}_{Plus}$ ] :=  $\sigma_{rs}$  /@  $\mathcal{E}$ ;
mj_ -> j_ = Identity; mj_ -> k_[0] = 0;
mj_ -> k_[ $\mathcal{E}_{Plus}$ ] := Simp[mj_ -> k_ /@  $\mathcal{E}$ ];
mis_, i_ -> j_ , k_[ $\mathcal{E}$ ] := mj_ -> k_@mis, i_ -> j_@ $\mathcal{E}$ ;
Si_[ $\mathcal{E}_{Plus}$ ] := Simp[Si /@  $\mathcal{E}$ ];
 $\Delta_{is\_}$ [ $\mathcal{E}_{Plus}$ ] := Simp[ $\Delta_{is}$  /@  $\mathcal{E}$ ];

```

Implementing $CU = \mathcal{U}(\mathfrak{sl}_2^{YE})$

Verify σ and Δ ! Also Generalize Δ to $\Delta_{i,j_1,j_2,\dots}$.

```

In[ ]:=
DeclareAlgebra[CU, Generators -> {y, a, x}, Centrals -> {t}];
B[aCU, yCU] = -y yCU; B[xCU, aCU] = -y xCU;
B[xCU, yCU] = 2 e aCU - t 1CU;
(S@yCU = -yCU; S@aCU = -aCU; S@xCU = -xCU);
Si_[CU, Centrals] = {ti -> -ti};
 $\Delta$ @yCU = CU@y1 + CU@y2;  $\Delta$ @aCU = CU@a1 + CU@a2;  $\Delta$ @xCU = CU@x1 + CU@x2;
 $\Delta_{i_ -> j_ , k_}$ [CU, Centrals] = {ti -> tj + tk};

```

Implementing $QU = \mathcal{U}_q(\mathfrak{sl}_2^{YE})$

```

In[ ]:=
DeclareAlgebra[QU, Generators -> {y, a, x}, Centrals -> {t, T}];
B[aQU, yQU] = -y yQU; B[xQU, aQU] = -y QU@x;
B[xQU, yQU] := SS[qh - 1] QU@{y, x} + QQU[{a}, SS[(1 - T e-2 e a h) / h]];
(S@yQU := QQU[{a, y}, SS[-T-1 eh e a y]]; S@aQU = -aQU; S@xQU := QQU[{a, x}, SS[-eh e a x]]);
Si_[QU, Centrals] = {ti -> -ti, Ti -> Ti-1};
 $\Delta$ @yQU := QQU[{y1, a1}, SS[y1 + T1 e-h e a1 y2]];
 $\Delta$ @aQU = QU@a1 + QU@a2;  $\Delta$ @xQU := QQU[{a1, x1}, SS[x1 + e-h e a1 x2]];
 $\Delta_{i_ -> j_ , k_}$ [QU, Centrals] = {ti -> tj + tk, Ti -> Tj Tk};

```

tSW

Logoi from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

Goal. In either U , compute $F = e^{-\eta y} e^{\xi x} e^{\eta y} e^{-\xi x}$. First compute $G = e^{\xi x} y e^{-\xi x}$, a finite sum. Now F satisfies the ODE $\partial_\eta F = \partial_\eta (e^{-\eta y} e^{\eta G}) = -yF + FG$ with initial conditions $F(\eta = 0) = 1$. So we set it up and solve:

```
In[*]:=
SWxy[U_, kk_] :=
  SWxy[U, kk] = Block[{ $U = U, $k = kk, $p = kk }, Module[{ G, F, fs, f, bs, e, b, es },
    G = Simp[Table[ $\xi^k / k!$ , {k, 0, $k + 1}].NestList[Simp[B[xU, #]] &, yU, $k + 1]];
    fs = Flatten@Table[f1,i,j,k[ $\eta$ ], {1, 0, $k}, {i, 0, 1}, {j, 0, 1}, {k, 0, 1}];
    F = fs.(bs = fs /. fL_,i_,j_,k_[ $\eta$ ]  $\Rightarrow e^L U @ \{y^i, a^j, x^k\}$ );
    es = Flatten[
      Table[Coefficient[e, b] == 0, {e, {F - 1U /.  $\eta \rightarrow 0$ , F  $\boxtimes$  G - yU  $\boxtimes$  F -  $\partial_\eta F$ }}, {b, bs}]];
    F = F /. DSolve[es, fs,  $\eta$ ][[1]];
     $\mathbb{E}$ [0,
       $\xi x + \eta y + (U /. \{CU \rightarrow -t \eta \xi, QU \rightarrow \eta \xi (1 - T) / \hbar\})$ ,
      F + 0$k /. {e $\rightarrow$  1, U $\rightarrow$  Times}
    ] /. (v :  $\eta$  |  $\xi$  | t | T | y | a | x)  $\rightarrow$  v1
  ]];
tSWxy,i_,j_ $\rightarrow$ k_ := SWxy[$U, $k] /. { $\xi_1 \rightarrow \xi_i, \eta_1 \rightarrow \eta_j, (v : t | T | y | a | x)_1 \rightarrow v_k$ };
tSWxa,i_,j_ $\rightarrow$ k_ :=  $\mathbb{E}[\alpha_j a_k, e^{-y \alpha_j} \xi_i x_k, 1]$ ;
tSWay,i_,j_ $\rightarrow$ k_ :=  $\mathbb{E}[\alpha_i a_k, e^{-y \alpha_i} \eta_j y_k, 1]$ ;
```

Exponentials as needed.

Task. Define $\text{Exp}_{U_i,k}[\xi, P]$ which computes $e^{\xi \mathcal{O}(P)}$ to ϵ^k in the algebra U_i , where ξ is a scalar, X is x_i or y_i , and P is an ϵ -dependent near-docile element, giving the answer in $\mathbb{E}$ -form. Should satisfy $U @ \text{Exp}_{U_i,k}[\xi, P] == \mathbb{S}_U[e^{\xi X}, x \rightarrow \mathcal{O}(P)]$.

Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\xi \mathcal{O}(P)} = \mathcal{O}(e^{\xi P_0} F(\xi))$, then $F(\xi = 0) = 1$ and we have:

$$\mathcal{O}(e^{\xi P_0} (P_0 F(\xi) + \partial_\xi F)) = \mathcal{O}(\partial_\xi e^{\xi P_0} F(\xi)) = \partial_\xi \mathcal{O}(e^{\xi P_0} F(\xi)) = \partial_\xi e^{\xi \mathcal{O}(P)} = e^{\xi \mathcal{O}(P)} \mathcal{O}(P) = \mathcal{O}(e^{\xi P_0} F(\xi)) \mathcal{O}(P).$$

This is an ODE for F . Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ .

```

In[*]:= (* Bug: The first line is valid only if  $\mathbb{Q}(e^{P_0}) = e^{\mathbb{Q}(P_0)}$ . *)
(* Bug:  $\xi$  must be a symbol. *)
ExpUi, 0[ $\xi$ _, P_] := Module[{LQ = Normal@P /.  $\epsilon \rightarrow 0$ },
  E[ $\xi$  LQ /. (x | y)i → 0,  $\xi$  LQ /. (t | a)i → 0, 1]];
ExpUi, k[ $\xi$ _, P_] := Block[{$U = U, $k = k},
  Module[{P0,  $\varphi$ ,  $\varphi$ s, F, j, rhs, at0, at $\xi$ },
    P0 = Normal@P /.  $\epsilon \rightarrow 0$ ;
     $\varphi$ s =
      Flatten@Table[ $\varphi_{j1, j2, j3}[\xi]$ , {j2, 0, k}, {j1, 0, 2 k + 1 - j2}, {j3, 0, 2 k + 1 - j2 - j1}];
    F = Normal@Last@ExpUi, k-1[ $\xi$ , P] +  $\epsilon^k \varphi$ s. ( $\varphi$ s /.  $\varphi_{js\_}[\xi] \Rightarrow \text{Times} @@ \{y_i, a_i, x_i\}^{\{js\}}$ );
    rhs = Normal@
      Last@mi, j→i[E[ $\xi$  P0 /. (x | y)i → 0,  $\xi$  P0 /. (t | a)i → 0, F + 0k] mi→j@E[0, 0, P + 0k]];
    at0 = (# == 0) & /@ Flatten@CoefficientList[F - 1 /.  $\xi \rightarrow 0$ , {yi, ai, xi}];
    at $\xi$  = (# == 0) & /@ Flatten@CoefficientList[( $\partial_{\xi}$ F) + P0 F - rhs, {yi, ai, xi}];
    E[ $\xi$  P0 /. (x | y)i → 0,  $\xi$  P0 /. (t | a)i → 0, F + 0k] /.
    DSolve[And@@(at0 ∪ at $\xi$ ),  $\varphi$ s,  $\xi$ ][[1]]]

```

```

In[*]:= $k = 1

```

```

Out[*]=

```

```

1

```

```

In[*]:= $Basis = {aQU, xQU, yQU};
Module[{a, b}, Table[{a, b} → CE[B[a, b]], {a, $Basis}, {b, $Basis}]] // MatrixForm

```

```

Out[*]//MatrixForm=

$$\begin{pmatrix} \{QU[a], QU[a]\} \rightarrow 0 & \{QU[a], QU[x]\} \rightarrow \gamma QU[x] & \{QU[x], QU[x]\} \rightarrow 0 \\ \{QU[x], QU[a]\} \rightarrow -\gamma QU[x] & \{QU[x], QU[x]\} \rightarrow 0 & \{QU[x], QU[x]\} \rightarrow 0 \\ \{QU[y], QU[a]\} \rightarrow \gamma QU[y] & \{QU[y], QU[x]\} \rightarrow \left(-\frac{1}{\hbar} + \frac{T}{\hbar}\right) QU[] - 2 T \in QU[a] - \gamma \in \hbar QU[y, x] \end{pmatrix}$$


```

```

In[*]:= $Basis = {aQU, xQU, yQU};
Module[{a, b},
  DeleteCases[
    Flatten[Table[{a, b, c} → CE[B[a, b] + B[b, a]], {a, $Basis}, {b, $Basis}]], _ → 0]
]
Module[{a, b, c},
  DeleteCases[Flatten[Table[{a, b, c} → CE[B[a, B[b, c]] + B[b, B[c, a]] + B[c, B[a, b]]],
    {a, $Basis}, {b, $Basis}, {c, $Basis}]], _ → 0]
]
$ABasis = {aQU, xQU, yQU, QU[y, y], QU[y, a], QU[y, x], QU[a, a], QU[a, x], QU[x, x]};
Module[{a, b, c},
  DeleteCases[Flatten[Table[{a, b, c} → CE[a ⊗ CE[b ⊗ c] - CE[a ⊗ b] ⊗ c],
    {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]], _ → 0]
]

Out[*]=
{}

Out[*]=
{}

Out[*]=
{}

In[*]:= Module[{a, b, c},
  Table[{a, b, c} → CE[a ⊗ CE[b ⊗ c]], {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]
];

In[*]:= B[xQU, yQU] // CE
Out[*]=

$$\left(\frac{1}{\hbar} - \frac{T}{\hbar}\right) QU[] + 2 T \in QU[a] + \gamma \in \hbar QU[y, x]$$


In[*]:= B[xQU, B[xQU, yQU]] // CE
Out[*]=

$$(\gamma \in -3 T \gamma \in) QU[x]$$


In[*]:= B[xQU, B[xQU, B[xQU, yQU]]] // CE
Out[*]=
0

In[*]:= B[xQU, B[xQU, B[xQU, B[xQU, yQU]]]] // CE
Out[*]=
0

In[*]:= B[xQU, B[xQU, B[xQU, B[xQU, B[xQU, yQU]]]]] // Expand
Out[*]=
0

In[*]:= B[xQU, B[xQU, B[xQU, B[xQU, B[xQU, B[xQU, yQU]]]]]] // Expand
Out[*]=
0

```

```

In[*]:= adPower[x_, k_, Env_] [c_ * y_QU] := CE[c adPower[x, k, Env] [y]];
adPower[x_, k_, Env_] [y_Plus] := adPower[x, k, Env] /@ y;
adPower[_ , 0, Env_] [y_] := y;
adPower[x_, k_, Env_] [0] = 0;
adPower[x_, k_, Env_] [y_QU] :=
  adPower[x, k, Env] [y] = CE@B[adPower[x, k - 1, Env] [y], x];
adPower[x_, k_] [y_] := adPower[x, k, $E] [y];

In[*]:= adPower[x_QU, 1] [y_QU]
Out[*]=

$$\left(-\frac{1}{\hbar} + \frac{T}{\hbar}\right) QU[] - 2 T \in QU[a] - \gamma \in \hbar QU[y, x]$$


In[*]:= adPower[x_QU, 2] [y_QU]
Out[*]=

$$(\gamma \in -3 T \gamma \in) QU[x]$$


In[*]:= adPower[x_QU, 3] [y_QU]
Out[*]=
0

In[*]:= adPower[x_QU, 4] [y_QU]
Out[*]=
0

In[*]:= adPower[x_QU, 5] [y_QU]
Out[*]=
0

In[*]:= NilQ[x_QU] = NilQ[y_QU] = True; NilQ[a_QU] = False;
PotQ[c_ * x_QU] := ¬ NilQ[x]; (* Potent Q *)
NilQ[c_ * x_QU] := NilQ[x];
PotQ[x_Plus] := And @@ (PotQ /@ (List @@ x));
NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));

In[*]:= Ad[c_ . a_QU] [z_] := z /. w_QU => ec γ (Count[w,y]-Count[w,x]) w;
Ad[c_ . QU[]] [z_] := z;

In[*]:= Ad[x_] [y_] /; NilQ[x] := Expand@Module[{k = 0, s = 0},
  While[adPower[x, k] [y] != 0,
    s += adPower[x, k] [y] / k!;
    ++k
  ];
  s
]

In[*]:= Ad[x_QU] [y_QU]
Out[*]=

$$-\frac{QU[]}{\hbar} + \frac{T QU[]}{\hbar} - 2 T \in QU[a] + \frac{1}{2} \gamma \in QU[x] - \frac{3}{2} T \gamma \in QU[x] + QU[y] - \gamma \in \hbar QU[y, x]$$


```

```

In[*]:= Ad[c aQU] /@ {QU[], xQU, yQU, QU[x, y]}
Out[*]=
{QU[], e-c γ QU[x], ec γ QU[y], QU[x, y]}

In[*]:= Ad[(α1 + α2 + ε fa[λ]) aQU] /@ {QU[], xQU, yQU, QU[x, y]}
Out[*]=
{QU[], e-γ (α1+α2+ε fa[λ]) QU[x], eγ (α1+α2+ε fa[λ]) QU[y], QU[x, y]}

In[*]:= Normal@Series[e-γ (α1+α2+ε fa[λ]), {ε, 0, $k}]
Out[*]=
e-(α1+α2) γ - e-(α1+α2) γ γ ε fa[λ]

In[*]:= CE[
  Ad[(α1 + α2 + ε fa[λ]) aQU] /@ {QU[], xQU, yQU, QU[x, y]} /. eε -> Normal@Series[eε, {ε, 0, $k}]]
Out[*]=
{QU[], (e-α1 γ-α2 γ - e-α1 γ-α2 γ γ ε fa[λ]) QU[x], (eα1 γ+α2 γ + eα1 γ+α2 γ γ ε fa[λ]) QU[y], QU[x, y]}

In[*]:= Ad[aQU] [Ad[xQU] [yQU]]
Out[*]=
-  $\frac{QU[]}{\hbar} + \frac{T QU[]}{\hbar} - 2 T \in QU[a] + \frac{1}{2} e^{-\gamma} \gamma \in QU[x] - \frac{3}{2} e^{-\gamma} T \gamma \in QU[x] + e^{\gamma} QU[y] - \gamma \in \hbar QU[y, x]$ 

In[*]:= λTangent[] = 0;
λTangent[xs___, x_] := CE[Ad[x] [λTangent[xs]] + ∂λ x];
λTangent[xs_List] := λTangent@@xs

In[*]:= λTangent[λ yQU, λ aQU, λ xQU]
Out[*]=

$$\left( -\frac{e^{\gamma \lambda} \lambda}{\hbar} + \frac{e^{\gamma \lambda} T \lambda}{\hbar} \right) QU[] + (1 - 2 e^{\gamma \lambda} T \in \lambda) QU[a] +$$


$$\left( 1 + \gamma \lambda + \frac{1}{2} e^{\gamma \lambda} \gamma \in \lambda^2 - \frac{3}{2} e^{\gamma \lambda} T \gamma \in \lambda^2 \right) QU[x] + e^{\gamma \lambda} QU[y] - e^{\gamma \lambda} \gamma \in \lambda \hbar QU[y, x]$$


In[*]:= $k = 1
Out[*]=
1

```


In[*]:= lhs = λTangent[λ η1 yqu, α1 aqu, λ ξ1 xqu, λ η2 yqu, α2 aqu, λ ξ2 xqu]

Out[*]=

$$\begin{aligned} & \left(-\frac{e^{\alpha_1 \gamma} \eta_1 \lambda \xi_1}{\hbar} + \frac{e^{\alpha_1 \gamma} \tau \eta_1 \lambda \xi_1}{\hbar} + \frac{\eta_2 \lambda \xi_1}{\hbar} - \frac{\tau \eta_2 \lambda \xi_1}{\hbar} + \frac{e^{\alpha_1 \gamma} \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1^2}{2 \hbar} - \right. \\ & \quad \frac{2 e^{\alpha_1 \gamma} \tau \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1^2}{\hbar} + \frac{3 e^{\alpha_1 \gamma} \tau^2 \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1^2}{2 \hbar} - \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \eta_1 \lambda \xi_2}{\hbar} + \\ & \quad \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \eta_1 \lambda \xi_2}{\hbar} - \frac{e^{\alpha_2 \gamma} \eta_2 \lambda \xi_2}{\hbar} + \frac{e^{\alpha_2 \gamma} \tau \eta_2 \lambda \xi_2}{\hbar} + \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1 \xi_2}{\hbar} - \\ & \quad \frac{4 e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1 \xi_2}{\hbar} + \frac{3 e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau^2 \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1 \xi_2}{\hbar} - \\ & \quad \left. \frac{e^{\alpha_2 \gamma} \gamma \in \eta_2^2 \lambda^3 \xi_1 \xi_2}{2 \hbar} + \frac{2 e^{\alpha_2 \gamma} \tau \gamma \in \eta_2^2 \lambda^3 \xi_1 \xi_2}{\hbar} - \frac{3 e^{\alpha_2 \gamma} \tau^2 \gamma \in \eta_2^2 \lambda^3 \xi_1 \xi_2}{2 \hbar} \right) \text{QU}[] + \\ & (-2 e^{\alpha_1 \gamma} \tau \in \eta_1 \lambda \xi_1 + 2 \tau \in \eta_2 \lambda \xi_1 - 2 e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \in \eta_1 \lambda \xi_2 - 2 e^{\alpha_2 \gamma} \tau \in \eta_2 \lambda \xi_2) \text{QU}[a] + \\ & \left(e^{-\alpha_2 \gamma} \xi_1 + \frac{1}{2} e^{\alpha_1 \gamma - \alpha_2 \gamma} \gamma \in \eta_1 \lambda^2 \xi_1^2 - \frac{3}{2} e^{\alpha_1 \gamma - \alpha_2 \gamma} \tau \gamma \in \eta_1 \lambda^2 \xi_1^2 + \xi_2 + e^{\alpha_1 \gamma} \gamma \in \eta_1 \lambda^2 \xi_1 \xi_2 - \right. \\ & \quad 3 e^{\alpha_1 \gamma} \tau \gamma \in \eta_1 \lambda^2 \xi_1 \xi_2 - \gamma \in \eta_2 \lambda^2 \xi_1 \xi_2 + 3 \tau \gamma \in \eta_2 \lambda^2 \xi_1 \xi_2 + \frac{1}{2} e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \eta_1 \lambda^2 \xi_2^2 - \\ & \quad \left. \frac{3}{2} e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \gamma \in \eta_1 \lambda^2 \xi_2^2 + \frac{1}{2} e^{\alpha_2 \gamma} \gamma \in \eta_2 \lambda^2 \xi_2^2 - \frac{3}{2} e^{\alpha_2 \gamma} \tau \gamma \in \eta_2 \lambda^2 \xi_2^2 \right) \text{QU}[x] + \\ & \left(e^{\alpha_1 \gamma + \alpha_2 \gamma} \eta_1 + e^{\alpha_2 \gamma} \eta_2 - e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \eta_1 \eta_2 \lambda^2 \xi_1 + 3 e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \gamma \in \eta_1 \eta_2 \lambda^2 \xi_1 + \right. \\ & \quad \left. \frac{1}{2} e^{\alpha_2 \gamma} \gamma \in \eta_2^2 \lambda^2 \xi_1 - \frac{3}{2} e^{\alpha_2 \gamma} \tau \gamma \in \eta_2^2 \lambda^2 \xi_1 \right) \text{QU}[y] + \\ & (-e^{\alpha_1 \gamma} \gamma \in \eta_1 \lambda \xi_1 \hbar + \gamma \in \eta_2 \lambda \xi_1 \hbar - e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \eta_1 \lambda \xi_2 \hbar - e^{\alpha_2 \gamma} \gamma \in \eta_2 \lambda \xi_2 \hbar) \text{QU}[y, x] \end{aligned}$$

In[*]:= rhs = CE[λTangent[(ft0[λ] + e ft1[λ]) QU[], (fy0[λ] + e fy1[λ]) yqu, (α1 + α2 + e fa1[λ]) aqu, (fx0[λ] + e fx1[λ]) xqu] /. e^ε -> Normal@Series[e^ε, {ε, 0, \$k}]]

Out[*]=

$$\begin{aligned} & -e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \hbar \text{fx0}[\lambda] \text{QU}[y, x] \text{fy0}'[\lambda] + \\ & \text{QU}[a] \left(\epsilon \text{fa1}'[\lambda] - 2 e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \in \text{fx0}[\lambda] \text{fy0}'[\lambda] \right) + \text{QU}[x] \left(\gamma \in \text{fx0}[\lambda] \text{fa1}'[\lambda] + \text{fx0}'[\lambda] + \right. \\ & \quad \left. \epsilon \text{fx1}'[\lambda] + \frac{1}{2} e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \text{fx0}[\lambda]^2 \text{fy0}'[\lambda] - \frac{3}{2} e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \gamma \in \text{fx0}[\lambda]^2 \text{fy0}'[\lambda] \right) + \\ & \text{QU}[y] \left(e^{\alpha_1 \gamma + \alpha_2 \gamma} \text{fy0}'[\lambda] + e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \text{fa1}[\lambda] \text{fy0}'[\lambda] + e^{\alpha_1 \gamma + \alpha_2 \gamma} \epsilon \text{fy1}'[\lambda] \right) + \\ & \text{QU}[] \left(\text{ft0}'[\lambda] + \epsilon \text{ft1}'[\lambda] - \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} + \right. \\ & \quad \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \gamma \in \text{fa1}[\lambda] \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} + \\ & \quad \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \gamma \in \text{fa1}[\lambda] \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \epsilon \text{fx1}[\lambda] \text{fy0}'[\lambda]}{\hbar} + \\ & \quad \left. \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \in \text{fx1}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \epsilon \text{fx0}[\lambda] \text{fy1}'[\lambda]}{\hbar} + \frac{e^{\alpha_1 \gamma + \alpha_2 \gamma} \tau \in \text{fx0}[\lambda] \text{fy1}'[\lambda]}{\hbar} \right) \end{aligned}$$

In[*]:= **diff** = **CE**[**lhs** - **rhs**]

Out[*]=

$$\begin{aligned}
& \text{QU}[a] \left(-2 e^{a^1 \gamma} T \in \eta_1 \lambda \xi_1 + 2 T \in \eta_2 \lambda \xi_1 - 2 e^{a^1 \gamma + a^2 \gamma} T \in \eta_1 \lambda \xi_2 - \right. \\
& \quad \left. 2 e^{a^2 \gamma} T \in \eta_2 \lambda \xi_2 - \epsilon \text{fa1}'[\lambda] + 2 e^{a^1 \gamma + a^2 \gamma} T \in \text{fx0}[\lambda] \text{fy0}'[\lambda] \right) + \\
& \text{QU}[y, x] \left(-e^{a^1 \gamma} \gamma \in \eta_1 \lambda \xi_1 \hbar + \gamma \in \eta_2 \lambda \xi_1 \hbar - e^{a^1 \gamma + a^2 \gamma} \gamma \in \eta_1 \lambda \xi_2 \hbar - \right. \\
& \quad \left. e^{a^2 \gamma} \gamma \in \eta_2 \lambda \xi_2 \hbar + e^{a^1 \gamma + a^2 \gamma} \gamma \in \hbar \text{fx0}[\lambda] \text{fy0}'[\lambda] \right) + \\
& \text{QU}[x] \left(e^{-a^2 \gamma} \xi_1 + \frac{1}{2} e^{a^1 \gamma - a^2 \gamma} \gamma \in \eta_1 \lambda^2 \xi_1^2 - \frac{3}{2} e^{a^1 \gamma - a^2 \gamma} T \gamma \in \eta_1 \lambda^2 \xi_1^2 + \xi_2 + e^{a^1 \gamma} \gamma \in \eta_1 \lambda^2 \xi_1 \xi_2 - \right. \\
& \quad 3 e^{a^1 \gamma} T \gamma \in \eta_1 \lambda^2 \xi_1 \xi_2 - \gamma \in \eta_2 \lambda^2 \xi_1 \xi_2 + 3 T \gamma \in \eta_2 \lambda^2 \xi_1 \xi_2 + \frac{1}{2} e^{a^1 \gamma + a^2 \gamma} \gamma \in \eta_1 \lambda^2 \xi_2^2 - \\
& \quad \frac{3}{2} e^{a^1 \gamma + a^2 \gamma} T \gamma \in \eta_1 \lambda^2 \xi_2^2 + \frac{1}{2} e^{a^2 \gamma} \gamma \in \eta_2 \lambda^2 \xi_2^2 - \frac{3}{2} e^{a^2 \gamma} T \gamma \in \eta_2 \lambda^2 \xi_2^2 - \gamma \in \text{fx0}[\lambda] \text{fa1}'[\lambda] - \\
& \quad \text{fx0}'[\lambda] - \epsilon \text{fx1}'[\lambda] - \frac{1}{2} e^{a^1 \gamma + a^2 \gamma} \gamma \in \text{fx0}[\lambda]^2 \text{fy0}'[\lambda] + \frac{3}{2} e^{a^1 \gamma + a^2 \gamma} T \gamma \in \text{fx0}[\lambda]^2 \text{fy0}'[\lambda] \left. \right) + \text{QU}[y] \\
& \left(e^{a^1 \gamma + a^2 \gamma} \eta_1 + e^{a^2 \gamma} \eta_2 - e^{a^1 \gamma + a^2 \gamma} \gamma \in \eta_1 \eta_2 \lambda^2 \xi_1 + 3 e^{a^1 \gamma + a^2 \gamma} T \gamma \in \eta_1 \eta_2 \lambda^2 \xi_1 + \frac{1}{2} e^{a^2 \gamma} \gamma \in \eta_2^2 \lambda^2 \xi_1 - \right. \\
& \quad \left. \frac{3}{2} e^{a^2 \gamma} T \gamma \in \eta_2^2 \lambda^2 \xi_1 - e^{a^1 \gamma + a^2 \gamma} \text{fy0}'[\lambda] - e^{a^1 \gamma + a^2 \gamma} \gamma \in \text{fa1}[\lambda] \text{fy0}'[\lambda] - e^{a^1 \gamma + a^2 \gamma} \epsilon \text{fy1}'[\lambda] \right) + \\
& \text{QU}[] \left(-\frac{e^{a^1 \gamma} \eta_1 \lambda \xi_1}{\hbar} + \frac{e^{a^1 \gamma} T \eta_1 \lambda \xi_1}{\hbar} + \frac{\eta_2 \lambda \xi_1}{\hbar} - \frac{T \eta_2 \lambda \xi_1}{\hbar} + \frac{e^{a^1 \gamma} \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1^2}{2 \hbar} - \right. \\
& \quad \frac{2 e^{a^1 \gamma} T \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1^2}{\hbar} + \frac{3 e^{a^1 \gamma} T^2 \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1^2}{2 \hbar} - \frac{e^{a^1 \gamma + a^2 \gamma} \eta_1 \lambda \xi_2}{\hbar} + \\
& \quad \frac{e^{a^1 \gamma + a^2 \gamma} T \eta_1 \lambda \xi_2}{\hbar} - \frac{e^{a^2 \gamma} \eta_2 \lambda \xi_2}{\hbar} + \frac{e^{a^2 \gamma} T \eta_2 \lambda \xi_2}{\hbar} + \frac{e^{a^1 \gamma + a^2 \gamma} \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1 \xi_2}{\hbar} - \\
& \quad \frac{4 e^{a^1 \gamma + a^2 \gamma} T \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1 \xi_2}{\hbar} + \frac{3 e^{a^1 \gamma + a^2 \gamma} T^2 \gamma \in \eta_1 \eta_2 \lambda^3 \xi_1 \xi_2}{\hbar} - \frac{e^{a^2 \gamma} \gamma \in \eta_2^2 \lambda^3 \xi_1 \xi_2}{2 \hbar} + \\
& \quad \frac{2 e^{a^2 \gamma} T \gamma \in \eta_2^2 \lambda^3 \xi_1 \xi_2}{\hbar} - \frac{3 e^{a^2 \gamma} T^2 \gamma \in \eta_2^2 \lambda^3 \xi_1 \xi_2}{2 \hbar} - \text{ft0}'[\lambda] - \epsilon \text{ft1}'[\lambda] + \\
& \quad \frac{e^{a^1 \gamma + a^2 \gamma} \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \frac{e^{a^1 \gamma + a^2 \gamma} T \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} + \frac{e^{a^1 \gamma + a^2 \gamma} \gamma \in \text{fa1}[\lambda] \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \\
& \quad \frac{e^{a^1 \gamma + a^2 \gamma} T \gamma \in \text{fa1}[\lambda] \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} + \frac{e^{a^1 \gamma + a^2 \gamma} \epsilon \text{fx1}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \\
& \quad \left. \frac{e^{a^1 \gamma + a^2 \gamma} T \epsilon \text{fx1}[\lambda] \text{fy0}'[\lambda]}{\hbar} + \frac{e^{a^1 \gamma + a^2 \gamma} \epsilon \text{fx0}[\lambda] \text{fy1}'[\lambda]}{\hbar} - \frac{e^{a^1 \gamma + a^2 \gamma} T \epsilon \text{fx0}[\lambda] \text{fy1}'[\lambda]}{\hbar} \right)
\end{aligned}$$

In[*]:= **diff** /. **e** → **0**

Out[*]=

$$\begin{aligned}
& \text{QU}[x] \left(e^{-a^2 \gamma} \xi_1 + \xi_2 - \text{fx0}'[\lambda] \right) + \text{QU}[y] \left(e^{a^1 \gamma + a^2 \gamma} \eta_1 + e^{a^2 \gamma} \eta_2 - e^{a^1 \gamma + a^2 \gamma} \text{fy0}'[\lambda] \right) + \\
& \text{QU}[] \left(-\frac{e^{a^1 \gamma} \eta_1 \lambda \xi_1}{\hbar} + \frac{e^{a^1 \gamma} T \eta_1 \lambda \xi_1}{\hbar} + \frac{\eta_2 \lambda \xi_1}{\hbar} - \frac{T \eta_2 \lambda \xi_1}{\hbar} - \frac{e^{a^1 \gamma + a^2 \gamma} \eta_1 \lambda \xi_2}{\hbar} + \frac{e^{a^1 \gamma + a^2 \gamma} T \eta_1 \lambda \xi_2}{\hbar} - \right. \\
& \quad \frac{e^{a^2 \gamma} \eta_2 \lambda \xi_2}{\hbar} + \frac{e^{a^2 \gamma} T \eta_2 \lambda \xi_2}{\hbar} - \text{ft0}'[\lambda] + \frac{e^{a^1 \gamma + a^2 \gamma} \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} - \frac{e^{a^1 \gamma + a^2 \gamma} T \text{fx0}[\lambda] \text{fy0}'[\lambda]}{\hbar} \left. \right)
\end{aligned}$$

In[*]:= {sol1} = DSolve[Coefficient[diff /. $\epsilon \rightarrow 0$, x_{qu}] == 0 $\wedge$ fx0[0] == 0, fx0, λ]

Out[*]=

{ {fx0 $\rightarrow$ Function[{ λ }, $e^{-\alpha 2 \gamma \lambda} (\xi 1 + e^{\alpha 2 \gamma \xi 2})$] } }

In[*]:= CE[diff /. sol1 /. $\epsilon \rightarrow 0$]

Out[*]=

QU[y] ($e^{\alpha 1 \gamma + \alpha 2 \gamma} \eta 1 + e^{\alpha 2 \gamma} \eta 2 - e^{\alpha 1 \gamma + \alpha 2 \gamma} \text{fy0}'[\lambda]$) +
 QU[] $\left(-\frac{e^{\alpha 1 \gamma} \eta 1 \lambda \xi 1}{\hbar} + \frac{e^{\alpha 1 \gamma} T \eta 1 \lambda \xi 1}{\hbar} + \frac{\eta 2 \lambda \xi 1}{\hbar} - \frac{T \eta 2 \lambda \xi 1}{\hbar} - \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} \eta 1 \lambda \xi 2}{\hbar} + \right.$
 $\frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} T \eta 1 \lambda \xi 2}{\hbar} - \frac{e^{\alpha 2 \gamma} \eta 2 \lambda \xi 2}{\hbar} + \frac{e^{\alpha 2 \gamma} T \eta 2 \lambda \xi 2}{\hbar} - \text{ft0}'[\lambda] + \frac{e^{\alpha 1 \gamma} \lambda \xi 1 \text{fy0}'[\lambda]}{\hbar} -$
 $\left. \frac{e^{\alpha 1 \gamma} T \lambda \xi 1 \text{fy0}'[\lambda]}{\hbar} + \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} \lambda \xi 2 \text{fy0}'[\lambda]}{\hbar} - \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} T \lambda \xi 2 \text{fy0}'[\lambda]}{\hbar} \right)$

In[*]:= {sol2} = DSolve[Coefficient[CE[diff /. sol1 /. $\epsilon \rightarrow 0$], y_{qu}] == 0 $\wedge$ fy0[0] == 0, fy0, λ]

Out[*]=

{ {fy0 $\rightarrow$ Function[{ λ }, $e^{-\alpha 1 \gamma} (e^{\alpha 1 \gamma} \eta 1 + \eta 2) \lambda$] } }

In[*]:= CE[diff /. sol1 /. sol2 /. $\epsilon \rightarrow 0$]

Out[*]=

QU[] $\left(\frac{2 \eta 2 \lambda \xi 1}{\hbar} - \frac{2 T \eta 2 \lambda \xi 1}{\hbar} - \text{ft0}'[\lambda] \right)$

In[*]:= {sol3} = DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. $\epsilon \rightarrow 0$], QU[] == 0 $\wedge$ ft0[0] == 0, ft0, λ]

Out[*]=

{ {ft0 $\rightarrow$ Function[{ λ }, $-\frac{(-1 + T) \eta 2 \lambda^2 \xi 1}{\hbar}$] } }

In[*]:= **CE[diff /. sol1 /. sol2 /. sol3]**

Out[*]=

$$\begin{aligned}
& 2 \gamma \in \eta 2 \lambda \xi 1 \hbar \text{QU}[y, x] + \text{QU}[a] (4 T \in \eta 2 \lambda \xi 1 - \epsilon \text{fa1}'[\lambda]) + \\
& \text{QU}[x] \left(-\frac{1}{2} e^{-\alpha 2 \gamma} \gamma \in \eta 2 \lambda^2 \xi 1^2 + \frac{3}{2} e^{-\alpha 2 \gamma} T \gamma \in \eta 2 \lambda^2 \xi 1^2 - 2 \gamma \in \eta 2 \lambda^2 \xi 1 \xi 2 + \right. \\
& \quad \left. 6 T \gamma \in \eta 2 \lambda^2 \xi 1 \xi 2 - e^{-\alpha 2 \gamma} \gamma \in \lambda \xi 1 \text{fa1}'[\lambda] - \gamma \in \lambda \xi 2 \text{fa1}'[\lambda] - \epsilon \text{fx1}'[\lambda] \right) + \\
& \text{QU}[y] \left(-e^{\alpha 1 \gamma + \alpha 2 \gamma} \gamma \in \eta 1 \eta 2 \lambda^2 \xi 1 + 3 e^{\alpha 1 \gamma + \alpha 2 \gamma} T \gamma \in \eta 1 \eta 2 \lambda^2 \xi 1 + \frac{1}{2} e^{\alpha 2 \gamma} \gamma \in \eta 2^2 \lambda^2 \xi 1 - \right. \\
& \quad \left. \frac{3}{2} e^{\alpha 2 \gamma} T \gamma \in \eta 2^2 \lambda^2 \xi 1 - e^{\alpha 1 \gamma + \alpha 2 \gamma} \gamma \in \eta 1 \text{fa1}[\lambda] - e^{\alpha 2 \gamma} \gamma \in \eta 2 \text{fa1}[\lambda] - e^{\alpha 1 \gamma + \alpha 2 \gamma} \epsilon \text{fy1}'[\lambda] \right) + \\
& \text{QU}[] \left(\frac{e^{\alpha 1 \gamma} \gamma \in \eta 1 \eta 2 \lambda^3 \xi 1^2}{2 \hbar} - \frac{2 e^{\alpha 1 \gamma} T \gamma \in \eta 1 \eta 2 \lambda^3 \xi 1^2}{\hbar} + \frac{3 e^{\alpha 1 \gamma} T^2 \gamma \in \eta 1 \eta 2 \lambda^3 \xi 1^2}{2 \hbar} + \right. \\
& \quad \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} \gamma \in \eta 1 \eta 2 \lambda^3 \xi 1 \xi 2}{\hbar} - \frac{4 e^{\alpha 1 \gamma + \alpha 2 \gamma} T \gamma \in \eta 1 \eta 2 \lambda^3 \xi 1 \xi 2}{\hbar} + \frac{3 e^{\alpha 1 \gamma + \alpha 2 \gamma} T^2 \gamma \in \eta 1 \eta 2 \lambda^3 \xi 1 \xi 2}{\hbar} - \\
& \quad \frac{e^{\alpha 2 \gamma} \gamma \in \eta 2^2 \lambda^3 \xi 1 \xi 2}{2 \hbar} + \frac{2 e^{\alpha 2 \gamma} T \gamma \in \eta 2^2 \lambda^3 \xi 1 \xi 2}{\hbar} - \frac{3 e^{\alpha 2 \gamma} T^2 \gamma \in \eta 2^2 \lambda^3 \xi 1 \xi 2}{2 \hbar} + \\
& \quad \frac{e^{\alpha 1 \gamma} \gamma \in \eta 1 \lambda \xi 1 \text{fa1}[\lambda]}{\hbar} - \frac{e^{\alpha 1 \gamma} T \gamma \in \eta 1 \lambda \xi 1 \text{fa1}[\lambda]}{\hbar} + \frac{\gamma \in \eta 2 \lambda \xi 1 \text{fa1}[\lambda]}{\hbar} - \\
& \quad \frac{T \gamma \in \eta 2 \lambda \xi 1 \text{fa1}[\lambda]}{\hbar} + \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} \gamma \in \eta 1 \lambda \xi 2 \text{fa1}[\lambda]}{\hbar} - \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} T \gamma \in \eta 1 \lambda \xi 2 \text{fa1}[\lambda]}{\hbar} + \\
& \quad \frac{e^{\alpha 2 \gamma} \gamma \in \eta 2 \lambda \xi 2 \text{fa1}[\lambda]}{\hbar} - \frac{e^{\alpha 2 \gamma} T \gamma \in \eta 2 \lambda \xi 2 \text{fa1}[\lambda]}{\hbar} + \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} \epsilon \eta 1 \text{fx1}[\lambda]}{\hbar} - \\
& \quad \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} T \epsilon \eta 1 \text{fx1}[\lambda]}{\hbar} + \frac{e^{\alpha 2 \gamma} \epsilon \eta 2 \text{fx1}[\lambda]}{\hbar} - \frac{e^{\alpha 2 \gamma} T \epsilon \eta 2 \text{fx1}[\lambda]}{\hbar} - \epsilon \text{ft1}'[\lambda] + \\
& \quad \left. \frac{e^{\alpha 1 \gamma} \epsilon \lambda \xi 1 \text{fy1}'[\lambda]}{\hbar} - \frac{e^{\alpha 1 \gamma} T \epsilon \lambda \xi 1 \text{fy1}'[\lambda]}{\hbar} + \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} \epsilon \lambda \xi 2 \text{fy1}'[\lambda]}{\hbar} - \frac{e^{\alpha 1 \gamma + \alpha 2 \gamma} T \epsilon \lambda \xi 2 \text{fy1}'[\lambda]}{\hbar} \right)
\end{aligned}$$