

Pensieve header: A solvable algebra which is not a semi-direct product A+N where N is nilpotent, A is Abelian, and A acts on N semi-simply.

```
In[*]:= A = {{1, 0, 0}, {0, 0, 1}, {0, 0, 0}};
X = {{0, 0, 1}, {0, 0, 0}, {0, 0, 0}};
Y = {{0, -1, 0}, {0, 0, 0}, {0, 0, 0}};
```

```
In[*]:= MatrixForm /@ {A, X, Y}
```

```
Out[*]=
{

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
}$$

```

```
In[*]:= Comm[a_, b_] := a.b - b.a
```

```
Comm[A, X] === X
Comm[A, Y] === X + Y
Comm[X, Y] === 0 A
```

```
Out[*]=
True
```

```
Out[*]=
True
```

```
Out[*]=
True
```

```
In[*]:= Permutations[{a, x, y}]
```

```
Out[*]=
{{a, x, y}, {a, y, x}, {x, a, y}, {x, y, a}, {y, a, x}, {y, x, a}}
```

```
In[*]:= MatrixForm /@
```

```
((p -> Dot@@ (MatrixExp /@ (p /. {a -> α A, x -> ξ X, y -> η Y}))) /@ Permutations[{a, x, y}])
```

```
Out[*]=
{

$$\begin{pmatrix} e^\alpha & -e^\alpha \eta & e^\alpha \xi \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^\alpha & -e^\alpha \eta & e^\alpha \xi \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^\alpha & -e^\alpha \eta & \xi \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix},$$


$$\begin{pmatrix} e^\alpha & -\eta & -\alpha \eta + \xi \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^\alpha & -\eta & -\alpha \eta + e^\alpha \xi \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} e^\alpha & -\eta & -\alpha \eta + \xi \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}
}$$

```