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In[*]:=  $\mathbb{E}_{\{1,2\} \rightarrow \{1,2,3,4\}} [99 \mathbf{a1}_{\$[1]} \alpha_{1_1} + 11 \mathbf{y12}_{\$[3]} \eta_{12_2} + 7 \mathbf{x12}_{\$[1]} \xi_{12_1}] // \mathbb{E}_{\{1,2,3,4\} \rightarrow \{1,2\}} [e^{b_{1_1} - \alpha_{1_1}} \eta_{12_3} \xi_{12_1}]$ 
» { { $\mathbf{a1}_{\$[1]}$ ,  $\mathbf{x12}_{\$[1]}$ ,  $\mathbf{y12}_{\$[3]}$ ,  $\alpha_{1_1}$ ,  $\eta_{12_{\$[3]}}$ ,  $\xi_{12_{\$[1]}}$ },  $F \mathbf{a1}_{\$[1]} F \alpha_{1_1} + F \mathbf{y12}_{\$[3]} F \eta_{12_{\$[3]}} + F \mathbf{x12}_{\$[1]} F \xi_{12_{\$[1]}}$ ,  $99 \mathbf{a1}_{\$[1]} \alpha_{1_1} + 11 \mathbf{y12}_{\$[3]} \eta_{12_2} + 7 \mathbf{x12}_{\$[1]} \xi_{12_1} + e^{b_{1_1} - \alpha_{1_1}} \eta_{12_{\$[3]}} \xi_{12_{\$[1]}}$  }
» Zip2 { { $\mathbf{a1}_{\$[1]}$ ,  $\alpha_{1_1}$ },  $e^{b_{1_1} - \alpha_{1_1}} F \mathbf{x12}_{\$[1]} F \mathbf{y12}_{\$[3]} + F \mathbf{a1}_{\$[1]} F \alpha_{1_1} + F \mathbf{y12}_{\$[3]} F \eta_{12_{\$[3]}} + F \mathbf{x12}_{\$[1]} F \xi_{12_{\$[1]}}$ ,  $99 \mathbf{a1}_{\$[1]} \alpha_{1_1} + 11 \mathbf{y12}_{\$[3]} \eta_{12_2} + 7 \mathbf{x12}_{\$[1]} \xi_{12_1}$  }
» F { { $\emptyset$ ,  $1$ }, { $1$ ,  $\emptyset$ } }
» Y {  $99 \alpha_{1_1}$ ,  $\emptyset$  }
» { Zip3, {  $\mathbf{x12}_{\$[1]}$ ,  $\mathbf{y12}_{\$[3]}$ ,  $\eta_{12_{\$[3]}}$ ,  $\xi_{12_{\$[1]}}$  },  $\mathcal{CF}[4] + \mathcal{CF}[1] F \mathbf{x12}_{\$[1]} F \mathbf{y12}_{\$[3]} + \mathcal{CF}[3] F \mathbf{y12}_{\$[3]} F \eta_{12_{\$[3]}} + \mathcal{CF}[2] F \mathbf{x12}_{\$[1]} F \xi_{12_{\$[1]}}$ ,  $\mathbb{E}[\mathcal{CE}[1] \mathbf{x12}_{\$[1]} + \mathcal{CE}[2] \mathbf{y12}_{\$[3]}]$  } }
» { Zip3, {  $\mathbf{a1}_{\$[1]}$ ,  $\alpha_{1_1}$  },  $F \mathbf{a1}_{\$[1]} F \alpha_{1_1}$ ,  $\mathbb{E}[77 e^{b_{1_1} - \alpha_{1_1}} \eta_{12_2} \xi_{12_1}]$  }

Out[*]=
 $\mathbb{E}_{\{1,2\} \rightarrow \{1,2\}} [77 e^{b_{1_1}} \eta_{12_2} \xi_{12_1}]$ 

In[*]:= vs = { $\mathbf{a1}_{\$[1]}$ ,  $\alpha_{1_1}$ } }

Out[*]=
{  $\mathbf{a1}_{\$[1]}$ ,  $\alpha_{1_1}$  }

In[*]:= Q =  $99 \mathbf{a1}_{\$[1]} \alpha_{1_1} + 11 \mathbf{y12}_{\$[3]} \eta_{12_2} + 7 \mathbf{x12}_{\$[1]} \xi_{12_1}$ 

Out[*]=
 $99 \mathbf{a1}_{\$[1]} \alpha_{1_1} + 11 \mathbf{y12}_{\$[3]} \eta_{12_2} + 7 \mathbf{x12}_{\$[1]} \xi_{12_1}$ 

In[*]:= MatrixForm[F = { { $\emptyset$ ,  $1$ }, { $1$ ,  $\emptyset$ } } ]

Out[*]//MatrixForm=

$$\begin{pmatrix} \emptyset & 1 \\ 1 & \emptyset \end{pmatrix}$$


In[*]:= Y = {  $99 \alpha_{1_1}$ ,  $\emptyset$  }

Out[*]=
{  $99 \alpha_{1_1}$ ,  $\emptyset$  }

In[*]:= Y.F.Y / 2

Out[*]=
 $\emptyset$ 

In[*]:= Y.vs + Y.F.Y / 2

Out[*]=
 $99 \mathbf{a1}_{\$[1]} \alpha_{1_1}$ 

In[*]:= Thread[vs → vs + F.Y]

Out[*]=
{  $\mathbf{a1}_{\$[1]} \rightarrow \mathbf{a1}_{\$[1]}$ ,  $\alpha_{1_1} \rightarrow 99 \alpha_{1_1} + \alpha_{1_1}$  }

In[*]:= E[Q - Y.vs + Y.F.Y / 2, P]

Out[*]=
 $\mathbb{E}[11 \mathbf{y12}_{\$[3]} \eta_{12_2} + 7 \mathbf{x12}_{\$[1]} \xi_{12_1}, P]$ 

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$\{\mathcal{F}, \mathbb{E}[Q - Y \cdot vs + Y \cdot F \cdot Y / 2, P] /. \text{Thread}[vs \rightarrow vs + F \cdot Y]\}$

$Out[*]=$

$\{\mathcal{F}, \mathbb{E}[11 y_{12_{\$[3]}} \eta_{12_2} + 7 x_{12_{\$[1]}} \xi_{12_1}, P]\}$

$In[*]:= \{\mathbf{a}_{1_{\$[1]}}, \alpha_{1_{\$[1]}}\}^*$

$Out[*]=$

$\{\alpha_{1_{\$[1]}}, \mathbf{a}_{1_{\$[1]}}\}$