

```

DeclareAlgebra[O_Symbol, opts__Rule] := Module[
{gp, sr, g, cp, M, pow, k = 0,
gs = Generators /. {opts},
cs = Centrals /. {opts} /. Centrals → {}
},
U[Generators] = gs;
(#_j = U@#) & /@ gs;
gp = Alternatives @@ gs; gp = gp | gp_ ; (* gens *)
sr = Flatten@Table[{g → ++k, g_i_ → {i, k}}, {g, gs}]; (* sorting → *)
cp = Alternatives @@ cs; (* cents *)
SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
CE[ε_] := Collect[ε, _U, Expand] /. e^x_ := eExpand[x] /. $trim;
U_i_[ε_] := ε /. {t : cp => t_i, u_U => (#i &) /@ u};
U_i_[NCM[]] = pow[ε_, 0] = U@{} = 1_U = U[];
B[U@(x_)_i_, U@(y_)_i_] := U_i_@B[U@x, U@y];
B[U@(x_)_i_, U@(y_)_j_] /; i != j := 0;
B[U@y_, U@x_] := CE[-B[U@x, U@y]];
x_ ⓧ (c_. 1_U) := CE[c x]; (c_. 1_U) ⓧ x_ := CE[c x];
(a_. U[xx___, x_]) ⓧ (b_. U[y_, yy___]) := If[OrderedQ[{x, y} /. sr],
CE@M[a b /. $trim, U[xx, x, y, yy]],
U@xx ⓧ CE@M[a b /. $trim, U@y ⓧ U@x + B[U@x, U@y, $E]] ⓧ U@yy];
U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ⓧ U@{r}];
U@{c_. * l : gp, r___} := CE[c U[l] ⓧ U@{r}];
U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
U@{l_, r___} := U@{Expand[l], r};
U[ε_NonCommutativeMultiply] := U /@ ε;
O_U[specs___, poly_] := Module[{sp, null, vs, us},
sp = Replace[{specs}, l_List => l_null, {1}];
vs = Join @@ (First /@ sp);
us = Join @@ (sp /. l_s_ => (l /. x_i_ => x_s));
CE[Total[
CoefficientRules[poly, vs] /. (p_ → c_) => c U@{us^p}
]] /. x_null => x];
O_U[specs___, E[l_, Q_, P_]] := O_U[specs, SS@Normal[P eL+Q]];
pow[ε_, n_] := pow[ε, n - 1] ⓧ ε;
S_U[ε_, ss__Rule] := CE@Total[
CoefficientRules[ε, First /@ {ss}] /. (p_ → c_) => c NCM @@ MapThread[pow, {Last /@ {ss}, p}]]];
σ_rs___[c_. * u_U] := (c /. (t : cp)_j_ => t_j/.{rs}) U[List @@ (u /. v_j_ => v_j/.{rs})];
m_j_→k_[c_. * u_U] :=
CE[ ((c /. (t : cp)_j_ → t_k) DeleteCases[u, _j|k]) ⓧ U @@ Cases[u, w_j_ => w_k] ⓧ U @@ Cases[u, _k] ];
U /: c_. * u_U * v_U := CE[c u ⓧ v];
S_i_[c_. * u_U] :=
CE[ ((c /. S_i[U, Centrals]) DeleteCases[u, _i]) ⓧ U_i[NCM @@ Reverse@Cases[u, x_i_ => S@U@x]]];
Δ_i_→j_,k_[c_. * u_U] :=
CE[ ((c /. Δ_i_→j_,k[U, Centrals]) DeleteCases[u, _i]) ⓧ
(NCM @@ Cases[u, x_i_ => σ_1_→j_,2_→k_@Δ@U@x] /. NCM[] → U[]) ];

```