

Hidden T_EX

A Timing Game

Headers

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Pensieve header: Verifying 2-parameter type SA_n quantum groups following Ian Martin, Alexander Tsymbaliuk, Two-Parameter Quantum Groups and R-Matrices: Classical Types and Orthogonal bases for two-parameter quantum groups.

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```
In[ ]:= SetDirectory@"C:\\drorbn\\AcademicPensieve\\Projects\\SolvablePBW";
<< KnotTheory`
```

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Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.
Read more at <http://katlas.org/wiki/KnotTheory>.

nb2tex

```
In[ ]:= Get["../nb2tex/nb2tex.m"];
$NotebookName = FileBaseName@FileNameTake@NotebookFileName[EvaluationNotebook[]]
nb2tex[$NotebookName, PDFWidth → 7.5, PDFFolder → "Snips" <> $NotebookName];
texfile = OpenAppend[$NotebookName <> ".tex"];
WriteLine[texfile, "\\end{document}"]; Close[texfile];
Run@
  ("\"C:\\Users\\drorb\\AppData\\Local\\Programs\\MiKTeX\\miktex\\bin\\x64\\pdflatex.exe\"
   \" <> $NotebookName <> ".tex");
```

Time: 5.31 seconds Out[]:=

TwoParameterQuantumGroups

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Initialization / Utilities

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```
In[*]:=
$k = 1; $U = QU; $E := {$k};
$trim := {e^{k-} /; k > $k -> 0};
SetAttributes[{SS, SST}, HoldAll];
q_{h-} := e^{Y e^{h-}};
(* Upper to lower and lower to Upper: *)
U2l = {B_{i-}^{p-} -> e^{-p h Y} b_i, B_{-}^{p-} -> e^{-p h Y} b, T_{i-}^{p-} -> e^{p h} t_i, T_{-}^{p-} -> e^{p h} t, A_{i-}^{p-} -> e^{p Y} a_i, A_{-}^{p-} -> e^{p Y} a};
l2U = {e^{c_{-} b_i + d_{-}} -> B_i^{-c/(h Y)} e^d, e^{c_{-} b + d_{-}} -> B^{-c/(h Y)} e^d,
  e^{c_{-} t_i + d_{-}} -> T_i^{c/h} e^d, e^{c_{-} t + d_{-}} -> T^{c/h} e^d,
  e^{c_{-} a_i + d_{-}} -> A_i^{c/Y} e^d, e^{c_{-} a + d_{-}} -> A^{c/Y} e^d,
  e^{\mathcal{E}} -> e^{Expand@E}};
SS[E_, op_] := Collect[Normal@Series[E, {e, 0, $k}], e, op];
SS[E_] := SS[E, Together];
SST[E_, op___] := SS[E /. U2l, op];
Simp[E_, op_] := Collect[E, _CU | _QU, op];
Simp[E_] := Simp[E, SS[#, Expand] &];
SimpT[E_] := Collect[E, _CU | _QU, SST[#, Expand] &];
Kd /: Kd_{i-, j-} := If[i === j, 1, 0];
c_Integer_{k_Integer} := c + O[e]^{k+1};
```

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DeclareAlgebra

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```
In[*]:=
Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z_] := (x \otimes y) \otimes z;
0 \otimes _ = _ \otimes 0 = 0;
(x_Plus) \otimes y_ := (# \otimes y) & /@ x; x_ \otimes (y_Plus) := (x \otimes #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x \otimes y - y \otimes x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

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```
In[*]:=
DeclareAlgebra[U_Symbol, opts__Rule] := Module[
  {gp, sr, g, cp, M, pow, k = 0,
   gs = Generators /. {opts},
   cs = Centrals /. {opts} /. Centrals -> {}},
  ],
  U[Generators] = gs;
  (#_ = U@#) & /@ gs;
  gp = Alternatives @@ gs; gp = gp | gp; (* gens *)
  sr = Flatten@Table[{g -> ++k, g_i -> {i, k}}, {g, gs}]; (* sorting -> *)
  cp = Alternatives @@ cs; (* cents *)
```

```

SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
CE[ε_] := Collect[ε, _U, Expand] /. e^x_ := eExpand[x] /. $trim;
U_i[ε_] := ε /. {t : cp => t_i, u_U => (#i &) /@ u};
U_i[NCM[]] = pow[ε_, 0] = U@{} = 1_U = U[];
B[U@(x_)_i_, U@(y_)_i_] := U_i@B[U@x, U@y];
B[U@(x_)_i_, U@(y_)_j_] /; i != j := 0;
B[U@y_, U@x_] := CE[-B[U@x, U@y]];
x_ ⓧ (c_. 1_U) := CE[c x]; (c_. 1_U) ⓧ x_ := CE[c x];
(a_. U[xx___, x_]) ⓧ (b_. U[y_, yy___]) := If[OrderedQ[{x, y} /. sr],
  CE@M[a b /. $trim, U[xx, x, y, yy]],
  U@xx ⓧ CE@M[a b /. $trim, U@y ⓧ U@x + B[U@x, U@y, $E]] ⓧ U@yy];
U@{c_. * (l : gp)n_, r___} /; FreeQ[c, gp] := CE[c U@Table[L, {n}] ⓧ U@{r}];
U@{c_. * l : gp, r___} := CE[c U[L] ⓧ U@{r}];
U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
U@{L_Plus, r___} := CE[U@{#, r} & /@ L];
U@{L_, r___} := U@{Expand[L], r};
U[ε_NonCommutativeMultiply] := U /@ ε;
O_U[specs___, poly_] := Module[{sp, null, vs, us},
  sp = Replace[{specs}, L_List => L_null, {1}];
  vs = Join@@ (First /@ sp);
  us = Join@@ (sp /. L_s_ => (L /. x_i_ => x_s));
  CE[Total[
    CoefficientRules[poly, vs] /. (p_ -> c_) => c U@ (usp)
  ] / . x_null => x];
O_U[specs___, E[L_, Q_, P_]] := O_U[specs, SS@Normal[P eL+Q]];
pow[ε_, n_] := pow[ε, n - 1] ⓧ ε;
S_U[ε_, ss___Rule] := CE@Total[
  CoefficientRules[ε, First /@ {ss}] /.
    (p_ -> c_) => c NCM@@ MapThread[pow, {Last /@ {ss}, p}]];
σrs [c_. * u_U] := (c /. (t : cp)_j_ => tj /. {rs}) U[List@@ (u /. v_j_ => vj /. {rs})];
mj->k [c_. * u_U] := CE[(c /. (t : cp)_j_ => tj) DeleteCases[u, _j|k] ⓧ
  U@@ Cases[u, w_j_ => wk] ⓧ U@@ Cases[u, _k]];
U /: c_. * u_U * v_U := CE[c u ⓧ v];
S_i [c_. * u_U] := CE[(c /. S_i[U, Centrals]) DeleteCases[u, _i] ⓧ
  U_i[NCM@@ Reverse@Cases[u, x_i_ => S@U@x]]];
Δi->j,k [c_. * u_U] := CE[(c /. Δi->j,k[U, Centrals]) DeleteCases[u, _i] ⓧ
  (NCM@@ Cases[u, x_i_ => σ1->j, 2->k@Δ@U@x] /. NCM[] -> U[])];

```

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Implementing $QU = U_{r,s}[A_n]$

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Conventions:

$$\{e_i, f_i, \omega_i^{\pm 1}, (\omega_i')^{\pm 1}, r, s\} \leftrightarrow \{x_{i,i+1}, y_{i,i+1}, e^{\pm v_i}, e^{\pm w_i}, e^r, e^s\}$$

We are assuming that $r_i = r$ and $s_i = s$.

```
In[*]:= $n = 3;
Flatten@{
  Table[xi,j, {i, 1, $n}, {j, i + 1, $n + 1}],
  Table[yi,j, {i, 1, $n}, {j, i + 1, $n + 1}],
  Table[{vi, wi}, {i, $n}]
}

Time: 0.00 seconds Out[ ]=
{x1,2, x1,3, x1,4, x2,3, x2,4, x3,4, y1,2, y1,3, y1,4, y2,3, y2,4, y3,4, v1, w1, v2, w2, v3, w3}
```

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```
In[*]:= $n = 3;
DeclareAlgebra[QU,
  Generators → Flatten@{
    Table[xi,j, {i, 1, $n}, {j, i + 1, $n + 1}],
    Table[yi,j, {i, 1, $n}, {j, i + 1, $n + 1}],
    Table[{vi, wi}, {i, $n}]
  },
  Centrals → {}
];
```

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```
In[*]:= Ringel[i_, j_] := Which[
  i + 1 < j ∨ i > j, 0,
  i + 1 == j, -1,
  i == j, 1
]
```

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```
Do[
  B[QU@vi, QU@vj] = B[QU@vi, QU@wj] = B[QU@wi, QU@wj] = 0;
  B[QU@vi, QU@xj,j+1] = (Ringel[j, i] r - Ringel[i, j] s) QU[xj,j+1, vi]
,
  {i, 1, $n}, {j, 1, $n}
];
```

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```

B[aQU, xQU] = -2 QU@x;
B[bQU, xQU] = QU@x;
B[bQU, yQU] = -2 QU@y;
B[aQU, yQU] = QU@y;
B[aQU, zQU] = -QU@z;
B[bQU, zQU] = -QU@z;
B[yQU, xQU] = -zQU + ε ħ QU[y, x];
B[zQU, xQU] = -ε ħ QU[z, x];
B[bQU, aQU] = 0;
B[zQU, yQU] = ε ħ QU[z, y];
B[aQU, XQU] = 2 XQU;
B[bQU, XQU] = -XQU;
B[aQU, YQU] = -YQU;
B[bQU, YQU] = 2 YQU;
B[aQU, ZQU] = ZQU;
B[bQU, ZQU] = ZQU;

B[xQU, XQU] = -2 ε ħ QU@{X, x} +  $\frac{(1-s) QU[]}{\hbar}$  + 2 s ε QU[a];
B[yQU, YQU] = -2 ε ħ QU@{Y, y} +  $\frac{(1-t) QU[]}{\hbar}$  + 2 t ε QU[b];
B[zQU, ZQU] = -2 ε ħ QU@{Z, z} +  $\frac{(1-s t) QU[]}{\hbar}$  + 2 s t ε QU[a] + 2 s t ε QU[b];

B[yQU, ZQU] = XQU - ε ħ QU@{Z, y};
B[xQU, ZQU] = -ε ħ QU@{Z, x} - (s + s ε ħ) QU[Y] + 2 s ε ħ QU[Y, a];
B[zQU, XQU] = 2 ε yQU - ε ħ QU@{X, z};
B[zQU, YQU] = -2 t ε QU[x] - ε ħ QU[Y, z];
B[yQU, XQU] = ε ħ QU[X, y];
B[xQU, YQU] = ε ħ QU[Y, x];
B[XQU, YQU] = 2 ε ZQU - ε ħ QU[X, Y];
B[XQU, ZQU] = ε ħ QU[X, Z];
B[ZQU, YQU] = ε ħ QU[Y, Z];

```

In[]:= QU@Generators

Time: 0.02 seconds Out[]:=

{X, Y, Z, b, a, z, y, x}

```

In[*]:= $Basis = QU /@ QU@Generators;
Module[{a, b}, Table[{a, b} → CE[B[a, b]], {a, $Basis}, {b, $Basis}]] // MatrixForm
Time: 0.02 seconds Out|=

$$\left( \begin{array}{ll}
\{QU[X], QU[X]\} \rightarrow \emptyset & \{QU[X], QU[Y]\} \rightarrow 2 \in QU[Z] - \in QU[X] \\
\{QU[Y], QU[X]\} \rightarrow -2 \in QU[Z] + \in QU[X, Y] & \{QU[Y], QU[Y]\} \rightarrow \emptyset \\
\{QU[Z], QU[X]\} \rightarrow -\in QU[X, Z] & \{QU[Z], QU[Y]\} \rightarrow \in QU[Y, Z] \\
\{QU[b], QU[X]\} \rightarrow -QU[X] & \{QU[b], QU[Y]\} \rightarrow 2 QU[Y] \\
\{QU[a], QU[X]\} \rightarrow 2 QU[X] & \{QU[a], QU[Y]\} \rightarrow -QU[Y] \\
\{QU[z], QU[X]\} \rightarrow 2 \in QU[y] - \in QU[X, z] & \{QU[z], QU[Y]\} \rightarrow -2 t \in QU[x] - \in \\
\{QU[y], QU[X]\} \rightarrow \in QU[X, y] & \{QU[y], QU[Y]\} \rightarrow (1 - t) QU[] + 2 t \in QU[t] \\
\{QU[x], QU[X]\} \rightarrow (1 - s) QU[] + 2 s \in QU[a] - 2 \in QU[X, x] & \{QU[x], QU[Y]\} \rightarrow \in QU[Y, x]
\end{array} \right)$$

In[*]:= Module[{a, b, c},
DeleteCases[Flatten[Table[{a, b, c} → CE[B[a, B[b, c]] + B[b, B[c, a]] + B[c, B[a, b]]],
{a, $Basis}, {b, $Basis}, {c, $Basis}]], _ → 0]
]
Time: 0.80 seconds Out|=
{}

```