Pensieve header: An isolated attempt to compute the multiplication tensor of QU.

The implementation of QU is modified from pensieve://Projects/SL2Portfolio/SL2PortfolioProgram.nb.

Initialization / Utilities

```
In[∘]:=
$k = 1; $U = QU; $E := {$k};
$trim := {ϵ^k_. /; k > $k → 0};
SetAttributes[{SS, SST}, HoldAll];
q_ℏ_ := 𝕖^(γ ϵ ℏ);
(* Upper to lower and lower to Upper: *)
U2l = {B_i_^p_. → 𝕖^(-p ℏ γ b_i), B^p_. → 𝕖^(-p ℏ γ b), T_i_^p_. → 𝕖^(p ℏ t_i), T^p_. → 𝕖^(p ℏ t), 𝒜_i_^p_. → 𝕖^(p γ α_i), 𝒜^p_. → 𝕖^(p γ α)};
l2U = {𝕖^(c_. b_i_ + d_.) :→ B_i^(-c/(ℏ γ)) 𝕖^d, 𝕖^(c_. b + d_.) :→ B^(-c/(ℏ γ)) 𝕖^d,
    𝕖^(c_. t_i_ + d_.) :→ T_i^(c/ℏ) 𝕖^d, 𝕖^(c_. t + d_.) :→ T^(c/ℏ) 𝕖^d,
    𝕖^(c_. α_i_ + d_.) :→ 𝒜_i^(c/γ) 𝕖^d, 𝕖^(c_. α + d_.) :→ 𝒜^(c/γ) 𝕖^d,
    𝕖^ℰ_ :→ 𝕖^(Expand@ℰ)};
SS[ℰ_, op_] := Collect[Normal@Series[ℰ, {ϵ, 0, $k}], ϵ, op];
SS[ℰ_] := SS[ℰ, Together];
SST[ℰ_, op___] := SS[ℰ /. U2l, op];
Simp[ℰ_, op_] := Collect[ℰ, _CU | _QU, op];
Simp[ℰ_] := Simp[ℰ, SS[#, Expand] &];
SimpT[ℰ_] := Collect[ℰ, _CU | _QU, SST[#, Expand] &];
Kδ /: Kδ_{i_,j_} := If[i === j, 1, 0];
c_Integer_k_Integer := c + O[ϵ]^(k+1);
```

```
In[∘]:=
CF[ℰ_] := ExpandDenominator@
    ExpandNumerator@Together[Expand[ℰ] //. 𝕖^x_ 𝕖^y_ :→ 𝕖^(x+y) /. 𝕖^x_ :→ 𝕖^CF[x]];
```

```
In[∘]:=
Unprotect[SeriesData];
SeriesData /: CF[sd_SeriesData] := MapAt[CF, sd, 3];
SeriesData /: Expand[sd_SeriesData] := MapAt[Expand, sd, 3];
SeriesData /: Simplify[sd_SeriesData] := MapAt[Simplify, sd, 3];
SeriesData /: Together[sd_SeriesData] := MapAt[Together, sd, 3];
SeriesData /: Collect[sd_SeriesData, specs__] := MapAt[Collect[#, specs] &, sd, 3];
Protect[SeriesData];
```

Self-Pair (SP):

```
In[∘]:=
SP_{}[P_] := P; SP_{ξ_→x_,ps___}[P_] := Expand[P // SP_{ps}] /. f_. ξ^d_. :→ ∂_{x,d} f
```

DeclareAlgebra

```
In[=]:= Unprotect[NonCommutativeMultiply]; Attributes[NonCommutativeMultiply] = {};
(NCM = NonCommutativeMultiply)[x_] := x;
NCM[x_, y_, z__] := (x ** y) ** z;
0 ** _ = _ ** 0 = 0;
(x_Plus) ** y_ := (# ** y) & /@ x; x_ ** (y_Plus) := (x ** #) & /@ y;
B[x_, x_] = 0; B[x_, y_] := x ** y - y ** x;
B[x_, y_, e_] := B[x, y, e] = B[x, y];
```

```
In[=]:= DeclareAlgebra[U_Symbol, opts__Rule] := Module[
  {gp, sr, g, cp, M, pow, k = 0,
   gs = Generators /. {opts},
   cs = Centrals /. {opts} /. Centrals → {}
  },
  U[Generators] = gs;
  (#_U = U@#) & /@ gs;
  gp = Alternatives @@ gs; gp = gp | gp_; (* gens *)
  sr = Flatten@Table[{g → ++k, g_i_ → {i, k}}, {g, gs}]; (* sorting → *)
  cp = Alternatives @@ cs; (* cents *)
  SetAttributes[M, HoldRest]; M[0, _] = 0; M[a_, x_] := a x;
  CE[ℰ_] := Collect[ℰ, _U, Expand] /. e^x_ :> e^Expand[x] /. $trim;
  U_i_ [ℰ_] := ℰ /. {t : cp :> t_i, u_U :> (#_i &) /@ u};
  U_i_ [NCM[]] = pow[ℰ_, 0] = U@{} = 1_U = U[];
  B[U@(x_)_i_, U@(y_)_i_] := U_i@B[U@x, U@y];
  B[U@(x_)_i_, U@(y_)_j_] /; i =!= j := 0;
  B[U@y_, U@x_] := CE[-B[U@x, U@y]];
  x_ ** (c_. 1_U) := CE[c x]; (c_. 1_U) ** x_ := CE[c x];
  (a_. U[xx___, x_]) ** (b_. U[y_, yy___]) := If[OrderedQ[{x, y} /. sr],
    CE@M[a b /. $trim, U[xx, x, y, yy]],
    U@xx ** CE@M[a b /. $trim, U@y ** U@x + B[U@x, U@y, $E]] ** U@yy ];
  U@{c_. * (l : gp)^n_, r___} /; FreeQ[c, gp] := CE[c U@Table[l, {n}] ** U@{r}];
  U@{c_. * l : gp, r___} := CE[c U[l] ** U@{r}];
  U@{c_, r___} /; FreeQ[c, gp] := CE[c U@{r}];
  U@{l_Plus, r___} := CE[U@{#, r} & /@ l];
  U@{l_, r___} := U@{Expand[l], r};
  U[ℰ_NonCommutativeMultiply] := U /@ ℰ;
  O_U[specs___, poly_] := Module[{sp, null, vs, us},
    sp = Replace[{specs}, l_List :> l_null, {1}];
    vs = Join @@ (First /@ sp);
    us = Join @@ (sp /. l_s_ :> (l /. x_i_ :> x_s));
    CE[Total[
       CoefficientRules[poly, vs] /. (p_ → c_) :> c U@(us^p)
     ]] /. x_null :> x];
```

```
𝕆_U[specs___, 𝔼[L_, Q_, P_]] := 𝕆_U[specs, SS@Normal[P e^(L+Q)]];
pow[ℰ_, n_] := pow[ℰ, n - 1] ⊠ ℰ;
𝕊_U[ℰ_, ss___Rule] := CE@Total[
   CoefficientRules[ℰ, First /@ {ss}] /.
    (p_ → c_) :→ c NCM @@ MapThread[pow, {Last /@ {ss}, p}]];
σ_rs___[c_. * u_U] := (c /. (t : cp)_j_ :→ t_(j/.{rs})) U[List @@ (u /. v_j_ :→ v_(j/.{rs}))];
m_(j_→k_)[c_. * u_U] := CE[((c /. (t : cp)_j → t_k) DeleteCases[u, _(j|k)]) ⊠
   U @@ Cases[u, w_j_ :→ w_k] ⊠ U @@ Cases[u, _k]];
U /: c_. * u_U * v_U := CE[c u ⊠ v];
S_i_[c_. * u_U] := CE[((c /. S_i[U, Centrals]) DeleteCases[u, _i]) ⊠
   U_i[NCM @@ Reverse@Cases[u, x_i_ :→ S@U@x]]];
Δ_(i_→j_,k_)[c_. * u_U] := CE[((c /. Δ_(i→j,k)[U, Centrals]) DeleteCases[u, _i]) ⊠
   (NCM @@ Cases[u, x_i_ :→ σ_(1→j,2→k)@Δ@U@x] /. NCM[] → U[])]; ]
```

Meta-Operations

In[∘]:=
```
σ_rs___[ℰ_Plus] := σ_rs /@ ℰ;
m_(j_→j_) = Identity; m_(j_→k_)[0] = 0;
m_(j_→k_)[ℰ_Plus] := Simp[m_(j→k) /@ ℰ];
m_(is___,i_,j_→k_)[ℰ_] := m_(j→k)@m_(is,i→j)@ℰ;
S_i_[ℰ_Plus] := Simp[S_i /@ ℰ];
Δ_is___[ℰ_Plus] := Simp[Δ_is /@ ℰ];
```

Implementing CU = $\mathcal{U}(sl_2^{\gamma\epsilon})$

Verify σ and Δ! Also Generalize Δ to $\Delta_{i,j_1,j_2,\ldots}$.

In[∘]:=
```
DeclareAlgebra[CU, Generators → {y, a, x}, Centrals → {t}];
B[a_CU, y_CU] = -γ y_CU; B[x_CU, a_CU] = -γ x_CU;
B[x_CU, y_CU] = 2 ϵ a_CU - t 1_CU;
(S@y_CU = -y_CU; S@a_CU = -a_CU; S@x_CU = -x_CU;)
S_i_[CU, Centrals] = {t_i → -t_i};
Δ@y_CU = CU@y_1 + CU@y_2; Δ@a_CU = CU@a_1 + CU@a_2; Δ@x_CU = CU@x_1 + CU@x_2;
Δ_(i_→j_,k_)[CU, Centrals] = {t_i → t_j + t_k};
```

Implementing QU = $\mathcal{U}_q(sl_2^{\gamma\epsilon})$

In[∘]:=
```
ℏ = γ = 1;
DeclareAlgebra[QU, Generators → {y, a, x}, Centrals → {t, T}];
B[a_QU, y_QU] = -γ y_QU; B[x_QU, a_QU] = -γ QU@x;
B[x_QU, y_QU] := SS[q_ℏ - 1] QU@{y, x} + 𝕆_QU[{a}, SS[(1 - T e^(-2 ϵ a ℏ)) / ℏ]];
(S@y_QU := 𝕆_QU[{a, y}, SS[-T^-1 e^(ℏ ϵ a) y]]; S@a_QU = -a_QU; S@x_QU := 𝕆_QU[{a, x}, SS[-e^(ℏ ϵ a) x]];)
S_i_ [QU, Centrals] = {t_i → -t_i, T_i → T_i^-1};
Δ@y_QU := 𝕆_QU[{y_1, a_1}_1, {y_2}_2, SS[y_1 + T_1 e^(-ℏ ϵ a_1) y_2]];
Δ@a_QU = QU@a_1 + QU@a_2; Δ@x_QU := 𝕆_QU[{a_1, x_1}_1, {x_2}_2, SS[x_1 + e^(-ℏ ϵ a_1) x_2]];
Δ_(i_→j_,k_) [QU, Centrals] = {t_i → t_j + t_k, T_i → T_j T_k};
```

In[∘]:=
```
B[x_QU, y_QU]
```

Out[∘]=

$(1 - T)\,QU[\,] + 2\,T\,\epsilon\,QU[a] + \epsilon\,QU[y, x]$

tSW

Logoi from Pensieve://Talks/Toulouse-1705/DogmaDemo.nb and from Pensieve://Talks/Sydney-1708/ExtraDetails@@.nb.

Goal. In either U, compute $F = e^{-\eta y}\, e^{\xi x}\, e^{\eta y}\, e^{-\xi x}$. First compute $G = e^{\xi x}\, y\, e^{-\xi x}$, a finite sum. Now F satisfies the ODE $\partial_\eta F = \partial_\eta\left(e^{-\eta y}\, e^{\eta G}\right) = -yF + FG$ with initial conditions $F(\eta = 0) = 1$. So we set it up and solve:

In[∘]:=
```
SW_xy[U_, kk_] :=
  SW_xy[U, kk] = Block[{$U = U, $k = kk, $p = kk}, Module[{G, F, fs, f, bs, e, b, es},
     G = Simp[Table[ξ^k / k!, {k, 0, $k + 1}].NestList[Simp[B[x_U , #]] &, y_U, $k + 1]];
     fs = Flatten@Table[f_(l,i,j,k)[η], {l, 0, $k}, {i, 0, 1}, {j, 0, 1}, {k, 0, 1}];
     F = fs.(bs = fs /. f_(l_,i_,j_,k_)[η] :→ ϵ^l U@{y^i, a^j, x^k});
     es = Flatten[
       Table[Coefficient[e, b] == 0, {e, {F - 1_U /. η → 0, F ⊠ G - y_U ⊠ F - ∂_η F}}, {b, bs}]];
     F = F /. DSolve[es, fs, η][[1]];
     𝔼[0,
       ξ x + η y + (U /. {CU → -t η ξ, QU → η ξ (1 - T) / ℏ}),
       F + 0_$k /. {e- → 1, U → Times}
      ] /. (v : η | ξ | t | T | y | a | x) → v_1
    ]];
tSW_(xy_,i_,j_→k_) := SW_xy[$U, $k] /. {ξ_1 → ξ_i, η_1 → η_j, (v : t | T | y | a | x)_1 → v_k};
tSW_(xa,i_,j_→k_) := 𝔼[α_j a_k, e^(-γ α_j) ξ_i x_k, 1];
tSW_(ay,i_,j_→k_) := 𝔼[α_i a_k, e^(-γ α_i) η_j y_k, 1];
```

Exponentials as needed.

Task. Define $\mathrm{Exp}_{U_i,k}[\xi, P]$ which computes $e^{\xi\,\mathbb{O}(P)}$ to ϵ^k in the algebra U_i, where ξ is a scalar, X is x_i or y_i, and P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form. Should satisfy

$U@\mathrm{Exp}_{U_i,k}[\xi, P] == \mathbb{S}_U[e^{\xi x},\ x \to \mathbb{O}(P)]$.
Methodology. If $P_0 := P_{\epsilon=0}$ and $e^{\xi\,\mathbb{O}(P)} = \mathbb{O}(e^{\xi P_0} F(\xi))$, then $F(\xi=0) = 1$ and we have:
$\mathbb{O}(e^{\xi P_0}(P_0 F(\xi) + \partial_\xi F)) = \mathbb{O}(\partial_\xi e^{\xi P_0} F(\xi)) = \partial_\xi \mathbb{O}(e^{\xi P_0} F(\xi)) = \partial_\xi e^{\xi\,\mathbb{O}(P)} = e^{\xi\,\mathbb{O}(P)}\,\mathbb{O}(P) = \mathbb{O}(e^{\xi P_0} F(\xi))\,\mathbb{O}(P)$.
This is an ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k \varphi$ we find that $F_0 = 1$ and solve for φ.

In[∘]:=
```
(* Bug: The first line is valid only if O(e^P0) == e^O(P0). *)
(* Bug: ξ must be a symbol. *)
Exp_{U_i_,0}[ξ_, P_] := Module[{LQ = Normal@P /. ϵ → 0},
   𝔼[ξ LQ /. (x | y)_i → 0, ξ LQ /. (t | a)_i → 0, 1]];
Exp_{U_i_,k_}[ξ_, P_] := Block[{$U = U, $k = k},
  Module[{P0, φ, φs, F, j, rhs, at0, atξ},
   P0 = Normal@P /. ϵ → 0;
   φs =
    Flatten@Table[φ_{j1,j2,j3}[ξ], {j2, 0, k}, {j1, 0, 2 k + 1 - j2}, {j3, 0, 2 k + 1 - j2 - j1}];
   F = Normal@Last@Exp_{U_i,k-1}[ξ, P] + ϵ^k φs.(φs /. φ_{js__}[ξ] :→ Times @@ {y_i, a_i, x_i}^{js});
   rhs = Normal@
     Last@m_{i,j→i}[𝔼[ξ P0 /. (x | y)_i → 0, ξ P0 /. (t | a)_i → 0, F + O_k] m_{i→j}@𝔼[0, 0, P + O_k]];
   at0 = (# == 0) & /@ Flatten@CoefficientList[F - 1 /. ξ → 0, {y_i, a_i, x_i}];
   atξ = (# == 0) & /@ Flatten@CoefficientList[(∂_ξ F) + P0 F - rhs, {y_i, a_i, x_i}];
   𝔼[ξ P0 /. (x | y)_i → 0, ξ P0 /. (t | a)_i → 0, F + O_k] /.
    DSolve[And @@ (at0 ⋃ atξ), φs, ξ][[1]] ]]
```

In[∘]:=
```
$k = 1
```
Out[∘]=
```
1
```

In[∘]:=
```
$Basis = {a_QU, x_QU, y_QU};
Module[{a, b}, Table[{a, b} → CE[B[a, b]], {a, $Basis}, {b, $Basis}]] // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \{QU[a], QU[a]\} \to 0 & \{QU[a], QU[x]\} \to \gamma\, QU[x] & \\ \{QU[x], QU[a]\} \to -\gamma\, QU[x] & \{QU[x], QU[x]\} \to 0 & \{QU[x], \\ \{QU[y], QU[a]\} \to \gamma\, QU[y] & \{QU[y], QU[x]\} \to \left(-\frac{1}{\hbar} + \frac{T}{\hbar}\right) QU[] - 2\, T\, \epsilon\, QU[a] - \gamma\, \epsilon\, \hbar\, QU[y, x] & \end{pmatrix}$$

```
In[•]:= $Basis = {a_QU, x_QU, y_QU};
Module[{a, b},
 DeleteCases[
  Flatten[Table[{a, b, c} → CE[B[a, b] + B[b, a]], {a, $Basis}, {b, $Basis}]], _ → 0]
]
Module[{a, b, c},
 DeleteCases[Flatten[Table[{a, b, c} → CE[B[a, B[b, c]] + B[b, B[c, a]] + B[c, B[a, b]]],
    {a, $Basis}, {b, $Basis}, {c, $Basis}]], _ → 0]
]
$ABasis = {a_QU, x_QU, y_QU, QU[y, y], QU[y, a], QU[y, x], QU[a, a], QU[a, x], QU[x, x]};
Module[{a, b, c},
 DeleteCases[Flatten[Table[{a, b, c} → CE[a ⊠ CE[b ⊠ c] - CE[a ⊠ b] ⊠ c],
    {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]], _ → 0]
]
Out[•]=
{}
Out[•]=
{}
Out[•]=
{}
```

```
In[•]:= Module[{a, b, c},
  Table[{a, b, c} → CE[a ⊠ CE[b ⊠ c]], {a, $ABasis}, {b, $ABasis}, {c, $ABasis}]
 ];
```

```
In[•]:= B[x_QU, y_QU] // CE
Out[•]=
```

$$\left(\frac{1}{\hbar}-\frac{T}{\hbar}\right)\mathrm{QU}[\,]+2\,T\,\epsilon\,\mathrm{QU}[a]+\gamma\,\epsilon\,\hbar\,\mathrm{QU}[y,x]$$

```
In[•]:= B[x_QU, B[x_QU, y_QU]] // CE
Out[•]=
```

$$(\gamma\,\epsilon-3\,T\,\gamma\,\epsilon)\,\mathrm{QU}[x]$$

```
In[•]:= B[x_QU, B[x_QU, B[x_QU, y_QU]]] // CE
Out[•]=
0
```

```
In[•]:= B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]] // CE
Out[•]=
0
```

```
In[•]:= B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]]] // Expand
Out[•]=
0
```

```
In[•]:= B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, B[x_QU, y_QU]]]]]] // Expand
Out[•]=
0
```

In[∘]:=
```
adPower[x_, k_, Env_][c_ * y_QU] := CE[c adPower[x, k, Env][y]];
adPower[x_, k_, Env_][y_Plus] := adPower[x, k, Env] /@ y;
adPower[_, 0, Env_][y_] := y;
adPower[x_, k_, Env_][0] = 0;
adPower[x_, k_, Env_][y_QU] :=
  adPower[x, k, Env][y] = CE@B[adPower[x, k - 1, Env][y], x];
adPower[x_, k_][y_] := adPower[x, k, $E][y];
```

In[∘]:= adPower[x_{QU}, 1][y_{QU}]

Out[∘]=

$$\left(-\frac{1}{\hbar}+\frac{T}{\hbar}\right) QU[\,] - 2\,T\,\epsilon\,QU[a] - \gamma\,\epsilon\,\hbar\,QU[y, x]$$

In[∘]:= adPower[x_{QU}, 2][y_{QU}]

Out[∘]=

$$(\gamma\,\epsilon - 3\,T\,\gamma\,\epsilon)\,QU[x]$$

In[∘]:= adPower[x_{QU}, 3][y_{QU}]

Out[∘]=

0

In[∘]:= adPower[x_{QU}, 4][y_{QU}]

Out[∘]=

0

In[∘]:= adPower[x_{QU}, 5][y_{QU}]

Out[∘]=

0

In[∘]:=
```
NilQ[x_QU] = NilQ[y_QU] = True; NilQ[a_QU] = False;
PotQ[c_. * x_QU] := ¬ NilQ[x]; (* Potent Q *)
NilQ[c_ * x_QU] := NilQ[x];
PotQ[x_Plus] := And @@ (PotQ /@ (List @@ x));
NilQ[x_Plus] := And @@ (NilQ /@ (List @@ x));
```

In[∘]:=
```
Ad[0] = Identity;
Ad[c_. a_QU][z_] := z /. w_QU :> e^(c γ (Count[w,y]-Count[w,x])) w;
Ad[c_. QU[]][z_] := z;
```

In[∘]:=
```
Ad[x_][y_] /; NilQ[x] := Expand@Module[{k = 0, s = 0},
   While[adPower[x, k][y] =!= 0,
    s += adPower[x, k][y] / k!;
    ++k
   ];
   s
  ]
```

In[ ]:=
```
Ad[x_QU][y_QU]
```
Out[ ]=
$$-QU[] + T\,QU[] - 2\,T\,\epsilon\,QU[a] + \frac{1}{2}\,\epsilon\,QU[x] - \frac{3}{2}\,T\,\epsilon\,QU[x] + QU[y] - \epsilon\,QU[y, x]$$

In[ ]:=
```
$k
```
Out[ ]=
1

In[ ]:=
```
Ad[c a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]}
```
Out[ ]=
$$\{QU[],\ e^{-c\,\gamma}\,QU[x],\ e^{c\,\gamma}\,QU[y],\ QU[x, y]\}$$

In[ ]:=
```
Ad[(α1 + α2 + ϵ fa[λ]) a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]}
```
Out[ ]=
$$\left\{QU[],\ e^{-\gamma\,(\alpha 1+\alpha 2+\epsilon\,fa[\lambda])}\,QU[x],\ e^{\gamma\,(\alpha 1+\alpha 2+\epsilon\,fa[\lambda])}\,QU[y],\ QU[x, y]\right\}$$

In[ ]:=
```
Normal@Series[e^(-γ (α1+α2+ϵ fa[λ])), {ϵ, 0, $k}]
```
Out[ ]=
$$e^{-((\alpha 1+\alpha 2)\,\gamma)} - e^{-((\alpha 1+\alpha 2)\,\gamma)}\,\gamma\,\epsilon\,fa[\lambda]$$

In[ ]:=
```
CE[
 Ad[(α1 + α2 + ϵ fa[λ]) a_QU] /@ {QU[], x_QU, y_QU, QU[x, y]} /. e^ℰ_ :> Normal@Series[e^ℰ, {ϵ, 0, $k}]]
```
Out[ ]=
$$\left\{QU[],\ \left(e^{-\alpha 1\,\gamma-\alpha 2\,\gamma} - e^{-\alpha 1\,\gamma-\alpha 2\,\gamma}\,\gamma\,\epsilon\,fa[\lambda]\right)QU[x],\ \left(e^{\alpha 1\,\gamma+\alpha 2\,\gamma} + e^{\alpha 1\,\gamma+\alpha 2\,\gamma}\,\gamma\,\epsilon\,fa[\lambda]\right)QU[y],\ QU[x, y]\right\}$$

In[ ]:=
```
Ad[a_QU][Ad[x_QU][y_QU]]
```
Out[ ]=
$$-\frac{QU[]}{\hbar} + \frac{T\,QU[]}{\hbar} - 2\,T\,\epsilon\,QU[a] + \frac{1}{2}\,e^{-\gamma}\,\gamma\,\epsilon\,QU[x] - \frac{3}{2}\,e^{-\gamma}\,T\,\gamma\,\epsilon\,QU[x] + e^{\gamma}\,QU[y] - \gamma\,\epsilon\,\hbar\,QU[y, x]$$

In[ ]:=
```
λTangent[] = 0;
λTangent[xs___, x_] := CE[Ad[x][λTangent[xs]] + ∂_λ x];
λTangent[xs_List] := λTangent @@ xs
```

In[ ]:=
```
λTangent[λ y_QU, λ a_QU, λ x_QU]
```
Out[ ]=
$$\left(-\frac{e^{\gamma\,\lambda}\,\lambda}{\hbar} + \frac{e^{\gamma\,\lambda}\,T\,\lambda}{\hbar}\right)QU[] + \left(1 - 2\,e^{\gamma\,\lambda}\,T\,\epsilon\,\lambda\right)QU[a] +$$
$$\left(1 + \gamma\,\lambda + \frac{1}{2}\,e^{\gamma\,\lambda}\,\gamma\,\epsilon\,\lambda^2 - \frac{3}{2}\,e^{\gamma\,\lambda}\,T\,\gamma\,\epsilon\,\lambda^2\right)QU[x] + e^{\gamma\,\lambda}\,QU[y] - e^{\gamma\,\lambda}\,\gamma\,\epsilon\,\lambda\,\hbar\,QU[y, x]$$

In[ ]:=
```
Seepage_U_[{}][c_] = c;
Seepage_U_[ξs_List][P_] := Module[{gs = U[Generators], lx},
  lx = gs[[Length@ξs]];
  Collect[P, lx, (Ad[Last[ξs] lx_U][O_U[gs, Seepage_U[Most@ξs]@#]] /. U → Times) &]
 ]
```

In[ ]:= $\text{Seepage}_{QU}[\{\eta, \alpha, \xi\}][y]$

Out[ ]=

$$e^{\alpha} y - e^{\alpha} \xi + e^{\alpha} T \xi - 2 a e^{\alpha} T \epsilon \xi - e^{\alpha} x y \epsilon \xi + \frac{1}{2} e^{\alpha} x \epsilon \xi^2 - \frac{3}{2} e^{\alpha} T x \epsilon \xi^2$$

In[ ]:= $\mathbb{O}_{QU}[\{y, a, x\}, x\, y]$

Out[ ]=

QU[y, x]

In[ ]:= $\text{Ad}[\text{QU}[x]][y_{QU}]$

Out[ ]=

$Aborted

In[ ]:= $\text{Expand}[\text{Seepage}_{QU}[\{\eta\}][7] \mathbin{/.} \epsilon \to 0]$

Out[ ]=

7

In[ ]:= $\text{Expand}[\text{Seepage}_{QU}[\{\eta, \alpha, \xi\}][7\, x] \mathbin{/.} \epsilon \to 0]$

Out[ ]=

7 x

In[ ]:= $\text{Expand}[\text{Seepage}_{QU}[\{\eta, \alpha, \xi\}][7\, a] \mathbin{/.} \epsilon \to 0]$
$\text{Expand}[\text{Seepage}_{QU}[\{\eta, \alpha, \xi\}][7\, y] \mathbin{/.} \epsilon \to 0]$
$\text{Expand}[\text{Seepage}_{QU}[\{\eta, \alpha, \xi\}][7\, a + y] \mathbin{/.} \epsilon \to 0]$

Out[ ]=

$7 a + 7 x \xi$

Out[ ]=

$Aborted

Out[ ]=

$Aborted

In[ ]:= $k = 1

Out[ ]=

1

In[∘]:= **lhs = λTangent[λ η1 y_QU, α1 a_QU, λ ξ1 x_QU, λ η2 y_QU, α2 a_QU, λ ξ2 x_QU]**

Out[∘]=

$$\Big(-e^{\alpha1}\,\eta1\,\lambda\,\xi1+e^{\alpha1}\,T\,\eta1\,\lambda\,\xi1+\eta2\,\lambda\,\xi1-T\,\eta2\,\lambda\,\xi1+\frac{1}{2}\,e^{\alpha1}\,\epsilon\,\eta1\,\eta2\,\lambda^3\,\xi1^2-2\,e^{\alpha1}\,T\,\epsilon\,\eta1\,\eta2\,\lambda^3\,\xi1^2+\frac{3}{2}\,e^{\alpha1}\,T^2\,\epsilon\,\eta1\,\eta2\,\lambda^3\,\xi1^2-e^{\alpha1+\alpha2}\,\eta1\,\lambda\,\xi2+e^{\alpha1+\alpha2}\,T\,\eta1\,\lambda\,\xi2-e^{\alpha2}\,\eta2\,\lambda\,\xi2+e^{\alpha2}\,T\,\eta2\,\lambda\,\xi2+e^{\alpha1+\alpha2}\,\epsilon\,\eta1\,\eta2\,\lambda^3\,\xi1\,\xi2-4\,e^{\alpha1+\alpha2}\,T\,\epsilon\,\eta1\,\eta2\,\lambda^3\,\xi1\,\xi2+3\,e^{\alpha1+\alpha2}\,T^2\,\epsilon\,\eta1\,\eta2\,\lambda^3\,\xi1\,\xi2-\frac{1}{2}\,e^{\alpha2}\,\epsilon\,\eta2^2\,\lambda^3\,\xi1\,\xi2+2\,e^{\alpha2}\,T\,\epsilon\,\eta2^2\,\lambda^3\,\xi1\,\xi2-\frac{3}{2}\,e^{\alpha2}\,T^2\,\epsilon\,\eta2^2\,\lambda^3\,\xi1\,\xi2\Big)\,QU[\,]+$$
$$\left(-2\,e^{\alpha1}\,T\,\epsilon\,\eta1\,\lambda\,\xi1+2\,T\,\epsilon\,\eta2\,\lambda\,\xi1-2\,e^{\alpha1+\alpha2}\,T\,\epsilon\,\eta1\,\lambda\,\xi2-2\,e^{\alpha2}\,T\,\epsilon\,\eta2\,\lambda\,\xi2\right)\,QU[a]+$$
$$\Big(e^{-\alpha2}\,\xi1+\frac{1}{2}\,e^{\alpha1-\alpha2}\,\epsilon\,\eta1\,\lambda^2\,\xi1^2-\frac{3}{2}\,e^{\alpha1-\alpha2}\,T\,\epsilon\,\eta1\,\lambda^2\,\xi1^2+\xi2+e^{\alpha1}\,\epsilon\,\eta1\,\lambda^2\,\xi1\,\xi2-3\,e^{\alpha1}\,T\,\epsilon\,\eta1\,\lambda^2\,\xi1\,\xi2-\epsilon\,\eta2\,\lambda^2\,\xi1\,\xi2+3\,T\,\epsilon\,\eta2\,\lambda^2\,\xi1\,\xi2+\frac{1}{2}\,e^{\alpha1+\alpha2}\,\epsilon\,\eta1\,\lambda^2\,\xi2^2-\frac{3}{2}\,e^{\alpha1+\alpha2}\,T\,\epsilon\,\eta1\,\lambda^2\,\xi2^2+\frac{1}{2}\,e^{\alpha2}\,\epsilon\,\eta2\,\lambda^2\,\xi2^2-\frac{3}{2}\,e^{\alpha2}\,T\,\epsilon\,\eta2\,\lambda^2\,\xi2^2\Big)\,QU[x]+$$
$$\Big(e^{\alpha1+\alpha2}\,\eta1+e^{\alpha2}\,\eta2-e^{\alpha1+\alpha2}\,\epsilon\,\eta1\,\eta2\,\lambda^2\,\xi1+3\,e^{\alpha1+\alpha2}\,T\,\epsilon\,\eta1\,\eta2\,\lambda^2\,\xi1+\frac{1}{2}\,e^{\alpha2}\,\epsilon\,\eta2^2\,\lambda^2\,\xi1-\frac{3}{2}\,e^{\alpha2}\,T\,\epsilon\,\eta2^2\,\lambda^2\,\xi1\Big)\,QU[y]+$$
$$\left(-e^{\alpha1}\,\epsilon\,\eta1\,\lambda\,\xi1+\epsilon\,\eta2\,\lambda\,\xi1-e^{\alpha1+\alpha2}\,\epsilon\,\eta1\,\lambda\,\xi2-e^{\alpha2}\,\epsilon\,\eta2\,\lambda\,\xi2\right)\,QU[y,x]$$

In[∘]:= **seep = Seepage_QU[{fy0[λ] + ϵ fy1[λ], α1 + α2 + ϵ fa1[λ], fx0[λ] + ϵ fx1[λ]}][
∂_λ((ft0[λ] + ϵ ft1[λ]) t + (fy0[λ] + ϵ fy1[λ]) y + ϵ fa1[λ] a + (fx0[λ] + ϵ fx1[λ]) x)]**

Out[∘]=

$$a\,\epsilon\,fa1'[\lambda]+x\,\epsilon\,fx0[\lambda]\,fa1'[\lambda]+t\,ft0'[\lambda]+t\,\epsilon\,ft1'[\lambda]+x\,(fx0'[\lambda]+\epsilon\,fx1'[\lambda])+e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,y\,fy0'[\lambda]-e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,fx0[\lambda]\,fy0'[\lambda]+e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,T\,fx0[\lambda]\,fy0'[\lambda]-2\,a\,e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,T\,\epsilon\,fx0[\lambda]\,fy0'[\lambda]-e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,x\,y\,\epsilon\,fx0[\lambda]\,fy0'[\lambda]+\frac{1}{2}\,e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,x\,\epsilon\,fx0[\lambda]^2\,fy0'[\lambda]-\frac{3}{2}\,e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,T\,x\,\epsilon\,fx0[\lambda]^2\,fy0'[\lambda]-e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,\epsilon\,fx1[\lambda]\,fy0'[\lambda]+e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,T\,\epsilon\,fx1[\lambda]\,fy0'[\lambda]+e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,y\,\epsilon\,fy1'[\lambda]-e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,\epsilon\,fx0[\lambda]\,fy1'[\lambda]+e^{\alpha1+\alpha2+\epsilon\,fa1[\lambda]}\,T\,\epsilon\,fx0[\lambda]\,fy1'[\lambda]$$

In[∘]:= **rhs = $\mathbb{O}_{QU}$[{y, a, x}, seep] /. $e^{\mathcal{E}_-}$:→ Normal@Series[$e^{\mathcal{E}}$, {ϵ, 0, $k}]**

Out[∘]=

$$-\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]\,\mathrm{QU}[y,x]\,\mathrm{fy0}'[\lambda]+\mathrm{QU}[a]\left(\epsilon\,\mathrm{fa1}'[\lambda]-2\,T\,\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[x]\left(\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fa1}'[\lambda]+\mathrm{fx0}'[\lambda]+\epsilon\,\mathrm{fx1}'[\lambda]+\frac{1}{2}\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]^2\,\mathrm{fy0}'[\lambda]-\frac{3}{2}T\,\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]^2\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[y]\left(\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fy0}'[\lambda]+\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fy1}'[\lambda]\right)+\mathrm{QU}[\,]\left(t\,\mathrm{ft0}'[\lambda]+t\,\epsilon\,\mathrm{ft1}'[\lambda]-\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]+T\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx1}[\lambda]\,\mathrm{fy0}'[\lambda]+T\,\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx1}[\lambda]\,\mathrm{fy0}'[\lambda]-\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]\,\mathrm{fy1}'[\lambda]+T\,\epsilon\left(e^{\alpha 1+\alpha 2}+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\right)\mathrm{fx0}[\lambda]\,\mathrm{fy1}'[\lambda]\right)$$

In[∘]:= **diff = CE[lhs − rhs]**

Out[∘]=

$$\mathrm{QU}[y,x]\left(-e^{\alpha 1}\epsilon\,\eta 1\,\lambda\,\xi 1+\epsilon\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\epsilon\,\eta 2\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[a]\left(-2\,e^{\alpha 1}T\,\epsilon\,\eta 1\,\lambda\,\xi 1+2\,T\,\epsilon\,\eta 2\,\lambda\,\xi 1-2\,e^{\alpha 1+\alpha 2}T\,\epsilon\,\eta 1\,\lambda\,\xi 2-2\,e^{\alpha 2}T\,\epsilon\,\eta 2\,\lambda\,\xi 2-\epsilon\,\mathrm{fa1}'[\lambda]+2\,e^{\alpha 1+\alpha 2}T\,\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[x]\left(e^{-\alpha 2}\xi 1+\frac{1}{2}e^{\alpha 1-\alpha 2}\epsilon\,\eta 1\,\lambda^2\,\xi 1^2-\frac{3}{2}e^{\alpha 1-\alpha 2}T\,\epsilon\,\eta 1\,\lambda^2\,\xi 1^2+\xi 2+e^{\alpha 1}\epsilon\,\eta 1\,\lambda^2\,\xi 1\,\xi 2-3\,e^{\alpha 1}T\,\epsilon\,\eta 1\,\lambda^2\,\xi 1\,\xi 2-\epsilon\,\eta 2\,\lambda^2\,\xi 1\,\xi 2+3\,T\,\epsilon\,\eta 2\,\lambda^2\,\xi 1\,\xi 2+\frac{1}{2}e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\lambda^2\,\xi 2^2-\frac{3}{2}e^{\alpha 1+\alpha 2}T\,\epsilon\,\eta 1\,\lambda^2\,\xi 2^2+\frac{1}{2}e^{\alpha 2}\epsilon\,\eta 2\,\lambda^2\,\xi 2^2-\frac{3}{2}e^{\alpha 2}T\,\epsilon\,\eta 2\,\lambda^2\,\xi 2^2-\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fa1}'[\lambda]-\mathrm{fx0}'[\lambda]-\epsilon\,\mathrm{fx1}'[\lambda]-\frac{1}{2}e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]^2\,\mathrm{fy0}'[\lambda]+\frac{3}{2}e^{\alpha 1+\alpha 2}T\,\epsilon\,\mathrm{fx0}[\lambda]^2\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[y]\left(e^{\alpha 1+\alpha 2}\eta 1+e^{\alpha 2}\eta 2-e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\eta 2\,\lambda^2\,\xi 1+3\,e^{\alpha 1+\alpha 2}T\,\epsilon\,\eta 1\,\eta 2\,\lambda^2\,\xi 1+\frac{1}{2}e^{\alpha 2}\epsilon\,\eta 2^2\,\lambda^2\,\xi 1-\frac{3}{2}e^{\alpha 2}T\,\epsilon\,\eta 2^2\,\lambda^2\,\xi 1-e^{\alpha 1+\alpha 2}\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fy1}'[\lambda]\right)+\mathrm{QU}[\,]\left(-e^{\alpha 1}\eta 1\,\lambda\,\xi 1+e^{\alpha 1}T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1+\frac{1}{2}e^{\alpha 1}\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-2\,e^{\alpha 1}T\,\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1^2+\frac{3}{2}e^{\alpha 1}T^2\,\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1^2-e^{\alpha 1+\alpha 2}\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\eta 2\,\lambda\,\xi 2+e^{\alpha 2}T\,\eta 2\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-4\,e^{\alpha 1+\alpha 2}T\,\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2+3\,e^{\alpha 1+\alpha 2}T^2\,\epsilon\,\eta 1\,\eta 2\,\lambda^3\,\xi 1\,\xi 2-\frac{1}{2}e^{\alpha 2}\epsilon\,\eta 2^2\,\lambda^3\,\xi 1\,\xi 2+2\,e^{\alpha 2}T\,\epsilon\,\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-\frac{3}{2}e^{\alpha 2}T^2\,\epsilon\,\eta 2^2\,\lambda^3\,\xi 1\,\xi 2-t\,\mathrm{ft0}'[\lambda]-t\,\epsilon\,\mathrm{ft1}'[\lambda]+e^{\alpha 1+\alpha 2}\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}T\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fa1}[\lambda]\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}T\,\epsilon\,\mathrm{fa1}[\lambda]\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx1}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}T\,\epsilon\,\mathrm{fx1}[\lambda]\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy1}'[\lambda]-e^{\alpha 1+\alpha 2}T\,\epsilon\,\mathrm{fx0}[\lambda]\,\mathrm{fy1}'[\lambda]\right)$$

In[∘]:= `diff /. ϵ → 0`

Out[∘]=

$$\mathrm{QU}[x]\left(e^{-\alpha 2}\,\xi 1+\xi 2-\mathrm{fx0}'[\lambda]\right)+\mathrm{QU}[y]\left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[\,]\left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\,T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,T\,\eta 2\,\lambda\,\xi 2-t\,\mathrm{ft0}'[\lambda]+e^{\alpha 1+\alpha 2}\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\,\mathrm{fx0}[\lambda]\,\mathrm{fy0}'[\lambda]\right)$$

In[∘]:= `{sol1} = DSolve[Coefficient[diff /. ϵ → 0, x_QU] == 0 ∧ fx0[0] == 0, fx0, λ]`

Out[∘]=

$$\left\{\left\{\mathrm{fx0}\to\mathrm{Function}\left[\{\lambda\},\ e^{-\alpha 2}\,\lambda\left(\xi 1+e^{\alpha 2}\,\xi 2\right)\right]\right\}\right\}$$

In[∘]:= `CE[diff /. sol1 /. ϵ → 0]`

Out[∘]=

$$\mathrm{QU}[y]\left(e^{\alpha 1+\alpha 2}\,\eta 1+e^{\alpha 2}\,\eta 2-e^{\alpha 1+\alpha 2}\,\mathrm{fy0}'[\lambda]\right)+\mathrm{QU}[\,]\left(-e^{\alpha 1}\,\eta 1\,\lambda\,\xi 1+e^{\alpha 1}\,T\,\eta 1\,\lambda\,\xi 1+\eta 2\,\lambda\,\xi 1-T\,\eta 2\,\lambda\,\xi 1-e^{\alpha 1+\alpha 2}\,\eta 1\,\lambda\,\xi 2+e^{\alpha 1+\alpha 2}\,T\,\eta 1\,\lambda\,\xi 2-e^{\alpha 2}\,\eta 2\,\lambda\,\xi 2+e^{\alpha 2}\,T\,\eta 2\,\lambda\,\xi 2-t\,\mathrm{ft0}'[\lambda]+e^{\alpha 1}\,\lambda\,\xi 1\,\mathrm{fy0}'[\lambda]-e^{\alpha 1}\,T\,\lambda\,\xi 1\,\mathrm{fy0}'[\lambda]+e^{\alpha 1+\alpha 2}\,\lambda\,\xi 2\,\mathrm{fy0}'[\lambda]-e^{\alpha 1+\alpha 2}\,T\,\lambda\,\xi 2\,\mathrm{fy0}'[\lambda]\right)$$

In[∘]:= `{sol2} = DSolve[Coefficient[CE[diff /. sol1 /. ϵ → 0], y_QU] == 0 ∧ fy0[0] == 0, fy0, λ]`

Out[∘]=

$$\left\{\left\{\mathrm{fy0}\to\mathrm{Function}\left[\{\lambda\},\ e^{-\alpha 1}\left(e^{\alpha 1}\,\eta 1+\eta 2\right)\lambda\right]\right\}\right\}$$

In[∘]:= `CE[diff /. sol1 /. sol2 /. ϵ → 0]`

Out[∘]=

$$\mathrm{QU}[\,]\left(2\,\eta 2\,\lambda\,\xi 1-2\,T\,\eta 2\,\lambda\,\xi 1-t\,\mathrm{ft0}'[\lambda]\right)$$

In[∘]:= `{sol3} = DSolve[Coefficient[CE[diff /. sol1 /. sol2 /. ϵ → 0], QU[]] == 0 ∧ ft0[0] == 0, ft0, λ]`

Out[∘]=

$$\left\{\left\{\mathrm{ft0}\to\mathrm{Function}\left[\{\lambda\},\ -\frac{(-1+T)\,\eta 2\,\lambda^2\,\xi 1}{t}\right]\right\}\right\}$$

In[•]:= **CE[diff /. sol1 /. sol2 /. sol3]**

Out[•]=

$$2 \in \eta2\, \lambda\, \xi1\, \mathrm{QU}[y, x] + \mathrm{QU}[a]\, (4\, T \in \eta2\, \lambda\, \xi1 - \in \mathrm{fa1}'[\lambda]) + \mathrm{QU}[x] \left(-\frac{1}{2}\, e^{-\alpha2} \in \eta2\, \lambda^2\, \xi1^2 + \frac{3}{2}\, e^{-\alpha2}\, T \in \eta2\, \lambda^2\, \xi1^2 - 2 \in \eta2\, \lambda^2\, \xi1\, \xi2 + 6\, T \in \eta2\, \lambda^2\, \xi1\, \xi2 - e^{-\alpha2} \in \lambda\, \xi1\, \mathrm{fa1}'[\lambda] - \in \lambda\, \xi2\, \mathrm{fa1}'[\lambda] - \in \mathrm{fx1}'[\lambda]\right) + \mathrm{QU}[y] \left(-e^{\alpha1+\alpha2} \in \eta1\, \eta2\, \lambda^2\, \xi1 + 3\, e^{\alpha1+\alpha2}\, T \in \eta1\, \eta2\, \lambda^2\, \xi1 + \frac{1}{2}\, e^{\alpha2} \in \eta2^2\, \lambda^2\, \xi1 - \frac{3}{2}\, e^{\alpha2}\, T \in \eta2^2\, \lambda^2\, \xi1 - e^{\alpha1+\alpha2} \in \eta1\, \mathrm{fa1}[\lambda] - e^{\alpha2} \in \eta2\, \mathrm{fa1}[\lambda] - e^{\alpha1+\alpha2} \in \mathrm{fy1}'[\lambda]\right) + \mathrm{QU}[\,] \left(\frac{1}{2}\, e^{\alpha1} \in \eta1\, \eta2\, \lambda^3\, \xi1^2 - 2\, e^{\alpha1}\, T \in \eta1\, \eta2\, \lambda^3\, \xi1^2 + \frac{3}{2}\, e^{\alpha1}\, T^2 \in \eta1\, \eta2\, \lambda^3\, \xi1^2 + e^{\alpha1+\alpha2} \in \eta1\, \eta2\, \lambda^3\, \xi1\, \xi2 - 4\, e^{\alpha1+\alpha2}\, T \in \eta1\, \eta2\, \lambda^3\, \xi1\, \xi2 + 3\, e^{\alpha1+\alpha2}\, T^2 \in \eta1\, \eta2\, \lambda^3\, \xi1\, \xi2 - \frac{1}{2}\, e^{\alpha2} \in \eta2^2\, \lambda^3\, \xi1\, \xi2 + 2\, e^{\alpha2}\, T \in \eta2^2\, \lambda^3\, \xi1\, \xi2 - \frac{3}{2}\, e^{\alpha2}\, T^2 \in \eta2^2\, \lambda^3\, \xi1\, \xi2 + e^{\alpha1} \in \eta1\, \lambda\, \xi1\, \mathrm{fa1}[\lambda] - e^{\alpha1}\, T \in \eta1\, \lambda\, \xi1\, \mathrm{fa1}[\lambda] + \in \eta2\, \lambda\, \xi1\, \mathrm{fa1}[\lambda] - T \in \eta2\, \lambda\, \xi1\, \mathrm{fa1}[\lambda] + e^{\alpha1+\alpha2} \in \eta1\, \lambda\, \xi2\, \mathrm{fa1}[\lambda] - e^{\alpha1+\alpha2}\, T \in \eta1\, \lambda\, \xi2\, \mathrm{fa1}[\lambda] + e^{\alpha2} \in \eta2\, \lambda\, \xi2\, \mathrm{fa1}[\lambda] - e^{\alpha2}\, T \in \eta2\, \lambda\, \xi2\, \mathrm{fa1}[\lambda] + e^{\alpha1+\alpha2} \in \eta1\, \mathrm{fx1}[\lambda] - e^{\alpha1+\alpha2}\, T \in \eta1\, \mathrm{fx1}[\lambda] + e^{\alpha2} \in \eta2\, \mathrm{fx1}[\lambda] - e^{\alpha2}\, T \in \eta2\, \mathrm{fx1}[\lambda] - t \in \mathrm{ft1}'[\lambda] + e^{\alpha1} \in \lambda\, \xi1\, \mathrm{fy1}'[\lambda] - e^{\alpha1}\, T \in \lambda\, \xi1\, \mathrm{fy1}'[\lambda] + e^{\alpha1+\alpha2} \in \lambda\, \xi2\, \mathrm{fy1}'[\lambda] - e^{\alpha1+\alpha2}\, T \in \lambda\, \xi2\, \mathrm{fy1}'[\lambda]\right)$$