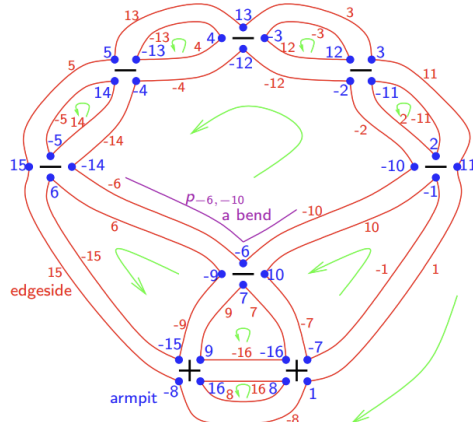
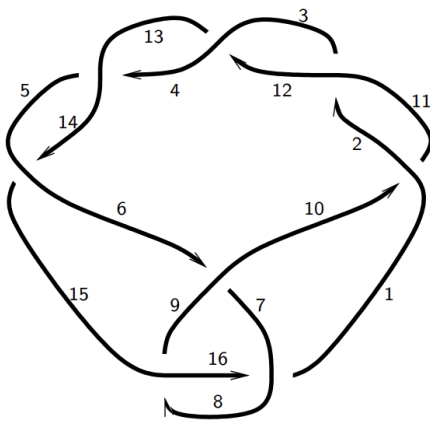


Pensieve header: The Alexander polynomial using a Seifert formula derived from an $F \times F$ matrix.
Continues AlexanderAsFxF.nb of pensieve://Projects/Signatures.

```
In[ ]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SeifertBox"];
Once[<< KnotTheory`]
```

```
In[ ]:= PD[epd_EPD] := PD@@epd /. {X[i_,j_] := X[j, i + 1, j + 1, i], X[i_,j_] := X[j, i, j + 1, i + 1]}
```

```
In[ ]:= Rot[{X_, φ_}] := {X, φ};
Rot[pd_PD] := Module[{n, xs, x, rots, Xp, Xm, front = {1}, k},
  n = Length@pd; rots = Table[0, {2 n + 1}];
  xs = Cases[pd, x_X := {Xp[x[[4]], x[[1]] PositiveQ@x},
    {Xm[x[[2]], x[[1]]} True];
  For[k = 1, k ≤ 2 n, ++k,
    If[FreeQ[front, -k],
      front = Flatten@Replace[front, k → (xs /. {
        Xp[k, l_] | Xm[l_, k] := {l + 1, k + 1, -l},
        Xp[l_, k] | Xm[k, l_] := (++rots[[l]]; {-l, k + 1, l + 1}),
        _Xp | _Xm := {}
      }], {1}],
      Cases[front, k | -k] /. {k, -k} := --rots[[k];
    ]
  ];
  {xs /. {Xp[i_, j_] := {+1, i, j}, Xm[i_, j_] := {-1, i, j}}, rots}];
Rot[K_] := Rot[PD[K]];
```



$PD[X[10, 1, 11, 2], X[2, 11, 3, 12], \dots] \quad \{X_-[-1, 11, 2, -10], X_-[-11, 3, 12, -2], \dots\}$
(diagram from Talks/CMS-2112)

```

In[*]:= Alex[K_] := Module[
  {ω$, XingsByArmpits, bends, faces, p, A$, is, Cs, φ, SeifertCycles, sc$, drops, keeps},
  XingsByArmpits = List@@PD[K] /.
    x : X[i_, j_, k_, l_] => If[PositiveQ[x], X+[-i, j, k, -l], X-[-j, k, l, -i]];
  bends = Times@@XingsByArmpits /. _[X][a_, b_, c_, d_] => pa,-d pb,-a pc,-b pd,-c;
  faces = bends /. px_,y_ py_,z_ => px,y,z;
  A = Table[0, Length@faces, Length@faces];
  Do[is = Position[faces, #][[1, 1]] & /@ List@@x;
    A[[is, is]] += If[Head[x] === X+,
      
$$\begin{pmatrix} \frac{(-1+\omega)(1+\omega)}{2\omega} & \frac{\omega}{2} & -\omega & \frac{1}{2\omega} \\ -\frac{1}{2\omega} & 0 & \frac{1}{2\omega} & 0 \\ \frac{1}{\omega} & -\frac{\omega}{2} & \frac{(-1+\omega)(1+\omega)}{2\omega} & -\frac{1}{2\omega} \\ -\frac{\omega}{2} & 0 & \frac{\omega}{2} & 0 \end{pmatrix}, \begin{pmatrix} -\frac{(-1+\omega)(1+\omega)}{2\omega} & \frac{\omega}{2} & -\frac{1}{\omega} & \frac{1}{2\omega} \\ -\frac{1}{2\omega} & 0 & \frac{1}{2\omega} & 0 \\ \omega & -\frac{\omega}{2} & -\frac{(-1+\omega)(1+\omega)}{2\omega} & -\frac{1}{2\omega} \\ -\frac{\omega}{2} & 0 & \frac{\omega}{2} & 0 \end{pmatrix}],
    {x, XingsByArmpits}];
  {Cs, φ} = Rot[PD@K]; φ[[1]] = -1;
  SeifertCycles = Times@@(Cs /. {1 | -1, i_, j_} => sci,j+1 scj,i+1) /.
    sci_,j_ scj_,k_ => sci,j,k /. sci_,j_,i_ => sci,j;
  drops = {-1} ∪ Cases[SeifertCycles, scis_ => -Total@φ[[{is}]] Min[{is}]];
  keeps = Complement[Range@Length@A, FirstPosition[#][faces][[1]] & /@ drops];
  Factor@Det[A[[keeps, keeps]] /. ω → T1/2]
]
Table[Simplify[Alex[K] / Alexander[K][T]], {K, AllKnots[{3, 11}]}]$$

```

... KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

... KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

