

Pensieve header: Analysis of k=2 invariants in CU.

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio"];
<< KnotTheory`
<< "SL2PortfolioProgram.m"
```

Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.  
Read more at <http://katlas.org/wiki/KnotTheory>.

```
In[*]:= $p = 5; $k = 2; $U = QU;
```

```
In[*]:= SC0[p_] := Collect[C0@0CU[{y, a, x}, p] /. {CU -> Times, \gamma | h -> 1}, \epsilon, Simplify];
SQ0[p_] := Collect[Q0@0QU[{y, a, x}, p] /. {QU -> Times, \gamma | h -> 1}, \epsilon, Simplify];
```

```
In[*]:= E[L_, Q_, P_]$_k := E[L, Q, Series[Normal@P, {\epsilon, 0, $k}]];
Ed_r[L_, Q_, P_]$_k := Ed_r@@E[L, Q, P]$_k;
E3@E[\omega_, L_, Q_, Ps_] := CF/@E[L, \omega^-1 Q, \omega^-1 (\omega^-4 \epsilon)^{-1+Range@Length@Ps}.Ps]$_k;
E4@E[L_, Q_, P_] := Module[
  {\omega = Normal[P]^-1 /. \epsilon -> 0, Ps = CoefficientList[P, \epsilon]},
  CF/@E[\omega, L, \omega Q, \omega^{-3+4 Range@Length@Ps} Ps]];
E3@Esp___[as___] := E3@E[as] /. E -> Esp;
E4@Esp___[as___] := E4@E[as] /. E -> Esp;
```

```
In[*]:= P[Knot[n_, k_]] := P[Knot[n, k]] = Module[{fname},
  fname = "../SL2Invariant/k=2/Data/" <> ToString[n] <> "_" <> ToString[k] <> ".m";
  Collect[E3[Get[fname][[2, 2]]][[3]] // Normal, \epsilon, Simplify]
];
CP[K_Knot] := CP[K] = CF@SD@P[K];
```

```
In[*]:= MatrixForm[
  AllKnots[{3, 8}] /. K_Knot -> {K, cp = Collect[CP@K /. {y -> 0, a -> -1/2}, {\epsilon, a}, Factor];
  \omega = (cp /. \epsilon -> 0)^-1, P1 = Expand[(\frac{T}{(T-1)^2})*\omega^3 Coefficient[cp, \epsilon]],
  P2 = Expand[\omega^5 Coefficient[cp, \epsilon^2]], {\omega, \partial_T \omega, P1, P2} /. T -> 1}]
```

Out[\*]//MatrixForm=

Knot [3, 1]	$\frac{1-T+T^2}{T}$	$-2 - \frac{1}{T^2} + \frac{2}{T} + 2T - T^2$
Knot [4, 1]	$-\frac{1-3T+T^2}{T}$	$0$
Knot [5, 1]	$\frac{1-T+T^2-T^3+T^4}{T^2}$	$-6 - \frac{2}{T^4} + \frac{4}{T^3} - \frac{5}{T^2} + \frac{6}{T} + 6T - 5T^2 + 4T^3 - 2T^4$
Knot [5, 2]	$\frac{2-3T+2T^2}{T}$	$-18 - \frac{5}{T^2} + \frac{14}{T} + 14T - 5T^2$
Knot [6, 1]	$-\frac{(-2+T)(-1+2T)}{T}$	$-10 - \frac{1}{T^2} + \frac{6}{T} + 6T - T^2$
Knot [6, 2]	$-\frac{1-3T+3T^2-3T^3+T^4}{T^2}$	$-16 - \frac{1}{T^4} + \frac{6}{T^3} - \frac{13}{T^2} + \frac{16}{T} + 16T - 13T^2 + 6T^3 - T^4$
Knot [6, 3]	$\frac{1-3T+5T^2-3T^3+T^4}{T^2}$	$0$
Knot [7, 1]	$\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$-12 - \frac{3}{T^6} + \frac{6}{T^5} - \frac{8}{T^4} + \frac{10}{T^3} - \frac{11}{T^2} + \frac{12}{T} + 12T - 11T^2 + 10T^3 - 8T^4 + 6$
Knot [7, 2]	$\frac{3-5T+3T^2}{T}$	$-60 - \frac{14}{T^2} + \frac{44}{T} + 44T - 14T^2$

Knot [7, 3]	$\frac{-2+1+3+1-3+1+2+1}{T^2}$	$56 + \frac{2}{T^4} - \frac{60}{T^3} + \frac{44}{T^2} - \frac{24}{T} - 52 T + 41 T^2 - 26 T^3 + 9 T^4$
Knot [7, 4]	$\frac{4-7 T+4 T^2}{T}$	$112 + \frac{24}{T^2} - \frac{80}{T} - 80 T + 24 T^2$
Knot [7, 5]	$\frac{2-4 T+5 T^2-4 T^3+2 T^4}{T^2}$	$-114 - \frac{9}{T^4} + \frac{34}{T^3} - \frac{70}{T^2} + \frac{102}{T} + 102 T - 70 T^2 + 34 T^3 - 9 T^4$
Knot [7, 6]	$-\frac{1-5 T+7 T^2-5 T^3+T^4}{T^2}$	$-78 - \frac{1}{T^4} + \frac{10}{T^3} - \frac{36}{T^2} + \frac{66}{T} + 66 T - 36 T^2 + 10 T^3 - T^4$
Knot [7, 7]	$\frac{1-5 T+9 T^2-5 T^3+T^4}{T^2}$	$22 + \frac{3}{T^2} - \frac{14}{T} - 14 T + 3 T^2$
Knot [8, 1]	$-\frac{3-7 T+3 T^2}{T}$	$-42 - \frac{5}{T^2} + \frac{26}{T} + 26 T - 5 T^2$
Knot [8, 2]	$-\frac{1-3 T+3 T^2-3 T^3+3 T^4-3 T^5+T^6}{T^3}$	$-50 - \frac{2}{T^6} + \frac{12}{T^5} - \frac{28}{T^4} + \frac{40}{T^3} - \frac{47}{T^2} + \frac{50}{T} + 50 T - 47 T^2 + 40 T^3 - 28 T^4 + 1:$
Knot [8, 3]	$-\frac{4-9 T+4 T^2}{T}$	$0$
Knot [8, 4]	$-\frac{2-5 T+5 T^2-5 T^3+2 T^4}{T^2}$	$-20 - \frac{3}{T^4} + \frac{14}{T^3} - \frac{25}{T^2} + \frac{24}{T} + 24 T - 25 T^2 + 14 T^3 - 3 T^4$
Knot [8, 5]	$-\frac{(1-T+T^2)(1-2 T+T^2-2 T^3+T^4)}{T^3}$	$92 + \frac{2}{T^6} - \frac{12}{T^5} + \frac{31}{T^4} - \frac{54}{T^3} + \frac{75}{T^2} - \frac{88}{T} - 88 T + 75 T^2 - 54 T^3 + 31 T^4 - 12$
Knot [8, 6]	$-\frac{2-6 T+7 T^2-6 T^3+2 T^4}{T^2}$	$-120 - \frac{5}{T^4} + \frac{30}{T^3} - \frac{73}{T^2} + \frac{108}{T} + 108 T - 73 T^2 + 30 T^3 - 5 T^4$
Knot [8, 7]	$\frac{1-3 T+5 T^2-5 T^3+5 T^4-3 T^5+T^6}{T^3}$	$50 + \frac{1}{T^6} - \frac{6}{T^5} + \frac{19}{T^4} - \frac{36}{T^3} + \frac{47}{T^2} - \frac{50}{T} - 50 T + 47 T^2 - 36 T^3 + 19 T^4 - 6$
Knot [8, 8]	$\frac{(2-2 T+T^2)(1-2 T+2 T^2)}{T^2}$	$56 + \frac{1}{T^4} - \frac{6}{T^3} + \frac{21}{T^2} - \frac{44}{T} - 44 T + 21 T^2 - 6 T^3 + T^4$
Knot [8, 9]	$-\frac{(-1+T-2 T^2+T^3)(-1+2 T-T^2+T^3)}{T^3}$	$0$
Knot [8, 10]	$\frac{(1-T+T^2)^3}{T^3}$	$82 + \frac{1}{T^6} - \frac{6}{T^5} + \frac{20}{T^4} - \frac{42}{T^3} + \frac{64}{T^2} - \frac{78}{T} - 78 T + 64 T^2 - 42 T^3 + 20 T^4 - 6$
Knot [8, 11]	$-\frac{(-2+T)(-1+2 T)(1-T+T^2)}{T^2}$	$-166 - \frac{5}{T^4} + \frac{34}{T^3} - \frac{92}{T^2} + \frac{146}{T} + 146 T - 92 T^2 + 34 T^3 - 5 T^4$
Knot [8, 12]	$\frac{1-7 T+13 T^2-7 T^3+T^4}{T^2}$	$0$
Knot [8, 13]	$\frac{2-7 T+11 T^2-7 T^3+2 T^4}{T^2}$	$68 + \frac{1}{T^4} - \frac{6}{T^3} + \frac{23}{T^2} - \frac{52}{T} - 52 T + 23 T^2 - 6 T^3 + T^4$
Knot [8, 14]	$-\frac{2-8 T+11 T^2-8 T^3+2 T^4}{T^2}$	$-250 - \frac{5}{T^4} + \frac{38}{T^3} - \frac{118}{T^2} + \frac{210}{T} + 210 T - 118 T^2 + 38 T^3 - 5 T^4$
Knot [8, 15]	$\frac{(1-T+T^2)(3-5 T+3 T^2)}{T^2}$	$-520 - \frac{21}{T^4} + \frac{106}{T^3} - \frac{269}{T^2} + \frac{444}{T} + 444 T - 269 T^2 + 106 T^3 - 21$
Knot [8, 16]	$\frac{1-4 T+8 T^2-9 T^3+8 T^4-4 T^5+T^6}{T^3}$	$-142 - \frac{1}{T^6} + \frac{8}{T^5} - \frac{30}{T^4} + \frac{68}{T^3} - \frac{108}{T^2} + \frac{134}{T} + 134 T - 108 T^2 + 68 T^3 - 30 T^4$
Knot [8, 17]	$-\frac{1-4 T+8 T^2-11 T^3+8 T^4-4 T^5+T^6}{T^3}$	$0$
Knot [8, 18]	$-\frac{(1-3 T+T^2)(1-T+T^2)^2}{T^3}$	$0$
Knot [8, 19]	$\frac{(1-T+T^2)(1-T^2+T^4)}{T^3}$	$6 + \frac{3}{T^6} - \frac{6}{T^5} + \frac{3}{T^4} + \frac{4}{T^3} - \frac{5}{T^2} - \frac{2}{T} - 2 T - 5 T^2 + 4 T^3 + 3 T^4 - 6 T^5 +$
Knot [8, 20]	$\frac{(1-T+T^2)^2}{T^2}$	$-16 - \frac{4}{T^2} + \frac{12}{T} + 12 T - 4 T^2$
Knot [8, 21]	$-\frac{(1-3 T+T^2)(1-T+T^2)}{T^2}$	$-72 - \frac{1}{T^4} + \frac{10}{T^3} - \frac{33}{T^2} + \frac{60}{T} + 60 T - 33 T^2 + 10 T^3 - T^4$