Task. Define $\text{Exp}_{U_i,k}[\xi, P]$ which computes $\mathbb{e}^{\xi\,\mathbb{O}(P)}$ to ϵ^k in the algebra U_i, where ξ is a scalar, X is x_i or y_i, and P is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form. Should satisfy

$$U@\text{Exp}_{U_i,k}[\xi, P] == \mathbb{S}_U\left[\mathbb{e}^{\xi x},\ x \to \mathbb{O}(P)\right].$$

Methodology. If $P_0 := P_{\epsilon=0}$ and $\mathbb{e}^{\xi\,\mathbb{O}(P)} = \mathbb{O}\left(\mathbb{e}^{\xi P_0}\, F(\xi)\right)$, then $F(\xi = 0) = 1$ and we have:

$$\mathbb{O}\left(\mathbb{e}^{\xi P_0}(P_0\, F(\xi) + \partial_\xi F\right) = \mathbb{O}\left(\partial_\xi \mathbb{e}^{\xi P_0}\, F(\xi)\right) = \qquad .$$

$$\partial_\xi \mathbb{O}\left(\mathbb{e}^{\xi P_0}\, F(\xi)\right) = \partial_\xi \mathbb{e}^{\xi\,\mathbb{O}(P)} = \mathbb{e}^{\xi\,\mathbb{O}(P)}\, \mathbb{O}(P) = \mathbb{O}\left(\mathbb{e}^{\xi P_0}\, F(\xi)\right) \mathbb{O}(P)$$

This is an ODE for F. Setting inductively $F_k = F_{k-1} + \epsilon^k\, \varphi$ we find that $F_0 = 1$ and solve for φ.