In[∘]:=
```
Once[
 SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Invariant"];
 << SL2Invariant.m
]
```

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: June 2018. Original version: July 1994.
```

In[∘]:= $R_{i,j}$

Out[∘]= $$\mathbb{E}\left[a_j\,b_i,\ x_j\,y_i,\ 1-\frac{1}{4}\left(x_j^2\,y_i^2\right)\epsilon+\left(\frac{1}{9}\,x_j^3\,y_i^3+\frac{1}{32}\,x_j^4\,y_i^4\right)\epsilon^2+O[\epsilon]^3\right]$$

In[∘]:= $\overline{R}_{i,j}$

Out[∘]= $$\mathbb{E}\Big[-a_j\,b_i,\ -\frac{x_j\,y_i}{B_i},\ 1+\frac{\left(-4\,a_j\,B_i\,x_j\,y_i-3\,x_j^2\,y_i^2\right)\epsilon}{4\,B_i^2}+\frac{1}{288\,B_i^4}\left(-144\,a_j^2\,B_i^3\,x_j\,y_i+144\,B_i^2\,x_j^2\,y_i^2-432\,a_j\,B_i^2\,x_j^2\,y_i^2+144\,a_j^2\,B_i^2\,x_j^2\,y_i^2-320\,B_i\,x_j^3\,y_i^3+216\,a_j\,B_i\,x_j^3\,y_i^3+81\,x_j^4\,y_i^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:= $\overline{R}_{i,j} \sim B_i \sim P_{i,j}$

Out[∘]= $$\mathbb{E}\Big[-a_j\,\alpha_j,\ -x_j\,\mathcal{A}_j\,\xi_j,\ 1+\frac{1}{2}\left(-2\,a_j\,x_j\,\mathcal{A}_j\,\xi_j-x_j^2\,\mathcal{A}_j^2\,\xi_j^2\right)\epsilon+\frac{1}{8}\left(-4\,a_j^2\,x_j\,\mathcal{A}_j\,\xi_j+2\,x_j^2\,\mathcal{A}_j^2\,\xi_j^2-8\,a_j\,x_j^2\,\mathcal{A}_j^2\,\xi_j^2+4\,a_j^2\,x_j^2\,\mathcal{A}_j^2\,\xi_j^2-4\,x_j^3\,\mathcal{A}_j^3\,\xi_j^3+4\,a_j\,x_j^3\,\mathcal{A}_j^3\,\xi_j^3+x_j^4\,\mathcal{A}_j^4\,\xi_j^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:= aS_j

Out[∘]= $$\mathbb{E}\Big[-a_j\,\alpha_j,\ -x_j\,\mathcal{A}_j\,\xi_j,\ 1+\frac{1}{2}\left(-2\,a_j\,x_j\,\mathcal{A}_j\,\xi_j-x_j^2\,\mathcal{A}_j^2\,\xi_j^2\right)\epsilon+\frac{1}{8}\left(-4\,a_j^2\,x_j\,\mathcal{A}_j\,\xi_j+2\,x_j^2\,\mathcal{A}_j^2\,\xi_j^2-8\,a_j\,x_j^2\,\mathcal{A}_j^2\,\xi_j^2+4\,a_j^2\,x_j^2\,\mathcal{A}_j^2\,\xi_j^2-4\,x_j^3\,\mathcal{A}_j^3\,\xi_j^3+4\,a_j\,x_j^3\,\mathcal{A}_j^3\,\xi_j^3+x_j^4\,\mathcal{A}_j^4\,\xi_j^4\right)\epsilon^2+O[\epsilon]^3\Big]$$

In[∘]:= $\left(R_{i,j}\,\overline{R}_{k,l}\right) \sim B_{i,j,k,l} \sim \left(bm_{i,k\to i}\,am_{j,l\to j}\right)$

Out[∘]= $\mathbb{E}\left[0,\ 0,\ 1+O[\epsilon]^3\right]$

In[∘]:= $\text{Define}\Big[iR_{i,j} = \mathbb{E}\Big[-a_j\,b_i,\ -x_j\,y_i\,/\,B_i,\ 1+\text{If}\Big[\$k == 0,\ 0,\ \text{Normal}@iR_{\{i,j\},\$k-1}[\![3]\!] - \Big(\Big(\left(iR_{\{i,j\},0}\right)_{\$k}\,R_{1,2}\,\left(iR_{\{3,4\},\$k-1}\right)_{\$k}\Big) \sim B_{i,j,1,2} \sim \left(bm_{i,1\to i}\,am_{j,2\to j}\right) \sim B_{i,j,3,4} \sim \left(bm_{i,3\to i}\,am_{j,4\to j}\right)\Big)[\![3]\!]\Big]\Big]\Big]$

In[∘]:= $\text{Block}[\{\$k = 0\},\ iR_{i,j}]$

Out[∘]= $$\mathbb{E}\left[-a_j\,b_i,\ -\frac{x_j\,y_i}{B_i},\ 1\right]$$

In[∘]:= **$k = 1**

Out[∘]= 1

In[∘]:= $\left(\mathbf{iR}_{\{i,j\},0}\right)_{\$k}\,\mathbf{R}_{1,2}\,\left(\mathbf{iR}_{\{3,4\},\$k-1}\right)_{\$k}$

Out[∘]= $\mathbb{E}\left[a_2\,b_1 - a_4\,b_3 - a_j\,b_i,\ x_2\,y_1 - \frac{x_4\,y_3}{B_3} - \frac{x_j\,y_i}{B_i},\ 1 - \frac{1}{4}\left(x_2^2\,y_1^2\right)\in + O[\in]^2\right]$

In[∘]:= $\left(\left(\mathbf{iR}_{\{i,j\},0}\right)_{\$k}\,\mathbf{R}_{1,2}\,\left(\mathbf{iR}_{\{3,4\},\$k-1}\right)_{\$k}\right)\sim\mathbf{B}_{i,j,1,2}\sim\left(\mathbf{bm}_{i,1\to i}\,\mathbf{am}_{j,2\to j}\right)$

Out[∘]= $\mathbb{E}\left[-a_4\,b_3,\ -\frac{x_4\,y_3}{B_3},\ 1 + \frac{1}{4}\left(4\,a_j\,x_j\,y_i + 3\,x_j^2\,y_i^2\right)\in + O[\in]^2\right]$

In[∘]:= $\left(\left(\left(\mathbf{iR}_{\{i,j\},0}\right)_{\$k}\,\mathbf{R}_{1,2}\,\left(\mathbf{iR}_{\{3,4\},\$k-1}\right)_{\$k}\right)\sim\mathbf{B}_{i,j,1,2}\sim\left(\mathbf{am}_{i,1\to i}\,\mathbf{bm}_{j,2\to j}\right)\sim\mathbf{B}_{i,j,3,4}\sim\left(\mathbf{am}_{i,3\to i}\,\mathbf{bm}_{j,3\to j}\right)\right)$

Out[∘]= $\mathbb{E}\left[0,\ 0,\ 1 + O[\in]^2\right]$

In[∘]:= **Block[{$k = 2}, iR$_{i,j}$]**

Out[∘]= $\mathbb{E}\Bigg[-a_j\,b_i,\ -\frac{x_j\,y_i}{B_i},\ 1 + \left(-\frac{a_j\,x_j\,y_i}{B_i} - \frac{3\,x_j^2\,y_i^2}{4\,B_i^2}\right)\in +$
$\Bigg(\frac{1}{288\,B_i^4}\left(-144\,a_j^2\,B_i^3\,x_j\,y_i + 144\,B_i^2\,x_j^2\,y_i^2 - 144\,a_j\,B_i^2\,x_j^2\,y_i^2 - 144\,a_j^2\,B_i^2\,x_j^2\,y_i^2 + 112\,B_i\,x_j^3\,y_i^3 - 216\,a_j\,B_i\,x_j^3\,y_i^3 -\right.$
$\left.81\,x_j^4\,y_i^4\right) - \frac{1}{16\,B_i^4}\left(16\,a_j\,B_i^2\,x_j^2\,y_i^2 - 16\,a_j^2\,B_i^2\,x_j^2\,y_i^2 + 24\,B_i\,x_j^3\,y_i^3 - 24\,a_j\,B_i\,x_j^3\,y_i^3 - 9\,x_j^4\,y_i^4\right)\Bigg)\in^2 + O[\in]^3\Bigg]$

In[∘]:= $\mathbf{iR}_{i,j} \equiv \overline{\mathbf{R}}_{i,j}$

Out[∘]= True