

Pensieve header: Reordering exponentials in sl_2 and in \mathfrak{g}_0 .

Prolog

Go;

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\PPSA"];
SetOptions[EvaluationNotebook[],
  NotebookEventActions -> {
    {"MenuCommand", "Save"} -> Get["MakeSnips.m"],
    PassEventsDown -> True
  }];
```

sl2matrices

$$y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \quad a = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix};$$

$$\{a.x - x.a == 2x, a.y - y.a == -2y, x.y - y.x == a\}$$

sl2matrices

{True, True, True}

Reordering_sl2

```
E[M_?MatrixQ] := MatrixExp[M];
Simplify[E[\eta_1 y].E[\alpha_1 a].E[\xi_1 x].E[\eta_2 y].E[\alpha_2 a].E[\xi_2 x] == E[\eta_0 y].E[\alpha_0 a].E[\xi_0 x] /.
  {\eta_0 -> \eta_1 + \frac{e^{-2\alpha_1} \eta_2}{1 + \eta_2 \xi_1}, \alpha_0 -> \alpha_1 + \alpha_2 + \text{Log}[1 + \eta_2 \xi_1], \xi_0 -> \frac{\xi_2 + \xi_1 (e^{-2\alpha_2} + \eta_2 \xi_2)}{1 + \eta_2 \xi_1}}]]
```

Reordering_sl2

True

g0matrices

$$y = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}; \quad c = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\{a.x - x.a == 2x, a.y - y.a == -2y, x.y - y.x == c, x.c == c.x, y.c == c.y, a.c == c.a\}$$

g0matrices

{True, True, True, True, True, True}

Reordering_g0

```
Simplify[
  E[\eta_1 y].E[\alpha_1 a].E[\xi_1 x].E[\eta_2 y].E[\alpha_2 a].E[\xi_2 x] == E[\eta_0 y].E[\alpha_0 a].E[\xi_0 x].E[\gamma_0 c] /.
  {\eta_0 -> \eta_1 + e^{-2\alpha_1} \eta_2, \alpha_0 -> \alpha_1 + \alpha_2, \xi_0 -> e^{-2\alpha_2} \xi_1 + \xi_2, \gamma_0 -> \eta_2 \xi_1}]]
```

Reordering_g0

True

Making {Snips/ReorderingDemo.txt} ...