

Pensieve header: Comparing frakg_1 with Roland's picture.

```

SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1606"];
<< OneCo.m

bas = LBasis[1] /. x_ -> x
{U[],  $\epsilon$  U[], U[c], U[u], U[w],  $\epsilon$  U[c],  $\epsilon$  U[u],  $\epsilon$  U[w], U[u, w],  $\epsilon$  U[c, c],  $\epsilon$  U[c, u],
 $\epsilon$  U[c, w],  $\epsilon$  U[u, w],  $\epsilon$  U[c, u, w],  $\epsilon$  U[u, u, w],  $\epsilon$  U[u, w, w],  $\epsilon$  U[u, u, w, w]}

Length@bas
17

u1 = f3 U[c] +  $\epsilon$  f6 U[c] + f4 U[u] +  $\epsilon$  f7 U[u] + f5 U[w] +  $\epsilon$  f8 U[w] +
 $\epsilon$  f10 U[c, c] +  $\epsilon$  f11 U[c, u] +  $\epsilon$  f12 U[c, w] + f9 U[u, w] +  $\epsilon$  f13 U[u, w];
w1 = g3 U[c] +  $\epsilon$  g6 U[c] + g4 U[u] +  $\epsilon$  g7 U[u] + g5 U[w] +  $\epsilon$  g8 U[w] +
 $\epsilon$  g10 U[c, c] +  $\epsilon$  g11 U[c, u] +  $\epsilon$  g12 U[c, w] + g9 U[u, w] +  $\epsilon$  g13 U[u, w];
c1 = h1 U[] +  $\epsilon$  h2 U[] + h3 U[c] +  $\epsilon$  h6 U[c] + h4 U[u] +  $\epsilon$  h7 U[u] + h5 U[w] +  $\epsilon$  h8 U[w] +
 $\epsilon$  h10 U[c, c] +  $\epsilon$  h11 U[c, u] +  $\epsilon$  h12 U[c, w] + h9 U[u, w] +  $\epsilon$  h13 U[u, w];
eqs1 = Simp[{
  B[w1, c1] - w1,
  B[c1, u1] - u1,
  B[w1, u1] - ( $\epsilon$  u1 ** w1 + (1 - t - 0  $\epsilon$  t) U[] - 2  $\epsilon$  t c1)
}]

```

$$\begin{aligned}
& \{ (-b g_5 h_4 - b e g_8 h_4 + b g_4 h_5 + b e g_7 h_5 + b e g_{11} h_5 - b e g_5 h_7 + b e g_4 h_8 - b e g_5 h_{11}) U[] + \\
& (-g_3 - e g_6 + 2 e g_5 h_4 - b e g_{12} h_4 - 2 e g_4 h_5 + b e g_{11} h_5 - b e g_5 h_{11} + b e g_4 h_{12}) U[c] + \\
& (-g_4 - e g_7 - g_4 h_3 - e g_7 h_3 + g_3 h_4 + e g_6 h_4 - b g_9 h_4 - 2 e g_9 h_4 - e g_{10} h_4 - b e g_{13} h_4 - \\
& \quad e g_4 h_6 + e g_3 h_7 - b e g_9 h_7 + b g_4 h_9 + 2 e g_4 h_9 + b e g_7 h_9 + e g_4 h_{10} + b e g_4 h_{13}) U[u] + \\
& (-g_5 - e g_8 + g_5 h_3 + e g_8 h_3 - g_3 h_5 - e g_6 h_5 + b g_9 h_5 - e g_{10} h_5 + b e g_{13} h_5 + e g_5 h_6 - \\
& \quad e g_3 h_8 + b e g_9 h_8 - b g_5 h_9 - b e g_8 h_9 + e g_5 h_{10} - b e g_5 h_{13}) U[w] - e g_{10} U[c, c] + \\
& (-e g_{11} - e g_{11} h_3 + 2 e g_9 h_4 + 2 e g_{10} h_4 - 2 e g_4 h_9 + b e g_{11} h_9 - 2 e g_4 h_{10} + e g_3 h_{11} - b e g_9 h_{11}) \\
& \quad U[c, u] + \\
& (-e g_{12} + e g_{12} h_3 - 2 e g_9 h_5 - 2 e g_{10} h_5 + 2 e g_5 h_9 - b e g_{12} h_9 + 2 e g_5 h_{10} - e g_3 h_{12} + b e g_9 h_{12}) \\
& \quad U[c, w] + (e g_{11} h_4 - e g_4 h_{11}) U[u, u] + \\
& (-g_9 - e g_{13} + e g_{12} h_4 - e g_{11} h_5 + e g_5 h_{11} - e g_4 h_{12}) U[u, w] + (-e g_{12} h_5 + e g_5 h_{12}) U[w, w], \\
& (b f_5 h_4 + b e f_8 h_4 - b f_4 h_5 - b e f_7 h_5 - b e f_{11} h_5 + b e f_5 h_7 - b e f_4 h_8 + b e f_5 h_{11}) U[] + \\
& (-f_3 - e f_6 - 2 e f_5 h_4 + b e f_{12} h_4 + 2 e f_4 h_5 - b e f_{11} h_5 + b e f_5 h_{11} - b e f_4 h_{12}) U[c] + \\
& (-f_4 - e f_7 + f_4 h_3 + e f_7 h_3 - f_3 h_4 - e f_6 h_4 + b f_9 h_4 + 2 e f_9 h_4 + e f_{10} h_4 + b e f_{13} h_4 + \\
& \quad e f_4 h_6 - e f_3 h_7 + b e f_9 h_7 - b f_4 h_9 - 2 e f_4 h_9 - b e f_7 h_9 - e f_4 h_{10} - b e f_4 h_{13}) U[u] + \\
& (-f_5 - e f_8 - f_5 h_3 - e f_8 h_3 + f_3 h_5 + e f_6 h_5 - b f_9 h_5 + e f_{10} h_5 - b e f_{13} h_5 - e f_5 h_6 + \\
& \quad e f_3 h_8 - b e f_9 h_8 + b f_5 h_9 + b e f_8 h_9 - e f_5 h_{10} + b e f_5 h_{13}) U[w] - e f_{10} U[c, c] + \\
& (-e f_{11} + e f_{11} h_3 - 2 e f_9 h_4 - 2 e f_{10} h_4 + 2 e f_4 h_9 - b e f_{11} h_9 + 2 e f_4 h_{10} - e f_3 h_{11} + b e f_9 h_{11}) \\
& \quad U[c, u] + \\
& (-e f_{12} - e f_{12} h_3 + 2 e f_9 h_5 + 2 e f_{10} h_5 - 2 e f_5 h_9 + b e f_{12} h_9 - 2 e f_5 h_{10} + e f_3 h_{12} - b e f_9 h_{12}) \\
& \quad U[c, w] + (-e f_{11} h_4 + e f_4 h_{11}) U[u, u] + \\
& (-f_9 - e f_{13} - e f_{12} h_4 + e f_{11} h_5 - e f_5 h_{11} + e f_4 h_{12}) U[u, w] + (e f_{12} h_5 - e f_5 h_{12}) U[w, w], \\
& (-1 + t + b f_5 g_4 + b e f_5 g_4 + b e f_8 g_4 - b f_4 g_5 - b e f_7 g_5 - \\
& \quad b e f_{11} g_5 + b e f_5 g_7 - b e f_4 g_8 + b e f_5 g_{11} + 2 t e h_1) U[] + \\
& (-2 e f_5 g_4 + b e f_{12} g_4 + 2 e f_4 g_5 - b e f_{11} g_5 + b e f_5 g_{11} - b e f_4 g_{12} + 2 t e h_3) U[c] + \\
& (f_4 g_3 + e f_4 g_3 + e f_7 g_3 - f_3 g_4 - e f_6 g_4 + b f_9 g_4 + 2 e f_9 g_4 + b e f_9 g_4 + e f_{10} g_4 + b e f_{13} g_4 + \\
& \quad e f_4 g_6 - e f_3 g_7 + b e f_9 g_7 - b f_4 g_9 - 2 e f_4 g_9 - b e f_7 g_9 - e f_4 g_{10} - b e f_4 g_{13} + 2 t e h_4) \\
& \quad U[u] + (-f_5 g_3 - e f_5 g_3 - e f_8 g_3 + f_3 g_5 + e f_6 g_5 - b f_9 g_5 + e f_{10} g_5 - b e f_{13} g_5 - e f_5 g_6 + \\
& \quad e f_3 g_8 - b e f_9 g_8 + b f_5 g_9 + b e f_5 g_9 + b e f_8 g_9 - e f_5 g_{10} + b e f_5 g_{13} + 2 t e h_5) U[w] - \\
& e f_3 g_3 U[c, c] + (-e f_4 g_3 + e f_{11} g_3 - e f_3 g_4 - 2 e f_9 g_4 - 2 e f_{10} g_4 + 2 e f_4 g_9 - \\
& \quad b e f_{11} g_9 + 2 e f_4 g_{10} - e f_3 g_{11} + b e f_9 g_{11}) U[c, u] + \\
& (-e f_5 g_3 - e f_{12} g_3 - e f_3 g_5 + 2 e f_9 g_5 + 2 e f_{10} g_5 - 2 e f_5 g_9 + b e f_{12} g_9 - 2 e f_5 g_{10} + \\
& \quad e f_3 g_{12} - b e f_9 g_{12}) U[c, w] + (-e f_4 g_4 - e f_{11} g_4 + e f_4 g_{11}) U[u, u] + \\
& (-e f_5 g_4 - e f_{12} g_4 - e f_4 g_5 + e f_{11} g_5 + b e f_9 g_9 - e f_5 g_{11} + e f_4 g_{12} + 2 t e h_9) U[u, w] + \\
& (-e f_5 g_5 + e f_{12} g_5 - e f_5 g_{12}) U[w, w] + (-e f_9 g_3 - e f_3 g_9) U[c, u, w] + \\
& (-e f_9 g_4 - e f_4 g_9) U[u, u, w] + (-e f_9 g_5 - e f_5 g_9) U[u, w, w] - e f_9 g_9 U[u, u, w, w] \}
\end{aligned}$$

```

eqs2 = DeleteCases[
  Cases[eqs1, a_*_U -> {(a /. e -> 0) == 0, Coefficient[a, e] == 0}, ∞] // Flatten, True]
{-b g5 h4 + b g4 h5 == 0, -b g8 h4 + b g7 h5 + b g11 h5 - b g5 h7 + b g4 h8 - b g5 h11 == 0,
 -g3 == 0, -g6 + 2 g5 h4 - b g12 h4 - 2 g4 h5 + b g11 h5 - b g5 h11 + b g4 h12 == 0,
 -g4 - g4 h3 + g3 h4 - b g9 h4 + b g4 h9 == 0, -g7 - g7 h3 + g6 h4 - 2 g9 h4 - g10 h4 - b g13 h4 - g4 h6 +
   g3 h7 - b g9 h7 + 2 g4 h9 + b g7 h9 + g4 h10 + b g4 h13 == 0, -g5 + g5 h3 - g3 h5 + b g9 h5 - b g5 h9 == 0,
 -g8 + g8 h3 - g6 h5 - g10 h5 + b g13 h5 + g5 h6 - g3 h8 + b g9 h8 - b g8 h9 + g5 h10 - b g5 h13 == 0,
 -g10 == 0, -g11 - g11 h3 + 2 g9 h4 + 2 g10 h4 - 2 g4 h9 + b g11 h9 - 2 g4 h10 + g3 h11 - b g9 h11 == 0,
 -g12 + g12 h3 - 2 g9 h5 - 2 g10 h5 + 2 g5 h9 - b g12 h9 + 2 g5 h10 - g3 h12 + b g9 h12 == 0,
 g11 h4 - g4 h11 == 0, -g9 == 0, -g13 + g12 h4 - g11 h5 + g5 h11 - g4 h12 == 0, -g12 h5 + g5 h12 == 0,
 b f5 h4 - b f4 h5 == 0, b f8 h4 - b f7 h5 - b f11 h5 + b f5 h7 - b f4 h8 + b f5 h11 == 0,
 -f3 == 0, -f6 - 2 f5 h4 + b f12 h4 + 2 f4 h5 - b f11 h5 + b f5 h11 - b f4 h12 == 0,
 -f4 + f4 h3 - f3 h4 + b f9 h4 - b f4 h9 == 0, -f7 + f7 h3 - f6 h4 + 2 f9 h4 + f10 h4 + b f13 h4 + f4 h6 -
   f3 h7 + b f9 h7 - 2 f4 h9 - b f7 h9 - f4 h10 - b f4 h13 == 0, -f5 - f5 h3 + f3 h5 - b f9 h5 + b f5 h9 == 0,
 -f8 - f8 h3 + f6 h5 + f10 h5 - b f13 h5 - f5 h6 + f3 h8 - b f9 h8 + b f8 h9 - f5 h10 + b f5 h13 == 0,
 -f10 == 0, -f11 + f11 h3 - 2 f9 h4 - 2 f10 h4 + 2 f4 h9 - b f11 h9 + 2 f4 h10 - f3 h11 + b f9 h11 == 0,
 -f12 - f12 h3 + 2 f9 h5 + 2 f10 h5 - 2 f5 h9 + b f12 h9 - 2 f5 h10 + f3 h12 - b f9 h12 == 0,
 -f11 h4 + f4 h11 == 0, -f9 == 0, -f13 - f12 h4 + f11 h5 - f5 h11 + f4 h12 == 0,
 f12 h5 - f5 h12 == 0, -1 + t + b f5 g4 - b f4 g5 == 0,
 b f5 g4 + b f8 g4 - b f7 g5 - b f11 g5 + b f5 g7 - b f4 g8 + b f5 g11 + 2 t h1 == 0,
 -2 f5 g4 + b f12 g4 + 2 f4 g5 - b f11 g5 + b f5 g11 - b f4 g12 + 2 t h3 == 0,
 f4 g3 - f3 g4 + b f9 g4 - b f4 g9 == 0, f4 g3 + f7 g3 - f6 g4 + 2 f9 g4 + b f9 g4 + f10 g4 +
   b f13 g4 + f4 g6 - f3 g7 + b f9 g7 - 2 f4 g9 - b f7 g9 - f4 g10 - b f4 g13 + 2 t h4 == 0,
 -f5 g3 + f3 g5 - b f9 g5 + b f5 g9 == 0, -f5 g3 - f8 g3 + f6 g5 + f10 g5 - b f13 g5 - f5 g6 +
   f3 g8 - b f9 g8 + b f5 g9 + b f8 g9 - f5 g10 + b f5 g13 + 2 t h5 == 0, -f3 g3 == 0,
 -f4 g3 + f11 g3 - f3 g4 - 2 f9 g4 - 2 f10 g4 + 2 f4 g9 - b f11 g9 + 2 f4 g10 - f3 g11 + b f9 g11 == 0,
 -f5 g3 - f12 g3 - f3 g5 + 2 f9 g5 + 2 f10 g5 - 2 f5 g9 + b f12 g9 - 2 f5 g10 + f3 g12 - b f9 g12 == 0,
 -f4 g4 - f11 g4 + f4 g11 == 0, -f5 g4 - f12 g4 - f4 g5 + f11 g5 + b f9 g9 - f5 g11 + f4 g12 + 2 t h9 == 0,
 -f5 g5 + f12 g5 - f5 g12 == 0, -f9 g3 - f3 g9 == 0, -f9 g4 - f4 g9 == 0, -f9 g5 - f5 g9 == 0, -f9 g9 == 0}

vars = Union@Cases[eqs2, f_ | g_ | h_, ∞]
{f3, f4, f5, f6, f7, f8, f9, f10, f11, f12, f13, g3, g4, g5, g6, g7, g8,
 g9, g10, g11, g12, g13, h1, h3, h4, h5, h6, h7, h8, h9, h10, h11, h12, h13}

```

sol = Solve[eqs2, vars]

Solve::svars: Equations may not give solutions for all "solve" variables. >>

$$\left\{ \left\{ \begin{aligned} f_3 \rightarrow 0, f_5 \rightarrow 0, f_6 \rightarrow 0, f_8 \rightarrow 0, f_9 \rightarrow 0, f_{10} \rightarrow 0, f_{12} \rightarrow 0, f_{13} \rightarrow 0, g_3 \rightarrow 0, g_4 \rightarrow 0, g_5 \rightarrow \frac{-1+t}{b f_4}, \\ g_6 \rightarrow 0, g_7 \rightarrow 0, g_9 \rightarrow 0, g_{10} \rightarrow 0, g_{11} \rightarrow 0, g_{12} \rightarrow \frac{-2 f_4 - b f_4 + 2 t f_4 + 3 b t f_4 + 2 b f_{11} - 2 b t f_{11}}{2 b^2 f_4^2}, \\ g_{13} \rightarrow 0, h_1 \rightarrow \frac{-f_7 + t f_7 - f_{11} + t f_{11} + b f_4^2 g_8}{2 t f_4}, h_3 \rightarrow \frac{2 - b - 2 t + 3 b t}{4 b t}, \\ h_4 \rightarrow 0, h_5 \rightarrow 0, h_6 \rightarrow -\frac{-2 + b + 2 t + b t - 4 b^3 t h_{13}}{4 b^2 t}, h_7 \rightarrow 0, h_8 \rightarrow 0, \\ h_9 \rightarrow \frac{2 - b - 2 t - b t}{4 b^2 t}, h_{10} \rightarrow \frac{-2 + b + 2 t + b t}{4 b^2 t}, h_{11} \rightarrow 0, h_{12} \rightarrow 0 \end{aligned} \right\}, \right. \\ \left. \left\{ \begin{aligned} f_3 \rightarrow 0, f_4 \rightarrow 0, f_6 \rightarrow 0, f_7 \rightarrow 0, f_9 \rightarrow 0, f_{10} \rightarrow 0, f_{11} \rightarrow 0, \\ f_{12} \rightarrow \frac{f_5 (-2 + b + 2 t - 3 b t + 2 b^2 f_5 g_{11})}{2 b (-1 + t)}, f_{13} \rightarrow 0, g_3 \rightarrow 0, g_4 \rightarrow \frac{1 - t}{b f_5}, g_5 \rightarrow 0, g_6 \rightarrow 0, \\ g_8 \rightarrow 0, g_9 \rightarrow 0, g_{10} \rightarrow 0, g_{12} \rightarrow 0, g_{13} \rightarrow 0, h_1 \rightarrow \frac{-f_5 + t f_5 - f_8 + t f_8 - b f_5^2 g_7 - b f_5^2 g_{11}}{2 t f_5}, \\ h_3 \rightarrow \frac{2 + b - 2 t - 3 b t}{4 b t}, h_4 \rightarrow 0, h_5 \rightarrow 0, h_6 \rightarrow \frac{2 + b - 2 t + b t + 4 b^3 t h_{13}}{4 b^2 t}, h_7 \rightarrow 0, \\ h_8 \rightarrow 0, h_9 \rightarrow \frac{2 + b - 2 t + b t}{4 b^2 t}, h_{10} \rightarrow \frac{-2 - b + 2 t - b t}{4 b^2 t}, h_{11} \rightarrow 0, h_{12} \rightarrow 0 \end{aligned} \right\} \right\}$$

(*further={f4→(t-1)/b, f7|11→0, g8→0, h2|13→0}*)

further = {f4 → (t - 1) / b, f7|11 → 0, g8 → 0, h2|13 → 0};

u1 /. sol[[1]] /. further

$$\frac{(-1+t) U[u]}{b}$$

Collect[w1 /. sol[[1]] /. further, _U, Simplify]

$$U[w] + \frac{(-2 - b + 2 t + 3 b t) \epsilon U[c, w]}{2 b (-1 + t)}$$

Collect[c1 /. sol[[1]] /. further, _U, Simplify]

$$\frac{(b^2 (-1 + 3 t) - 2 (-1 + t) \epsilon - b (-2 + \epsilon + t (2 + \epsilon))) U[c]}{4 b^2 t} + \frac{(-2 + b + 2 t + b t) \epsilon U[c, c]}{4 b^2 t} - \frac{(-2 + b + 2 t + b t) U[u, w]}{4 b^2 t}$$

Simp[eqs1 /. sol[[1]] /. further] // Simplify

$$\{0, 0, 0\}$$

Collect[{c1, u1, w1} /. sol[[1]] /. further /. \epsilon → 0, _U, Simplify]

$$\left\{ \frac{(2 - b - 2 t + 3 b t) U[c]}{4 b t} - \frac{(-2 + b + 2 t + b t) U[u, w]}{4 b^2 t}, \frac{(-1 + t) U[u]}{b}, U[w] \right\}$$