

Pensieve header: The invariant of the trefoil, using data from CC4NOE-1t.nb.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1606"];
<< NOE-1t.m
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Rp[i_, j_] :=
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$$\mathbb{E}\left[1, b_i c_j, u_i w_j, -\frac{1}{2} c_i (-1 + t_i)^2 - \frac{1}{2} c_i^2 (-1 + t_i)^2 + \frac{1}{2} c_i c_j (-2 - t_i + t_j^2) - \frac{1}{2} c_j u_i w_i + c_i (1 - t_i) u_i w_i - \frac{1}{2} u_i^2 w_i^2 + u_i w_j + \frac{1}{2} c_j t_i u_i w_j + c_i (-2 + t_i) t_i u_i w_j + \frac{1}{2} c_i (1 + t_j) u_j w_j + (-1 + t_i) u_i^2 w_i w_j - \frac{1}{2} (-2 + t_i) t_i u_i^2 w_j^2\right];$$

```
Rm[i_, j_] := E[1, -b_i c_j, -\frac{u_i w_j}{t_i}, \frac{1}{2} c_i (-1 + t_i)^2 + \frac{1}{2} c_i^2 (-1 + t_i)^2 + \frac{1}{2} c_i c_j (2 + t_i - t_j^2) + \frac{1}{2} c_j u_i w_i +
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$$c_i (-1 + t_i) u_i w_i + \frac{1}{2} u_i^2 w_i^2 + \left(\frac{1}{2} - \frac{1}{2 t_i}\right) u_i w_j + c_i \left(-\frac{5}{2} + \frac{3}{2 t_i} + t_i\right) u_i w_j + \frac{c_j (1 + t_i - t_j^2) u_i w_j}{2 t_i} +$$

$$\frac{1}{2} c_i (-1 - t_j) u_j w_j + \left(1 - \frac{3}{2 t_i}\right) u_i^2 w_i w_j + \left(\frac{1}{2} + \frac{1}{t_i^2} - \frac{3}{2 t_i}\right) u_i^2 w_j^2 - \frac{(1 + t_j) u_i u_j w_j^2}{2 t_i}];$$

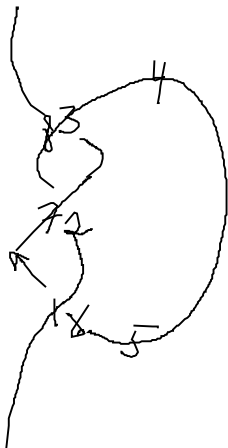
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ur[i_] := E[\frac{1}{t_i^{1/4}}, 0, 0, \frac{c_i t_i}{4} + \frac{u_i w_i}{8}]; nr[i_] := E[t_i^{1/4}, 0, 0, -\frac{1}{4} c_i t_i^3 - \frac{1}{8} t_i^2 u_i w_i];
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ul[i_] := E[t_i^{1/4}, 0, 0, \frac{1}{4} c_i t_i (4 + t_i) - \frac{1}{8} t_i^2 u_i w_i]; nl[i_] := E[\frac{1}{t_i^{1/4}}, 0, 0, c_i \left(-\frac{1}{4} - \frac{1}{t_i}\right) + \frac{u_i w_i}{8}];
```

```
kp[i_] := E[\sqrt{t_i}, b_i c_i, \frac{u_i w_i}{\sqrt{t_i}}, \frac{1}{2} c_i^2 (-3 + t_i) t_i^2 + \frac{1}{4} c_i t_i^2 (2 + 5 t_i - 3 t_i^2) + c_i t_i u_i w_i - \frac{3}{4} t_i^2 u_i w_i];
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km[i_] :=
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$$\mathbb{E}\left[\frac{1}{\sqrt{t_i}}, -b_i c_i, -\frac{u_i w_i}{\sqrt{t_i}}, \frac{1}{4} c_i \left(3 - \frac{2}{t_i^2} - \frac{5}{t_i}\right) - \frac{c_i^2 (-3 + t_i)}{2 t_i^2} + \left(\frac{3}{4} - \frac{2}{t_i^2}\right) u_i w_i + \frac{c_i (2 - t_i) u_i w_i}{t_i^2} - \frac{(-1 + t_i) u_i^2 w_i^2}{2 t_i^2}\right];$$



$$z = \text{Rp}[1, 6] \text{Rp}[7, 2] \text{Rp}[3, 8] \text{nr}[4] u_1[5] km[9] km[10] km[11]$$

$$E \left[\frac{t_4^{1/4} t_5^{1/4}}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}}, b_7 c_2 + b_1 c_6 + b_3 c_8 - b_9 c_9 - b_{10} c_{10} - b_{11} c_{11}, \frac{t_4^{1/4} t_5^{1/4} u_7 w_2}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}} + \right. \\ \left. \frac{t_4^{1/4} t_5^{1/4} u_1 w_6}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}} + \frac{t_4^{1/4} t_5^{1/4} u_3 w_8}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}} - \frac{t_4^{1/4} t_5^{1/4} u_9 w_9}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}} - \frac{t_4^{1/4} t_5^{1/4} u_{10} w_{10}}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}} - \frac{t_4^{1/4} t_5^{1/4} u_{11} w_{11}}{\sqrt{t_9} \sqrt{t_{10}} \sqrt{t_{11}}}, \right. \\ \frac{3 c_{11} t_4 t_5}{4 t_9^2 t_{10}^2} + \frac{3 c_{10} t_4 t_5}{4 t_9^2 t_{11}^2} + \frac{3 c_9 t_4 t_5}{4 t_{10}^2 t_{11}^2} - \frac{c_1 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_1^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_5 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_1 c_6 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} - \\ \frac{c_7 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_2 c_7 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_7^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3 c_8 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_9 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{3 c_9^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_{10} t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{3 c_{10}^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_{11} t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{3 c_{11}^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1 t_1 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1^2 t_1 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_1 c_6 t_1 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_1 t_1^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_1^2 t_1^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_2 c_7 t_2^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_3 t_3 t_4 t_5}{t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{c_3^2 t_3 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3 c_8 t_3 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3 t_3^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3^2 t_3^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_4 t_4^3 t_5}{4 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_5 t_4 t_5^2}{4 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1 c_6 t_4 t_5 t_6^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_7 t_4 t_5 t_7}{t_9^2 t_{10}^2 t_{11}^2} - \\ \frac{c_2 c_7 t_4 t_5 t_7}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_7^2 t_4 t_5 t_7}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_7 t_4 t_5 t_7^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_7^2 t_4 t_5 t_7^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_3 c_8 t_4 t_5 t_8^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{5 c_9 t_4 t_5}{4 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_9^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{5 c_{10} t_4 t_5}{4 t_9^2 t_{10}^2 t_{11}^2} - \\ \frac{c_{10}^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{5 c_{11} t_4 t_5}{4 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_{11}^2 t_4 t_5}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1 t_4 t_5 u_1 w_1}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_6 t_4 t_5 u_1 w_1}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_1 t_1 t_4 t_5 u_1 w_1}{t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_1^2 w_1^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_7 t_4 t_5 u_2 w_2}{2 t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{c_7 t_2 t_4 t_5 u_2 w_2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{t_4 t_5 u_7 w_2}{t_9^2 t_{10}^2 t_{11}^2} + \frac{c_2 t_4 t_5 t_7 u_7 w_2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{2 c_7 t_4 t_5 t_7 u_7 w_2}{t_9^2 t_{10}^2 t_{11}^2} + \frac{c_7 t_4 t_5 t_7^2 u_7 w_2}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_4 t_5 t_7 u_7^2 w_2^2}{t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 t_7^2 u_7^2 w_2^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{c_3 t_4 t_5 u_3 w_3}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_8 t_4 t_5 u_3 w_3}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_3 t_3 t_4 t_5 u_3 w_3}{t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_3^2 w_3^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4^2 t_5 u_4 w_4}{8 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5^2 u_5 w_5}{8 t_9^2 t_{10}^2 t_{11}^2} + \frac{t_4 t_5 u_1 w_6}{t_9^2 t_{10}^2 t_{11}^2} - \frac{2 c_1 t_1 t_4 t_5 u_1 w_6}{t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{c_6 t_1 t_4 t_5 u_1 w_6}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1 t_1^2 t_4 t_5 u_1 w_6}{t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1 t_4 t_5 u_6 w_6}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_1 t_4 t_5 t_6 u_6 w_6}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_1^2 w_1 w_6}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_1 t_4 t_5 u_1^2 w_1 w_6}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_1 t_4 t_5 u_1^2 w_6^2}{t_9^2 t_{10}^2 t_{11}^2} - \\ \frac{t_1^2 t_4 t_5 u_1^2 w_6^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{c_2 t_4 t_5 u_7 w_7}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_7 t_4 t_5 u_7 w_7}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_7 t_4 t_5 t_7 u_7 w_7}{t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_7^2 w_2 w_7}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_4 t_5 t_7 u_7^2 w_2 w_7}{t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_7^2 w_7^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{t_4 t_5 u_3 w_8}{t_9^2 t_{10}^2 t_{11}^2} - \frac{2 c_3 t_3 t_4 t_5 u_3 w_8}{t_9^2 t_{10}^2 t_{11}^2} + \frac{c_8 t_3 t_4 t_5 u_3 w_8}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_3 t_3^2 t_4 t_5 u_3 w_8}{t_9^2 t_{10}^2 t_{11}^2} + \frac{c_3 t_4 t_5 u_8 w_8}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{c_3 t_4 t_5 t_8 u_8 w_8}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_3^2 w_3 w_8}{t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{t_3 t_4 t_5 u_3^2 w_3 w_8}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_3 t_4 t_5 u_3^2 w_8^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_3^2 t_4 t_5 u_3^2 w_8^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{3 t_4 t_5 u_9 w_9}{4 t_{10}^2 t_{11}^2} - \frac{2 t_4 t_5 u_9 w_9}{t_9^2 t_{10}^2 t_{11}^2} + \frac{2 c_9 t_4 t_5 u_9 w_9}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_9 t_4 t_5 u_9 w_9}{t_9^2 t_{10}^2 t_{11}^2} + \\ \frac{t_4 t_5 u_9^2 w_9^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_9^2 w_9^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{3 t_4 t_5 u_{10} w_{10}}{4 t_9^2 t_{11}^2} - \frac{2 t_4 t_5 u_{10} w_{10}}{t_9^2 t_{10}^2 t_{11}^2} + \frac{2 c_{10} t_4 t_5 u_{10} w_{10}}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_{10} t_4 t_5 u_{10} w_{10}}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_4 t_5 u_{10}^2 w_{10}^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \\ \frac{t_4 t_5 u_{10}^2 w_{10}^2}{2 t_9^2 t_{10}^2 t_{11}^2} + \frac{3 t_4 t_5 u_{11} w_{11}}{4 t_9^2 t_{10}^2} - \frac{2 t_4 t_5 u_{11} w_{11}}{t_9^2 t_{10}^2 t_{11}^2} + \frac{2 c_{11} t_4 t_5 u_{11} w_{11}}{t_9^2 t_{10}^2 t_{11}^2} - \frac{c_{11} t_4 t_5 u_{11} w_{11}}{t_9^2 t_{10}^2 t_{11}^2} + \frac{t_4 t_5 u_{11}^2 w_{11}^2}{2 t_9^2 t_{10}^2 t_{11}^2} - \frac{t_4 t_5 u_{11}^2 w_{11}^2}{2 t_9^2 t_{10}^2 t_{11}^2} \left. \right]$$

$$\text{Do}[z = z // m[1, k, 1], \{k, 2, 11\}]; z$$

$$E \left[-1 + \frac{1}{t_1} + t_1, 0, 0, \right. \\ -16 + \frac{9 c_1}{2} - \frac{2 c_1}{t_1^4} + \frac{1}{t_1^3} + \frac{11 c_1}{2 t_1^3} - \frac{4}{t_1^2} - \frac{8 c_1}{t_1^2} + \frac{10}{t_1} + \frac{4 c_1}{t_1} + 18 t_1 - 10 c_1 t_1 - 14 t_1^2 + 8 c_1 t_1^2 + 7 t_1^3 - \frac{3}{2} c_1 t_1^3 - 2 t_1^4 - 2 c_1 t_1^4 + \\ \left. 2 c_1 t_1^5 - \frac{1}{2} c_1 t_1^6 - 4 u_1 w_1 + \frac{2 u_1 w_1}{t_1^4} - \frac{7 u_1 w_1}{2 t_1^3} + \frac{9 u_1 w_1}{2 t_1^2} + \frac{u_1 w_1}{2 t_1} + 6 t_1 u_1 w_1 - 2 t_1^2 u_1 w_1 - \frac{1}{2} t_1^3 u_1 w_1 + \frac{3}{2} t_1^4 u_1 w_1 - \frac{1}{2} t_1^5 u_1 w_1 \right]$$

The Invariant of the Trefoil

$$\text{FromCoefficientRules}[\text{CoefficientRules}[\text{Last}@z, \{c_1, u_1, w_1\}] /. \{(e_ \rightarrow c_) \Rightarrow (e \rightarrow \text{Simplify}[c])\}, \{c_1, u_1, w_1\}]$$

$$- \frac{(1 - t_1 + t_1^2)^2 (-1 + 2 t_1 - 3 t_1^2 + 2 t_1^3)}{t_1^3} - \frac{c_1 (1 - t_1 + t_1^2)^3 (4 + t_1 - 5 t_1^2 - t_1^3 + t_1^4)}{2 t_1^4} - \frac{(1 - t_1 + t_1^2)^3 (-4 - 5 t_1 + t_1^3) u_1 w_1}{2 t_1^4}$$