

Pensieve header: Normally Ordered Exponentials at 1-Co in the t variables.

For pragmatic reasons, $\mathbb{E}[\omega, L, Q, P]$ means $\omega^{-1}(1 + \epsilon \omega^{-4} P) \text{Exp}[L + \omega^{-1} Q]$, where ω is an ϵ -free scalar, L is linear and contains only c 's and b 's, Q is a balanced quadratic in the u 's and the w 's and contains no c 's and b 's, and P is a balanced quartic polynomial in the c 's, u 's, and w 's.

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DP[P_, x_ → D_α, y_ → D_β][f_] :=
  Total[CoefficientRules[P, {x, y}] /. ((m_, n_) → c_) ⇒ c D[f, {α, m}, {β, n}]]
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E /: Simplify[E[ω_, L_, Q_, P_]] := E[Expand@Together[ω /. b_L_ ⇒ Log[t_L]],
  Expand[L], Expand@Together[Q /. b_L_ ⇒ Log[t_L]], Expand@Together[P /. b_L_ ⇒ Log[t_L]]];
E /: E[ω1_, L1_, Q1_, P1_] ≡ E[ω2_, L2_, Q2_, P2_] := (ω1 == ω2 ∧ L1 == L2 ∧ Q1 == Q2 ∧ P1 == P2);
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```
E /: E[ω1_, L1_, Q1_, P1_] E[ω2_, L2_, Q2_, P2_] :=
  Simplify@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

```
NO[u_i_, c_j_, k_] [E[ω_, L_, Q_, P_]] := With[{q = e^{-y} β u_k + γ c_k}, Simplify[
  E[ω, γ c_k + (L /. c_j → 0), ω e^{-y} β u_k + (Q /. u_i → 0),
  e^{-q} DP[P, c_j → D_γ, u_i → D_β][e^q]] /. {γ → ∂_{c_j} L, β → ω^{-1} ∂_{u_i} Q}]]
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NO[w_i_, c_j_, k_] [E[ω_, L_, Q_, P_]] := With[{q = e^y β w_k + γ c_k}, Simplify[
  E[ω, γ c_k + (L /. c_j → 0), ω e^y β w_k + (Q /. w_i → 0),
  e^{-q} DP[P, c_j → D_γ, w_i → D_β][e^q]] /. {γ → ∂_{c_j} L, β → ω^{-1} ∂_{w_i} Q}]]
```

```
NO[w_i_, u_j_, k_] [E[ω_, L_, Q_, P_]] := With[{q = (1 - t_k) v α β + v β u_k + v δ u_k w_k + v α w_k}, Simplify[
  E[v^{-1} ω, L, v^{-1} ω q + v^{-1} (Q /. w_i | u_j → 0), v^{-4} e^{-q} DP[P, w_i → D_α, u_j → D_β][e^q] + ω^4 Δ] /. {
  v → (1 + (t_k - 1) δ)^{-1},
  Δ → -\frac{1}{2} (-1 + t_k) (\alpha^2 \beta^2 + 4 \alpha \beta \delta (1 + (-1 + t_k) \delta) + 2 \delta^2 (1 + (-1 + t_k) \delta)^2) +
  2 (1 + (-1 + t_k) \delta)^2 (\alpha \beta + \delta + (-1 + t_k) \delta^2) c_k - \beta (1 + 2 (-1 + t_k) \delta) (\alpha \beta + 2 \delta (1 + (-1 + t_k) \delta)) u_k +
  2 \beta \delta (1 + (-1 + t_k) \delta)^2 c_k u_k - \frac{1}{2} \beta^2 \delta (2 + 3 (-1 + t_k) \delta) u_k^2 + \alpha (\alpha \beta + 2 \delta (1 + (-1 + t_k) \delta)) w_k +
  2 \alpha \delta (1 + (-1 + t_k) \delta)^2 c_k w_k - 2 (-1 + t_k) \delta^2 (\alpha \beta + \delta (1 + (-1 + t_k) \delta)) u_k w_k + 2 \delta^2 (1 + (-1 + t_k) \delta)^2
  c_k u_k w_k - \beta \delta^2 (1 + 2 (-1 + t_k) \delta) u_k^2 w_k + \frac{1}{2} \alpha^2 \delta (2 + (-1 + t_k) \delta) w_k^2 + \alpha \delta^2 u_k w_k^2 - \frac{1}{2} (-1 + t_k) \delta^4 u_k^2 w_k^2
  } /. {\alpha → ω^{-1} (\partial_{w_i} Q /. u_j → 0), \beta → ω^{-1} (\partial_{u_j} Q /. w_i → 0), \delta → ω^{-1} \partial_{w_i, u_j} Q}]]];
```

```
m[i_, j_, kk_] [Z_] := Module[{x, y},
  Z // ReplaceAll[{b_{i|j} → b_{kk}, t_{i|j} → t_{kk}}] // NO[w_i, c_j, x] // NO[w_x, u_j, y] //
  ReplaceAll[{c_{x|y} → c_x, w_j → w_y}] // NO[u_i, c_x, x] //
  ReplaceAll[{c_{i|x} → c_{kk}, u_{x|y} → u_{kk}, w_y → w_{kk}, b_{x|y} → b_{kk}, t_{x|y} → t_{kk}}] // Simplify]
```

```
Q0 = E[1, 0, u_1 w_1 + u_2 w_3, 0];
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```
t1 = Q0 // m[1, 2, 4]
```

$$\mathbb{E}\left[1, 0, 2 u_4 w_3 - t_4 u_4 w_3 + u_4 w_4, -2 u_4 w_3 + 2 c_4 u_4 w_3 - \frac{5}{2} u_4^2 w_3^2 + \frac{3}{2} t_4 u_4^2 w_3^2 - u_4^2 w_3 w_4\right]$$

```
Q0 = E[e^{u_1 w_1 + u_3 w_2}];
```

```
Q0 = E[Exp[Sum[a_{i,j} u_i w_j + l_{i,j} b_i c_j, {i, 3}, {j, 3}]]]
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```
E[1, b_1 c_1 l_{1,1} + b_1 c_2 l_{1,2} + b_1 c_3 l_{1,3} + b_2 c_1 l_{2,1} + b_2 c_2 l_{2,2} + b_2 c_3 l_{2,3} + b_3 c_1 l_{3,1} + b_3 c_2 l_{3,2} + b_3 c_3 l_{3,3},
  u_1 w_1 a_{1,1} + u_1 w_2 a_{1,2} + u_1 w_3 a_{1,3} + u_2 w_1 a_{2,1} + u_2 w_2 a_{2,2} + u_2 w_3 a_{2,3} + u_3 w_1 a_{3,1} + u_3 w_2 a_{3,2} + u_3 w_3 a_{3,3}, 0]
```

$$Q0 = \mathbb{E}[\text{Exp}[\text{Sum}[a_{i,j} u_i w_j + l_{i,j} b_i c_j, \{i, 3\}, \{j, 3\}]]]$$

$$\mathbb{E}[1, b_1 c_1 l_{1,1} + b_1 c_2 l_{1,2} + b_1 c_3 l_{1,3} + b_2 c_1 l_{2,1} + b_2 c_2 l_{2,2} + b_2 c_3 l_{2,3} + b_3 c_1 l_{3,1} + b_3 c_2 l_{3,2} + b_3 c_3 l_{3,3}, u_1 w_1 a_{1,1} + u_1 w_2 a_{1,2} + u_1 w_3 a_{1,3} + u_2 w_1 a_{2,1} + u_2 w_2 a_{2,2} + u_2 w_3 a_{2,3} + u_3 w_1 a_{3,1} + u_3 w_2 a_{3,2} + u_3 w_3 a_{3,3}, 0]$$

SeedRandom[6];

$$Q0 = \mathbb{E}[1, \text{Sum}[\text{RandomInteger}[\{-2, 2\}] b_i c_j, \{i, 3\}, \{j, 3\}],$$

$$\text{Sum}[\text{RandomInteger}[\{-2, 2\}] u_i w_j, \{i, 3\}, \{j, 3\}], 0]$$

$$t1 = Q0 // m[1, 2, 4] // m[4, 3, 5]$$

$$t2 = Q0 // m[2, 3, 4] // m[1, 4, 5]$$

$$t3 = (t1 \equiv t2)$$

$$\mathbb{E}[1, -2 b_1 c_2 + b_2 c_2 - b_3 c_2 + 2 b_2 c_3 + 2 b_3 c_3, u_1 w_1 - u_2 w_1 - u_3 w_1 - u_1 w_2 + 2 u_2 w_2 + u_1 w_3 + u_2 w_3 - u_3 w_3, 0]$$

$$\mathbb{E}\left[1 + \frac{1}{t_5^2} - \frac{1}{t_5} + 3 t_5^2 - 5 t_5^3 + 2 t_5^4, 2 b_5 c_5, 3 u_5 w_5 + \frac{3 u_5 w_5}{t_5^4} - \frac{2 u_5 w_5}{t_5^3} - 3 t_5 u_5 w_5 - t_5^2 u_5 w_5 + t_5^3 u_5 w_5, \right.$$

$$118 - 374 c_5 + \frac{1}{t_5^8} - \frac{2 c_5}{t_5^8} - \frac{3}{t_5^7} + \frac{6 c_5}{t_5^7} + \frac{5}{t_5^6} - \frac{12 c_5}{t_5^6} - \frac{5}{t_5^5} + \frac{14 c_5}{t_5^5} + \frac{15}{t_5^4} - \frac{40 c_5}{t_5^4} - \frac{45}{t_5^3} + \frac{110 c_5}{t_5^3} + \frac{78}{t_5^2} - \frac{212 c_5}{t_5^2} - \frac{84}{t_5} + \frac{268 c_5}{t_5} -$$

$$292 t_5 + 768 c_5 t_5 + 540 t_5^2 - 1396 c_5 t_5^2 - 634 t_5^3 + 1784 c_5 t_5^3 + 635 t_5^4 - 1860 c_5 t_5^4 - 1017 t_5^5 + 2526 c_5 t_5^5 + 1810 t_5^6 -$$

$$4074 c_5 t_5^6 - 2236 t_5^7 + 5092 c_5 t_5^7 + 1935 t_5^8 - 4560 c_5 t_5^8 - 1771 t_5^9 + 3870 c_5 t_5^9 + 2403 t_5^{10} - 4582 c_5 t_5^{10} - 2921 t_5^{11} +$$

$$5378 c_5 t_5^{11} + 2312 t_5^{12} - 4300 c_5 t_5^{12} - 1100 t_5^{13} + 2088 c_5 t_5^{13} + 288 t_5^{14} - 560 c_5 t_5^{14} - 32 t_5^{15} + 64 c_5 t_5^{15} + 3190 u_5 w_5 -$$

$$590 c_5 u_5 w_5 + \frac{8 u_5 w_5}{t_5^{10}} - \frac{2 c_5 u_5 w_5}{t_5^{10}} - \frac{20 u_5 w_5}{t_5^9} + \frac{4 c_5 u_5 w_5}{t_5^9} + \frac{26 u_5 w_5}{t_5^8} - \frac{2 c_5 u_5 w_5}{t_5^8} - \frac{18 u_5 w_5}{t_5^7} - \frac{4 c_5 u_5 w_5}{t_5^7} + \frac{108 u_5 w_5}{t_5^6} -$$

$$\frac{20 c_5 u_5 w_5}{t_5^6} - \frac{326 u_5 w_5}{t_5^5} + \frac{76 c_5 u_5 w_5}{t_5^5} + \frac{506 u_5 w_5}{t_5^4} - \frac{92 c_5 u_5 w_5}{t_5^4} - \frac{470 u_5 w_5}{t_5^3} + \frac{28 c_5 u_5 w_5}{t_5^3} + \frac{758 u_5 w_5}{t_5^2} - \frac{84 c_5 u_5 w_5}{t_5^2} -$$

$$\frac{1936 u_5 w_5}{t_5} + \frac{396 c_5 u_5 w_5}{t_5} - 3230 t_5 u_5 w_5 + 280 c_5 t_5 u_5 w_5 + 2958 t_5^2 u_5 w_5 - 20 c_5 t_5^2 u_5 w_5 - 4650 t_5^3 u_5 w_5 + 504 c_5 t_5^3 u_5 w_5 +$$

$$7252 t_5^4 u_5 w_5 - 936 c_5 t_5^4 u_5 w_5 - 7302 t_5^5 u_5 w_5 + 176 c_5 t_5^5 u_5 w_5 + 5012 t_5^6 u_5 w_5 + 940 c_5 t_5^6 u_5 w_5 - 3776 t_5^7 u_5 w_5 -$$

$$924 c_5 t_5^7 u_5 w_5 + 4028 t_5^8 u_5 w_5 + 498 c_5 t_5^8 u_5 w_5 - 3482 t_5^9 u_5 w_5 - 1080 c_5 t_5^9 u_5 w_5 + 2004 t_5^{10} u_5 w_5 + 1900 c_5 t_5^{10} u_5 w_5 -$$

$$970 t_5^{11} u_5 w_5 - 1648 c_5 t_5^{11} u_5 w_5 + 500 t_5^{12} u_5 w_5 + 756 c_5 t_5^{12} u_5 w_5 - 192 t_5^{13} u_5 w_5 - 176 c_5 t_5^{13} u_5 w_5 + 32 t_5^{14} u_5 w_5 +$$

$$16 c_5 t_5^{14} u_5 w_5 - \frac{137}{2} u_5^2 w_5^2 + \frac{11 u_5^2 w_5^2}{2 t_5^{12}} - \frac{19 u_5^2 w_5^2}{2 t_5^{11}} - \frac{u_5^2 w_5^2}{t_5^{10}} + \frac{14 u_5^2 w_5^2}{t_5^9} + \frac{57 u_5^2 w_5^2}{t_5^8} - \frac{179 u_5^2 w_5^2}{t_5^7} + \frac{169 u_5^2 w_5^2}{t_5^6} - \frac{20 u_5^2 w_5^2}{t_5^5} +$$

$$\frac{385 u_5^2 w_5^2}{2 t_5^4} - \frac{1525 u_5^2 w_5^2}{2 t_5^3} + \frac{904 u_5^2 w_5^2}{t_5^2} - \frac{265 u_5^2 w_5^2}{t_5} - \frac{871}{2} t_5 u_5^2 w_5^2 + 486 t_5^2 u_5^2 w_5^2 + 512 t_5^3 u_5^2 w_5^2 - \frac{2551}{2} t_5^4 u_5^2 w_5^2 + \frac{2201}{2} t_5^5 u_5^2 w_5^2 -$$

$$816 t_5^6 u_5^2 w_5^2 + 1034 t_5^7 u_5^2 w_5^2 - 1331 t_5^8 u_5^2 w_5^2 + 1167 t_5^9 u_5^2 w_5^2 - 638 t_5^{10} u_5^2 w_5^2 + 192 t_5^{11} u_5^2 w_5^2 - 20 t_5^{12} u_5^2 w_5^2 - 2 t_5^{13} u_5^2 w_5^2]$$

$$\begin{aligned}
& \mathbb{E} \left[1 + \frac{1}{t_5} - \frac{1}{t_5} + 3 t_5^2 - 5 t_5^3 + 2 t_5^4, 2 b_5 c_5, 3 u_5 w_5 + \frac{3 u_5 w_5}{t_5^4} - \frac{2 u_5 w_5}{t_5^3} - 3 t_5 u_5 w_5 - t_5^2 u_5 w_5 + t_5^3 u_5 w_5, \right. \\
& 118 - 374 c_5 + \frac{1}{t_5^8} - \frac{2 c_5}{t_5^8} - \frac{3}{t_5^7} + \frac{6 c_5}{t_5^7} + \frac{5}{t_5^6} - \frac{12 c_5}{t_5^6} - \frac{5}{t_5^5} + \frac{14 c_5}{t_5^5} + \frac{15}{t_5^4} - \frac{40 c_5}{t_5^4} - \frac{45}{t_5^3} + \frac{110 c_5}{t_5^3} + \frac{78}{t_5^2} - \frac{212 c_5}{t_5^2} - \frac{84}{t_5} + \frac{268 c_5}{t_5} - \\
& 292 t_5 + 768 c_5 t_5 + 540 t_5^2 - 1396 c_5 t_5^2 - 634 t_5^3 + 1784 c_5 t_5^3 + 635 t_5^4 - 1860 c_5 t_5^4 - 1017 t_5^5 + 2526 c_5 t_5^5 + 1810 t_5^6 - \\
& 4074 c_5 t_5^6 - 2236 t_5^7 + 5092 c_5 t_5^7 + 1935 t_5^8 - 4560 c_5 t_5^8 - 1771 t_5^9 + 3870 c_5 t_5^9 + 2403 t_5^{10} - 4582 c_5 t_5^{10} - 2921 t_5^{11} + \\
& 5378 c_5 t_5^{11} + 2312 t_5^{12} - 4300 c_5 t_5^{12} - 1100 t_5^{13} + 2088 c_5 t_5^{13} + 288 t_5^{14} - 560 c_5 t_5^{14} - 32 t_5^{15} + 64 c_5 t_5^{15} + 3190 u_5 w_5 - \\
& 590 c_5 u_5 w_5 + \frac{8 u_5 w_5}{t_5^{10}} - \frac{2 c_5 u_5 w_5}{t_5^{10}} - \frac{20 u_5 w_5}{t_5^9} + \frac{4 c_5 u_5 w_5}{t_5^9} + \frac{26 u_5 w_5}{t_5^8} - \frac{2 c_5 u_5 w_5}{t_5^8} - \frac{18 u_5 w_5}{t_5^7} - \frac{4 c_5 u_5 w_5}{t_5^7} + \frac{108 u_5 w_5}{t_5^6} - \\
& \frac{20 c_5 u_5 w_5}{t_5^6} - \frac{326 u_5 w_5}{t_5^5} + \frac{76 c_5 u_5 w_5}{t_5^5} + \frac{506 u_5 w_5}{t_5^4} - \frac{92 c_5 u_5 w_5}{t_5^4} - \frac{470 u_5 w_5}{t_5^3} + \frac{28 c_5 u_5 w_5}{t_5^3} + \frac{758 u_5 w_5}{t_5^2} - \frac{84 c_5 u_5 w_5}{t_5^2} - \\
& \frac{1936 u_5 w_5}{t_5} + \frac{396 c_5 u_5 w_5}{t_5} - 3230 t_5 u_5 w_5 + 280 c_5 t_5 u_5 w_5 + 2958 t_5^2 u_5 w_5 - 20 c_5 t_5^2 u_5 w_5 - 4650 t_5^3 u_5 w_5 + 504 c_5 t_5^3 u_5 w_5 + \\
& 7252 t_5^4 u_5 w_5 - 936 c_5 t_5^4 u_5 w_5 - 7302 t_5^5 u_5 w_5 + 176 c_5 t_5^5 u_5 w_5 + 5012 t_5^6 u_5 w_5 + 940 c_5 t_5^6 u_5 w_5 - 3776 t_5^7 u_5 w_5 - \\
& 924 c_5 t_5^7 u_5 w_5 + 4028 t_5^8 u_5 w_5 + 498 c_5 t_5^8 u_5 w_5 - 3482 t_5^9 u_5 w_5 - 1080 c_5 t_5^9 u_5 w_5 + 2004 t_5^{10} u_5 w_5 + 1900 c_5 t_5^{10} u_5 w_5 - \\
& 970 t_5^{11} u_5 w_5 - 1648 c_5 t_5^{11} u_5 w_5 + 500 t_5^{12} u_5 w_5 + 756 c_5 t_5^{12} u_5 w_5 - 192 t_5^{13} u_5 w_5 - 176 c_5 t_5^{13} u_5 w_5 + 32 t_5^{14} u_5 w_5 + \\
& 16 c_5 t_5^{14} u_5 w_5 - \frac{137}{2} u_5^2 w_5^2 + \frac{11 u_5^2 w_5^2}{2 t_5^{12}} - \frac{19 u_5^2 w_5^2}{2 t_5^{11}} - \frac{u_5^2 w_5^2}{t_5^{10}} + \frac{14 u_5^2 w_5^2}{t_5^9} + \frac{57 u_5^2 w_5^2}{t_5^8} - \frac{179 u_5^2 w_5^2}{t_5^7} + \frac{169 u_5^2 w_5^2}{t_5^6} - \frac{20 u_5^2 w_5^2}{t_5^5} + \\
& \frac{385 u_5^2 w_5^2}{2 t_5^4} - \frac{1525 u_5^2 w_5^2}{2 t_5^3} + \frac{904 u_5^2 w_5^2}{t_5^2} - \frac{265 u_5^2 w_5^2}{t_5} - \frac{871}{2} t_5 u_5^2 w_5^2 + 486 t_5^2 u_5^2 w_5^2 + 512 t_5^3 u_5^2 w_5^2 - \frac{2551}{2} t_5^4 u_5^2 w_5^2 + \frac{2201}{2} t_5^5 u_5^2 w_5^2 - \\
& 816 t_5^6 u_5^2 w_5^2 + 1034 t_5^7 u_5^2 w_5^2 - 1331 t_5^8 u_5^2 w_5^2 + 1167 t_5^9 u_5^2 w_5^2 - 638 t_5^{10} u_5^2 w_5^2 + 192 t_5^{11} u_5^2 w_5^2 - 20 t_5^{12} u_5^2 w_5^2 - 2 t_5^{13} u_5^2 w_5^2]
\end{aligned}$$

True

Simplify[t3 /. b5 → Log[t5]]

$$\frac{(-1 + 2 t_5 + 3 t_5^2 - 7 t_5^3 + 3 t_5^4) u_5 w_5}{t_5 (-1 + 2 t_5)} == 0 \&\&$$

$$\frac{(-1 + t_5) (-2 + 3 t_5) u_5 w_5 (-3 + u_5 w_5 - 11 t_5^2 u_5 w_5 + 15 t_5^3 u_5 w_5 + t_5 (6 - u_5 w_5))}{t_5} == 0$$

$$Q0 = \mathbb{E}[1, \text{Sum}[l_{i,j} b_i c_j, \{i, 3\}, \{j, 3\}], \text{Sum}[a_{i,j} u_i w_j, \{i, 3\}, \{j, 3\}], 0]$$

$$t1 = Q0 // m[1, 2, 4] // m[4, 3, 5]$$

$$t2 = Q0 // m[2, 3, 4] // m[1, 4, 5]$$

$$t3 = (t1 \equiv t2)$$

$$\mathbb{E}[1, b_1 c_1 l_{1,1} + b_1 c_2 l_{1,2} + b_1 c_3 l_{1,3} + b_2 c_1 l_{2,1} + b_2 c_2 l_{2,2} + b_2 c_3 l_{2,3} + b_3 c_1 l_{3,1} + b_3 c_2 l_{3,2} + b_3 c_3 l_{3,3}, u_1 w_1 a_{1,1} + u_1 w_2 a_{1,2} + u_1 w_3 a_{1,3} + u_2 w_1 a_{2,1} + u_2 w_2 a_{2,2} + u_2 w_3 a_{2,3} + u_3 w_1 a_{3,1} + u_3 w_2 a_{3,2} + u_3 w_3 a_{3,3}, 0]$$

$$\mathbb{E}\left[1 - t_5^{1+1,2+1,3+1,2+1,3,2} a_{2,1} + t_5^{1+1,2+1,3+1,2+1,3,2} a_{2,1} - t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{3,1} + t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{3,1} - t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,2} a_{3,1} + 2 t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,2} a_{3,1} - t_5^{2+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,2} a_{3,1} - t_5^{1+1,3+1,2,3+1,3,3} a_{3,2} + t_5^{1+1,3+1,2,3+1,3,3} a_{3,2} + t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,1} a_{3,2} - 2 t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,1} a_{3,2} + t_5^{2+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,1} a_{3,2}, \dots, 2 c_5 t_5^{\dots} a_{2,1} - 2 \dots a_{2,1} + \dots + 2 \dots a_{3,3}^2\right]$$

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$$\mathbb{E}\left[1 - t_5^{1+1,2+1,3+1,2+1,3,2} a_{2,1} + t_5^{1+1,2+1,3+1,2+1,3,2} a_{2,1} - t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{3,1} + t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{3,1} - t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,2} a_{3,1} + 2 t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,2} a_{3,1} - t_5^{2+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,2} a_{3,1} - t_5^{1+1,3+1,2,3+1,3,3} a_{3,2} + t_5^{1+1,3+1,2,3+1,3,3} a_{3,2} + t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,1} a_{3,2} - 2 t_5^{1+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,1} a_{3,2} + t_5^{2+1,2+1,3+1,2,2+1,2,3+1,3,2+1,3,3} a_{2,1} a_{3,2}, \dots, 2 c_5 t_5^{\dots} a_{2,1} - 2 \dots a_{2,1} + \dots + 2 \dots a_{3,3}^2\right]$$

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True

$$Rp[1, 2] Rm[4, 3]$$

$$\mathbb{E}\left[b_1 c_2 - b_4 c_3 + \frac{(-1 + e^{b_1}) u_1 w_2}{b_1} + \frac{(-1 + e^{-b_4}) u_4 w_3}{b_4}\right]$$

$$Rp[1, 2] Rm[4, 3] // m[1, 4, 1] // m[2, 3, 2]$$

$\mathbb{E}[0]$

$$Rm[1, 2] Rp[4, 3] // m[4, 1, 4] // m[2, 3, 2]$$

$\mathbb{E}[0]$

$$t1 = Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[3, 5, a] // m[1, 6, b] // m[2, 4, c]$$

$$\mathbb{E}\left[\frac{b_a^2 b_b (c_b + c_c) + (-1 + e^{b_a}) b_b u_a (w_b + w_c) + b_a (b_b^2 c_c + (-1 + e^{b_b}) u_b w_c)}{b_a b_b}\right]$$

$$t2 = Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[1, 3, a] // m[2, 5, b] // m[4, 6, c]$$

$$\mathbb{E}\left[\frac{b_a^2 b_b (c_b + c_c) + (-1 + e^{b_a}) b_b u_a (w_b + w_c) + b_a (b_b^2 c_c + (-1 + e^{b_b}) u_b w_c)}{b_a b_b}\right]$$

$$t1 \equiv t2$$

True

t3 = Rm[12, 1] Rm[2, 7] Rm[8, 3] Rm[4, 11] Rp[16, 5] Rp[6, 13] Rp[14, 9] Rp[10, 15]

$$\mathbb{E} \left[-b_{12} c_1 - b_8 c_3 + b_{16} c_5 - b_2 c_7 + b_{14} c_9 - b_4 c_{11} + b_6 c_{13} + b_{10} c_{15} + \frac{(-1 + e^{-b_{12}}) u_{12} w_1}{b_{12}} + \frac{(-1 + e^{-b_8}) u_8 w_3}{b_8} + \frac{(-1 + e^{b_{16}}) u_{16} w_5}{b_{16}} + \frac{(-1 + e^{-b_2}) u_2 w_7}{b_2} + \frac{(-1 + e^{b_{14}}) u_{14} w_9}{b_{14}} + \frac{(-1 + e^{-b_4}) u_4 w_{11}}{b_4} + \frac{(-1 + e^{b_6}) u_6 w_{13}}{b_6} + \frac{(-1 + e^{b_{10}}) u_{10} w_{15}}{b_{10}} \right]$$

t3 // m[1, 2, 1]

$$\mathbb{E} \left[-b_{12} c_1 - b_8 c_3 + b_{16} c_5 - b_1 c_7 + b_{14} c_9 - b_4 c_{11} + b_6 c_{13} + b_{10} c_{15} + \frac{(-1 + e^{-b_{12}}) u_{12} w_1}{b_{12}} + \frac{(-1 + e^{-b_8}) u_8 w_3}{b_8} + \frac{(-1 + e^{b_{16}}) u_{16} w_5}{b_{16}} + \frac{(-1 + e^{-b_1}) u_1 w_7}{b_1} - \frac{(-1 + e^{-b_1}) (-1 + e^{-b_{12}}) u_{12} w_7}{b_{12}} + \frac{(-1 + e^{b_{14}}) u_{14} w_9}{b_{14}} + \frac{(-1 + e^{-b_4}) u_4 w_{11}}{b_4} + \frac{(-1 + e^{b_6}) u_6 w_{13}}{b_6} + \frac{(-1 + e^{b_{10}}) u_{10} w_{15}}{b_{10}} \right]$$

t3 // m[1, 2, 1] // m[1, 3, 1]

$$\mathbb{E} \left[-b_8 c_1 - b_{12} c_1 + b_{16} c_5 - b_1 c_7 + b_{14} c_9 - b_4 c_{11} + b_6 c_{13} + b_{10} c_{15} + \frac{(-1 + e^{-b_8}) u_8 w_1}{b_8} + \frac{e^{-b_8} (-1 + e^{-b_{12}}) u_{12} w_1}{b_{12}} + \frac{(-1 + e^{b_{16}}) u_{16} w_5}{b_{16}} + \frac{e^{b_8} (-1 + e^{-b_1}) u_1 w_7}{b_1} - \frac{(-1 + e^{-b_1}) (-1 + e^{-b_{12}}) u_{12} w_7}{b_{12}} + \frac{(-1 + e^{b_{14}}) u_{14} w_9}{b_{14}} + \frac{(-1 + e^{-b_4}) u_4 w_{11}}{b_4} + \frac{(-1 + e^{b_6}) u_6 w_{13}}{b_6} + \frac{(-1 + e^{b_{10}}) u_{10} w_{15}}{b_{10}} \right]$$

Do[t3 = t3 // m[1, kk, 1], {kk, 2, 16}]; t3

$$\frac{e^{3 b_1} \mathbb{E}[\emptyset]}{1 - 4 e^{b_1} + 8 e^{2 b_1} - 11 e^{3 b_1} + 8 e^{4 b_1} - 4 e^{5 b_1} + e^{6 b_1}}$$

Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[1, 4, 4] // m[4, 5, 5] // m[5, 2, 2] // m[2, 3, 3] // m[3, 6, 6] // m[6, 1, 1]

$$\frac{\mathbb{E} \left[3 b_1 c_1 + \frac{(1 - e^{-3 b_1}) u_1 w_1}{b_1} \right]}{1 + e^{b_1} (-1 + e^{b_1})}$$