Pensieve header: Normally Ordered Exponentials at 0-Co in the t variables.

"NO" for "Normal Order".

For pragmatic reasons, $\mathbb{E}[\omega, L, Q]$ means $\omega^{-1}\,\mathrm{Exp}\left[L+\omega^{-1}Q\right]$, where ω is a scalar, L is linear and contains only c's and b's, and Q is a balanced quadratic in the u's and the w's and contains no c's and b's. $\mathbb{E}$ is also a casting operator: $\mathbb{E}\left[\omega^{-1}\,\mathrm{Exp}\left[L+\omega^{-1}Q\right]\right]$ returns $\mathbb{E}[\omega, L, Q]$, meaning $\mathbb{E}[\omega\,\mathrm{Exp}[L+Q]]$ returns $\mathbb{E}\left[\omega^{-1}, L, \omega^{-1}Q\right]$.

```
𝔼 /: Simplify[𝔼[ω_, L_, Q_]] :=
  𝔼[Expand@Together@ω, Expand[L], Expand@Together[Q /. b_ι_ :> Log[t_ι]]];
𝔼 /: 𝔼[ω1_, L1_, Q1_] ≡ 𝔼[ω2_, L2_, Q2_] := (ω1 == ω2 ⋀ L1 == L2 ⋀ Q1 == Q2);
𝔼[ω_. ℯ^F_] := Simplify@𝔼[ω^-1, F /. u_ | w_ | α | β → 0, ω^-1 F /. c_ → 0];
𝔼[ω_] /; FreeQ[ω, ℯ-] := 𝔼[ω^-1, 0, 0];
𝔼[ℰ_] := 𝔼[Factor[ℰ]];
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𝔼 /: 𝔼[ω1_, L1_, Q1_] 𝔼[ω2_, L2_, Q2_] := Simplify@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2];
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```
Rp[i_, j_] := 𝔼[ℯ^(b_i c_j + u_i w_j)]; Rm[i_, j_] := 𝔼[ℯ^(-b_i c_j - t_i^-1 u_i w_j)];
```

```
NO[u_i_, c_j_, k_][𝔼[ω_, L_, Q_]] := 𝔼[1, L /. c_j → 0, ω^-1 Q /. u_i → 0] (
    𝔼[ω^-1 ℯ^(ℯ^-γ β u_k + γ c_k)] /. {γ → ∂_(c_j) L, β → ω^-1 ∂_(u_i) Q});
NO[w_i_, c_j_, k_][𝔼[ω_, L_, Q_]] := 𝔼[1, L /. c_j → 0, ω^-1 Q /. w_i → 0] (
    𝔼[ω^-1 ℯ^(ℯ^γ β w_k + γ c_k)] /. {γ → ∂_(c_j) L, β → ω^-1 ∂_(w_i) Q});
NO[w_i_, u_j_, k_][𝔼[ω_, L_, Q_]] := 𝔼[1, L, ω^-1 Q /. w_i | u_j → 0] (
    𝔼[ν ω^-1 ℯ^((1-t_k) ν α β + ν β u_k + ν δ u_k w_k + ν α w_k)] /. ν → (1 + (t_k - 1) δ)^-1 /. {
      α → ω^-1 (∂_(w_i) Q /. u_j → 0), β → ω^-1 (∂_(u_j) Q /. w_i → 0), δ → ω^-1 ∂_(w_i, u_j) Q});
```

```
m[i_, j_, k_][Z_] := Module[{a}, Simplify[
   (Z /. {b_i|j → b_k, t_i|j → t_k} // NO[w_i, c_j, a] // NO[u_i, c_a, a] // NO[w_a, u_j, a]) /.
    {c_i → c_k, w_j → w_k, y_a :> y_k}]]
```

```
σ[i_, j_][ℰ_] := ℰ /. {b_i → b_j, t_i → t_j, c_i → c_j, u_i → u_j, w_i → w_j}
```

```
Q0 = 𝔼[ℯ^(u1 w1 + u2 w3)];
t1 = Q0 // m[1, 2, 4]
```

$$\mathbb{E}[1,\ 0,\ 2\,u_4\,w_3 - t_4\,u_4\,w_3 + u_4\,w_4]$$

```
NO[u1, c1, 1][𝔼[ℯ^(b2 c1 + w2 u1 + u3 w4)]]
```

$$\mathbb{E}\left[1,\ b_2\,c_1,\ \frac{u_1\,w_2}{t_2} + u_3\,w_4\right]$$

```
Q0 = 𝔼[Exp[Sum[a_{i,j} u_i w_j + l_{i,j} b_i c_j, {i, 3}, {j, 3}]]]
```

$$\mathbb{E}\left[1,\ b_1 c_1 l_{1,1} + b_1 c_2 l_{1,2} + b_1 c_3 l_{1,3} + b_2 c_1 l_{2,1} + b_2 c_2 l_{2,2} + b_2 c_3 l_{2,3} + b_3 c_1 l_{3,1} + b_3 c_2 l_{3,2} + b_3 c_3 l_{3,3},\ u_1 w_1 a_{1,1} + u_1 w_2 a_{1,2} + u_1 w_3 a_{1,3} + u_2 w_1 a_{2,1} + u_2 w_2 a_{2,2} + u_2 w_3 a_{2,3} + u_3 w_1 a_{3,1} + u_3 w_2 a_{3,2} + u_3 w_3 a_{3,3}\right]$$

```
Q0 // m[1, 2, 1]
```

$$\mathbb{E}\Big[1 - t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, a_{2,1} + t_1^{1+l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, a_{2,1},\ b_1 c_1 l_{1,1} + b_1 c_1 l_{1,2} + b_1 c_3 l_{1,3} + b_1 c_1 l_{2,1} + b_1 c_1 l_{2,2} + b_1 c_3 l_{2,3} + b_3 c_1 l_{3,1} + b_3 c_1 l_{3,2} + b_3 c_3 l_{3,3},\ u_1 w_1 a_{1,1} + t_1^{-l_{1,2}-l_{2,2}}\, t_3^{-l_{3,2}}\, u_1 w_1 a_{1,2} + t_1^{-l_{1,2}-l_{2,2}}\, t_3^{-l_{3,2}}\, u_1 w_3 a_{1,3} + t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_1 w_1 a_{2,1} - u_1 w_1 a_{1,2} a_{2,1} + t_1\, u_1 w_1 a_{1,2} a_{2,1} - u_1 w_3 a_{1,3} a_{2,1} + t_1\, u_1 w_3 a_{1,3} a_{2,1} + u_1 w_1 a_{2,2} + u_1 w_1 a_{1,1} a_{2,2} - t_1\, u_1 w_1 a_{1,1} a_{2,2} + u_1 w_3 a_{2,3} + u_1 w_3 a_{1,1} a_{2,3} - t_1\, u_1 w_3 a_{1,1} a_{2,3} + t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_1 a_{3,1} + t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_1 a_{2,2} a_{3,1} - t_1^{1+l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_1 a_{2,2} a_{3,1} + t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_3 a_{2,3} a_{3,1} - t_1^{1+l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_3 a_{2,3} a_{3,1} + u_3 w_1 a_{3,2} - t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_1 a_{2,1} a_{3,2} + t_1^{1+l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_1 a_{2,1} a_{3,2} + u_3 w_3 a_{3,3} - t_1^{l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_3 a_{2,1} a_{3,3} + t_1^{1+l_{1,2}+l_{2,2}}\, t_3^{l_{3,2}}\, u_3 w_3 a_{2,1} a_{3,3}\Big]$$

```
t1 = Q0 // m[1, 2, 1] // m[1, 3, 1]
```

$$\mathbb{E}\Big[1 - t_1^{l_{1,2}+l_{2,2}+l_{3,2}}\, a_{2,1} + t_1^{1+l_{1,2}+l_{2,2}+l_{3,2}}\, a_{2,1} - t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{3,1} + t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{3,1} - t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,2} a_{3,1} + 2\, t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,2} a_{3,1} - t_1^{2+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,2} a_{3,1} - t_1^{l_{1,3}+l_{2,3}+l_{3,3}}\, a_{3,2} + t_1^{1+l_{1,3}+l_{2,3}+l_{3,3}}\, a_{3,2} + t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,1} a_{3,2} - 2\, t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,1} a_{3,2} + t_1^{2+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,1} a_{3,2},\ b_1 c_1 l_{1,1} + b_1 c_1 l_{1,2} + b_1 c_1 l_{1,3} + b_1 c_1 l_{2,1} + b_1 c_1 l_{2,2} + b_1 c_1 l_{2,3} + b_1 c_1 l_{3,1} + b_1 c_1 l_{3,2} + b_1 c_1 l_{3,3},\ u_1 w_1 a_{1,1} + t_1^{-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1 w_1 a_{1,2} + t_1^{-l_{1,2}-l_{1,3}-l_{2,2}-l_{2,3}-l_{3,2}-l_{3,3}}\, u_1 w_1 a_{1,3} + t_1^{l_{1,2}+l_{2,2}+l_{3,2}}\, u_1 w_1 a_{2,1} - u_1 w_1 a_{1,2} a_{2,1} + t_1\, u_1 w_1 a_{1,2} a_{2,1} - t_1^{-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1 w_1 a_{1,3} a_{2,1} + t_1^{1-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1 w_1 a_{1,3} a_{2,1} + u_1 w_1 a_{2,2} + u_1 w_1 a_{1,1} a_{2,2} - t_1\, u_1 w_1 a_{1,1} a_{2,2} + t_1^{-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1 w_1 a_{2,3} + t_1^{-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1 w_1 a_{1,1} a_{2,3} - t_1^{1-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1 w_1 a_{1,1} a_{2,3} + t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1 w_1 a_{3,1} - u_1 w_1 a_{1,3} a_{3,1} + t_1\, u_1 w_1 a_{1,3} a_{3,1} + t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1 w_1 a_{2,2} a_{3,1} - t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1 w_1 a_{2,2} a_{3,1} - u_1 w_1 a_{1,3} a_{2,2} a_{3,1} + 2\, t_1\, u_1 w_1 a_{1,3} a_{2,2} a_{3,1} - t_1^2\, u_1 w_1 a_{1,3} a_{2,2} a_{3,1} + u_1 w_1 a_{1,2} a_{2,3} a_{3,1} - 2\, t_1\, u_1 w_1 a_{1,2} a_{2,3} a_{3,1} + t_1^2\, u_1 w_1 a_{1,2} a_{2,3} a_{3,1} + t_1^{l_{1,3}+l_{2,3}+l_{3,3}}\, u_1 w_1 a_{3,2} - t_1^{-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1 w_1 a_{1,3} a_{3,2} + t_1^{1-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1 w_1 a_{1,3} a_{3,2} - t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1 w_1 a_{2,1} a_{3,2} + t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1 w_1 a_{2,1} a_{3,2} + u_1 w_1 a_{1,3} a_{2,1} a_{3,2} - 2\, t_1\, u_1 w_1 a_{1,3} a_{2,1} a_{3,2} + t_1^2\, u_1 w_1 a_{1,3} a_{2,1} a_{3,2} - u_1 w_1 a_{2,3} a_{3,2} + t_1\, u_1 w_1 a_{2,3} a_{3,2} - u_1 w_1 a_{1,1} a_{2,3} a_{3,2} + 2\, t_1\, u_1 w_1 a_{1,1} a_{2,3} a_{3,2} - t_1^2\, u_1 w_1 a_{1,1} a_{2,3} a_{3,2} + u_1 w_1 a_{3,3} + u_1 w_1 a_{1,1} a_{3,3} - t_1\, u_1 w_1 a_{1,1} a_{3,3} + t_1^{-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1 w_1 a_{1,2} a_{3,3} - t_1^{1-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1 w_1 a_{1,2} a_{3,3} - u_1 w_1 a_{1,2} a_{2,1} a_{3,3} + 2\, t_1\, u_1 w_1 a_{1,2} a_{2,1} a_{3,3} - t_1^2\, u_1 w_1 a_{1,2} a_{2,1} a_{3,3} + u_1 w_1 a_{2,2} a_{3,3} - t_1\, u_1 w_1 a_{2,2} a_{3,3} + u_1 w_1 a_{1,1} a_{2,2} a_{3,3} - 2\, t_1\, u_1 w_1 a_{1,1} a_{2,2} a_{3,3} + t_1^2\, u_1 w_1 a_{1,1} a_{2,2} a_{3,3}\Big]$$

```
t2 = Q0 // m[2, 3, 2] // m[1, 2, 1]
```

$$\mathbb{E}\Big[1 - t_1^{l_{1,2}+l_{2,2}+l_{3,2}}\, a_{2,1} + t_1^{1+l_{1,2}+l_{2,2}+l_{3,2}}\, a_{2,1} - t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{3,1} + t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{3,1} - t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,2}\, a_{3,1} + 2\, t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,2}\, a_{3,1} - t_1^{2+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,2}\, a_{3,1} - t_1^{l_{1,3}+l_{2,3}+l_{3,3}}\, a_{3,2} + t_1^{1+l_{1,3}+l_{2,3}+l_{3,3}}\, a_{3,2} + t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,1}\, a_{3,2} - 2\, t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,1}\, a_{3,2} + t_1^{2+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, a_{2,1}\, a_{3,2},$$
$$b_1\, c_1\, l_{1,1} + b_1\, c_1\, l_{1,2} + b_1\, c_1\, l_{1,3} + b_1\, c_1\, l_{2,1} + b_1\, c_1\, l_{2,2} + b_1\, c_1\, l_{2,3} + b_1\, c_1\, l_{3,1} + b_1\, c_1\, l_{3,2} + b_1\, c_1\, l_{3,3},$$
$$u_1\, w_1\, a_{1,1} + t_1^{-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1\, w_1\, a_{1,2} + t_1^{-l_{1,2}-l_{1,3}-l_{2,2}-l_{2,3}-l_{3,2}-l_{3,3}}\, u_1\, w_1\, a_{1,3} + t_1^{l_{1,2}+l_{2,2}+l_{3,2}}\, u_1\, w_1\, a_{2,1} - u_1\, w_1\, a_{1,2}\, a_{2,1} + t_1\, u_1\, w_1\, a_{1,2}\, a_{2,1} - t_1^{-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1\, w_1\, a_{1,3}\, a_{2,1} + t_1^{1-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1\, w_1\, a_{1,3}\, a_{2,1} + u_1\, w_1\, a_{2,2} + u_1\, w_1\, a_{1,1}\, a_{2,2} - t_1\, u_1\, w_1\, a_{1,1}\, a_{2,2} + t_1^{-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1\, w_1\, a_{2,3} + t_1^{-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1\, w_1\, a_{1,1}\, a_{2,3} - t_1^{1-l_{1,3}-l_{2,3}-l_{3,3}}\, u_1\, w_1\, a_{1,1}\, a_{2,3} + t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1\, w_1\, a_{3,1} - u_1\, w_1\, a_{1,3}\, a_{3,1} + t_1\, u_1\, w_1\, a_{1,3}\, a_{3,1} + t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1\, w_1\, a_{2,2}\, a_{3,1} - t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1\, w_1\, a_{2,2}\, a_{3,1} - u_1\, w_1\, a_{1,3}\, a_{2,2}\, a_{3,1} + 2\, t_1\, u_1\, w_1\, a_{1,3}\, a_{2,2}\, a_{3,1} - t_1^2\, u_1\, w_1\, a_{1,3}\, a_{2,2}\, a_{3,1} + u_1\, w_1\, a_{1,2}\, a_{2,3}\, a_{3,1} - 2\, t_1\, u_1\, w_1\, a_{1,2}\, a_{2,3}\, a_{3,1} + t_1^2\, u_1\, w_1\, a_{1,2}\, a_{2,3}\, a_{3,1} + t_1^{l_{1,3}+l_{2,3}+l_{3,3}}\, u_1\, w_1\, a_{3,2} - t_1^{-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1\, w_1\, a_{1,3}\, a_{3,2} + t_1^{1-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1\, w_1\, a_{1,3}\, a_{3,2} - t_1^{l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1\, w_1\, a_{2,1}\, a_{3,2} + t_1^{1+l_{1,2}+l_{1,3}+l_{2,2}+l_{2,3}+l_{3,2}+l_{3,3}}\, u_1\, w_1\, a_{2,1}\, a_{3,2} + u_1\, w_1\, a_{1,3}\, a_{2,1}\, a_{3,2} - 2\, t_1\, u_1\, w_1\, a_{1,3}\, a_{2,1}\, a_{3,2} + t_1^2\, u_1\, w_1\, a_{1,3}\, a_{2,1}\, a_{3,2} - u_1\, w_1\, a_{2,3}\, a_{3,2} + t_1\, u_1\, w_1\, a_{2,3}\, a_{3,2} - u_1\, w_1\, a_{1,1}\, a_{2,3}\, a_{3,2} + 2\, t_1\, u_1\, w_1\, a_{1,1}\, a_{2,3}\, a_{3,2} - t_1^2\, u_1\, w_1\, a_{1,1}\, a_{2,3}\, a_{3,2} + u_1\, w_1\, a_{3,3} + u_1\, w_1\, a_{1,1}\, a_{3,3} - t_1\, u_1\, w_1\, a_{1,1}\, a_{3,3} + t_1^{-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1\, w_1\, a_{1,2}\, a_{3,3} - t_1^{1-l_{1,2}-l_{2,2}-l_{3,2}}\, u_1\, w_1\, a_{1,2}\, a_{3,3} - u_1\, w_1\, a_{1,2}\, a_{2,1}\, a_{3,3} + 2\, t_1\, u_1\, w_1\, a_{1,2}\, a_{2,1}\, a_{3,3} - t_1^2\, u_1\, w_1\, a_{1,2}\, a_{2,1}\, a_{3,3} + u_1\, w_1\, a_{2,2}\, a_{3,3} - t_1\, u_1\, w_1\, a_{2,2}\, a_{3,3} + u_1\, w_1\, a_{1,1}\, a_{2,2}\, a_{3,3} - 2\, t_1\, u_1\, w_1\, a_{1,1}\, a_{2,2}\, a_{3,3} + t_1^2\, u_1\, w_1\, a_{1,1}\, a_{2,2}\, a_{3,3}\Big]$$

```
t1 ≡ t2
```

True

```
Rp[1, 2] Rm[4, 3]
```

$$\mathbb{E}\Big[1,\ b_1\, c_2 - b_4\, c_3,\ u_1\, w_2 - \frac{u_4\, w_3}{t_4}\Big]$$

```
Rp[1, 2] Rm[4, 3] // m[1, 4, 1] // m[2, 3, 2]
```

$$\mathbb{E}[1, 0, 0]$$

```
Rm[1, 2] Rp[4, 3] // m[4, 1, 4] // m[2, 3, 2]
```

$$\mathbb{E}[1, 0, 0]$$

```
t1 = Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[3, 5, x] // m[1, 6, y] // m[2, 4, z]
```

$$\mathbb{E}[1,\ b_x\, c_y + b_x\, c_z + b_y\, c_z,\ u_x\, w_y + u_x\, w_z + u_y\, w_z]$$

```
t2 = Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[1, 3, x] // m[2, 5, y] // m[4, 6, z]
```

$$\mathbb{E}[1,\ b_x\, c_y + b_x\, c_z + b_y\, c_z,\ u_x\, w_y + u_x\, w_z + u_y\, w_z]$$

```
t1 ≡ t2
```

True

```
t3 = Rm[12, 1] Rm[2, 7] Rm[8, 3] Rm[4, 11] Rp[16, 5] Rp[6, 13] Rp[14, 9] Rp[10, 15]
```

$$\mathbb{E}\Big[1,\ -b_{12}\, c_1 - b_8\, c_3 + b_{16}\, c_5 - b_2\, c_7 + b_{14}\, c_9 - b_4\, c_{11} + b_6\, c_{13} + b_{10}\, c_{15},\ -\frac{u_{12}\, w_1}{t_{12}} - \frac{u_8\, w_3}{t_8} + u_{16}\, w_5 - \frac{u_2\, w_7}{t_2} + u_{14}\, w_9 - \frac{u_4\, w_{11}}{t_4} + u_6\, w_{13} + u_{10}\, w_{15}\Big]$$

```
t3 // m[1, 2, 1]
```

$$\mathbb{E}\left[1,\ -b_{12}\,c_1 - b_8\,c_3 + b_{16}\,c_5 - b_1\,c_7 + b_{14}\,c_9 - b_4\,c_{11} + b_6\,c_{13} + b_{10}\,c_{15},\right.$$
$$\left. -\frac{u_{12}\,w_1}{t_{12}} - \frac{u_8\,w_3}{t_8} + u_{16}\,w_5 - \frac{u_1\,w_7}{t_1} - \frac{u_{12}\,w_7}{t_{12}} + \frac{u_{12}\,w_7}{t_1\,t_{12}} + u_{14}\,w_9 - \frac{u_4\,w_{11}}{t_4} + u_6\,w_{13} + u_{10}\,w_{15}\right]$$

```
t3 // m[1, 2, 1] // m[1, 3, 1]
```

$$\mathbb{E}\left[1,\ -b_8\,c_1 - b_{12}\,c_1 + b_{16}\,c_5 - b_1\,c_7 + b_{14}\,c_9 - b_4\,c_{11} + b_6\,c_{13} + b_{10}\,c_{15},\right.$$
$$\left. -\frac{u_8\,w_1}{t_8} - \frac{u_{12}\,w_1}{t_8\,t_{12}} + u_{16}\,w_5 - \frac{t_8\,u_1\,w_7}{t_1} - \frac{u_{12}\,w_7}{t_{12}} + \frac{u_{12}\,w_7}{t_1\,t_{12}} + u_{14}\,w_9 - \frac{u_4\,w_{11}}{t_4} + u_6\,w_{13} + u_{10}\,w_{15}\right]$$

```
t4 = t3;
Do[t4 = t4 // m[1, kk, 1], {kk, 2, 16}]; t4
```

$$\mathbb{E}\left[11 - \frac{1}{t_1^3} + \frac{4}{t_1^2} - \frac{8}{t_1} - 8\,t_1 + 4\,t_1^2 - t_1^3,\ 0,\ 0\right]$$

```
Rp[1, 2] Rp[3, 4] Rp[5, 6] // m[1, 4, 4] // m[4, 5, 5] // m[5, 2, 2] // m[2, 3, 3] //
  m[3, 6, 6] // m[6, 1, 1]
```

$$\mathbb{E}\left[1 - t_1 + t_1^2,\ 3\,b_1\,c_1,\ \frac{u_1\,w_1}{t_1^3} + \frac{u_1\,w_1}{t_1} + t_1\,u_1\,w_1\right]$$