

Pensieve header: Finding the most general R using low algebra, now with a balanced basis.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OneCo-1606"];
<< OneCo.m
```

## Finding R

```
bas = Cases[BLBasis[{j, k}], e * U[...]]
```

```
{e U[c_j], e U[c_k], e U[c_j, c_j], e U[c_j, c_k], e U[c_k, c_k], e U[u_j, w_j], e U[u_j, w_k], e U[u_k, w_j],
  e U[u_k, w_k], e U[c_j, u_j, w_j], e U[c_j, u_j, w_k], e U[c_j, u_k, w_j], e U[c_j, u_k, w_k],
  e U[c_k, u_j, w_j], e U[c_k, u_j, w_k], e U[c_k, u_k, w_j], e U[c_k, u_k, w_k], e U[u_j, u_j, w_j, w_j],
  e U[u_j, u_j, w_j, w_k], e U[u_j, u_j, w_k, w_k], e U[u_j, u_k, w_j, w_j], e U[u_j, u_k, w_j, w_k],
  e U[u_j, u_k, w_k, w_k], e U[u_k, u_k, w_j, w_j], e U[u_k, u_k, w_j, w_k], e U[u_k, u_k, w_k, w_k]}
```

```
Sum[f_i[b_j, b_k] bas[[i]], {i, Length@bas}]
```

```
e U[c_j] f_1[b_j, b_k] + e U[c_k] f_2[b_j, b_k] + e U[c_j, c_j] f_3[b_j, b_k] +
  e U[c_j, c_k] f_4[b_j, b_k] + e U[c_k, c_k] f_5[b_j, b_k] + e U[u_j, w_j] f_6[b_j, b_k] +
  e U[u_j, w_k] f_7[b_j, b_k] + e U[u_k, w_j] f_8[b_j, b_k] + e U[u_k, w_k] f_9[b_j, b_k] +
  e U[c_j, u_j, w_j] f_10[b_j, b_k] + e U[c_j, u_j, w_k] f_11[b_j, b_k] + e U[c_j, u_k, w_j] f_12[b_j, b_k] +
  e U[c_j, u_k, w_k] f_13[b_j, b_k] + e U[c_k, u_j, w_j] f_14[b_j, b_k] + e U[c_k, u_j, w_k] f_15[b_j, b_k] +
  e U[c_k, u_k, w_j] f_16[b_j, b_k] + e U[c_k, u_k, w_k] f_17[b_j, b_k] + e U[u_j, u_j, w_j, w_j] f_18[b_j, b_k] +
  e U[u_j, u_j, w_j, w_k] f_19[b_j, b_k] + e U[u_j, u_j, w_k, w_k] f_20[b_j, b_k] +
  e U[u_j, u_k, w_j, w_j] f_21[b_j, b_k] + e U[u_j, u_k, w_j, w_k] f_22[b_j, b_k] +
  e U[u_j, u_k, w_k, w_k] f_23[b_j, b_k] + e U[u_k, u_k, w_j, w_j] f_24[b_j, b_k] +
  e U[u_k, u_k, w_j, w_k] f_25[b_j, b_k] + e U[u_k, u_k, w_k, w_k] f_26[b_j, b_k]
```

```
 $\rho_0[j_, k_] := e U[c_j] f_1[b_j, b_k] + e U[c_k] f_2[b_j, b_k] + e U[c_j, c_j] f_3[b_j, b_k] +
  e U[c_j, c_k] f_4[b_j, b_k] + e U[c_k, c_k] f_5[b_j, b_k] + e U[u_j, w_j] f_6[b_j, b_k] +
  e U[u_j, w_k] f_7[b_j, b_k] + e U[u_k, w_j] f_8[b_j, b_k] + e U[u_k, w_k] f_9[b_j, b_k] +
  e U[c_j, u_j, w_j] f_{10}[b_j, b_k] + e U[c_j, u_j, w_k] f_{11}[b_j, b_k] + e U[c_j, u_k, w_j] f_{12}[b_j, b_k] +
  e U[c_j, u_k, w_k] f_{13}[b_j, b_k] + e U[c_k, u_j, w_j] f_{14}[b_j, b_k] + e U[c_k, u_j, w_k] f_{15}[b_j, b_k] +
  e U[c_k, u_k, w_j] f_{16}[b_j, b_k] + e U[c_k, u_k, w_k] f_{17}[b_j, b_k] + e U[u_j, u_j, w_j, w_j] f_{18}[b_j, b_k] +
  e U[u_j, u_j, w_j, w_k] f_{19}[b_j, b_k] + e U[u_j, u_j, w_k, w_k] f_{20}[b_j, b_k] +
  e U[u_j, u_k, w_j, w_j] f_{21}[b_j, b_k] + e U[u_j, u_k, w_j, w_k] f_{22}[b_j, b_k] +
  e U[u_j, u_k, w_k, w_k] f_{23}[b_j, b_k] + e U[u_k, u_k, w_j, w_j] f_{24}[b_j, b_k] +
  e U[u_k, u_k, w_j, w_k] f_{25}[b_j, b_k] + e U[u_k, u_k, w_k, w_k] f_{26}[b_j, b_k]$ 
```

```
R[j_, k_][ $\mathcal{E}$ ] := Simp[ $\mathcal{E}$  // Ea[j, k] // (# + B[\rho_0[j, k], #]) &]
```

```
U[c_1] // R[1, 2] // Simp // Short
```

```
U[c_1] + e U[u_2, w_1] f_8[b_1, b_2] + <<18>> + U[u_1, u_2, w_1, w_2] (<<1>>) +
  U[u_1, u_2, w_2, w_2]  $\left( -\frac{e f_{13}[b_1, b_2]}{b_1} + \text{<<11>>} + \frac{2 e^{b_1} e b_2 f_{26}[b_1, b_2]}{b_1} \right)$ 
```

$U[c_1] // R[1, 2] // R[1, 3] // R[2, 3]$

$$U[c_1] + e^{-b_2} \in U[u_3, w_1] f_8[b_1, b_3] + \dots 65 \dots +$$

$$U[u_1, u_3, w_3, w_3] \left( -\frac{e f_{13}[b_1, b_3]}{b_1} + \frac{e^{b_1} \in f_{13}[b_1, b_3]}{b_1} + \frac{e^{b_2} \in f_{17}[b_1, b_3]}{b_1} - \frac{e^{b_1+b_2} \in f_{17}[b_1, b_3]}{b_1} + \frac{e f_{17}[b_2, b_3]}{b_1} - \frac{e^{b_1} \in f_{17}[b_2, b_3]}{b_1} + e f_{22}[b_1, b_3] - e^{b_1} \in f_{22}[b_1, b_3] - e^{b_2} \in f_{23}[b_1, b_3] - \frac{e b_2 f_{23}[b_2, b_3]}{b_1} + \frac{e^{b_1} \in b_2 f_{23}[b_2, b_3]}{b_1} - \frac{2 e^{b_2} \in b_3 f_{26}[b_1, b_3]}{b_1} + \frac{2 e^{b_1+b_2} \in b_3 f_{26}[b_1, b_3]}{b_1} - \frac{2 e b_3 f_{26}[b_2, b_3]}{b_1} + \frac{2 e^{b_1} \in b_3 f_{26}[b_2, b_3]}{b_1} \right)$$

large output | show less | show more | show all | set size limit...

$R3[\mathcal{E}_] :=$

$Simp[(\mathcal{E} // R[1, 2] // R[1, 3] // R[2, 3]) - (\mathcal{E} // R[2, 3] // R[1, 3] // R[1, 2])]$

$R3s = Flatten[Table[U@y_i \to R3[U@y_i], \{y, \{c, u, w\}\}, \{i, 3\}]]$

$$\{U[c_1] \to U[u_3, w_1] (-e f_8[b_1, b_3] + e^{-b_2} \in f_8[b_1, b_3]) + \dots 57 \dots +$$

$$U[u_1, u_3, w_3, w_3] \left( -\frac{e f_{17}[b_1, b_3]}{b_1} + \frac{e^{b_1} \in f_{17}[b_1, b_3]}{b_1} + \frac{e^{b_2} \in f_{17}[b_1, b_3]}{b_1} - \frac{e^{b_1+b_2} \in f_{17}[b_1, b_3]}{b_1} + \frac{e f_{17}[b_2, b_3]}{b_1} - \frac{e^{b_1} \in f_{17}[b_2, b_3]}{b_1} + e f_{23}[b_1, b_3] - e^{b_2} \in \dots 1 \dots - \dots 1 \dots + \frac{e^{b_1} \in \dots 2 \dots \dots 1 \dots}{b_1} + \frac{2 e b_3 f_{26}[b_1, b_3]}{b_1} - \frac{2 e^{b_1} \in b_3 f_{26}[b_1, b_3]}{b_1} - \frac{2 e^{b_2} \in b_3 f_{26}[b_1, b_3]}{b_1} + \frac{2 e^{b_1+b_2} \in b_3 f_{26}[b_1, b_3]}{b_1} - \frac{2 e b_3 f_{26}[b_2, b_3]}{b_1} + \frac{2 e^{b_1} \in b_3 f_{26}[b_2, b_3]}{b_1} \right), \dots 7 \dots, U[w_3] \to \dots 1 \dots \}$$

large output | show less | show more | show all | set size limit...

$R3s /. e \to 0$

$\{U[c_1] \to 0, U[c_2] \to 0, U[c_3] \to 0, U[u_1] \to 0,$   
 $U[u_2] \to 0, U[u_3] \to 0, U[w_1] \to 0, U[w_2] \to 0, U[w_3] \to 0\}$

**bas3 = Cases[LBasis[{1, 2, 3}], e \* U[...]]**

{ $\in U[c_1]$ ,  $\in U[c_2]$ ,  $\in U[c_3]$ ,  $\in U[u_1]$ ,  $\in U[u_2]$ ,  $\in U[u_3]$ ,  $\in U[w_1]$ ,  $\in U[w_2]$ ,  $\in U[w_3]$ ,  
 $\in U[c_1, c_1]$ ,  $\in U[c_1, c_2]$ ,  $\in U[c_1, c_3]$ ,  $\in U[c_1, u_1]$ ,  $\in U[c_1, u_2]$ ,  $\in U[c_1, u_3]$ ,  
 $\in U[c_1, w_1]$ ,  $\in U[c_1, w_2]$ ,  $\in U[c_1, w_3]$ ,  $\in U[c_2, c_2]$ ,  $\in U[c_2, c_3]$ ,  $\in U[c_2, u_1]$ ,  
 $\in U[c_2, u_2]$ ,  $\in U[c_2, u_3]$ ,  $\in U[c_2, w_1]$ ,  $\in U[c_2, w_2]$ ,  $\in U[c_2, w_3]$ ,  $\in U[c_3, c_3]$ ,  
 $\in U[c_3, u_1]$ ,  $\in U[c_3, u_2]$ ,  $\in U[c_3, u_3]$ ,  $\in U[c_3, w_1]$ ,  $\in U[c_3, w_2]$ ,  $\in U[c_3, w_3]$ ,  
 $\in U[u_1, w_1]$ ,  $\in U[u_1, w_2]$ ,  $\in U[u_1, w_3]$ ,  $\in U[u_2, w_1]$ ,  $\in U[u_2, w_2]$ ,  $\in U[u_2, w_3]$ ,  
 $\in U[u_3, w_1]$ ,  $\in U[u_3, w_2]$ ,  $\in U[u_3, w_3]$ ,  $\in U[c_1, u_1, w_1]$ ,  $\in U[c_1, u_1, w_2]$ ,  $\in U[c_1, u_1, w_3]$ ,  
 $\in U[c_1, u_2, w_1]$ ,  $\in U[c_1, u_2, w_2]$ ,  $\in U[c_1, u_2, w_3]$ ,  $\in U[c_1, u_3, w_1]$ ,  $\in U[c_1, u_3, w_2]$ ,  
 $\in U[c_1, u_3, w_3]$ ,  $\in U[c_2, u_1, w_1]$ ,  $\in U[c_2, u_1, w_2]$ ,  $\in U[c_2, u_1, w_3]$ ,  $\in U[c_2, u_2, w_1]$ ,  
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 $\in U[c_3, u_1, w_1]$ ,  $\in U[c_3, u_1, w_2]$ ,  $\in U[c_3, u_1, w_3]$ ,  $\in U[c_3, u_2, w_1]$ ,  $\in U[c_3, u_2, w_2]$ ,  
 $\in U[c_3, u_2, w_3]$ ,  $\in U[c_3, u_3, w_1]$ ,  $\in U[c_3, u_3, w_2]$ ,  $\in U[c_3, u_3, w_3]$ ,  $\in U[u_1, u_1, w_1]$ ,  
 $\in U[u_1, u_1, w_2]$ ,  $\in U[u_1, u_1, w_3]$ ,  $\in U[u_1, u_2, w_1]$ ,  $\in U[u_1, u_2, w_2]$ ,  $\in U[u_1, u_2, w_3]$ ,  
 $\in U[u_1, u_3, w_1]$ ,  $\in U[u_1, u_3, w_2]$ ,  $\in U[u_1, u_3, w_3]$ ,  $\in U[u_1, w_1, w_1]$ ,  $\in U[u_1, w_1, w_2]$ ,  
 $\in U[u_1, w_1, w_3]$ ,  $\in U[u_1, w_2, w_2]$ ,  $\in U[u_1, w_2, w_3]$ ,  $\in U[u_1, w_3, w_3]$ ,  $\in U[u_2, u_2, w_1]$ ,  
 $\in U[u_2, u_2, w_2]$ ,  $\in U[u_2, u_2, w_3]$ ,  $\in U[u_2, u_3, w_1]$ ,  $\in U[u_2, u_3, w_2]$ ,  $\in U[u_2, u_3, w_3]$ ,  
 $\in U[u_2, w_1, w_1]$ ,  $\in U[u_2, w_1, w_2]$ ,  $\in U[u_2, w_1, w_3]$ ,  $\in U[u_2, w_2, w_2]$ ,  $\in U[u_2, w_2, w_3]$ ,  
 $\in U[u_2, w_3, w_3]$ ,  $\in U[u_3, u_3, w_1]$ ,  $\in U[u_3, u_3, w_2]$ ,  $\in U[u_3, u_3, w_3]$ ,  $\in U[u_3, w_1, w_1]$ ,  
 $\in U[u_3, w_1, w_2]$ ,  $\in U[u_3, w_1, w_3]$ ,  $\in U[u_3, w_2, w_2]$ ,  $\in U[u_3, w_2, w_3]$ ,  $\in U[u_3, w_3, w_3]$ ,  
 $\in U[u_1, u_1, w_1, w_1]$ ,  $\in U[u_1, u_1, w_1, w_2]$ ,  $\in U[u_1, u_1, w_1, w_3]$ ,  $\in U[u_1, u_1, w_2, w_2]$ ,  
 $\in U[u_1, u_1, w_2, w_3]$ ,  $\in U[u_1, u_1, w_3, w_3]$ ,  $\in U[u_1, u_2, w_1, w_1]$ ,  $\in U[u_1, u_2, w_1, w_2]$ ,  
 $\in U[u_1, u_2, w_1, w_3]$ ,  $\in U[u_1, u_2, w_2, w_2]$ ,  $\in U[u_1, u_2, w_2, w_3]$ ,  $\in U[u_1, u_2, w_3, w_3]$ ,  
 $\in U[u_1, u_3, w_1, w_1]$ ,  $\in U[u_1, u_3, w_1, w_2]$ ,  $\in U[u_1, u_3, w_1, w_3]$ ,  $\in U[u_1, u_3, w_2, w_2]$ ,  
 $\in U[u_1, u_3, w_2, w_3]$ ,  $\in U[u_1, u_3, w_3, w_3]$ ,  $\in U[u_2, u_2, w_1, w_1]$ ,  $\in U[u_2, u_2, w_1, w_2]$ ,  
 $\in U[u_2, u_2, w_1, w_3]$ ,  $\in U[u_2, u_2, w_2, w_2]$ ,  $\in U[u_2, u_2, w_2, w_3]$ ,  $\in U[u_2, u_2, w_3, w_3]$ ,  
 $\in U[u_2, u_3, w_1, w_1]$ ,  $\in U[u_2, u_3, w_1, w_2]$ ,  $\in U[u_2, u_3, w_1, w_3]$ ,  $\in U[u_2, u_3, w_2, w_2]$ ,  
 $\in U[u_2, u_3, w_2, w_3]$ ,  $\in U[u_2, u_3, w_3, w_3]$ ,  $\in U[u_3, u_3, w_1, w_1]$ ,  $\in U[u_3, u_3, w_1, w_2]$ ,  
 $\in U[u_3, u_3, w_1, w_3]$ ,  $\in U[u_3, u_3, w_2, w_2]$ ,  $\in U[u_3, u_3, w_2, w_3]$ ,  $\in U[u_3, u_3, w_3, w_3]$ }

```
Errors = Union[DeleteCases[
  Flatten[Table[Simplify@Coefficient[U@yi /. R3s, e],
    {y, {u, w}}, {i, 3}, {e, bas3}]] /. {b1 → x, b2 → y, b3 → z},
  0] // SortBy[LeafCount]
```

$$\left\{ -2(-1 + e^x) x f_{24}[x, z], e^x(-1 + e^y) z f_{12}[x, z], e^x(-1 + e^y) z f_{16}[x, z], \right.$$

$$e^x(-1 + e^y) z f_{21}[x, z], -e^x \left( \dots 1 \dots \right) z f_{12}[x, z], \dots 124 \dots,$$

$$\frac{\dots 1 \dots}{x^2}, \dots 1 \dots, \frac{\dots 1 \dots}{x y^2}, \frac{1}{x^3} e^{-x-y} z \left( -2(-1 + e^x) (-1 + e^{2y}) + \right.$$

$$\left. \dots 1 \dots - \dots 1 \dots + x \left( \dots 91 \dots + 2 \dots 2 \dots \dots 1 \dots \right) \right), \frac{1}{x^3} e^{-x-y} z$$

$$\left( 2(-1 + e^x) (-1 + e^y) + (-1 + e^x) x^2 \left( 2(-1 + e^x) f_{10}[x, z] - e^y f_{11}[x, y] + f_{11}[x, z] \right) + \right.$$

$$\left. x^3 \left( \dots 1 \dots \right) + x \left( 1 + \dots 85 \dots + z^2 f_{25}[y, z] - 2 e^x z^2 f_{25}[y, z] + e^{2x} z^2 f_{25}[y, z] \right) \right\}$$

large output
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```
DeleteFile["Errors.m"];
Errors >> "Errors.m"
```

From ErrorCorrection.nb:

$$\begin{aligned}
\mathbf{EC} = & \{f_1[x_, z_] \Rightarrow x f_6[x, z] + x^2 f_{18}[x, z] - g_7[z], \\
& f_2[x_, y_] \Rightarrow y f_9[x, y] + y^2 f_{26}[x, y] - g_8[x], f_3[x_, z_] \Rightarrow x^2 f_{18}[x, z], \\
& f_4[x_, z_] \Rightarrow x f_{14}[x, z] - g_5[z], f_5[x_, y_] \Rightarrow y^2 f_{26}[x, y], \\
& f_7[x_, z_] \Rightarrow \frac{(1 - e^x) g_9[z]}{x} + g_{10}[x], f_8[x_, z_] \Rightarrow 0, \\
& f_{10}[y_, z_] \Rightarrow 2 y f_{18}[y, z], f_{11}[x_, y_] \Rightarrow -g_2[x], f_{12}[x_, z_] \Rightarrow 0, \\
& f_{13}[x_, z_] \Rightarrow x f_{22}[x, z] - g_4[z], f_{14}[x_, y_] \Rightarrow y f_{22}[x, y] - g_6[x], \\
& f_{15}[x_, y_] \Rightarrow \frac{1}{(-1 + e^y) x^2 y} \left( -x + e^x x + e^y x - e^{x+y} x - y + e^x y + e^y y - e^{x+y} y - \right. \\
& \quad x y + e^{x+y} x y + e^y x y g_5[y] - e^{x+y} x y g_5[y] + x y^2 g_6[y] - e^x x y^2 g_6[y] - \\
& \quad e^y x y g_7[y] + e^{x+y} x y g_7[y] - x y g_8[y] + e^x x y g_8[y] - x y g_9[y] + e^x x y g_9[y] + \\
& \quad \left. e^y x y g_9[y] - e^{x+y} x y g_9[y] - x y^2 g_{10}[y] + e^x x y^2 g_{10}[y] + x y h_1[] - e^x x y h_1[] \right), \\
& f_{16}[x_, z_] \Rightarrow 0, f_{17}[x_, z_] \Rightarrow 2 z f_{26}[x, z], f_{19}[x_, y_] \Rightarrow -g_3[x], \\
& f_{20}[x_, y_] \Rightarrow \frac{1}{2(-1 + e^y) x^3} \left( -2 + 2 e^x + 2 e^y - 2 e^{x+y} - x - e^x x + e^y x + e^{x+y} x + x^3 g_3[x] - \right. \\
& \quad e^x x^3 g_3[x] - e^y x^3 g_3[x] + e^{x+y} x^3 g_3[x] + x y^2 g_3[y] - 2 e^x x y^2 g_3[y] + e^{2x} x y^2 g_3[y] - \\
& \quad e^x x g_7[x] + e^{2x} x g_7[x] + e^{x+y} x g_7[x] - e^{2x+y} x g_7[x] - e^y x g_7[y] + 2 e^{x+y} x g_7[y] - \\
& \quad e^{2x+y} x g_7[y] - x g_8[x] + e^x x g_8[x] + e^y x g_8[x] - e^{x+y} x g_8[x] - x g_8[y] + \\
& \quad 2 e^x x g_8[y] - e^{2x} x g_8[y] - x g_9[x] + 2 e^x x g_9[x] - e^{2x} x g_9[x] + e^y x g_9[x] - \\
& \quad 2 e^{x+y} x g_9[x] + e^{2x+y} x g_9[x] - x g_9[y] + 2 e^x x g_9[y] - e^{2x} x g_9[y] + e^y x g_9[y] - \\
& \quad 2 e^{x+y} x g_9[y] + e^{2x+y} x g_9[y] - x^2 g_{10}[x] + e^x x^2 g_{10}[x] + e^y x^2 g_{10}[x] - e^{x+y} x^2 g_{10}[x] - \\
& \quad \left. x y g_{10}[y] + 2 e^x x y g_{10}[y] - e^{2x} x y g_{10}[y] + x h_1[] - 2 e^x x h_1[] + e^{2x} x h_1[] \right), \\
& f_{21}[x_, z_] \Rightarrow 0, f_{23}[x_, z_] \Rightarrow -\frac{(-1 + e^x) g_1[z]}{x}, f_{24}[x_, z_] \Rightarrow 0, \\
& f_{25}[x_, y_] \Rightarrow 0, g_1[y_] \Rightarrow \frac{1}{2(-1 + e^y) y^2} \\
& \quad \left( -2 + 2 e^y - y - e^y y - 2 y^3 g_3[y] + 2 e^y y g_5[y] + 2 y^2 g_6[y] + y h_1[] - e^y y h_1[] \right), \\
& g_2[y_] \Rightarrow \frac{e^y g_7[y] + g_8[y] + g_9[y] - e^y g_9[y] + y g_{10}[y]}{y}, g_4[y_] \Rightarrow \frac{g_5[y] - h_1[]}{y} \};
\end{aligned}$$

**R3s = Simplify[Flatten[Table[U@y<sub>i</sub> → R3[U@y<sub>i</sub>], {y, {c, u, w}}, {i, 3}]] // . EC]**

$$\begin{aligned}
& \{U[c_1] \rightarrow 0, U[c_2] \rightarrow 0, U[c_3] \rightarrow 0, U[u_1] \rightarrow 0, \\
& U[u_2] \rightarrow 0, U[u_3] \rightarrow 0, U[w_1] \rightarrow 0, U[w_2] \rightarrow 0, U[w_3] \rightarrow 0\}
\end{aligned}$$