Cheat Sheet OneCo-1606

http://drorbn.net/AcademicPensieve/Projects/OneCo-1606/
Restarts Projects/OneCo-1604; modified November 10, 2016; continued 2016-11

$\mathfrak{g}_0$ **bi-local exponentiation relations.** In $\mathfrak{g}_0 = \mathcal{D}\mathbb{I}\mathbb{I} = \mathcal{D}\text{ii} = \mathcal{D}\text{II} = \mathcal{D}\text{II} := \langle b,c,u,w\rangle/([b,\cdot] = 0,\ [c,u] = u,\ [c,w] = -w,\ [u,w] = b)$ with $\deg(b,c,u,w) = (1,0,1,0)$ and $a_{12} = b_1c_2 + u_1w_2$: (verifications in `G0.nb`)

1. The Yang-Baxter element,
$$\exp_{\mathcal{U}}(a_{12}) = \exp\left(b_1c_2 + \frac{e^{b_1}-1}{b_1}u_1w_2\right)/\!\!/m_1^{u_1}/\!\!/m_2^{c_2w_2}$$
2. ✓ $[c, uw] = 0$
 ✓ $e^{\beta u}e^{\alpha c} = e^{\alpha c}e^{e^{-\alpha}\beta u}$ and $e^{\beta w}e^{\alpha c} = e^{\alpha c}e^{e^{\alpha}\beta w}$
3. ✓ $[w, e^{\gamma u}] = -b\gamma e^{\gamma u}$ and $[u, e^{\gamma w}] = b\gamma e^{\gamma w}$
4. With $M_{uw} = M_{uw}(\gamma) := e^{\gamma uw}/\!\!/m^{uw} = \sum_{k\geq 0}\frac{\gamma^k}{k!}u^kw^k$,
 ✓ $[u, M_{uw}] = b\gamma u M_{uw}$ and $[w, M_{uw}] = -b\gamma M_{uw}w$
 ✓ $M_{uw}^{-1}(\gamma(\alpha))\partial_\alpha M_{uw}(\gamma(\alpha)) = \frac{\partial_\alpha\gamma(\alpha)}{1-b\gamma(\alpha)}uw$
5. With $M_{wu} = M_{wu}(\delta) := e^{\delta uw}/\!\!/m^{wu} = \sum_{k\geq 0}\frac{\delta^k}{k!}w^ku^k$,
 ✓ $[u, M_{wu}] = b\delta M_{wu}u$ and $[w, M_{wu}] = -b\delta w M_{wu}$
 ✓ $M_{wu}^{-1}(\alpha\delta)\partial_\alpha M_{wu}(\alpha\delta) = \frac{\delta}{1+b\alpha\delta}wu = \frac{\delta}{1+b\alpha\delta}(uw-b)$
6. ✓ $M_{wu}(\delta) = \frac{1}{1+b\delta}M_{uw}\left(\frac{\delta}{1+b\delta}\right)$
7. ✓ The hard core uw relation. $e^{\alpha w}e^{\beta u} = e^{-b\alpha\beta}e^{\beta u}e^{\alpha w}$
 with $\nu = (1+b\delta)^{-1}$, $e^{\alpha w}M_{wu}(\delta)e^{\beta u} = \nu e^{-b\nu\alpha\beta}e^{\nu\beta u}M_{uw}(\nu\delta)e^{\nu\alpha w}$

✓ **1-Smidgen sl_2 / $\mathfrak{g}_1$ bi-local exponentiation relations.** With $\epsilon^2 = 0$, in $\mathfrak{g}_1 := \mathbb{Q}[\epsilon]\langle b,c,u,w\rangle/([b,\cdot] = 0,\ [c,u] = u,\ [c,w] = -w,\ [u,w] = b - 2\epsilon c)$ with $\deg(b,c,u,w,\epsilon) = (1,0,1,0,1)$ and $a_{12} = (b_1 - \epsilon c_1)c_2 + u_1w_2$: (verifications in `G1.nb`)

1. c relations: $e^{\beta u}e^{\alpha c} = e^{\alpha c}e^{e^{-\alpha}\beta u}$ and $e^{\beta w}e^{\alpha c} = e^{\alpha c}e^{e^{\alpha}\beta w}$
2. With $\nu = (1+b\delta)^{-1}$ and Λ as below,
$$\mathbb{O}\left(e^{\alpha w+\beta u+\delta uw} \mid wu\right) = \mathbb{O}\left(\nu(1+\epsilon\nu\Lambda)e^{\nu(-b\alpha\beta+\alpha w+\beta u+\delta uw)} \mid cuw\right)$$

(160805) Λ for Λόγος, "a principle of order and knowledge": $\Lambda = -\frac{1}{2}b\nu\left(\alpha^2\beta^2\nu^2 + 4\alpha\beta\delta\nu + 2\delta^2\right) - \frac{1}{2}\beta^2\delta\nu^3u^2(3b\delta+2) - \frac{1}{2}b\delta^4\nu^3u^2w^2 - \beta\delta^2\nu^3u^2w(2b\delta+1) - \beta\nu^2u(2b\delta+1)(\alpha\beta\nu+2\delta) - 2b\delta^2\nu^2uw(\alpha\beta\nu+\delta) + \frac{1}{2}\alpha^2\delta\nu^3w^2(b\delta+2) + 2c(\alpha\beta\nu+\delta) + 2\beta c\delta\nu u + 2c\delta^2\nu uw + 2\alpha c\delta\nu w + \alpha\delta^2\nu^3uw^2 + \alpha\nu^2w(\alpha\beta\nu+2\delta)$.

(160801a) Roland's $sm_q sl(2)$ formulas, BBS:VanDerVeen-160731, with $q = e^\epsilon$, $t = e^b$: b central, $[w,c] = w$, $[c,u] = u$, $wu - quw = 1 - te^{2\epsilon c}$ (at $\epsilon^2 = 0$: $[w,u] = \epsilon uw + 1 - t - 2\epsilon tc$), $R = \sum_{m,n}\frac{u^n(b+\epsilon c)^m\otimes c^mw^n}{m![n]_q!} \to \sum_{m,n}\frac{u^n(b+\epsilon c)^m\otimes c^mw^n}{m!n!}\left(1 - \frac{\epsilon}{2}\binom{n}{2}\right)$. Also, $\Delta(b,c,u,w) = (b_1+b_2, c_1+c_2, t_2e^{\epsilon c_2}u_1 + u_2, e^{\epsilon c_2}w_1 + w_2)$ and $S(b,c,u,w) = (-b,-c,-t^{-1}ue^{-\epsilon c}, -we^{-\epsilon c})$. Verified `VdVAlgebraAt1-Testing.nb`.

(160801b) $sm_0 sl(2)$ formulas, $t = e^b$: b central, $[w,c] = w$, $[c,u] = u$, $wu - uw = 1 - t$, $R = \sum_{m,n}\frac{u^nb^m\otimes c^mw^n}{m!n!}$ (verified `VdVAlgebraAt0.nb`). Also, $\Delta(b,c,u,w) = (b_1+b_2, c_1+c_2, t_2u_1 + u_2, w_1 + w_2)$ and $S(b,c,u,w) = (-b,-c,-t^{-1}u,-w)$ (unverified).

(160730) Lessons from Roland: • There is an additional grading, with $ht(b,c,u,w) = (0,0,1,-1)$. • Rescale $u \to \frac{b}{e^b-1}u$. • A simple R-matrix for 1-co.

(160725) Challenge: In $\mathfrak{g}_0$, understand $e^{\beta uw+\alpha u+\gamma w}/\!\!/m_x^{uw}$ and $e^{\beta uw+\alpha u+\gamma w}/\!\!/m_x^{wu}$.

(160628) Figure out duality in $\mathfrak{g}_1$!

(160622b) The $(b-\epsilon c)$-scapegoated 1-co low algebra $\mathfrak{g}_1$ "1-smidgen sl_2" ($\epsilon^2 = 0$, b central) with $a_{12} = (b_1 - \epsilon c_1)c_2 + u_1w_2$, $[w,c] = w$ $[c,u] = u$ $[u,w] = b - 2\epsilon c$. Also $ad(-a_{12}) = \{c_1 \mapsto u_1w_2,$ $c_2 \mapsto -u_1w_2,$ $u_1 \mapsto \epsilon u_1c_2,$ $u_2 \mapsto -(b_1-\epsilon c_1)u_2 + (b_2 - 2\epsilon c_2)u_1,$ $w_1 \mapsto -b_1w_2 - \epsilon w_1c_2 + 2\epsilon c_1w_2,$ $w_2 \mapsto (b_1 - \epsilon c_1)w_2\}$.

Claim. Over $\mathbb{Q}[\![\epsilon, b_i]\!]$ the following generate a sub-Lie algebra, sub-meta-monoid, and contains the a_{ij}'s:
$$\{1, c_i, u_i, w_i, u_iw_j\} \quad\text{and}$$
$$\epsilon\{c_ic_j, c_iu_j, c_iw_j, c_iu_jw_k, u_iu_jw_k, u_iw_jw_k, u_iu_jw_kw_l\}$$

(160621) 1-co low algebra ($\epsilon^2 = 0$): $a_{12} = I = b_1c_2 + u_1w_2 \in \mathfrak{b}_\epsilon^*\otimes\mathfrak{b}_\epsilon$

$[w,c] = w$	$[b,u] = -\epsilon u$	$\delta c = 0$	$\delta w = \epsilon(c\wedge w)$
$[b,c] = 0$	$[b,w] = \epsilon w$	$[c,u] = u$	$[u,w] = b - \epsilon c$

(verification in pensieve://2016-06)

Also, $ad(-a_{12}) = \{u_1 \mapsto \epsilon u_1c_2,\ u_2 \mapsto -b_1u_2 + b_2u_1 - \epsilon u_1c_2,\ b_1 \mapsto -\epsilon u_1w_2,\ b_2 \mapsto \epsilon u_1w_2,\ w_1 \mapsto -b_1w_2 - \epsilon w_1c_2 + \epsilon c_1w_2,\ w_2 \mapsto b_1w_2,\ c_1 \mapsto u_1w_2,\ c_2 \mapsto -u_1w_2\}$.

Recycling.

(160622a) Scapegoated 1-co low algebra ($\epsilon^2 = s^2 = \epsilon s = 0$, b central):
$$a_{12} = I = (b_1 + s_1)c_2 + u_1w_2 \in \mathfrak{b}_\epsilon^*\otimes\mathfrak{b}_\epsilon$$

$[w,c] = w$	$[s,u] = -\epsilon u$	$\delta c = 0$	$\delta w = \epsilon(c\wedge w)$
$[s,c] = 0$	$[s,w] = \epsilon w$	$[c,u] = u$	$[u,w] = b + s - \epsilon c$

Also, $ad(-a_{12}) = \{$
 $u_1 \mapsto \epsilon u_1c_2,$ $u_2 \mapsto -(b_1+s_1)u_2 + (b_2+s_2)u_1 - \epsilon u_1c_2,$
 $s_1 \mapsto -\epsilon u_1w_2,$ $s_2 \mapsto \epsilon u_1w_2,$ $c_1 \mapsto u_1w_2,$ $c_2 \mapsto -u_1w_2,$
 $w_1 \mapsto -(b_1+s_1)w_2 - \epsilon w_1c_2 + \epsilon c_1w_2,$ $w_2 \mapsto (b_1+s_1)w_2,$
$\}$.

(160629) Let $A = (A_0 = \langle 1\rangle)\oplus A_{>0}$ be a graded unital algebra over an augmented ring $\eta\colon R \to \mathbb{Q}$, and let $T\colon A^{\otimes n} \to A^{\otimes n}$ be such that ???. Then there is a unique