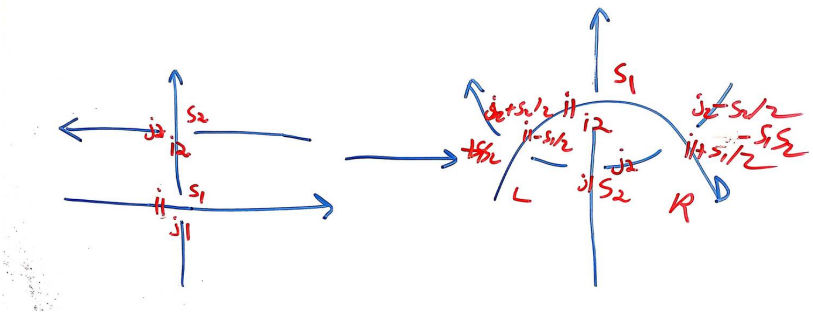
Pensieve header: A program to fix UO sequences in virtual tangle diagrams (with profiling).

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\OU"];
        << "../Profile/Profile.m"
```

```
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.

This version: April 2020. Original version: July 1994.
```

```
In[•]:= SetAttributes[VD, Orderless]
```

```
In[•]:= vd = VD[X_-1[4, 1], EOS[5], X_-1[3, 6], X_-1[7, 2], EOS[8]]
Out[•]= VD[EOS[5], EOS[8], X_-1[3, 6], X_-1[4, 1], X_-1[7, 2]]

In[•]:= js = Cases[vd, X_[_, j_] :> j] ⋂ Cases[vd, X_[i_, _] :> i - 1]
Out[•]= {2, 6}

In[•]:= j1 = First[js]
Out[•]= 2

In[•]:= {{s1, i1, j1}} = Cases[vd, X_s_[i_, j1] :> {s, i, j1}]
Out[•]= {{-1, 7, 2}}

In[•]:= {{s2, i2, j2}} = Cases[vd, X_s_[j1 + 1, j_] :> {s, j1 + 1, j}]
Out[•]= {{-1, 3, 6}}

In[•]:= Complement[vd, VD[X_s1[i1, j1], X_s2[i2, j2]]]
Out[•]= VD[EOS[5], EOS[8], X_-1[4, 1]]
```




```
In[•]:= out = Union[
          Complement[vd, VD[X_s1[i1, j1], X_s2[i2, j2]]],
          VD[X_s2[j1, j2], X_s1[i1, i2], X_s1 s2[i1 - .3 s1, j2 + .3 s2], X_-s1 s2[i1 + .3 s1, j2 - .3 s2]]
         ]
Out[•]= VD[EOS[5], EOS[8], X_-1[2, 6], X_-1[4, 1], X_-1[6.7, 6.3], X_-1[7, 3], X_1[7.3, 5.7]]
```

```
In[•]:= Tidy[vd_VD] := PP_Tidy@Module[{ps = Union @@ (List @@@ vd)},
           Replace[vd, Thread[ps → Range@Length@ps], {2}] ]
```

```
In[•]:= Tidy[out]
Out[•]= VD[EOS[5], EOS[12], X_1[2, 7], X_1[4, 1], X_1[9, 8], X_1[10, 3], X_1[11, 6]]
```

```
In[•]:= γ[vd_VD] := PP_γ@Module[{js, s1, i1, j1, s2, i2, j2},
          js = Cases[vd, X_[_, j_] :> j] ⋂ Cases[vd, X_[i_, _] :> i - 1];
          If[Length[js] == 0, vd,
           j1 = First[js];
           {{s1, i1, j1}} = Cases[vd, X_s_[i_, j1] :> {s, i, j1}];
           {{s2, i2, j2}} = Cases[vd, X_s_[j1 + 1, j_] :> {s, j1 + 1, j}];
           Tidy@Union[
             Complement[vd, VD[X_s1[i1, j1], X_s2[i2, j2]]],
             VD[X_s2[j1, j2], X_s1[i1, i2],
              X_s1s2[i1 - .3 s1, j2 + .3 s2], X_-s1s2[i1 + .3 s1, j2 - .3 s2]]
            ]
          ]]
```

```
In[•]:= Γ[vd_VD] := PP_Γ@FixedPoint[γ, vd, 2^8]
```

```
In[•]:= Γ[T_] /; Head[T] =!= VD := Γ[VD[T]]
```

```
vd // γ
Out[•]= VD[EOS[5], EOS[12], X_1[2, 7], X_1[4, 1], X_1[9, 8], X_1[10, 3], X_1[11, 6]]
```

```
vd // γ // γ
Out[•]= VD[EOS[7], EOS[16], X_1[1, 10], X_1[4, 11],
 X_1[5, 2], X_1[13, 12], X_1[14, 3], X_1[6, 9], X_1[15, 8]]
```

```
vd // γ // γ // γ
Out[•]= VD[EOS[7], EOS[20], X_1[1, 10], X_1[3, 12], X_1[5, 2],
 X_1[15, 14], X_1[16, 13], X_1[17, 4], X_1[6, 9], X_1[18, 11], X_1[19, 8]]
```

```
vd // γ // γ // γ // γ
Out[•]= VD[EOS[9], EOS[24], X_1[1, 12], X_1[2, 15], X_1[5, 16], X_1[6, 3],
 X_1[19, 18], X_1[20, 17], X_1[21, 4], X_1[7, 14], X_1[8, 11], X_1[22, 13], X_1[23, 10]]
```

```
vd // Γ // Short
Out[•]//Short= VD[EOS[261], EOS[1032], X_1[1, 264], X_1[2, 267], X_1[3, 270], <<507>>,
  X_1[1027, 274], X_1[1028, 271], X_1[1029, 268], X_1[1030, 265], X_1[1031, 262]]
```

```
In[•]:= VPB[n_, {σs___}] := VPB[n, σs];
```

```
In[•]:= VD /: vd1_VD ** vd2_VD := PP"**"@Module[{es1, es2, m2},
           es1 = Cases[vd1, EOS[i_] :> i];
           m2 = Max[es2 = Cases[vd2, EOS[i_] :> i]];
           Tidy[vd1 ⋃ Replace[DeleteCases[vd2, _EOS],
              i_ :> i / m2 - 1 + es1[[1 + Count[es2, e_ /; i > e]]], {2}]]
         ]
```

```
In[•]:= vd
Out[•]= VD[EOS[5], EOS[8], X_1[3, 6], X_1[4, 1], X_1[7, 2]]

In[•]:= vd ** vd
Out[•]= VD[EOS[9], EOS[14], X_1[3, 10], X_1[4, 1], X_1[7, 12], X_1[8, 5], X_1[11, 2], X_1[13, 6]]

In[•]:= vd ** vd ** vd
Out[•]= VD[EOS[13], EOS[20], X_1[3, 14], X_1[4, 1], X_1[7, 16],
          X_1[8, 5], X_1[11, 18], X_1[12, 9], X_1[15, 2], X_1[17, 6], X_1[19, 10]]
```

```
In[•]:= VD[VPB[n_]] := VD @@ (EOS /@ Range[n]);
        VD[VPB[n_, σ_{i_,j_}]] := Tidy@Append[VD @@ (EOS /@ Range[n]), X_{+1}[i - 0.5, j - 0.5]];
        VD[VPB[n_, σ̄_{i_,j_}]] := Tidy@Append[VD @@ (EOS /@ Range[n]), X_{-1}[i - 0.5, j - 0.5]];
        VD[VPB[n_, σ_, σs__]] := VD[VPB[n, σ]] ** VD[VPB[n, σs]]

In[•]:= VD[VPB[5, σ̄_{4,2}]]
Out[•]= VD[EOS[1], EOS[3], EOS[4], EOS[6], EOS[7], X_1[5, 2]]

In[•]:= vd1 = VD[VPB[5, σ_{2,3}]]
Out[•]= VD[EOS[1], EOS[3], EOS[5], EOS[6], EOS[7], X_1[2, 4]]

In[•]:= vd2 = VD[VPB[5, σ_{3,4}]]
Out[•]= VD[EOS[1], EOS[2], EOS[4], EOS[6], EOS[7], X_1[3, 5]]

In[•]:= VD[VPB[5, σ_{2,3}, σ_{3,4}]]
Out[•]= VD[EOS[1], EOS[3], EOS[6], EOS[8], EOS[9], X_1[2, 4], X_1[5, 7]]

        VD[VPB[5, σ_{2,3}, σ_{3,4}]] // Γ
Out[•]= VD[EOS[1], EOS[5], EOS[8], EOS[12], EOS[13], X_1[4, 9], X_1[2, 11], X_1[3, 7], X_1[6, 10]]

        VPB[3, σ_{1,2}, σ_{1,3}, σ_{2,3}] // Γ
Out[•]= VD[EOS[5], EOS[8], EOS[13], X_1[3, 10], X_1[1, 12], X_1[2, 7], X_1[4, 9], X_1[6, 11]]

        VPB[3, σ_{2,3}, σ_{1,3}, σ_{1,2}] // Γ
Out[•]= VD[EOS[3], EOS[6], EOS[9], X_1[1, 8], X_1[2, 5], X_1[4, 7]]
```

```
In[•]:= R2ReduceB[vd_VD] := PP_R2ReduceB@Module[{R2s, R2},
          R2s = Cases[vd, X_s_[i_, j_] :> X_-s[i + 1, j + 1]] ⋂ (List @@ vd);
          If[Length[R2s] == 0, vd,
           R2 = First@R2s;
           Tidy@Complement[vd, VD[R2, R2 /. X_s_[i_, j_] :> X_-s[i - 1, j - 1]]]
          ]]

In[•]:= R2ReduceC[vd_VD] := PP_R2ReduceC@Module[{R2s, R2},
          R2s = Cases[vd, X_s_[i_, j_] :> X_-s[i + 1, j - 1]] ⋂ (List @@ vd);
          If[Length[R2s] == 0, vd,
           R2 = First@R2s;
           Tidy@Complement[vd, VD[R2, R2 /. X_s_[i_, j_] :> X_-s[i - 1, j + 1]]]
          ]]

In[•]:= R2Reduce[vd_VD] := PP_R2Reduce@FixedPoint[R2ReduceB @* R2ReduceC, vd]

In[•]:= R1Reduce1[vd_VD] := PP_R1Reduce1@Tidy@DeleteCases[vd, X_[i_, j_] /; Abs[i - j] == 1]

In[•]:= R12Reduce[vd_VD] := PP_R12Reduce@FixedPoint[R2ReduceB @* R2ReduceC @* R1Reduce1, vd]

In[•]:= VPB[3, σ_1,2, σ_1,3, σ_2,3] // Γ // R2Reduce
Out[•]= VD[EOS[3], EOS[6], EOS[9], X_1[1, 8], X_1[2, 5], X_1[4, 7]]

In[•]:= VPB[2, σ_1,2, σ_2,1] // VD
Out[•]= VD[EOS[3], EOS[6], X_1[1, 4], X_1[5, 2]]

In[•]:= VPB[2, σ_1,2, σ_2,1] // VD // Γ
Out[•]= VD[EOS[7], EOS[10], X_-1[3, 4], X_1[1, 6], X_1[2, 9], X_1[8, 5]]

In[•]:= VPB[2, σ_1,2, σ_2,1] // VD // Γ // R2Reduce
Out[•]= VD[EOS[7], EOS[10], X_-1[3, 4], X_1[1, 6], X_1[2, 9], X_1[8, 5]]

In[•]:= VPB[2, σ_1,2, σ_2,1] // VD // Γ // R12Reduce
Out[•]= VD[EOS[5], EOS[8], X_1[1, 4], X_1[2, 7], X_1[6, 3]]

In[•]:= Test1[n_, m_] := Module[{gens, i, j, k, l},
          gens = Flatten@Table[{σ_i,j, σ̄_i,j}, {i, n}, {j, DeleteCases[Range@n, i]}];
          Table[
           {i, j, k} = ijk;
           R12Reduce[Γ[VPB[n, Sequence @@ p, σ_i,j, σ_i,k, σ_j,k, Sequence @@ q]]] ==
            R12Reduce[Γ[VPB[n, Sequence @@ p, σ_j,k, σ_i,k, σ_i,j, Sequence @@ q]]],
           {l, 0, m - 3}, {p, Tuples[gens, l]}, {q, Tuples[gens, m - 3 - l]},
           {ijk, Join @@ (Permutations /@ Subsets[Range[n], {3}])}
          ]]
```

In[•]:=
```
Test1[3, 3]
```

Out[•]=
```
{{{{True, True, True, True, True, True}}}}
```

In[•]:=
```
Union@Flatten@Test1[4, 5]
```

Out[•]=
```
{True}
```

In[•]:=
```
Union@Flatten@Test1[5, 4]
```

Out[•]=
```
{True}
```

In[•]:=
```
Test2[n_, m_] := Module[{gens, s, r = 0, ij, ijk, ijkl, perm, i, j, k, l, tests},
  gens = Flatten@Table[{σ_{i,j}, σ̄_{i,j}}, {i, n}, {j, DeleteCases[Range@n, i]}];
  tests = Flatten[{
      Table[{i, j} = ij; {
        T[VPB[n, Join[p, {σ_{i,j}, σ̄_{i,j}}, q]], VPB[n, Join[p, q]]],
        T[VPB[n, Join[p, {σ̄_{i,j}, σ_{i,j}}, q]], VPB[n, Join[p, q]]]
       },
       {s, 0, m - 2}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
       {ij, Join @@ (Permutations /@ Subsets[Range[n], {2}])}
      ],
      Table[{i, j, k} = ijk; {
        T[VPB[n, Join[p, {σ_{i,j}, σ_{i,k}, σ_{j,k}}, q]], VPB[n, Join[p, {σ_{j,k}, σ_{i,k}, σ_{i,j}}, q]]],
        T[VPB[n, Join[p, {σ̄_{j,i}, σ_{i,k}, σ_{j,k}}, q]], VPB[n, Join[p, {σ_{j,k}, σ_{i,k}, σ̄_{j,i}}, q]]],
        T[VPB[n, Join[p, {σ_{i,j}, σ_{i,k}, σ̄_{k,j}}, q]], VPB[n, Join[p, {σ̄_{k,j}, σ_{i,k}, σ_{i,j}}, q]]],
        T[VPB[n, Join[p, {σ_{i,j}, σ̄_{k,i}, σ̄_{k,j}}, q]], VPB[n, Join[p, {σ̄_{k,j}, σ̄_{k,i}, σ_{i,j}}, q]]],
        T[VPB[n, Join[p, {σ̄_{j,i}, σ̄_{k,i}, σ_{j,k}}, q]], VPB[n, Join[p, {σ_{j,k}, σ̄_{k,i}, σ̄_{j,i}}, q]]],
        T[VPB[n, Join[p, {σ̄_{j,i}, σ̄_{k,i}, σ̄_{k,j}}, q]], VPB[n, Join[p, {σ̄_{k,j}, σ̄_{k,i}, σ̄_{j,i}}, q]]]
       },
       {s, 0, m - 3}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
       {ijk, Join @@ (Permutations /@ Subsets[Range[n], {3}])}
      ],
      Table[{i, j, k, l} = ijkl[[perm]]; {
        T[VPB[n, Join[p, {σ_{i,j}, σ_{k,l}}, q]], VPB[n, Join[p, {σ_{k,l}, σ_{i,j}}, q]]],
        T[VPB[n, Join[p, {σ̄_{i,j}, σ_{k,l}}, q]], VPB[n, Join[p, {σ_{k,l}, σ̄_{i,j}}, q]]],
        T[VPB[n, Join[p, {σ_{i,j}, σ̄_{k,l}}, q]], VPB[n, Join[p, {σ̄_{k,l}, σ_{i,j}}, q]]],
        T[VPB[n, Join[p, {σ̄_{i,j}, σ̄_{k,l}}, q]], VPB[n, Join[p, {σ̄_{k,l}, σ̄_{i,j}}, q]]]
       },
       {s, 0, m - 2}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
       {ijkl, Subsets[Range[n], {4}]}, {perm, {{1, 2, 3, 4}, {1, 3, 2, 4}, {1, 4, 2, 3}}}
      ]
     }];
  Cases[tests, T[b1_, b2_] /; R12Reduce[Γ[b1]] =!= R12Reduce[Γ[b2]]]
 ]
```

In[•]:=
```
Test2[3, 3]
```

Out[•]=
```
{}
```

```
In[•]:= Test2[3, 4]
Out[•]= {}

In[•]:= Test2[3, 5]
Out[•]= {}

In[•]:= Test2[4, 2]
Out[•]= {}

In[•]:= Test2[4, 3]
Out[•]= {}

        VPB[3, σ̄1,2, σ1,3, σ2,3] // Γ // R2Reduce
Out[•]= VD[EOS[5], EOS[8], EOS[13], X-1[2, 7], X-1[3, 12], X1[1, 10], X1[4, 9], X1[6, 11]]

        VPB[3, σ2,3, σ1,3, σ̄1,2] // Γ // R2Reduce
Out[•]= VD[EOS[3], EOS[6], EOS[9], X-1[2, 5], X1[1, 8], X1[4, 7]]

In[•]:= AllVPBInvariants[n_, m_] := Module[{gens, k},
          gens = Flatten@Table[{σi,j, σ̄i,j}, {i, n}, {j, DeleteCases[Range@n, i]}];
          Flatten@Table[VPB[n, Sequence @@ p] → R12Reduce@Γ@VPB[n, Sequence @@ p],
            {k, 0, m}, {p, Tuples[gens, k]}]]

In[•]:= AllVPBInvariants[2, 2] // Column
        VPB[2] → VD[EOS[1], EOS[2]]
        VPB[2, σ1,2] → VD[EOS[2], EOS[4], X1[1, 3]]
        VPB[2, σ̄1,2] → VD[EOS[2], EOS[4], X-1[1, 3]]
        VPB[2, σ2,1] → VD[EOS[2], EOS[4], X1[3, 1]]
        VPB[2, σ̄2,1] → VD[EOS[2], EOS[4], X-1[3, 1]]
        VPB[2, σ1,2, σ1,2] → VD[EOS[3], EOS[6], X1[1, 4], X1[2, 5]]
        VPB[2, σ1,2, σ̄1,2] → VD[EOS[1], EOS[2]]
        VPB[2, σ1,2, σ2,1] → VD[EOS[7], EOS[10], X-1[3, 4], X1[1, 6], X1[2, 9], X1[8, 5]]
        VPB[2, σ1,2, σ̄2,1] → VD[EOS[7], EOS[10], X-1[1, 4], X-1[8, 5], X1[2, 9], X1[3, 6]]
        VPB[2, σ̄1,2, σ1,2] → VD[EOS[1], EOS[2]]
Out[•]= VPB[2, σ̄1,2, σ̄1,2] → VD[EOS[3], EOS[6], X-1[1, 4], X-1[2, 5]]
        VPB[2, σ̄1,2, σ2,1] → VD[EOS[7], EOS[10], X-1[2, 9], X-1[3, 6], X1[1, 4], X1[8, 5]]
        VPB[2, σ̄1,2, σ̄2,1] → VD[EOS[7], EOS[10], X-1[1, 6], X-1[2, 9], X-1[8, 5], X1[3, 4]]
        VPB[2, σ2,1, σ1,2] → VD[EOS[3], EOS[10], X-1[6, 7], X1[1, 8], X1[4, 9], X1[5, 2]]
        VPB[2, σ2,1, σ̄1,2] → VD[EOS[3], EOS[10], X-1[1, 8], X-1[4, 7], X1[5, 2], X1[6, 9]]
        VPB[2, σ2,1, σ2,1] → VD[EOS[3], EOS[6], X1[4, 1], X1[5, 2]]
        VPB[2, σ2,1, σ̄2,1] → VD[EOS[1], EOS[2]]
        VPB[2, σ̄2,1, σ1,2] → VD[EOS[3], EOS[10], X-1[5, 2], X-1[6, 9], X1[1, 8], X1[4, 7]]
        VPB[2, σ̄2,1, σ̄1,2] → VD[EOS[3], EOS[10], X-1[1, 8], X-1[4, 9], X-1[5, 2], X1[6, 7]]
        VPB[2, σ̄2,1, σ2,1] → VD[EOS[1], EOS[2]]
        VPB[2, σ̄2,1, σ̄2,1] → VD[EOS[3], EOS[6], X-1[4, 1], X-1[5, 2]]
```

```
In[•]:= VPBGenerators[n_] :=
          VPBGenerators[n] = Flatten@Table[{σ_{i,j}, σ̄_{i,j}}, {i, n}, {j, DeleteCases[Range@n, i]}];

In[•]:= VPBGenerators[5]

Out[•]= {σ_{1,2}, σ̄_{1,2}, σ_{1,3}, σ̄_{1,3}, σ_{1,4}, σ̄_{1,4}, σ_{1,5}, σ̄_{1,5}, σ_{2,1}, σ̄_{2,1}, σ_{2,3}, σ̄_{2,3},
         σ_{2,4}, σ̄_{2,4}, σ_{2,5}, σ̄_{2,5}, σ_{3,1}, σ̄_{3,1}, σ_{3,2}, σ̄_{3,2}, σ_{3,4}, σ̄_{3,4}, σ_{3,5}, σ̄_{3,5}, σ_{4,1}, σ̄_{4,1},
         σ_{4,2}, σ̄_{4,2}, σ_{4,3}, σ̄_{4,3}, σ_{4,5}, σ̄_{4,5}, σ_{5,1}, σ̄_{5,1}, σ_{5,2}, σ̄_{5,2}, σ_{5,3}, σ̄_{5,3}, σ_{5,4}, σ̄_{5,4}}

In[•]:= (*CountOUForms[n_,m_]:=Module[{k},
          Length@Union@Flatten@Table[
              R12Reduce@Γ@VPB[n,Sequence@@p], {k,0,m},{p,Tuples[VPBGenerators[n],k]}]]*)

In[•]:= ProudFollowers[n_, σ_{i_,j_}] := ProudFollowers[n, σ_{i,j}] = PP_ProudFollowers@Module[{p, q, s},
            Flatten@{σ_{i,j}, σ_{j,i}, σ̄_{j,i},
              Table[{σ_{p,q}, σ_{q,p}, σ̄_{p,q}, σ̄_{q,p}}, {p, {i, j}}, {q, Complement[Range[n], {i, j}]}],
              Table[{σ_{p,q}, σ̄_{p,q}},
                {p, Complement[Range[i + 1, n], {j}]}, {q, Complement[Range[n], {i, j, p}]}]
             }];
        ProudFollowers[n_, σ̄_{i_,j_}] := ProudFollowers[n, σ̄_{i,j}] = ProudFollowers[n, σ_{i,j}] /. σ_{i,j} → σ̄_{i,j}

In[•]:= ProudFollowers[5, σ_{2,3}]

Out[•]= {σ_{2,3}, σ_{3,2}, σ̄_{3,2}, σ_{2,1}, σ_{1,2}, σ̄_{2,1}, σ̄_{1,2}, σ_{2,4}, σ_{4,2}, σ̄_{2,4}, σ̄_{4,2}, σ_{2,5}, σ_{5,2}, σ̄_{2,5}, σ̄_{5,2}, σ_{3,1}, σ_{1,3}, σ̄_{3,1},
         σ̄_{1,3}, σ_{3,4}, σ_{4,3}, σ̄_{3,4}, σ̄_{4,3}, σ_{3,5}, σ_{5,3}, σ̄_{3,5}, σ̄_{5,3}, σ_{4,1}, σ̄_{4,1}, σ_{4,5}, σ̄_{4,5}, σ_{5,1}, σ̄_{5,1}, σ_{5,4}, σ̄_{5,4}}

In[•]:= ProudFollowers[5, σ̄_{2,3}]

Out[•]= {σ̄_{2,3}, σ_{3,2}, σ̄_{3,2}, σ_{2,1}, σ_{1,2}, σ̄_{2,1}, σ̄_{1,2}, σ_{2,4}, σ_{4,2}, σ̄_{2,4}, σ̄_{4,2}, σ_{2,5}, σ_{5,2}, σ̄_{2,5}, σ̄_{5,2}, σ_{3,1}, σ_{1,3}, σ̄_{3,1},
         σ̄_{1,3}, σ_{3,4}, σ_{4,3}, σ̄_{3,4}, σ̄_{4,3}, σ_{3,5}, σ_{5,3}, σ̄_{3,5}, σ̄_{5,3}, σ_{4,1}, σ̄_{4,1}, σ_{4,5}, σ̄_{4,5}, σ_{5,1}, σ̄_{5,1}, σ_{5,4}, σ̄_{5,4}}

In[•]:= ProudVPBs[n_, 0] := {VPB[n]};
        ProudVPBs[n_, 1] := VPB[n, #] & /@ VPBGenerators[n];
        ProudVPBs[n_, m_] /; m > 1 := PP_ProudVPBs[
          Flatten[ProudVPBs[n, m - 1] /.
            VPB[n, σs___, σ_] :↦ (VPB[n, σs, σ, #] & /@ ProudFollowers[n, σ])]]

In[•]:= ProudVPBs[2, 2]

Out[•]= {VPB[2, σ_{1,2}, σ_{1,2}], VPB[2, σ_{1,2}, σ_{2,1}], VPB[2, σ_{1,2}, σ̄_{2,1}], VPB[2, σ̄_{1,2}, σ̄_{1,2}],
         VPB[2, σ̄_{1,2}, σ_{2,1}], VPB[2, σ̄_{1,2}, σ̄_{2,1}], VPB[2, σ_{2,1}, σ_{2,1}], VPB[2, σ_{2,1}, σ_{1,2}],
         VPB[2, σ_{2,1}, σ̄_{1,2}], VPB[2, σ̄_{2,1}, σ̄_{2,1}], VPB[2, σ̄_{2,1}, σ_{1,2}], VPB[2, σ̄_{2,1}, σ̄_{1,2}]}
```

In[•]:=
```
ProudVPBs[3, 3]
```

Out[•]=
$$\{\text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \sigma_{1,2}], \text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \sigma_{2,1}],$$
$$\text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \overline{\sigma}_{2,1}], \text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \sigma_{1,3}], \text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \sigma_{3,1}],$$
$$\text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \overline{\sigma}_{1,3}], \text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \overline{\sigma}_{3,1}], \text{VPB}[3, \sigma_{1,2}, \sigma_{1,2}, \sigma_{2,3}],$$
$$\cdots 1436 \cdots, \text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \sigma_{1,3}], \text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \sigma_{3,1}],$$
$$\text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{1,3}], \text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{3,1}], \text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \sigma_{2,3}],$$
$$\text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \sigma_{3,2}], \text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{2,3}], \text{VPB}[3, \overline{\sigma}_{3,2}, \overline{\sigma}_{1,2}, \overline{\sigma}_{3,2}]\}$$

large output | show less | show more | show all | set size limit...

In[•]:=
```
CountOUForms[n_, m_] := PP_CountOUForms@Module[{k},
   Length@Union@Flatten@Table[R12Reduce@Γ@vpb, {k, 0, m}, {vpb, ProudVPBs[n, k]}]]
```

In[•]:=
```
Timing@CountOUForms[2, 1]
```
Out[•]= {0., 5}

In[•]:=
```
Timing@CountOUForms[2, 2]
```
Out[•]= {0.015625, 17}

In[•]:=
```
Timing@CountOUForms[2, 3]
```
Out[•]= {0.046875, 53}

In[•]:=
```
Timing@CountOUForms[2, 4]
```
Out[•]= {0.328125, 161}

In[•]:=
```
Timing@CountOUForms[2, 5]
```
Out[•]= {4.92188, 485}

In[•]:=
```
Timing@CountOUForms[2, 6]
```
Out[•]= {64.5313, 1457}

In[•]:=
```
FindSequenceFunction[{5, 17, 53, 161, 485, 1457}]
```
Out[•]= $-1 + 2 \times 3^{\#1}$ &

In[•]:=
```
FindLinearRecurrence[{5, 17, 53, 161, 485, 1457}]
```
Out[•]= {4, −3}

In[•]:=
```
Timing@CountOUForms[3, 1]
```
Out[•]= {0., 13}

In[•]:=
```
Timing@CountOUForms[3, 2]
```
Out[•]= {0.0625, 145}

```
In[•]:= Timing@CountOUForms[3, 3]
Out[•]= {1.03125, 1561}

In[•]:= Timing@CountOUForms[3, 4]
Out[•]= {20.1875, 16 717}

In[•]:= Timing@CountOUForms[3, 5]
Out[•]= {484.328, 178 873}

In[•]:= Timing@CountOUForms[3, 6]

        {17038.5`, 1 913 737}

In[•]:= 17038.5`^2 / 484.328125`
Out[•]= 599 409.

In[•]:= FindSequenceFunction[{13, 145, 1561, 16 717, 178 873, 1 913 737}]
Out[•]= FindSequenceFunction[{13, 145, 1561, 16 717, 178 873, 1 913 737}]

In[•]:= Timing@CountOUForms[4, 1]
Out[•]= {0.015625, 25}

In[•]:= Timing@CountOUForms[4, 2]
Out[•]= {0.1875, 529}

In[•]:= Timing@CountOUForms[4, 3]
Out[•]= {6.34375, 10 873}

In[•]:= Timing@CountOUForms[4, 4]
Out[•]= {243.438, 222 289}

In[•]:= Timing@CountOUForms[4, 5]
Out[•]= {9002.38, 4 540 201}

In[•]:= 9002.375`^2 / 243.4375`
Out[•]= 332 910.

In[•]:= {25, 529, 10 873, 222 289, 4 540 201}
Out[•]= {25, 529, 10 873, 222 289, 4 540 201}

In[•]:= Timing@CountOUForms[5, 1]
Out[•]= {0.015625, 41}

In[•]:= Timing@CountOUForms[5, 2]
Out[•]= {0.453125, 1361}
```

```
In[•]:= Timing@CountOUForms[5, 3]
Out[•]= {24.5156, 43 121}

In[•]:= Timing@CountOUForms[5, 4]
Out[•]= {1459.64, 1 351 481}

In[•]:= 1459.640625`^2 / 24.515625`
Out[•]= 86 905.8

In[•]:= Timing@CountOUForms[5, 5]

In[•]:= {41, 1361, 43 121, 1 351 481}
Out[•]= {41, 1361, 43 121, 1 351 481}

In[•]:= Timing@CountOUForms[6, 1]
Out[•]= {0., 61}

In[•]:= Timing@CountOUForms[6, 2]
Out[•]= {1.64063, 2881}

In[•]:= Timing@CountOUForms[6, 3]
Out[•]= {77.2969, 127 021}

In[•]:= Timing@CountOUForms[6, 4]
Out[•]= {6666.34, 5 484 721}

In[•]:= FindSequenceFunction@{61, 2881, 127 021, 5 484 721}
Out[•]= FindSequenceFunction[{61, 2881, 127 021, 5 484 721}]

In[•]:= Timing@CountOUForms[7, 1]
Out[•]= {0.015625, 85}

In[•]:= Timing@CountOUForms[7, 2]
Out[•]= {1.90625, 5377}

In[•]:= Timing@CountOUForms[7, 3]
Out[•]= {186.781, 310 633}

In[•]:= {85, 5377, 310 633}
Out[•]= {85, 5377, 310 633}

In[•]:= Timing@CountOUForms[8, 1]
Out[•]= {0.015625, 113}

In[•]:= Timing@CountOUForms[8, 2]
Out[•]= {3.25, 9185}
```

```
In[•]:= Timing@CountOUForms[8, 3]
Out[•]= {493., 668 081}

In[•]:= {113, 9185, 668 081}
Out[•]= {113, 9185, 668 081}

In[•]:= Timing@CountOUForms[9, 1]
Out[•]= {0.03125, 145}

In[•]:= Timing@CountOUForms[9, 2]
Out[•]= {5.9375, 14 689}

In[•]:= Timing@CountOUForms[9, 3]
Out[•]= {1362.98, 1 307 233}

In[•]:= {145, 14 689}
Out[•]= {145, 14 689}
```

CountOUForms[n,1]:

```
In[•]:= n // FindSequenceFunction@{1, 5, 13, 25, 41, 61, 85, 113, 145}
Out[•]= 1 - 2 n + 2 n^2
```

CountOUForms[n,2]:

```
In[•]:= n // FindSequenceFunction@{1, 17, 145, 529, 1361, 2881, 5377, 9185, 14 689}
Out[•]= 1 + 12 n - 18 n^2 + 4 n^3 + 2 n^4
```

CountOUForms[n,3]:

```
In[•]:= n // FindSequenceFunction@{1, 53, 1561, 10 873, 43 121, 127 021, 310 633, 668 081, 1 307 233}
Out[•]= 1/3 (3 - 558 n + 1066 n^2 - 546 n^3 - 2 n^4 + 36 n^5 + 4 n^6)
```

Profiling

```
In[•]:= Timing@CountOUForms[3, 4]
Out[•]= {37.9531, 16 717}
```

In[•]:=
```
BeginProfile[]; Timing@CountOUForms[3, 4]
EndProfile[]; PrintProfile[]
```

Out[•]=
```
{61.2031, 16 717}
```

Out[•]=
```
ProfileRoot is root. Profiled time: 61.203
  (     1)   7.914/ 61.203 above CountOUForms
Tidy: called 207019 times, time in 17.301/17.301
  ( 50952)   3.600/  3.600 under **
  ( 68520)   3.674/  3.674 under CountOUForms
  ( 27187)   2.869/  2.869 under R1Reduce1
  (  3120)   0.343/  0.343 under R2ReduceB
  (  2760)   0.343/  0.343 under R2ReduceC
  ( 54480)   6.472/  6.472 under γ
γ: called 72049 times, time in 12.512/18.984
  ( 72049)  12.512/ 18.984 under Γ
  ( 54480)   6.472/  6.472 above Tidy
**: called 50952 times, time in 9.269/12.869
  ( 50952)   9.269/ 12.869 under CountOUForms
  ( 50952)   3.600/  3.600 above Tidy
CountOUForms: called 1 times, time in 7.914/61.203
  (     1)   7.914/ 61.203 under ProfileRoot
  ( 50952)   9.269/ 12.869 above **
  (     3)      0/     0 above ProudVPBs
  ( 17569)   3.375/ 14.226 above R12Reduce
  ( 68520)   3.674/  3.674 above Tidy
  ( 17569)   3.536/ 22.520 above Γ
Γ: called 17569 times, time in 3.536/22.52
  ( 17569)   3.536/ 22.520 under CountOUForms
  ( 72049)  12.512/ 18.984 above γ
R12Reduce: called 17569 times, time in 3.375/14.226
  ( 17569)   3.375/ 14.226 under CountOUForms
  ( 27187)   2.577/  5.446 above R1Reduce1
  ( 27187)   2.360/  2.703 above R2ReduceB
  ( 27187)   2.359/  2.702 above R2ReduceC
R1Reduce1: called 27187 times, time in 2.577/5.446
  ( 27187)   2.577/  5.446 under R12Reduce
  ( 27187)   2.869/  2.869 above Tidy
R2ReduceB: called 27187 times, time in 2.36/2.703
  ( 27187)   2.360/  2.703 under R12Reduce
  (  3120)   0.343/  0.343 above Tidy
R2ReduceC: called 27187 times, time in 2.359/2.702
  ( 27187)   2.359/  2.702 under R12Reduce
  (  2760)   0.343/  0.343 above Tidy
ProudVPBs: called 6 times, time in 0./0.
  (     3)      0/     0 under CountOUForms
  (     3)      0/     0 under ProudVPBs
  (     3)      0/     0 above ProudVPBs
```

```
In[•]:= BeginProfile[]; Timing@CountOUForms[4, 4]
        EndProfile[]; PrintProfile[]

Out[•]= {584.625, 222 289}

Out[•]= ProfileRoot is root. Profiled time: 584.625
          (      1)   86.916/ 584.625 above CountOUForms
        Tidy: called 2441705 times, time in 159.895/159.895
          ( 680224)   40.288/  40.288 under **
          ( 910832)   42.240/  42.240 under CountOUForms
          ( 295305)   24.240/  24.240 under R1Reduce1
          (  18520)    1.919/   1.919 under R2ReduceB
          (  15848)    1.841/   1.841 under R2ReduceC
          ( 520976)   49.367/  49.367 under γ
        γ: called 751585 times, time in 105.771/155.138
          ( 751585)  105.771/ 155.138 under Γ
          ( 520976)   49.367/  49.367 above Tidy
        **: called 680224 times, time in 104.761/145.049
          ( 680224)  104.761/ 145.049 under CountOUForms
          ( 680224)   40.288/  40.288 above Tidy
        CountOUForms: called 1 times, time in 86.916/584.625
          (      1)   86.916/ 584.625 under ProfileRoot
          ( 680224)  104.761/ 145.049 above **
          (      3)    0.282/   0.297 above ProudVPBs
          ( 230609)   34.066/ 123.018 above R12Reduce
          ( 910832)   42.240/  42.240 above Tidy
          ( 230609)   31.967/ 187.105 above Γ
        R12Reduce: called 230609 times, time in 34.066/123.018
          ( 230609)   34.066/ 123.018 under CountOUForms
          ( 295305)   20.305/  44.545 above R1Reduce1
          ( 295305)   20.376/  22.295 above R2ReduceB
          ( 295305)   20.271/  22.112 above R2ReduceC
        Γ: called 230609 times, time in 31.967/187.105
          ( 230609)   31.967/ 187.105 under CountOUForms
          ( 751585)  105.771/ 155.138 above γ
        R2ReduceB: called 295305 times, time in 20.376/22.295
          ( 295305)   20.376/  22.295 under R12Reduce
          (  18520)    1.919/   1.919 above Tidy
        R1Reduce1: called 295305 times, time in 20.305/44.545
          ( 295305)   20.305/  44.545 under R12Reduce
          ( 295305)   24.240/  24.240 above Tidy
        R2ReduceC: called 295305 times, time in 20.271/22.112
          ( 295305)   20.271/  22.112 under R12Reduce
          (  15848)    1.841/   1.841 above Tidy
        ProudVPBs: called 6 times, time in 0.297/0.312
          (      3)    0.282/   0.297 under CountOUForms
          (      3)    0.015/   0.015 under ProudVPBs
          (     12)       0/      0 above ProudFollowers
          (      3)    0.015/   0.015 above ProudVPBs
        ProudFollowers: called 12 times, time in 0./0.
          (     12)       0/      0 under ProudVPBs
```

In[•]:=
```
?? Profile
```

Out[•]=

Symbol

Profile[label,expr] evaluates expr while taking profiling data under the label label.

Definitions	Profile[Profile`private`label_, Profile`private`expr_] := Profile`private`expr
Attributes	{HoldFirst}
Full Name	Profile`Profile