

Pensieve header: Counting virtual pure braids: Cartesian product diagram generation, reasonable compression.

```
In[*]:= SetAttributes[{EROMake, EROPeek, EROAdjoin}, HoldFirst];
```

```
In[*]:= EROMake[er_, n_Integer] := er = Table[0, n];
```

```
In[*]:= EROPeek[er_, n_Integer] := If[er[[n]] == 0, n, er[[n]] = EROPeek[er, er[[n]]];
```

```
In[*]:= EROAdjoin[er_, new_UndirectedEdge] := EROAdjoin[er, new, Identity];
ERAdjoin[er_, n1_Integer  $\leftrightarrow$  n2_Integer, comp_] := Module[{m1, m2},
  m1 = EROPeek[er, n1]; m2 = EROPeek[er, n2];
  Switch[Order[{comp[m1], m1}, {comp[m2], m2}], 0, m1, 1, er[[m2]] = m1, -1, er[[m1]] = m2 ]
```

```
In[*]:=  $\sigma_{i,j}$  := 10 i + j;  $\bar{\sigma}_{i,j}$  := 100 + 10 i + j;
VPB[xs___Integer] := FromDigits[{xs}, 1000];
```

In[]:=

```

CountVPB[n_, m_] := CountVPB[n, {m, m}];
CountVPB[n_, {m1_, m2_}] :=
Module[{gens, diags, L, d2n, er, s, T, ij, ijk, ijk1, i, j, k, l, perm},
  gens = Flatten@Table[{ $\sigma_{i,j}$ ,  $\bar{\sigma}_{i,j}$ }, {i, n}, {j, DeleteCases[Range@n, i]}];
  L = Length[diags = Flatten[Table[VPB[n, #] & /@ Tuples[gens, s], {s, 0, m1}]]];
  diags = Join[diags, Flatten[Table[VPB[n, #] & /@ Tuples[gens, s], {s, m1 + 1, m2}]]];
  Print[Length@diags, " diagrams..."];
  d2n = Dispatch[Thread[diags  $\rightarrow$  Range@Length@diags]];
  EROMake[er, Length@diags];
  Print["Now relations..."];
  T[b1_, b2_] := EROAdjoin[er, (b1 /. d2n)  $\leftrightarrow$  (b2 /. d2n)];
  Do[{i, j} = ij; {
    T[VPB[n, Join[p, { $\sigma_{i,j}$ ,  $\bar{\sigma}_{i,j}$ }, q]], VPB[n, Join[p, q]]],
    T[VPB[n, Join[p, { $\bar{\sigma}_{i,j}$ ,  $\sigma_{i,j}$ }, q]], VPB[n, Join[p, q]]]
  },
  {s, 0, m2 - 2}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
  {ij, Join@@(Permutations /@ Subsets[Range[n], {2}])}
];
Do[{i, j, k} = ijk; {
  T[VPB[n, Join[p, { $\sigma_{i,j}$ ,  $\sigma_{i,k}$ ,  $\sigma_{j,k}$ }, q]], VPB[n, Join[p, { $\sigma_{j,k}$ ,  $\sigma_{i,k}$ ,  $\sigma_{i,j}$ }, q]]],
  T[VPB[n, Join[p, { $\bar{\sigma}_{j,i}$ ,  $\sigma_{i,k}$ ,  $\sigma_{j,k}$ }, q]], VPB[n, Join[p, { $\sigma_{j,k}$ ,  $\sigma_{i,k}$ ,  $\bar{\sigma}_{j,i}$ }, q]]],
  T[VPB[n, Join[p, { $\sigma_{i,j}$ ,  $\sigma_{i,k}$ ,  $\bar{\sigma}_{k,j}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,j}$ ,  $\sigma_{i,k}$ ,  $\sigma_{i,j}$ }, q]]],
  T[VPB[n, Join[p, { $\sigma_{i,j}$ ,  $\bar{\sigma}_{k,i}$ ,  $\bar{\sigma}_{k,j}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,j}$ ,  $\bar{\sigma}_{k,i}$ ,  $\sigma_{i,j}$ }, q]]],
  T[VPB[n, Join[p, { $\bar{\sigma}_{j,i}$ ,  $\bar{\sigma}_{k,i}$ ,  $\sigma_{j,k}$ }, q]], VPB[n, Join[p, { $\sigma_{j,k}$ ,  $\bar{\sigma}_{k,i}$ ,  $\bar{\sigma}_{j,i}$ }, q]]],
  T[VPB[n, Join[p, { $\bar{\sigma}_{j,i}$ ,  $\bar{\sigma}_{k,i}$ ,  $\bar{\sigma}_{k,j}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,j}$ ,  $\bar{\sigma}_{k,i}$ ,  $\bar{\sigma}_{j,i}$ }, q]]]
},
{s, 0, m2 - 3}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
{ijk, Join@@(Permutations /@ Subsets[Range[n], {3}])}
];
Do[{i, j, k, l} = ijk1; {
  T[VPB[n, Join[p, { $\sigma_{i,j}$ ,  $\sigma_{k,l}$ }, q]], VPB[n, Join[p, { $\sigma_{k,l}$ ,  $\sigma_{i,j}$ }, q]]],
  T[VPB[n, Join[p, { $\bar{\sigma}_{i,j}$ ,  $\sigma_{k,l}$ }, q]], VPB[n, Join[p, { $\sigma_{k,l}$ ,  $\bar{\sigma}_{i,j}$ }, q]]],
  T[VPB[n, Join[p, { $\sigma_{i,j}$ ,  $\bar{\sigma}_{k,l}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,l}$ ,  $\sigma_{i,j}$ }, q]]],
  T[VPB[n, Join[p, { $\bar{\sigma}_{i,j}$ ,  $\bar{\sigma}_{k,l}$ }, q]], VPB[n, Join[p, { $\bar{\sigma}_{k,l}$ ,  $\bar{\sigma}_{i,j}$ }, q]]]
},
{s, 0, m2 - 2}, {t, 0, s}, {p, Tuples[gens, t]}, {q, Tuples[gens, s - t]},
{ijk1, Join@@(Permutations /@ Subsets[Range[n], {4}])}
];
Count[Take[er, L], 0]
]

```

In[]:= VPB[4, $\sigma_{4,1}$, $\bar{\sigma}_{2,3}$]

Out[]:= 4 041 123

```
In[ ]:= CountVPB[2, 1]
```

```
5 diagrams...
```

```
Out[ ]:= 5
```

```
In[ ]:= CountVPB[2, 2]
```

```
21 diagrams...
```

```
Out[ ]:= 17
```

```
In[ ]:= Timing@CountVPB[2, 3]
```

```
85 diagrams...
```

```
Out[ ]:= {0., 53}
```

```
In[ ]:= Timing@CountVPB[2, {3, 4}]
```

```
341 diagrams...
```

```
Out[ ]:= {0., 53}
```

```
In[ ]:= Timing@CountVPB[2, 4]
```

```
341 diagrams...
```

```
Out[ ]:= {0., 161}
```

```
In[ ]:= Timing@CountVPB[3, 1]
```

```
13 diagrams...
```

```
Out[ ]:= {0., 13}
```

```
In[ ]:= Timing@CountVPB[3, 2]
```

```
157 diagrams...
```

```
Out[ ]:= {0., 145}
```

```
In[ ]:= Timing@CountVPB[3, 3]
```

```
1885 diagrams...
```

```
Out[ ]:= {0.015625, 1561}
```

```
In[ ]:= Timing@CountVPB[3, {3, 4}]
```

```
22621 diagrams...
```

```
Out[ ]:= {0.265625, 1561}
```

```
In[ ]:= Timing@CountVPB[3, 4]
```

```
22621 diagrams...
```

```
Out[ ]:= {0.28125, 16741}
```

In[]:= **Timing@CountVPB**[3, {4, 5}]

271453 diagrams...

Out[]:= {3.125, 16741}

In[]:= **Timing@CountVPB**[3, {4, 6}]

3257437 diagrams...

Out[]:= {123.203, 16717}

In[]:= **Timing@CountVPB**[3, {4, 7}]

39089245 diagrams...

Out[]:= {753.078, 16717}

In[]:= **Timing@CountVPB**[3, 5]

271453 diagrams...

Out[]:= {3.17188, 179401}

In[]:= **Timing@CountVPB**[3, {5, 6}]

3257437 diagrams...

Out[]:= {280.594, 179377}

In[]:= **Timing@CountVPB**[3, {5, 7}]

39089245 diagrams...

Out[]:= 178873

In[]:= **Timing@CountVPB**[4, 1]

25 diagrams...

Out[]:= {0., 25}

In[]:= **Timing@CountVPB**[4, 2]

601 diagrams...

Out[]:= {0., 529}

In[]:= **Timing@CountVPB**[4, 3]

14425 diagrams...

Out[]:= {0.265625, 10873}

In[]:= **Timing@CountVPB**[4, 4]

346201 diagrams...

Now relations...

Out[]:= {7.125, 222385}

In[*]:= **Timing@CountVPB**[4, {4, 5}]

8308825 diagrams...

Out[*]:= {229.563, 222385}

In[*]:= **Timing@CountVPB**[4, {4, 6}]

199411801 diagrams...

Now relations...

In[*]:= **Timing@CountVPB**[5, 1]

41 diagrams...

Out[*]:= {0., 41}

In[*]:= **Timing@CountVPB**[5, 2]

1641 diagrams...

Out[*]:= {0.03125, 1361}

In[*]:= **Timing@CountVPB**[5, 3]

65641 diagrams...

Out[*]:= {1.23438, 43121}