Extending the action to all SolKV:
Groupoid $SolKV^T$

Objects = pairs (F,T) — $F \in SolKV(n)$, T = a tree w/ n leaves

Mor = unique KRV element

$KRV(2) \longrightarrow Aut_{op}(SolKV)$

$G \longmapsto \begin{pmatrix} SolKV(n) \times Trees \longrightarrow SolKV(n) \times Trees \\ (F,T) \longmapsto (F \cdot G^T, T) \end{pmatrix}$

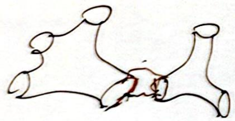
morphisms: the only thing it can be

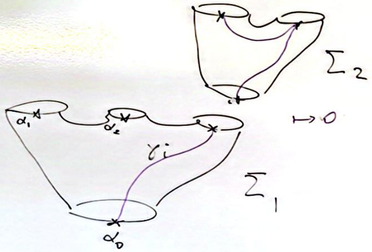
Anton's question:

$krv(n) \overset{?}{\cong} ogrt_1 \oplus t_n$

$krv(n)/t_n = ?$

Q: Do G-T expansions make an operad?



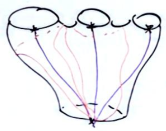


PA(n) = based paths

$PA = \cup PA(n)$

$\circ_i : PA(n) \times PA(m) \longrightarrow PA(n+m-1)$

Alternative: π_1 equipped w/ paths



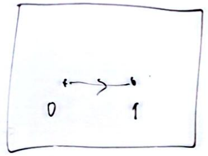
$\{\gamma_1, \ldots \gamma_n\}$

$\left(A = k\overline{\pi}_1,\ \{\cdot,\cdot\} : A \otimes A \to A \otimes A \right.$
$\mu : A \to A \otimes |A|$
$\left. |A| \otimes A \right)$

$\Sigma_1 \circ_i \Sigma_2 \rightsquigarrow A = A_1 \circ_i A_2,\ \{\cdot,\cdot\}^{tot},\ \mu^{tot}$

$A = k\overline{\pi}_1 \xrightarrow{\theta} \bigoplus_{i,j=0,\ldots n} 1_{\alpha_i}\, k\langle\langle x_1, \ldots x_n \rangle\rangle\, 1_{\alpha_j}$

$\{\cdot,\cdot\}_{st}$ μ_{st}



$\theta_1 \circ_i \theta_2 = ?$

Pavol wisdom :

$F \in \mathrm{TAut}(n)$ is a groupoid

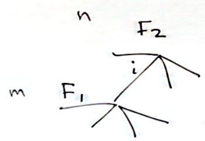
$$\begin{array}{c} \mathrm{TAut}(n) \\ s \downarrow \ \downarrow t \\ \mathrm{Lie}_n \oplus \mathrm{Cycl}(n) \\ \Psi \\ (a, u) \\ \uparrow \ \uparrow \end{array} \qquad \boxed{(a = x_1 + \dots x_n,\ u = 0)}$$

$$\mathcal{G} \subset (\mathrm{Lie}_n \oplus \mathrm{Cycl}_n) \times \mathrm{TAut}(n) \times (\mathrm{Lie}_n \oplus \mathrm{Cycl}_n)$$

$$((a, u), F, (b, v)) \text{ is } F(a) = b,\ F(u) + J(F) = v$$

Prop : this is a groupoid

? Define an operadic structure



$$\xrightarrow{s} \left(a_2(x_1, \dots, x_{i+m-1}, \dots x_{n+m-1}) \right)$$

$$u_2(x_1, \dots a_1(\ ), \dots x_{n+m-1}) + u_1(x_i \dots x_{i+k-1})$$

$$\uparrow \quad a_1(x_i, \dots, x_{i+m-1})$$

$$\xrightarrow{t} \quad a_2(x \dots, b_1(\ ), \dots x)$$

$$\downarrow \quad b_2(x, \dots b_1(\dots), \dots x)$$

$$x_1 \to g_1(x_1, x_2)\, x_1\, g_1(x_1, x_2)^{-1}$$

$$\downarrow$$

$$\tilde{g}_1\, g_1(x_1,x_2)\cancel{g_1^{-1}\tilde{g}_1}\, x_1\, \tilde{g}_1^{-1}$$

$$\tilde{F} \circ F : x_1 \mapsto \tilde{g}_1(\underline{a_1(x_1,x_2)}, x_3)\, x_1\, \tilde{g}_1^{-1}$$

$g_1(x_1,x_2)$

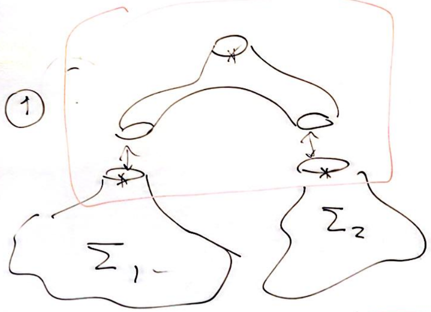
$$x_2 \mapsto \tilde{g}_1(a_1(x_1, x_2$$

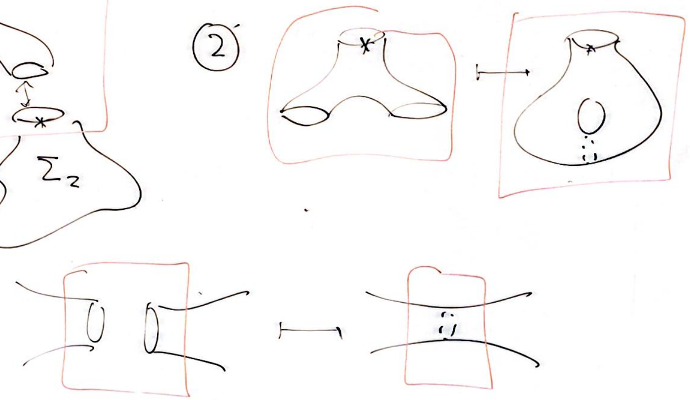
$$x_3 \mapsto \tilde{g}_2(a_1(x_1,x_2) \dots)\, x_3\, \tilde{g}_2^{-1}$$

g_1

Difficulties with TQFT:

① 2 different gluings ①





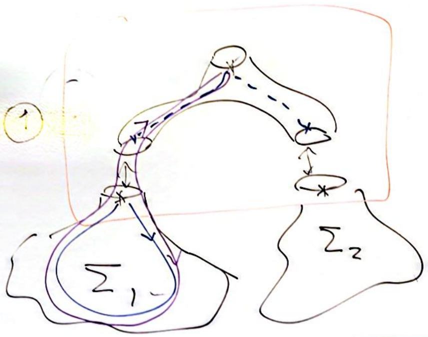
②

$\Sigma_1 = (g_1, n_1+1)$ $\Sigma_2 = (g_2, n_2+1)$

if $n_1 = 0$ or $g_2 = 0$

Difficulties with TQFT:

① 2 different gluings ①



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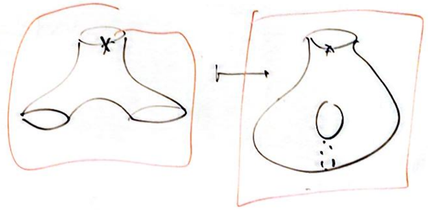
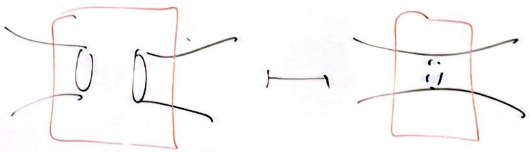
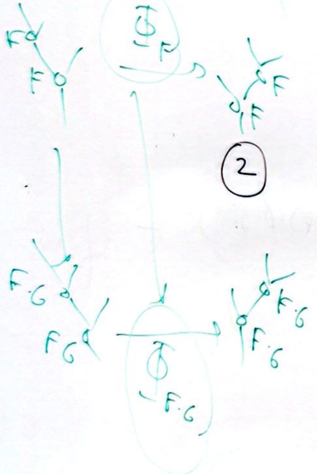




$$KRV(2) \xrightarrow{\cong} \mathrm{Auto}_P(\mathrm{SolKV}_{(2)})$$

↙ operad in groupoids

$$G \longmapsto \left(\mathrm{SolKV}_{(2)}(2) \to \mathrm{SolKV}_{(2)}(2) \atop F \longmapsto F \cdot G \right)$$



②

$$\Phi_{F\cdot G} = (G_{a_1}G)^{-1}\,\Phi_F\,(G_{a_2}G)$$

$\Sigma_1 = (g_1, n_1+1)$ $\Sigma_2 = (g_2, n_2+1)$

if $n_1 = 0$ or $g_2 = 0$

Questions

ANTON:

- GoTu & w-story

skew-symmetry of δ_{Turaev} $\longrightarrow$ $Ig = g \oplus g^*$

$\longrightarrow gl(N)[\varepsilon], \varepsilon^2 = 1$

- higher order analogs of $\delta_{Turaev} \Leftarrow$ Kontsevich δ

$\delta^{(2)}: \ker \delta_{Tu} \longrightarrow (X \; X)$

Zsuzsi:

- Does KV translate to a quantisation problem for Ig?

KV solution has more info than the quantisation. ← Marcy?

Construct $U_\hbar(Ig)$ using KV. ... $\Delta = J \Delta' J^{-1}$

Dror: GoTu is "secondary ops".

Z_{GoTu} should "secondary Z^w/Z^u"

* Is there an abstart general theory?

* A^{wm} should have a "classical subspace". $A^{wm} = \varphi(\ldots)$:

* Z^w would map uKO into $A^{wm}_{classical}$

* a kd, $A^{wgh} \longrightarrow A^{wm}_{classical}$ or two.

* There should be a meaningful Alexander relation on $A^{wm}_{classical}$.

Iva:

- TQFT / operad framework for expansions of GoTu or wF.

  - genus 0 or higher genus

- weight systems for vKO: expansions ⇠ ⇢ quantisations

  ↳ How are A^v related to $U(g \oplus g^*)$?

$GT \longleftrightarrow KV(g,n)$

Marcy:

$GT \not\longrightarrow Aut(\coprod \Gamma_{g,n})$

$GT \xrightarrow{\simeq} Aut(\coprod \Gamma_{0,n})$

$\exists \Lambda \subseteq GT_1 \longrightarrow Aut(\coprod \Gamma_{g,n})$

Hatcher-Lochak-Schneps

Nakamura-Schneps