(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`];
<< Rot.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

(Alt) In[ ]:=

```
{
  {r0,pxx[1, i_, j_], r0,pxx[-1, i_, j_]},
  {r1,ppx[1, i_, j_], r1,ppx[-1, i_, j_]},
  {r1,rest[1, i_, j_], r1,rest[-1, i_, j_]},
  γ1[φ_, k_]
 } = Get["px-data.m"]
```

(Alt) Out[ ]=

$$\left\{\left\{p_{3,j}\,x_{1,i}\,x_{2,i}-\frac{p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1},\ -\frac{p_{3,j}\,x_{1,i}\,x_{2,i}}{T_1^2\,T_2}+\frac{p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1\,T_2}\right\},\right.$$

$$\left\{p_{1,j}\,p_{2,i}\,x_{3,i}-p_{1,j}\,p_{2,j}\,x_{3,i},\ -\frac{p_{1,j}\,p_{2,i}\,x_{3,i}}{T_1}+\frac{p_{1,j}\,p_{2,j}\,x_{3,i}}{T_1}\right\},$$

$$\left\{\frac{T_2\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{-1+T_1\,T_2}-\frac{p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}-\frac{p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1}+\frac{p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+\right.$$
$$\frac{p_{3,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}+\frac{p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{-1+T_1}-$$
$$\frac{T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1}-\frac{p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{T_1\,(-1+T_1\,T_2)},$$
$$\frac{p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}}{T_1^2\,(-1+T_1\,T_2)}-\frac{(-1+T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}+\frac{(-T_1-T_2+T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{T_1^2\,T_2\,(-1+T_1\,T_2)}+$$
$$\frac{p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+$$
$$\frac{p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{3,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}}{T_1^2\,(-1+T_1\,T_2)}+\frac{p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{(-1+T_1)\,T_1\,T_2}-$$
$$\frac{p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{T_1^2\,T_2}-\frac{p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{(-1+T_2)\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}}{T_1\,T_2\,(-1+T_1\,T_2)}+$$
$$\frac{p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1^2\,T_2}-\frac{(-1+2\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1^2\,T_2^2}+\frac{p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{T_1^2\,T_2}+$$
$$\left.\frac{p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{T_1\,(-1+T_1\,T_2)}\right\},\ \left.-\frac{\varphi\,p_{3,k}\,x_{3,k}}{T_1\,(-1+T_1\,T_2)}\right\}$$

In[•]:=
```
r_{1,rest}[1, 4, 5]
```

Out[•]=

$$\frac{T_2\, p_{1,5}\, p_{2,5}\, x_{1,4}\, x_{2,4}}{-1+T_1\,T_2} - \frac{p_{1,5}\, p_{2,4}\, x_{1,5}\, x_{2,4}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} - \frac{p_{1,5}\, p_{2,5}\, x_{1,5}\, x_{2,4}}{(-1+T_1)\,T_1} + \frac{p_{1,4}\, p_{2,5}\, x_{1,4}\, x_{2,5}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{p_{3,5}\, x_{3,4}}{T_1\,(-1+T_1\,T_2)} - p_{1,5}\, p_{3,5}\, x_{1,4}\, x_{3,4} + \frac{p_{1,5}\, p_{3,4}\, x_{1,5}\, x_{3,4}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} + \frac{p_{1,5}\, p_{3,5}\, x_{1,5}\, x_{3,4}}{-1+T_1} - \frac{T_2\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,4}}{T_1} - \frac{p_{2,5}\, p_{3,4}\, x_{2,5}\, x_{3,4}}{T_1\,(-1+T_1\,T_2)} - \frac{p_{1,4}\, p_{3,5}\, x_{1,4}\, x_{3,5}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{T_2\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,5}}{T_1\,(-1+T_1\,T_2)}$$

(Alt) In[•]:=
```
{p*, x*, π*, ξ*} = {π, ξ, p, x};  (u_i__)* := (u*)_i;
```

(Alt) In[•]:=
```
Zip_{}[ℰ_] := ℰ;
Zip_{ξ_, ξs___}[ℰ_] := (Collect[ℰ // Zip_{ξs}, ξ*] /. f_. ξ*^d_. :> (D[f, {ξ*, d}])) /. ξ* → 0
```

(Alt) In[•]:=
```
px2g[ℰ_] := Module[{ps, xs, Q},
  ps = Union[Cases[ℰ, p__, ∞]]; xs = Union[Cases[ℰ, x__, ∞]];
  Q = Sum[p0* x0* g_{p0[[2]],x0[[2]],p0[[3]],x0[[3]]}, {p0, ps}, {x0, xs}];
  Expand[Zip_{ps∪xs}[ℰ e^Q] /. g_{α_,β_,i_,j_} :> If[α === β, g_{α,i,j}, 0]]
 ]
```

In[•]:=
```
px2g[p_{2,j}^2 x_{2,i} x_{2,j}]
```

Out[•]=

$$2\, g_{2,j,i}\, g_{2,j,j}$$

(Alt) In[•]:=
```
R_1[1, i_, j_] = px2g[r_{1,rest}[1, i, j]]
```

(Alt) Out[•]=

$$-\frac{g_{1,j,j}\, g_{2,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} + \frac{T_2\, g_{1,j,i}\, g_{2,j,i}}{-1+T_1\,T_2} - \frac{g_{1,j,j}\, g_{2,j,i}}{(-1+T_1)\,T_1} + \frac{g_{1,i,i}\, g_{2,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{g_{1,j,j}\, g_{3,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} - \frac{g_{2,j,j}\, g_{3,i,i}}{T_1\,(-1+T_1\,T_2)} + \frac{g_{3,j,i}}{T_1\,(-1+T_1\,T_2)} - g_{1,j,i}\, g_{3,j,i} + \frac{g_{1,j,j}\, g_{3,j,i}}{-1+T_1} - \frac{T_2\, g_{2,j,i}\, g_{3,j,i}}{T_1} - \frac{g_{1,i,i}\, g_{3,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{T_2\, g_{2,j,i}\, g_{3,j,j}}{T_1\,(-1+T_1\,T_2)}$$

(Alt) In[•]:=

```
R1[-1, i_, j_] = px2g[r1,rest[-1, i, j]]
```

(Alt) Out[•]=

$$\frac{g_{1,j,i}\,g_{2,i,i}}{T_1^2\,(-1+T_1\,T_2)}+\frac{g_{1,j,j}\,g_{2,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}-\frac{g_{1,i,i}\,g_{2,j,i}}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{g_{1,i,i}\,g_{2,j,i}}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}-$$
$$\frac{g_{1,j,i}\,g_{2,j,i}}{T_1^2\,(-1+T_1\,T_2)}+\frac{g_{1,j,i}\,g_{2,j,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{1,j,i}\,g_{2,j,i}}{T_1\,T_2\,(-1+T_1\,T_2)}+\frac{g_{1,j,j}\,g_{2,j,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{1,i,i}\,g_{2,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+$$
$$\frac{g_{1,j,i}\,g_{2,j,j}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{1,j,i}\,g_{3,i,i}}{T_1^2\,(-1+T_1\,T_2)}-\frac{g_{1,j,j}\,g_{3,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{g_{2,j,i}\,g_{3,i,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{2,j,i}\,g_{3,i,i}}{T_1\,T_2\,(-1+T_1\,T_2)}+$$
$$\frac{g_{2,j,j}\,g_{3,i,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{3,j,i}}{T_1\,(-1+T_1\,T_2)}+\frac{g_{1,i,i}\,g_{3,j,i}}{(-1+T_1)\,T_1\,T_2}-\frac{g_{1,j,i}\,g_{3,j,i}}{T_1^2\,T_2}+\frac{g_{2,i,i}\,g_{3,j,i}}{T_1^2\,T_2}+\frac{g_{2,j,i}\,g_{3,j,i}}{T_1^2\,T_2^2}-$$
$$\frac{2\,g_{2,j,i}\,g_{3,j,i}}{T_1^2\,T_2}-\frac{g_{2,j,j}\,g_{3,j,i}}{T_1^2\,T_2}+\frac{g_{1,i,i}\,g_{3,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{g_{1,j,i}\,g_{3,j,j}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{2,j,i}\,g_{3,j,j}}{T_1\,(-1+T_1\,T_2)}$$

In[•]:=

```
px2g[r0,pxx[1, i0, j0] r1,ppx[1, i1, j1]]
```

Out[•]=

$$g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}-\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}-g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}$$

(Alt) In[•]:=

```
θ[{1, i0_, j0_}, {1, i1_, j1_}] = px2g[r0,pxx[1, i0, j0] r1,ppx[1, i1, j1]]
θ[{1, i0_, j0_}, {-1, i1_, j1_}] = px2g[r0,pxx[1, i0, j0] r1,ppx[-1, i1, j1]]
θ[{-1, i0_, j0_}, {1, i1_, j1_}] = px2g[r0,pxx[-1, i0, j0] r1,ppx[1, i1, j1]]
θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = px2g[r0,pxx[-1, i0, j0] r1,ppx[-1, i1, j1]]
```

(Alt) Out[•]=

$$g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}-\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}-g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}$$

(Alt) Out[•]=

$$-\frac{g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}+\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2}+\frac{g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}-\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2}$$

(Alt) Out[•]=

$$-\frac{g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}+\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1\,T_2}+\frac{g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}-\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1\,T_2}$$

(Alt) Out[•]=

$$\frac{g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^3\,T_2}-\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}-\frac{g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^3\,T_2}+\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}$$

(Alt) In[•]:=

```
Γ1[φ_, k_] = px2g[γ1[φ, k]]
```

(Alt) Out[•]=

$$-\frac{\varphi\,g_{3,k,k}}{T_1\,(-1+T_1\,T_2)}$$

(Alt) In[ ]:=

```
T3 = T1 T2;
CF[ℰ_] := Factor@Together[ℰ];
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A]; gEval[ℰ_] := CF[ℰ /. g_{ν_,α_,β_} :> (G[[α, β]] /. T → T_ν)];
  z = gEval[Sum[Sum[θ[Cs[[k1]], Cs[[k2]]], {k2, 1, n}], {k1, 1, n}]];
  z += gEval[Sum[R1 @@ Cs[[k]], {k, 1, n}]];
  z += gEval[Sum[Γ1[φ[[k]], k], {k, 1, 2 n}]];
  {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // CF
 ];
```

(Alt) In[ ]:=

```
Timing[θ[Knot[3, 1]]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

(Alt) Out[ ]=

$$\left\{0.,\ \left\{\frac{1-T+T^2}{T},\ \frac{-1+T_1-T_1^2+T_2-T_1^2\,T_2+2\,T_1^3\,T_2-T_2^2-T_1\,T_2^2+T_1^2\,T_2^2-2\,T_1^3\,T_2^2+2\,T_1\,T_2^3-2\,T_1^2\,T_2^3+2\,T_1^3\,T_2^3}{T_1^2\,T_2\,(-1+T_1\,T_2)}\right\}\right\}$$

(Alt) In[ ]:=

```
Timing[θ[Knot[4, 1]]]
```

(Alt) Out[ ]=

$$\left\{0.015625,\ \left\{-\frac{1-3\,T+T^2}{T},\ -\frac{\left(1-3\,T_1+T_1^2\right)\,(1+T_1\,T_2)\,\left(1-3\,T_2+T_2^2\right)}{T_1^3\,T_2^2}\right\}\right\}$$

(Alt) In[•]:=
```
Timing[θ[Knot["K11n34"]]]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

(Alt) Out[•]=
$$\left\{0.015625,\ \left\{1,\ \frac{1}{T_1^7\,T_2^6\,(-1+T_1\,T_2)}\left(T_1^2-2\,T_1^3+T_1^4-2\,T_1\,T_2+2\,T_1^2\,T_2+2\,T_1^5\,T_2-2\,T_1^6\,T_2+T_2^2+2\,T_1\,T_2^2-2\,T_1^2\,T_2^2-2\,T_1^4\,T_2^2-2\,T_1^6\,T_2^2+2\,T_1^7\,T_2^2+T_1^8\,T_2^2-2\,T_2^3+T_1^4\,T_2^3+T_1^5\,T_2^3-2\,T_1^9\,T_2^3+T_2^4-2\,T_1^2\,T_2^4+T_1^3\,T_2^4+2\,T_1^4\,T_2^4+2\,T_1^6\,T_2^4+T_1^7\,T_2^4-2\,T_1^8\,T_2^4+T_1^{10}\,T_2^4+2\,T_1\,T_2^5+T_1^3\,T_2^5-4\,T_1^5\,T_2^5-4\,T_1^6\,T_2^5+T_1^8\,T_2^5+2\,T_1^{10}\,T_2^5-2\,T_1\,T_2^6-2\,T_1^2\,T_2^6+2\,T_1^4\,T_2^6-4\,T_1^5\,T_2^6+12\,T_1^6\,T_2^6-4\,T_1^7\,T_2^6+2\,T_1^8\,T_2^6-2\,T_1^{10}\,T_2^6-2\,T_1^{11}\,T_2^6+2\,T_1^2\,T_2^7+T_1^4\,T_2^7-4\,T_1^6\,T_2^7-4\,T_1^7\,T_2^7+T_1^9\,T_2^7+2\,T_1^{11}\,T_2^7+T_1^2\,T_2^8-2\,T_1^4\,T_2^8+T_1^5\,T_2^8+2\,T_1^6\,T_2^8+2\,T_1^8\,T_2^8+T_1^9\,T_2^8-2\,T_1^{10}\,T_2^8+T_1^{12}\,T_2^8-2\,T_1^3\,T_2^9+T_1^7\,T_2^9+T_1^8\,T_2^9-2\,T_1^{12}\,T_2^9+T_1^4\,T_2^{10}+2\,T_1^5\,T_2^{10}-2\,T_1^6\,T_2^{10}-2\,T_1^8\,T_2^{10}-2\,T_1^{10}\,T_2^{10}+2\,T_1^{11}\,T_2^{10}+T_1^{12}\,T_2^{10}-2\,T_1^6\,T_2^{11}+2\,T_1^7\,T_2^{11}+2\,T_1^{10}\,T_2^{11}-2\,T_1^{11}\,T_2^{11}+T_1^8\,T_2^{12}-2\,T_1^9\,T_2^{12}+T_1^{10}\,T_2^{12}\right)\right\}\right\}$$

(Alt) In[•]:=
```
Timing[θ[Knot["K11n42"]]]
```

(Alt) Out[•]=
$$\left\{0.03125,\ \left\{1,\ -\frac{1}{T_1^4\,T_2^3\,(-1+T_1\,T_2)}\left(T_1+T_1^2+T_2-2\,T_1\,T_2-2\,T_1^2\,T_2-2\,T_1^3\,T_2+T_1^4\,T_2+T_2^2-2\,T_1\,T_2^2+2\,T_1^2\,T_2^2+2\,T_1^3\,T_2^2-2\,T_1^4\,T_2^2+T_1^5\,T_2^2-2\,T_1\,T_2^3+2\,T_1^2\,T_2^3+2\,T_1^4\,T_2^3-2\,T_1^5\,T_2^3+T_1\,T_2^4-2\,T_1^2\,T_2^4+2\,T_1^3\,T_2^4+2\,T_1^4\,T_2^4-2\,T_1^5\,T_2^4+T_1^6\,T_2^4+T_1^2\,T_2^5-2\,T_1^3\,T_2^5-2\,T_1^4\,T_2^5-2\,T_1^5\,T_2^5+T_1^6\,T_2^5+T_1^4\,T_2^6+T_1^5\,T_2^6\right)\right\}\right\}$$

(Alt) In[•]:=
```
PD[GST48] = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
   X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
   X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
   X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
   X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
   X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
   X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
   X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
   X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
   X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
   X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
   X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];
```

(Alt) In[•]:=
```
AbsoluteTiming[gst48 = θ[GST48]]
```

(Alt) Out[•]=
$$\left\{14.5462,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\left(-1+T^3-2\,T^4+T^5+T^6-2\,T^7+T^8\right)}{T^8},\ -\frac{1}{T_1^{21}\,T_2^{20}\,(-1+T_1\,T_2)}\right.\right.$$

$$
\begin{aligned}
&\big(T_1^5 - 3\,T_1^6 + 4\,T_1^7 - 2\,T_1^8 - 2\,T_1^9 + 4\,T_1^{10} - 2\,T_1^{11} - 2\,T_1^{12} + 4\,T_1^{13} - 3\,T_1^{14} + T_1^{15} - 3\,T_1^5\,T_2 + 6\,T_1^6\,T_2 - 3\,T_1^7\,T_2 - \\
&\quad 6\,T_1^8\,T_2 + 12\,T_1^9\,T_2 - 6\,T_1^{10}\,T_2 - 6\,T_1^{11}\,T_2 + 12\,T_1^{12}\,T_2 - 6\,T_1^{13}\,T_2 - 3\,T_1^{14}\,T_2 + 6\,T_1^{15}\,T_2 - 3\,T_1^{16}\,T_2 - T_1^3\,T_2^2 + \\
&\quad 3\,T_1^4\,T_2^2 - T_1^6\,T_2^2 - 4\,T_1^7\,T_2^2 + 9\,T_1^8\,T_2^2 - 7\,T_1^9\,T_2^2 - 3\,T_1^{10}\,T_2^2 + 8\,T_1^{11}\,T_2^2 - 3\,T_1^{12}\,T_2^2 - 7\,T_1^{13}\,T_2^2 + 9\,T_1^{14}\,T_2^2 - \\
&\quad 4\,T_1^{15}\,T_2^2 - T_1^{16}\,T_2^2 + 3\,T_1^{18}\,T_2^2 - T_1^{19}\,T_2^2 - T_1^2\,T_2^3 + 6\,T_1^3\,T_2^3 - 10\,T_1^4\,T_2^3 + 3\,T_1^5\,T_2^3 + 2\,T_1^6\,T_2^3 - 3\,T_1^7\,T_2^3 + \\
&\quad 4\,T_1^8\,T_2^3 - 2\,T_1^9\,T_2^3 + 2\,T_1^{10}\,T_2^3 - T_1^{11}\,T_2^3 - T_1^{12}\,T_2^3 + 2\,T_1^{13}\,T_2^3 - 2\,T_1^{14}\,T_2^3 + 4\,T_1^{15}\,T_2^3 - 3\,T_1^{16}\,T_2^3 + 2\,T_1^{17}\,T_2^3 + \\
&\quad 3\,T_1^{18}\,T_2^3 - 10\,T_1^{19}\,T_2^3 + 6\,T_1^{20}\,T_2^3 - T_1^{21}\,T_2^3 + 3\,T_1^2\,T_2^4 - 10\,T_1^3\,T_2^4 + 3\,T_1^4\,T_2^4 + 17\,T_1^5\,T_2^4 - 19\,T_1^6\,T_2^4 + \\
&\quad 10\,T_1^7\,T_2^4 - 7\,T_1^8\,T_2^4 + 6\,T_1^9\,T_2^4 - T_1^{10}\,T_2^4 - 18\,T_1^{11}\,T_2^4 + 35\,T_1^{12}\,T_2^4 - 18\,T_1^{13}\,T_2^4 - T_1^{14}\,T_2^4 + 6\,T_1^{15}\,T_2^4 - 7\,T_1^{16}\,T_2^4 + \\
&\quad 10\,T_1^{17}\,T_2^4 - 19\,T_1^{18}\,T_2^4 + 17\,T_1^{19}\,T_2^4 + 3\,T_1^{20}\,T_2^4 - 10\,T_1^{21}\,T_2^4 + 3\,T_1^{22}\,T_2^4 + T_2^5 - 3\,T_1\,T_2^5 + 3\,T_1^3\,T_2^5 + 17\,T_1^4\,T_2^5 - \\
&\quad 38\,T_1^5\,T_2^5 + 22\,T_1^6\,T_2^5 + 7\,T_1^7\,T_2^5 - 11\,T_1^8\,T_2^5 - 6\,T_1^9\,T_2^5 + 14\,T_1^{10}\,T_2^5 + 11\,T_1^{11}\,T_2^5 - 31\,T_1^{12}\,T_2^5 + 9\,T_1^{13}\,T_2^5 + \\
&\quad T_1^{14}\,T_2^5 + 8\,T_1^{16}\,T_2^5 - 15\,T_1^{17}\,T_2^5 + 9\,T_1^{18}\,T_2^5 + 16\,T_1^{19}\,T_2^5 - 32\,T_1^{20}\,T_2^5 + 15\,T_1^{21}\,T_2^5 + 3\,T_1^{22}\,T_2^5 - 3\,T_1^{24}\,T_2^5 + \\
&\quad T_1^{25}\,T_2^5 - 3\,T_2^6 + 6\,T_1\,T_2^6 - T_1^2\,T_2^6 + 2\,T_1^3\,T_2^6 - 19\,T_1^4\,T_2^6 + 22\,T_1^5\,T_2^6 + 24\,T_1^6\,T_2^6 - 68\,T_1^7\,T_2^6 + 43\,T_1^8\,T_2^6 + \\
&\quad 9\,T_1^9\,T_2^6 - 29\,T_1^{10}\,T_2^6 + 2\,T_1^{11}\,T_2^6 - 12\,T_1^{12}\,T_2^6 + 28\,T_1^{13}\,T_2^6 - 42\,T_1^{14}\,T_2^6 + 26\,T_1^{15}\,T_2^6 - 29\,T_1^{16}\,T_2^6 - T_1^{17}\,T_2^6 + \\
&\quad 45\,T_1^{18}\,T_2^6 - 64\,T_1^{19}\,T_2^6 + 24\,T_1^{20}\,T_2^6 + 18\,T_1^{21}\,T_2^6 - 17\,T_1^{22}\,T_2^6 + 2\,T_1^{23}\,T_2^6 - T_1^{24}\,T_2^6 + 6\,T_1^{25}\,T_2^6 - 3\,T_1^{26}\,T_2^6 + \\
&\quad 4\,T_2^7 - 3\,T_1\,T_2^7 - 4\,T_1^2\,T_2^7 - 3\,T_1^3\,T_2^7 + 10\,T_1^4\,T_2^7 + 7\,T_1^5\,T_2^7 - 68\,T_1^6\,T_2^7 + 74\,T_1^7\,T_2^7 + 14\,T_1^8\,T_2^7 - 56\,T_1^9\,T_2^7 + \\
&\quad 14\,T_1^{10}\,T_2^7 + 55\,T_1^{11}\,T_2^7 - 23\,T_1^{12}\,T_2^7 + 11\,T_1^{13}\,T_2^7 + 51\,T_1^{14}\,T_2^7 - 33\,T_1^{15}\,T_2^7 + 41\,T_1^{16}\,T_2^7 + 28\,T_1^{17}\,T_2^7 - 60\,T_1^{18}\,T_2^7 + \\
&\quad 16\,T_1^{19}\,T_2^7 + 68\,T_1^{20}\,T_2^7 - 62\,T_1^{21}\,T_2^7 + 5\,T_1^{22}\,T_2^7 + 10\,T_1^{23}\,T_2^7 - 3\,T_1^{24}\,T_2^7 - 4\,T_1^{25}\,T_2^7 - 3\,T_1^{26}\,T_2^7 + 4\,T_1^{27}\,T_2^7 - \\
&\quad 2\,T_2^8 - 6\,T_1\,T_2^8 + 9\,T_1^2\,T_2^8 + 4\,T_1^3\,T_2^8 - 7\,T_1^4\,T_2^8 - 11\,T_1^5\,T_2^8 + 43\,T_1^6\,T_2^8 + 14\,T_1^7\,T_2^8 - 123\,T_1^8\,T_2^8 + 133\,T_1^9\,T_2^8 - \\
&\quad 36\,T_1^{10}\,T_2^8 - 89\,T_1^{11}\,T_2^8 + 136\,T_1^{12}\,T_2^8 - 127\,T_1^{13}\,T_2^8 + 31\,T_1^{14}\,T_2^8 - 31\,T_1^{15}\,T_2^8 + 16\,T_1^{16}\,T_2^8 - 33\,T_1^{17}\,T_2^8 - \\
&\quad 28\,T_1^{18}\,T_2^8 + 109\,T_1^{19}\,T_2^8 - 115\,T_1^{20}\,T_2^8 + 14\,T_1^{21}\,T_2^8 + 51\,T_1^{22}\,T_2^8 - 27\,T_1^{23}\,T_2^8 + T_1^{24}\,T_2^8 + 4\,T_1^{25}\,T_2^8 + 9\,T_1^{26}\,T_2^8 - \\
&\quad 6\,T_1^{27}\,T_2^8 - 2\,T_1^{28}\,T_2^8 - 2\,T_2^9 + 12\,T_1\,T_2^9 - 7\,T_1^2\,T_2^9 - 2\,T_1^3\,T_2^9 + 6\,T_1^4\,T_2^9 - 6\,T_1^5\,T_2^9 + 9\,T_1^6\,T_2^9 - 56\,T_1^7\,T_2^9 + \\
&\quad 133\,T_1^8\,T_2^9 - 149\,T_1^9\,T_2^9 - 10\,T_1^{10}\,T_2^9 + 224\,T_1^{11}\,T_2^9 - 314\,T_1^{12}\,T_2^9 + 67\,T_1^{13}\,T_2^9 + 111\,T_1^{14}\,T_2^9 - 124\,T_1^{15}\,T_2^9 + \\
&\quad 38\,T_1^{16}\,T_2^9 - 49\,T_1^{17}\,T_2^9 + 50\,T_1^{18}\,T_2^9 - 38\,T_1^{19}\,T_2^9 - 47\,T_1^{20}\,T_2^9 + 95\,T_1^{21}\,T_2^9 - 68\,T_1^{22}\,T_2^9 + 8\,T_1^{23}\,T_2^9 + 32\,T_1^{24}\,T_2^9 - \\
&\quad 19\,T_1^{25}\,T_2^9 - 2\,T_1^{26}\,T_2^9 - 7\,T_1^{27}\,T_2^9 + 12\,T_1^{28}\,T_2^9 - 2\,T_1^{29}\,T_2^9 + 4\,T_2^{10} - 6\,T_1\,T_2^{10} - 3\,T_1^2\,T_2^{10} + 2\,T_1^3\,T_2^{10} - T_1^4\,T_2^{10} + \\
&\quad 14\,T_1^5\,T_2^{10} - 29\,T_1^6\,T_2^{10} + 14\,T_1^7\,T_2^{10} - 36\,T_1^8\,T_2^{10} - 10\,T_1^9\,T_2^{10} + 240\,T_1^{10}\,T_2^{10} - 314\,T_1^{11}\,T_2^{10} + 74\,T_1^{12}\,T_2^{10} + \\
&\quad 431\,T_1^{13}\,T_2^{10} - 386\,T_1^{14}\,T_2^{10} + 200\,T_1^{15}\,T_2^{10} + 34\,T_1^{16}\,T_2^{10} - 37\,T_1^{17}\,T_2^{10} + 186\,T_1^{18}\,T_2^{10} - 186\,T_1^{19}\,T_2^{10} + \\
&\quad 136\,T_1^{20}\,T_2^{10} - 22\,T_1^{21}\,T_2^{10} - 12\,T_1^{22}\,T_2^{10} + 46\,T_1^{23}\,T_2^{10} - 93\,T_1^{24}\,T_2^{10} + 30\,T_1^{25}\,T_2^{10} + 11\,T_1^{26}\,T_2^{10} + 2\,T_1^{27}\,T_2^{10} - \\
&\quad 3\,T_1^{28}\,T_2^{10} - 6\,T_1^{29}\,T_2^{10} + 4\,T_1^{30}\,T_2^{10} - 2\,T_2^{11} - 6\,T_1\,T_2^{11} + 8\,T_1^2\,T_2^{11} - T_1^3\,T_2^{11} - 18\,T_1^4\,T_2^{11} + 11\,T_1^5\,T_2^{11} + \\
&\quad 2\,T_1^6\,T_2^{11} + 55\,T_1^7\,T_2^{11} - 89\,T_1^8\,T_2^{11} + 224\,T_1^9\,T_2^{11} - 314\,T_1^{10}\,T_2^{11} - 92\,T_1^{11}\,T_2^{11} + 764\,T_1^{12}\,T_2^{11} - 899\,T_1^{13}\,T_2^{11} + \\
&\quad 273\,T_1^{14}\,T_2^{11} + 176\,T_1^{15}\,T_2^{11} - 382\,T_1^{16}\,T_2^{11} + 391\,T_1^{17}\,T_2^{11} - 420\,T_1^{18}\,T_2^{11} + 75\,T_1^{19}\,T_2^{11} + 212\,T_1^{20}\,T_2^{11} - \\
&\quad 156\,T_1^{21}\,T_2^{11} - 46\,T_1^{22}\,T_2^{11} - 6\,T_1^{23}\,T_2^{11} + 65\,T_1^{24}\,T_2^{11} + 76\,T_1^{25}\,T_2^{11} - 107\,T_1^{26}\,T_2^{11} + 31\,T_1^{27}\,T_2^{11} - T_1^{28}\,T_2^{11} + \\
&\quad 8\,T_1^{29}\,T_2^{11} - 6\,T_1^{30}\,T_2^{11} - 2\,T_1^{31}\,T_2^{11} - 2\,T_2^{12} + 12\,T_1\,T_2^{12} - 3\,T_1^2\,T_2^{12} - T_1^3\,T_2^{12} + 35\,T_1^4\,T_2^{12} - 31\,T_1^5\,T_2^{12} - \\
&\quad 12\,T_1^6\,T_2^{12} - 23\,T_1^7\,T_2^{12} + 136\,T_1^8\,T_2^{12} - 314\,T_1^9\,T_2^{12} + 74\,T_1^{10}\,T_2^{12} + 764\,T_1^{11}\,T_2^{12} - 1304\,T_1^{12}\,T_2^{12} + 293\,T_1^{13}\,T_2^{12} + \\
&\quad 744\,T_1^{14}\,T_2^{12} - 996\,T_1^{15}\,T_2^{12} + 616\,T_1^{16}\,T_2^{12} - 380\,T_1^{17}\,T_2^{12} - 68\,T_1^{18}\,T_2^{12} + 589\,T_1^{19}\,T_2^{12} - 596\,T_1^{20}\,T_2^{12} - \\
&\quad 72\,T_1^{21}\,T_2^{12} + 294\,T_1^{22}\,T_2^{12} + 38\,T_1^{23}\,T_2^{12} - 64\,T_1^{24}\,T_2^{12} - 123\,T_1^{25}\,T_2^{12} + 60\,T_1^{26}\,T_2^{12} + 93\,T_1^{27}\,T_2^{12} - 69\,T_1^{28}\,T_2^{12} - \\
&\quad T_1^{29}\,T_2^{12} - 3\,T_1^{30}\,T_2^{12} + 12\,T_1^{31}\,T_2^{12} - 2\,T_1^{32}\,T_2^{12} + 4\,T_2^{13} - 6\,T_1\,T_2^{13} - 7\,T_1^2\,T_2^{13} + 2\,T_1^3\,T_2^{13} - 18\,T_1^4\,T_2^{13} + \\
&\quad 9\,T_1^5\,T_2^{13} + 28\,T_1^6\,T_2^{13} + 11\,T_1^7\,T_2^{13} - 127\,T_1^8\,T_2^{13} + 67\,T_1^9\,T_2^{13} + 431\,T_1^{10}\,T_2^{13} - 899\,T_1^{11}\,T_2^{13} + 293\,T_1^{12}\,T_2^{13} + \\
&\quad 1556\,T_1^{13}\,T_2^{13} - 1724\,T_1^{14}\,T_2^{13} + 887\,T_1^{15}\,T_2^{13} + 223\,T_1^{16}\,T_2^{13} - 480\,T_1^{17}\,T_2^{13} + 998\,T_1^{18}\,T_2^{13} - 905\,T_1^{19}\,T_2^{13} + \\
&\quad 212\,T_1^{20}\,T_2^{13} + 686\,T_1^{21}\,T_2^{13} - 294\,T_1^{22}\,T_2^{13} - 313\,T_1^{23}\,T_2^{13} + 146\,T_1^{24}\,T_2^{13} + 24\,T_1^{25}\,T_2^{13} + 123\,T_1^{26}\,T_2^{13} - \\
&\quad 238\,T_1^{27}\,T_2^{13} + 65\,T_1^{28}\,T_2^{13} + 45\,T_1^{29}\,T_2^{13} + 2\,T_1^{30}\,T_2^{13} - 7\,T_1^{31}\,T_2^{13} - 6\,T_1^{32}\,T_2^{13} + 4\,T_1^{33}\,T_2^{13} - 3\,T_2^{14} - 3\,T_1\,T_2^{14} + \\
&\quad 9\,T_1^2\,T_2^{14} - 2\,T_1^3\,T_2^{14} - T_1^4\,T_2^{14} + T_1^5\,T_2^{14} - 42\,T_1^6\,T_2^{14} + 51\,T_1^7\,T_2^{14} + 31\,T_1^8\,T_2^{14} + 111\,T_1^9\,T_2^{14} - 386\,T_1^{10}\,T_2^{14} + \\
&\quad 273\,T_1^{11}\,T_2^{14} + 744\,T_1^{12}\,T_2^{14} - 1724\,T_1^{13}\,T_2^{14} + 705\,T_1^{14}\,T_2^{14} + 482\,T_1^{15}\,T_2^{14} - 1315\,T_1^{16}\,T_2^{14} + 1061\,T_1^{17}\,T_2^{14} - \\
&\quad 855\,T_1^{18}\,T_2^{14} - 140\,T_1^{19}\,T_2^{14} + 809\,T_1^{20}\,T_2^{14} - 758\,T_1^{21}\,T_2^{14} - 370\,T_1^{22}\,T_2^{14} + 595\,T_1^{23}\,T_2^{14} + 58\,T_1^{24}\,T_2^{14} - \\
&\quad 229\,T_1^{25}\,T_2^{14} + T_1^{26}\,T_2^{14} + 95\,T_1^{27}\,T_2^{14} + 124\,T_1^{28}\,T_2^{14} - 151\,T_1^{29}\,T_2^{14} + 19\,T_1^{30}\,T_2^{14} - 2\,T_1^{31}\,T_2^{14} + 9\,T_1^{32}\,T_2^{14} - \\
&\quad 3\,T_1^{33}\,T_2^{14} - 3\,T_1^{34}\,T_2^{14} + T_2^{15} + 6\,T_1\,T_2^{15} - 4\,T_1^2\,T_2^{15} + 4\,T_1^3\,T_2^{15} + 6\,T_1^4\,T_2^{15} + 26\,T_1^6\,T_2^{15} - 33\,T_1^7\,T_2^{15} - 31\,T_1^8\,T_2^{15} - \\
&\quad 124\,T_1^9\,T_2^{15} + 200\,T_1^{10}\,T_2^{15} + 176\,T_1^{11}\,T_2^{15} - 996\,T_1^{12}\,T_2^{15} + 887\,T_1^{13}\,T_2^{15} + 482\,T_1^{14}\,T_2^{15} - 1534\,T_1^{15}\,T_2^{15} + \\
&\quad 1712\,T_1^{16}\,T_2^{15} - 619\,T_1^{17}\,T_2^{15} - 569\,T_1^{18}\,T_2^{15} + 1420\,T_1^{19}\,T_2^{15} - 914\,T_1^{20}\,T_2^{15} - 229\,T_1^{21}\,T_2^{15} + 992\,T_1^{22}\,T_2^{15} -
\end{aligned}
$$

$$257\,T_1^{23}\,T_2^{15} - 598\,T_1^{24}\,T_2^{15} + 440\,T_1^{25}\,T_2^{15} - 15\,T_1^{26}\,T_2^{15} - 50\,T_1^{27}\,T_2^{15} - 167\,T_1^{28}\,T_2^{15} + 92\,T_1^{29}\,T_2^{15} + 74\,T_1^{30}\,T_2^{15} - 49\,T_1^{31}\,T_2^{15} + 4\,T_1^{32}\,T_2^{15} - 4\,T_1^{33}\,T_2^{15} + 6\,T_1^{34}\,T_2^{15} + T_1^{35}\,T_2^{15} - 3\,T_1\,T_2^{16} - T_1^2\,T_2^{16} - 3\,T_1^3\,T_2^{16} - 7\,T_1^4\,T_2^{16} + 8\,T_1^5\,T_2^{16} - 29\,T_1^6\,T_2^{16} + 41\,T_1^7\,T_2^{16} + 16\,T_1^8\,T_2^{16} + 38\,T_1^9\,T_2^{16} + 34\,T_1^{10}\,T_2^{16} - 382\,T_1^{11}\,T_2^{16} + 616\,T_1^{12}\,T_2^{16} + 223\,T_1^{13}\,T_2^{16} - 1315\,T_1^{14}\,T_2^{16} + 1712\,T_1^{15}\,T_2^{16} - 720\,T_1^{16}\,T_2^{16} - 1180\,T_1^{17}\,T_2^{16} + 2146\,T_1^{18}\,T_2^{16} - 1310\,T_1^{19}\,T_2^{16} - 260\,T_1^{20}\,T_2^{16} + 1108\,T_1^{21}\,T_2^{16} - 545\,T_1^{22}\,T_2^{16} - 555\,T_1^{23}\,T_2^{16} + 792\,T_1^{24}\,T_2^{16} - 94\,T_1^{25}\,T_2^{16} - 350\,T_1^{26}\,T_2^{16} + 256\,T_1^{27}\,T_2^{16} - 24\,T_1^{28}\,T_2^{16} + 109\,T_1^{29}\,T_2^{16} - 189\,T_1^{30}\,T_2^{16} + 60\,T_1^{31}\,T_2^{16} + 17\,T_1^{32}\,T_2^{16} - 3\,T_1^{33}\,T_2^{16} - T_1^{34}\,T_2^{16} - 3\,T_1^{35}\,T_2^{16} + 2\,T_1^3\,T_2^{17} + 10\,T_1^4\,T_2^{17} - 15\,T_1^5\,T_2^{17} - T_1^6\,T_2^{17} + 28\,T_1^7\,T_2^{17} - 33\,T_1^8\,T_2^{17} - 49\,T_1^9\,T_2^{17} - 37\,T_1^{10}\,T_2^{17} + 391\,T_1^{11}\,T_2^{17} - 380\,T_1^{12}\,T_2^{17} - 480\,T_1^{13}\,T_2^{17} + 1061\,T_1^{14}\,T_2^{17} - 619\,T_1^{15}\,T_2^{17} - 1180\,T_1^{16}\,T_2^{17} + 2566\,T_1^{17}\,T_2^{17} - 1730\,T_1^{18}\,T_2^{17} - 591\,T_1^{19}\,T_2^{17} + 1520\,T_1^{20}\,T_2^{17} - 933\,T_1^{21}\,T_2^{17} - 265\,T_1^{22}\,T_2^{17} + 476\,T_1^{23}\,T_2^{17} + 123\,T_1^{24}\,T_2^{17} - 791\,T_1^{25}\,T_2^{17} + 681\,T_1^{26}\,T_2^{17} - 213\,T_1^{27}\,T_2^{17} - 82\,T_1^{28}\,T_2^{17} - 8\,T_1^{29}\,T_2^{17} + 74\,T_1^{30}\,T_2^{17} + 42\,T_1^{31}\,T_2^{17} - 59\,T_1^{32}\,T_2^{17} + 10\,T_1^{33}\,T_2^{17} + 2\,T_1^{34}\,T_2^{17} + 3\,T_1^2\,T_2^{18} + 3\,T_1^3\,T_2^{18} - 19\,T_1^4\,T_2^{18} + 9\,T_1^5\,T_2^{18} + 45\,T_1^6\,T_2^{18} - 60\,T_1^7\,T_2^{18} - 28\,T_1^8\,T_2^{18} + 50\,T_1^9\,T_2^{18} + 186\,T_1^{10}\,T_2^{18} - 420\,T_1^{11}\,T_2^{18} - 68\,T_1^{12}\,T_2^{18} + 998\,T_1^{13}\,T_2^{18} - 855\,T_1^{14}\,T_2^{18} - 569\,T_1^{15}\,T_2^{18} + 2146\,T_1^{16}\,T_2^{18} - 1730\,T_1^{17}\,T_2^{18} - 492\,T_1^{18}\,T_2^{18} + 2218\,T_1^{19}\,T_2^{18} - 1372\,T_1^{20}\,T_2^{18} - 146\,T_1^{21}\,T_2^{18} + 878\,T_1^{22}\,T_2^{18} - 163\,T_1^{23}\,T_2^{18} - 695\,T_1^{24}\,T_2^{18} + 872\,T_1^{25}\,T_2^{18} - 162\,T_1^{26}\,T_2^{18} - 458\,T_1^{27}\,T_2^{18} + 506\,T_1^{28}\,T_2^{18} - 208\,T_1^{29}\,T_2^{18} + 44\,T_1^{30}\,T_2^{18} - 100\,T_1^{31}\,T_2^{18} + 79\,T_1^{32}\,T_2^{18} - 19\,T_1^{33}\,T_2^{18} - 5\,T_1^{34}\,T_2^{18} + 3\,T_1^{35}\,T_2^{18} + 3\,T_1^{36}\,T_2^{18} - T_1^2\,T_2^{19} - 10\,T_1^3\,T_2^{19} + 17\,T_1^4\,T_2^{19} + 16\,T_1^5\,T_2^{19} - 64\,T_1^6\,T_2^{19} + 16\,T_1^7\,T_2^{19} + 109\,T_1^8\,T_2^{19} - 38\,T_1^9\,T_2^{19} - 186\,T_1^{10}\,T_2^{19} + 75\,T_1^{11}\,T_2^{19} + 589\,T_1^{12}\,T_2^{19} - 905\,T_1^{13}\,T_2^{19} - 140\,T_1^{14}\,T_2^{19} + 1420\,T_1^{15}\,T_2^{19} - 1310\,T_1^{16}\,T_2^{19} - 591\,T_1^{17}\,T_2^{19} + 2218\,T_1^{18}\,T_2^{19} - 2027\,T_1^{19}\,T_2^{19} + 155\,T_1^{20}\,T_2^{19} + 1033\,T_1^{21}\,T_2^{19} - 840\,T_1^{22}\,T_2^{19} - 49\,T_1^{23}\,T_2^{19} + 464\,T_1^{24}\,T_2^{19} + 37\,T_1^{25}\,T_2^{19} - 842\,T_1^{26}\,T_2^{19} + 972\,T_1^{27}\,T_2^{19} - 412\,T_1^{28}\,T_2^{19} - 44\,T_1^{29}\,T_2^{19} + 150\,T_1^{30}\,T_2^{19} - 21\,T_1^{31}\,T_2^{19} - 10\,T_1^{32}\,T_2^{19} - 42\,T_1^{33}\,T_2^{19} + 50\,T_1^{34}\,T_2^{19} - 13\,T_1^{35}\,T_2^{19} - 10\,T_1^{36}\,T_2^{19} - T_1^{37}\,T_2^{19} + 6\,T_1^3\,T_2^{20} + 3\,T_1^4\,T_2^{20} - 32\,T_1^5\,T_2^{20} + 24\,T_1^6\,T_2^{20} + 68\,T_1^7\,T_2^{20} - 115\,T_1^8\,T_2^{20} - 47\,T_1^9\,T_2^{20} + 136\,T_1^{10}\,T_2^{20} + 212\,T_1^{11}\,T_2^{20} - 596\,T_1^{12}\,T_2^{20} + 212\,T_1^{13}\,T_2^{20} + 809\,T_1^{14}\,T_2^{20} - 914\,T_1^{15}\,T_2^{20} - 260\,T_1^{16}\,T_2^{20} + 1520\,T_1^{17}\,T_2^{20} - 1372\,T_1^{18}\,T_2^{20} + 155\,T_1^{19}\,T_2^{20} + 1056\,T_1^{20}\,T_2^{20} - 1291\,T_1^{21}\,T_2^{20} + 674\,T_1^{22}\,T_2^{20} - 128\,T_1^{23}\,T_2^{20} - 56\,T_1^{24}\,T_2^{20} - 374\,T_1^{25}\,T_2^{20} + 603\,T_1^{26}\,T_2^{20} - 180\,T_1^{27}\,T_2^{20} - 504\,T_1^{28}\,T_2^{20} + 592\,T_1^{29}\,T_2^{20} - 340\,T_1^{30}\,T_2^{20} + 71\,T_1^{31}\,T_2^{20} - 39\,T_1^{32}\,T_2^{20} + 100\,T_1^{33}\,T_2^{20} - 60\,T_1^{34}\,T_2^{20} - 8\,T_1^{35}\,T_2^{20} + 19\,T_1^{36}\,T_2^{20} + 6\,T_1^{37}\,T_2^{20} - T_1^3\,T_2^{21} - 10\,T_1^4\,T_2^{21} + 15\,T_1^5\,T_2^{21} + 18\,T_1^6\,T_2^{21} - 62\,T_1^7\,T_2^{21} + 14\,T_1^8\,T_2^{21} + 95\,T_1^9\,T_2^{21} - 22\,T_1^{10}\,T_2^{21} - 156\,T_1^{11}\,T_2^{21} - 72\,T_1^{12}\,T_2^{21} + 686\,T_1^{13}\,T_2^{21} - 758\,T_1^{14}\,T_2^{21} - 229\,T_1^{15}\,T_2^{21} + 1108\,T_1^{16}\,T_2^{21} - 933\,T_1^{17}\,T_2^{21} - 146\,T_1^{18}\,T_2^{21} + 1033\,T_1^{19}\,T_2^{21} - 1291\,T_1^{20}\,T_2^{21} + 891\,T_1^{21}\,T_2^{21} - 152\,T_1^{22}\,T_2^{21} - 395\,T_1^{23}\,T_2^{21} + 328\,T_1^{24}\,T_2^{21} + 152\,T_1^{25}\,T_2^{21} - 52\,T_1^{26}\,T_2^{21} - 695\,T_1^{27}\,T_2^{21} + 1069\,T_1^{28}\,T_2^{21} - 559\,T_1^{29}\,T_2^{21} - 14\,T_1^{30}\,T_2^{21} + 166\,T_1^{31}\,T_2^{21} - 35\,T_1^{32}\,T_2^{21} - 12\,T_1^{33}\,T_2^{21} - 40\,T_1^{34}\,T_2^{21} + 52\,T_1^{35}\,T_2^{21} - 15\,T_1^{36}\,T_2^{21} - 10\,T_1^{37}\,T_2^{21} - T_1^{38}\,T_2^{21} + 3\,T_1^4\,T_2^{22} + 3\,T_1^5\,T_2^{22} - 17\,T_1^6\,T_2^{22} + 5\,T_1^7\,T_2^{22} + 51\,T_1^8\,T_2^{22} - 68\,T_1^9\,T_2^{22} - 12\,T_1^{10}\,T_2^{22} - 46\,T_1^{11}\,T_2^{22} + 294\,T_1^{12}\,T_2^{22} - 294\,T_1^{13}\,T_2^{22} - 370\,T_1^{14}\,T_2^{22} + 992\,T_1^{15}\,T_2^{22} - 545\,T_1^{16}\,T_2^{22} - 265\,T_1^{17}\,T_2^{22} + 878\,T_1^{18}\,T_2^{22} - 840\,T_1^{19}\,T_2^{22} + 674\,T_1^{20}\,T_2^{22} - 152\,T_1^{21}\,T_2^{22} - 206\,T_1^{22}\,T_2^{22} + 744\,T_1^{23}\,T_2^{22} - 390\,T_1^{24}\,T_2^{22} + 141\,T_1^{25}\,T_2^{22} - 385\,T_1^{26}\,T_2^{22} + 866\,T_1^{27}\,T_2^{22} - 464\,T_1^{28}\,T_2^{22} - 332\,T_1^{29}\,T_2^{22} + 614\,T_1^{30}\,T_2^{22} - 304\,T_1^{31}\,T_2^{22} + 60\,T_1^{32}\,T_2^{22} - 108\,T_1^{33}\,T_2^{22} + 85\,T_1^{34}\,T_2^{22} - 23\,T_1^{35}\,T_2^{22} - 3\,T_1^{36}\,T_2^{22} + 3\,T_1^{37}\,T_2^{22} + 3\,T_1^{38}\,T_2^{22} + 2\,T_1^6\,T_2^{23} + 10\,T_1^7\,T_2^{23} - 27\,T_1^8\,T_2^{23} + 8\,T_1^9\,T_2^{23} + 46\,T_1^{10}\,T_2^{23} - 6\,T_1^{11}\,T_2^{23} + 38\,T_1^{12}\,T_2^{23} - 313\,T_1^{13}\,T_2^{23} + 595\,T_1^{14}\,T_2^{23} - 257\,T_1^{15}\,T_2^{23} - 555\,T_1^{16}\,T_2^{23} + 476\,T_1^{17}\,T_2^{23} - 163\,T_1^{18}\,T_2^{23} - 49\,T_1^{19}\,T_2^{23} - 128\,T_1^{20}\,T_2^{23} - 395\,T_1^{21}\,T_2^{23} + 744\,T_1^{22}\,T_2^{23} - 1174\,T_1^{23}\,T_2^{23} + 198\,T_1^{24}\,T_2^{23} + 191\,T_1^{25}\,T_2^{23} - 109\,T_1^{26}\,T_2^{23} + 48\,T_1^{27}\,T_2^{23} - 668\,T_1^{28}\,T_2^{23} + 885\,T_1^{29}\,T_2^{23} - 489\,T_1^{30}\,T_2^{23} + 5\,T_1^{31}\,T_2^{23} + 19\,T_1^{32}\,T_2^{23} + 92\,T_1^{33}\,T_2^{23} + 51\,T_1^{34}\,T_2^{23} - 71\,T_1^{35}\,T_2^{23} + 10\,T_1^{36}\,T_2^{23} + 2\,T_1^{37}\,T_2^{23} - 3\,T_1^5\,T_2^{24} - T_1^6\,T_2^{24} - 3\,T_1^7\,T_2^{24} + T_1^8\,T_2^{24} + 32\,T_1^9\,T_2^{24} - 93\,T_1^{10}\,T_2^{24} + 65\,T_1^{11}\,T_2^{24} - 64\,T_1^{12}\,T_2^{24} + 146\,T_1^{13}\,T_2^{24} + 58\,T_1^{14}\,T_2^{24} - 598\,T_1^{15}\,T_2^{24} + 792\,T_1^{16}\,T_2^{24} + 123\,T_1^{17}\,T_2^{24} - 695\,T_1^{18}\,T_2^{24} + 464\,T_1^{19}\,T_2^{24} - 56\,T_1^{20}\,T_2^{24} + 328\,T_1^{21}\,T_2^{24} - 390\,T_1^{22}\,T_2^{24} + 198\,T_1^{23}\,T_2^{24} + 404\,T_1^{24}\,T_2^{24} - 140\,T_1^{25}\,T_2^{24} + 75\,T_1^{26}\,T_2^{24} - 655\,T_1^{27}\,T_2^{24} + 968\,T_1^{28}\,T_2^{24} - 310\,T_1^{29}\,T_2^{24} - 326\,T_1^{30}\,T_2^{24} + 364\,T_1^{31}\,T_2^{24} - 104\,T_1^{32}\,T_2^{24} + 133\,T_1^{33}\,T_2^{24} - 253\,T_1^{34}\,T_2^{24} + 84\,T_1^{35}\,T_2^{24} + 25\,T_1^{36}\,T_2^{24} - 3\,T_1^{37}\,T_2^{24} - T_1^{38}\,T_2^{24} - 3\,T_1^{39}\,T_2^{24} + T_1^5\,T_2^{25} + 6\,T_1^6\,T_2^{25} - 4\,T_1^7\,T_2^{25} + 4\,T_1^8\,T_2^{25} - 19\,T_1^9\,T_2^{25} + 30\,T_1^{10}\,T_2^{25} + 76\,T_1^{11}\,T_2^{25} - 123\,T_1^{12}\,T_2^{25} + 24\,T_1^{13}\,T_2^{25} - 229\,T_1^{14}\,T_2^{25} + 440\,T_1^{15}\,T_2^{25} - 94\,T_1^{16}\,T_2^{25} - 791\,T_1^{17}\,T_2^{25} + 872\,T_1^{18}\,T_2^{25} + 37\,T_1^{19}\,T_2^{25} - 374\,T_1^{20}\,T_2^{25} + 152\,T_1^{21}\,T_2^{25} + 141\,T_1^{22}\,T_2^{25} + 191\,T_1^{23}\,T_2^{25} - 140\,T_1^{24}\,T_2^{25} + 246\,T_1^{25}\,T_2^{25} - 674\,T_1^{26}\,T_2^{25} +$$

$$977\,T_1^{27}\,T_2^{25}-52\,T_1^{28}\,T_2^{25}-868\,T_1^{29}\,T_2^{25}+680\,T_1^{30}\,T_2^{25}-120\,T_1^{31}\,T_2^{25}+5\,T_1^{32}\,T_2^{25}-257\,T_1^{33}\,T_2^{25}+142\,T_1^{34}\,T_2^{25}+104\,T_1^{35}\,T_2^{25}-74\,T_1^{36}\,T_2^{25}+4\,T_1^{37}\,T_2^{25}-4\,T_1^{38}\,T_2^{25}+6\,T_1^{39}\,T_2^{25}+T_1^{40}\,T_2^{25}-3\,T_1^{6}\,T_2^{26}-3\,T_1^{7}\,T_2^{26}+9\,T_1^{8}\,T_2^{26}-2\,T_1^{9}\,T_2^{26}+11\,T_1^{10}\,T_2^{26}-107\,T_1^{11}\,T_2^{26}+60\,T_1^{12}\,T_2^{26}+123\,T_1^{13}\,T_2^{26}+T_1^{14}\,T_2^{26}-15\,T_1^{15}\,T_2^{26}-350\,T_1^{16}\,T_2^{26}+681\,T_1^{17}\,T_2^{26}-162\,T_1^{18}\,T_2^{26}-842\,T_1^{19}\,T_2^{26}+603\,T_1^{20}\,T_2^{26}-52\,T_1^{21}\,T_2^{26}-385\,T_1^{22}\,T_2^{26}-109\,T_1^{23}\,T_2^{26}+75\,T_1^{24}\,T_2^{26}-674\,T_1^{25}\,T_2^{26}+707\,T_1^{26}\,T_2^{26}+124\,T_1^{27}\,T_2^{26}-1276\,T_1^{28}\,T_2^{26}+1003\,T_1^{29}\,T_2^{26}+94\,T_1^{30}\,T_2^{26}-355\,T_1^{31}\,T_2^{26}-29\,T_1^{32}\,T_2^{26}+167\,T_1^{33}\,T_2^{26}+226\,T_1^{34}\,T_2^{26}-259\,T_1^{35}\,T_2^{26}+31\,T_1^{36}\,T_2^{26}-2\,T_1^{37}\,T_2^{26}+9\,T_1^{38}\,T_2^{26}-3\,T_1^{39}\,T_2^{26}-3\,T_1^{40}\,T_2^{26}+4\,T_1^{7}\,T_2^{27}-6\,T_1^{8}\,T_2^{27}-7\,T_1^{9}\,T_2^{27}+2\,T_1^{10}\,T_2^{27}+31\,T_1^{11}\,T_2^{27}+93\,T_1^{12}\,T_2^{27}-238\,T_1^{13}\,T_2^{27}+95\,T_1^{14}\,T_2^{27}-50\,T_1^{15}\,T_2^{27}+256\,T_1^{16}\,T_2^{27}-213\,T_1^{17}\,T_2^{27}-458\,T_1^{18}\,T_2^{27}+972\,T_1^{19}\,T_2^{27}-180\,T_1^{20}\,T_2^{27}-695\,T_1^{21}\,T_2^{27}+866\,T_1^{22}\,T_2^{27}+48\,T_1^{23}\,T_2^{27}-655\,T_1^{24}\,T_2^{27}+977\,T_1^{25}\,T_2^{27}+124\,T_1^{26}\,T_2^{27}-1524\,T_1^{27}\,T_2^{27}+1365\,T_1^{28}\,T_2^{27}+147\,T_1^{29}\,T_2^{27}-957\,T_1^{30}\,T_2^{27}+335\,T_1^{31}\,T_2^{27}+101\,T_1^{32}\,T_2^{27}+207\,T_1^{33}\,T_2^{27}-504\,T_1^{34}\,T_2^{27}+149\,T_1^{35}\,T_2^{27}+94\,T_1^{36}\,T_2^{27}+2\,T_1^{37}\,T_2^{27}-7\,T_1^{38}\,T_2^{27}-6\,T_1^{39}\,T_2^{27}+4\,T_1^{40}\,T_2^{27}-2\,T_1^{8}\,T_2^{28}+12\,T_1^{9}\,T_2^{28}-3\,T_1^{10}\,T_2^{28}-T_1^{11}\,T_2^{28}-69\,T_1^{12}\,T_2^{28}+65\,T_1^{13}\,T_2^{28}+124\,T_1^{14}\,T_2^{28}-167\,T_1^{15}\,T_2^{28}-24\,T_1^{16}\,T_2^{28}-82\,T_1^{17}\,T_2^{28}+506\,T_1^{18}\,T_2^{28}-412\,T_1^{19}\,T_2^{28}-504\,T_1^{20}\,T_2^{28}+1069\,T_1^{21}\,T_2^{28}-464\,T_1^{22}\,T_2^{28}-668\,T_1^{23}\,T_2^{28}+968\,T_1^{24}\,T_2^{28}-52\,T_1^{25}\,T_2^{28}-1276\,T_1^{26}\,T_2^{28}+1365\,T_1^{27}\,T_2^{28}+204\,T_1^{28}\,T_2^{28}-1248\,T_1^{29}\,T_2^{28}+726\,T_1^{30}\,T_2^{28}+270\,T_1^{31}\,T_2^{28}-224\,T_1^{32}\,T_2^{28}-267\,T_1^{33}\,T_2^{28}+196\,T_1^{34}\,T_2^{28}+189\,T_1^{35}\,T_2^{28}-173\,T_1^{36}\,T_2^{28}-T_1^{37}\,T_2^{28}-3\,T_1^{38}\,T_2^{28}+12\,T_1^{39}\,T_2^{28}-2\,T_1^{40}\,T_2^{28}-2\,T_1^{9}\,T_2^{29}-6\,T_1^{10}\,T_2^{29}+8\,T_1^{11}\,T_2^{29}-T_1^{12}\,T_2^{29}+45\,T_1^{13}\,T_2^{29}-151\,T_1^{14}\,T_2^{29}+92\,T_1^{15}\,T_2^{29}+109\,T_1^{16}\,T_2^{29}-8\,T_1^{17}\,T_2^{29}-208\,T_1^{18}\,T_2^{29}-44\,T_1^{19}\,T_2^{29}+592\,T_1^{20}\,T_2^{29}-559\,T_1^{21}\,T_2^{29}-332\,T_1^{22}\,T_2^{29}+885\,T_1^{23}\,T_2^{29}-310\,T_1^{24}\,T_2^{29}-868\,T_1^{25}\,T_2^{29}+1003\,T_1^{26}\,T_2^{29}+147\,T_1^{27}\,T_2^{29}-1248\,T_1^{28}\,T_2^{29}+896\,T_1^{29}\,T_2^{29}+114\,T_1^{30}\,T_2^{29}-478\,T_1^{31}\,T_2^{29}+75\,T_1^{32}\,T_2^{29}+119\,T_1^{33}\,T_2^{29}+166\,T_1^{34}\,T_2^{29}-269\,T_1^{35}\,T_2^{29}+94\,T_1^{36}\,T_2^{29}-T_1^{37}\,T_2^{29}+8\,T_1^{38}\,T_2^{29}-6\,T_1^{39}\,T_2^{29}-2\,T_1^{40}\,T_2^{29}+4\,T_1^{10}\,T_2^{30}-6\,T_1^{11}\,T_2^{30}-3\,T_1^{12}\,T_2^{30}+2\,T_1^{13}\,T_2^{30}+19\,T_1^{14}\,T_2^{30}+74\,T_1^{15}\,T_2^{30}-189\,T_1^{16}\,T_2^{30}+74\,T_1^{17}\,T_2^{30}+44\,T_1^{18}\,T_2^{30}+150\,T_1^{19}\,T_2^{30}-340\,T_1^{20}\,T_2^{30}-14\,T_1^{21}\,T_2^{30}+614\,T_1^{22}\,T_2^{30}-489\,T_1^{23}\,T_2^{30}-326\,T_1^{24}\,T_2^{30}+680\,T_1^{25}\,T_2^{30}+94\,T_1^{26}\,T_2^{30}-957\,T_1^{27}\,T_2^{30}+726\,T_1^{28}\,T_2^{30}+114\,T_1^{29}\,T_2^{30}-444\,T_1^{30}\,T_2^{30}+138\,T_1^{31}\,T_2^{30}+68\,T_1^{32}\,T_2^{30}+106\,T_1^{33}\,T_2^{30}-253\,T_1^{34}\,T_2^{30}+90\,T_1^{35}\,T_2^{30}+31\,T_1^{36}\,T_2^{30}+2\,T_1^{37}\,T_2^{30}-3\,T_1^{38}\,T_2^{30}-6\,T_1^{39}\,T_2^{30}+4\,T_1^{40}\,T_2^{30}-2\,T_1^{11}\,T_2^{31}+12\,T_1^{12}\,T_2^{31}-7\,T_1^{13}\,T_2^{31}-2\,T_1^{14}\,T_2^{31}-49\,T_1^{15}\,T_2^{31}+60\,T_1^{16}\,T_2^{31}+42\,T_1^{17}\,T_2^{31}-100\,T_1^{18}\,T_2^{31}-21\,T_1^{19}\,T_2^{31}+71\,T_1^{20}\,T_2^{31}+166\,T_1^{21}\,T_2^{31}-304\,T_1^{22}\,T_2^{31}+5\,T_1^{23}\,T_2^{31}+364\,T_1^{24}\,T_2^{31}-120\,T_1^{25}\,T_2^{31}-355\,T_1^{26}\,T_2^{31}+335\,T_1^{27}\,T_2^{31}+270\,T_1^{28}\,T_2^{31}-478\,T_1^{29}\,T_2^{31}+138\,T_1^{30}\,T_2^{31}+173\,T_1^{31}\,T_2^{31}-59\,T_1^{32}\,T_2^{31}-112\,T_1^{33}\,T_2^{31}+41\,T_1^{34}\,T_2^{31}+98\,T_1^{35}\,T_2^{31}-74\,T_1^{36}\,T_2^{31}-2\,T_1^{37}\,T_2^{31}-7\,T_1^{38}\,T_2^{31}+12\,T_1^{39}\,T_2^{31}-2\,T_1^{40}\,T_2^{31}-2\,T_1^{12}\,T_2^{32}-6\,T_1^{13}\,T_2^{32}+9\,T_1^{14}\,T_2^{32}+4\,T_1^{15}\,T_2^{32}+17\,T_1^{16}\,T_2^{32}-59\,T_1^{17}\,T_2^{32}+79\,T_1^{18}\,T_2^{32}-10\,T_1^{19}\,T_2^{32}-39\,T_1^{20}\,T_2^{32}-35\,T_1^{21}\,T_2^{32}+60\,T_1^{22}\,T_2^{32}+19\,T_1^{23}\,T_2^{32}-104\,T_1^{24}\,T_2^{32}+5\,T_1^{25}\,T_2^{32}-29\,T_1^{26}\,T_2^{32}+101\,T_1^{27}\,T_2^{32}-224\,T_1^{28}\,T_2^{32}+75\,T_1^{29}\,T_2^{32}+68\,T_1^{30}\,T_2^{32}-59\,T_1^{31}\,T_2^{32}-31\,T_1^{32}\,T_2^{32}-10\,T_1^{33}\,T_2^{32}+87\,T_1^{34}\,T_2^{32}-75\,T_1^{35}\,T_2^{32}+25\,T_1^{36}\,T_2^{32}+4\,T_1^{37}\,T_2^{32}+9\,T_1^{38}\,T_2^{32}-6\,T_1^{39}\,T_2^{32}-2\,T_1^{40}\,T_2^{32}+4\,T_1^{13}\,T_2^{33}-3\,T_1^{14}\,T_2^{33}-4\,T_1^{15}\,T_2^{33}-3\,T_1^{16}\,T_2^{33}+10\,T_1^{17}\,T_2^{33}-19\,T_1^{18}\,T_2^{33}-42\,T_1^{19}\,T_2^{33}+100\,T_1^{20}\,T_2^{33}-12\,T_1^{21}\,T_2^{33}-108\,T_1^{22}\,T_2^{33}+92\,T_1^{23}\,T_2^{33}+133\,T_1^{24}\,T_2^{33}-257\,T_1^{25}\,T_2^{33}+167\,T_1^{26}\,T_2^{33}+207\,T_1^{27}\,T_2^{33}-267\,T_1^{28}\,T_2^{33}+119\,T_1^{29}\,T_2^{33}+106\,T_1^{30}\,T_2^{33}-112\,T_1^{31}\,T_2^{33}-10\,T_1^{32}\,T_2^{33}+94\,T_1^{33}\,T_2^{33}-36\,T_1^{34}\,T_2^{33}-21\,T_1^{35}\,T_2^{33}+10\,T_1^{36}\,T_2^{33}-3\,T_1^{37}\,T_2^{33}-4\,T_1^{38}\,T_2^{33}-3\,T_1^{39}\,T_2^{33}+4\,T_1^{40}\,T_2^{33}-3\,T_1^{14}\,T_2^{34}+6\,T_1^{15}\,T_2^{34}-T_1^{16}\,T_2^{34}+2\,T_1^{17}\,T_2^{34}-5\,T_1^{18}\,T_2^{34}+50\,T_1^{19}\,T_2^{34}-60\,T_1^{20}\,T_2^{34}-40\,T_1^{21}\,T_2^{34}+85\,T_1^{22}\,T_2^{34}+51\,T_1^{23}\,T_2^{34}-253\,T_1^{24}\,T_2^{34}+142\,T_1^{25}\,T_2^{34}+226\,T_1^{26}\,T_2^{34}-504\,T_1^{27}\,T_2^{34}+196\,T_1^{28}\,T_2^{34}+166\,T_1^{29}\,T_2^{34}-253\,T_1^{30}\,T_2^{34}+41\,T_1^{31}\,T_2^{34}+87\,T_1^{32}\,T_2^{34}-36\,T_1^{33}\,T_2^{34}-60\,T_1^{34}\,T_2^{34}+46\,T_1^{35}\,T_2^{34}-3\,T_1^{36}\,T_2^{34}+2\,T_1^{37}\,T_2^{34}-T_1^{38}\,T_2^{34}+6\,T_1^{39}\,T_2^{34}-3\,T_1^{40}\,T_2^{34}+T_1^{15}\,T_2^{35}-3\,T_1^{16}\,T_2^{35}+3\,T_1^{18}\,T_2^{35}-13\,T_1^{19}\,T_2^{35}-8\,T_1^{20}\,T_2^{35}+52\,T_1^{21}\,T_2^{35}-23\,T_1^{22}\,T_2^{35}-71\,T_1^{23}\,T_2^{35}+84\,T_1^{24}\,T_2^{35}+104\,T_1^{25}\,T_2^{35}-259\,T_1^{26}\,T_2^{35}+149\,T_1^{27}\,T_2^{35}+189\,T_1^{28}\,T_2^{35}-269\,T_1^{29}\,T_2^{35}+90\,T_1^{30}\,T_2^{35}+98\,T_1^{31}\,T_2^{35}-75\,T_1^{32}\,T_2^{35}-21\,T_1^{33}\,T_2^{35}+46\,T_1^{34}\,T_2^{35}-2\,T_1^{35}\,T_2^{35}-15\,T_1^{36}\,T_2^{35}+3\,T_1^{37}\,T_2^{35}-3\,T_1^{39}\,T_2^{35}+T_1^{40}\,T_2^{35}+3\,T_1^{18}\,T_2^{36}-10\,T_1^{19}\,T_2^{36}+19\,T_1^{20}\,T_2^{36}-15\,T_1^{21}\,T_2^{36}-3\,T_1^{22}\,T_2^{36}+10\,T_1^{23}\,T_2^{36}+25\,T_1^{24}\,T_2^{36}-74\,T_1^{25}\,T_2^{36}+31\,T_1^{26}\,T_2^{36}+94\,T_1^{27}\,T_2^{36}-173\,T_1^{28}\,T_2^{36}+94\,T_1^{29}\,T_2^{36}+31\,T_1^{30}\,T_2^{36}-74\,T_1^{31}\,T_2^{36}+25\,T_1^{32}\,T_2^{36}+10\,T_1^{33}\,T_2^{36}-3\,T_1^{34}\,T_2^{36}-15\,T_1^{35}\,T_2^{36}+19\,T_1^{36}\,T_2^{36}-10\,T_1^{37}\,T_2^{36}+3\,T_1^{38}\,T_2^{36}-T_1^{19}\,T_2^{37}+6\,T_1^{20}\,T_2^{37}-10\,T_1^{21}\,T_2^{37}+3\,T_1^{22}\,T_2^{37}+2\,T_1^{23}\,T_2^{37}-3\,T_1^{24}\,T_2^{37}+4\,T_1^{25}\,T_2^{37}-$$

$$2\,T_1^{26}\,T_2^{37}+2\,T_1^{27}\,T_2^{37}-T_1^{28}\,T_2^{37}-T_1^{29}\,T_2^{37}+2\,T_1^{30}\,T_2^{37}-2\,T_1^{31}\,T_2^{37}+4\,T_1^{32}\,T_2^{37}-3\,T_1^{33}\,T_2^{37}+2\,T_1^{34}\,T_2^{37}+3\,T_1^{35}\,T_2^{37}-10\,T_1^{36}\,T_2^{37}+6\,T_1^{37}\,T_2^{37}-T_1^{38}\,T_2^{37}-T_1^{21}\,T_2^{38}+3\,T_1^{22}\,T_2^{38}-T_1^{24}\,T_2^{38}-4\,T_1^{25}\,T_2^{38}+9\,T_1^{26}\,T_2^{38}-7\,T_1^{27}\,T_2^{38}-3\,T_1^{28}\,T_2^{38}+8\,T_1^{29}\,T_2^{38}-3\,T_1^{30}\,T_2^{38}-7\,T_1^{31}\,T_2^{38}+9\,T_1^{32}\,T_2^{38}-4\,T_1^{33}\,T_2^{38}-T_1^{34}\,T_2^{38}+3\,T_1^{36}\,T_2^{38}-T_1^{37}\,T_2^{38}-3\,T_1^{24}\,T_2^{39}+6\,T_1^{25}\,T_2^{39}-3\,T_1^{26}\,T_2^{39}-6\,T_1^{27}\,T_2^{39}+12\,T_1^{28}\,T_2^{39}-6\,T_1^{29}\,T_2^{39}-6\,T_1^{30}\,T_2^{39}+12\,T_1^{31}\,T_2^{39}-6\,T_1^{32}\,T_2^{39}-3\,T_1^{33}\,T_2^{39}+6\,T_1^{34}\,T_2^{39}-3\,T_1^{35}\,T_2^{39}+T_1^{25}\,T_2^{40}-3\,T_1^{26}\,T_2^{40}+4\,T_1^{27}\,T_2^{40}-2\,T_1^{28}\,T_2^{40}-2\,T_1^{29}\,T_2^{40}+4\,T_1^{30}\,T_2^{40}-2\,T_1^{31}\,T_2^{40}-2\,T_1^{32}\,T_2^{40}+4\,T_1^{33}\,T_2^{40}-3\,T_1^{34}\,T_2^{40}+T_1^{35}\,T_2^{40}\Big)\Big\}\Big\}$$

NumberOfKnots[10, Alternating] = 123
NumberOfKnots[11, Alternating] = 367
NumberOfKnots[12, Alternating] = 1288
NumberOfKnots[13, Alternating] = 4878
NumberOfKnots[14, Alternating] = 19536
NumberOfKnots[15, Alternating] = 85263
NumberOfKnots[16, Alternating] = 379799

NumberOfKnots[10, NonAlternating] = 42
NumberOfKnots[11, NonAlternating] = 185
NumberOfKnots[12, NonAlternating] = 888
NumberOfKnots[13, NonAlternating] = 5110
NumberOfKnots[14, NonAlternating] = 27436
NumberOfKnots[15, NonAlternating] = 168030
NumberOfKnots[16, NonAlternating] = 1008906

(Alt) In[ ]:=
```
Monitor[
 tab14 = Table[K → θ@K, {K, AllKnots[{3, 14}] }],
 K]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

KnotTheory: Loading precomputed data in KnotTheory/13A.dts.

General: Further output of KnotTheory::loading will be suppressed during this calculation.

(Alt) Out[ ]=

(Alt) In[ ]:=
```
Put[tab14 /. {T1 → T1, T2 → T2}, "Data14.m"]
```

(Alt) In[ ]:=
```
dup14 = Map[First, Select[Gather[tab14, Last[#1] === Last[#2] &], Length[#] > 1 &], {2}]
```

(Alt) Out[ ]=

(Alt) In[•]:=
```
Length /@ dup14
```
(Alt) Out[•]=
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 4, 4, 4, 4, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2, 2, 2,
 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2,
 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2}
```

(Alt) In[•]:=
```
Total[(Length /@ dup14) - 1]
```
(Alt) Out[•]=
```
1118
```

(Alt) In[•]:=
```
dup14Q = Map[First,
  Select[Gather[tab14 /. {T1 → i, T2 → i}, Last[#1] === Last[#2] &], Length[#] > 1 &], {2}]
```
(Alt) Out[•]=

(Alt) In[•]:=
```
Total[(Length /@ dup14Q) - 1]
```
(Alt) Out[•]=
```
6452
```
(Alt) In[•]:=
```
dup14Q =
 Map[First, Select[Gather[tab14 /. {T1 → 22 / 7, T2 → 34 / 21}, Last[#1] === Last[#2] &],
   Length[#] > 1 &], {2}]
```
(Alt) Out[•]=

(Alt) In[•]:=
```
Total[(Length /@ dup14Q) - 1]
```
(Alt) Out[•]=
```
1118
```