

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensive\\Projects\\HigherRank"];
Once[<< KnotTheory`];
<< Rot.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[*]:= {
  {r0,pxx[1, i_, j_], r0,pxx[-1, i_, j_]},
  {r1,ppx[1, i_, j_], r1,ppx[-1, i_, j_]},
  {r1,rest[1, i_, j_], r1,rest[-1, i_, j_]},
  r1[phi_, k_]
} = Get["px-data.m"]
```

Out[*]=

$$\left\{ \left\{ p_{3,j} x_{1,i} x_{2,i} - \frac{p_{3,j} x_{1,j} x_{2,i}}{T_1}, -\frac{p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} \right\}, \right. \\ \left. \left\{ p_{1,j} p_{2,i} x_{3,i} - p_{1,j} p_{2,j} x_{3,i}, -\frac{p_{1,j} p_{2,i} x_{3,i}}{T_1} + \frac{p_{1,j} p_{2,j} x_{3,i}}{T_1} \right\}, \right. \\ \left\{ \frac{T_2 p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{-1 + T_1 T_2} - \frac{p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} - \frac{p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1} + \frac{p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \right. \\ \frac{p_{3,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - p_{1,j} p_{3,j} x_{1,i} x_{3,i} + \frac{p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{p_{1,j} p_{3,j} x_{1,j} x_{3,i}}{-1 + T_1} - \\ \frac{T_2 p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{T_1} - \frac{p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \frac{T_2 p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_1 T_2)}, \\ p_{1,j} p_{2,i} x_{1,i} x_{2,i} - \frac{(-1 + T_2) p_{1,i} p_{2,j} x_{1,i} x_{2,i}}{(-1 + T_1) T_2 (-1 + T_1 T_2)} + \frac{(-T_1 - T_2 + T_1 T_2) p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{T_1^2 T_2 (-1 + T_1 T_2)} + \\ \frac{p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \\ \frac{p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{T_1 (-1 + T_1 T_2)} - \frac{p_{3,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,j} p_{3,i} x_{1,i} x_{3,i}}{T_1^2 (-1 + T_1 T_2)} + \frac{p_{1,i} p_{3,j} x_{1,i} x_{3,i}}{(-1 + T_1) T_1 T_2} - \\ \frac{p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{T_1^2 T_2} - \frac{p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{(-1 + T_2) p_{2,j} p_{3,i} x_{2,i} x_{3,i}}{T_1 T_2 (-1 + T_1 T_2)} + \\ \frac{p_{2,i} p_{3,j} x_{2,i} x_{3,i}}{T_1^2 T_2} - \frac{(-1 + 2 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{T_1^2 T_2^2} + \frac{p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{T_1 (-1 + T_1 T_2)} - \frac{p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{T_1^2 T_2} + \\ \left. \frac{p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \frac{p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{T_1 (-1 + T_1 T_2)} - \frac{p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_1 T_2)} \right\}, -\frac{\varphi p_{3,k} x_{3,k}}{T_1 (-1 + T_1 T_2)} \left. \right\}$$

In[*]:= $r_{1,rest}[1, 4, 5]$

Out[*]=

$$\frac{T_2 p_{1,5} p_{2,5} x_{1,4} x_{2,4}}{-1 + T_1 T_2} - \frac{p_{1,5} p_{2,4} x_{1,5} x_{2,4}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} - \frac{p_{1,5} p_{2,5} x_{1,5} x_{2,4}}{(-1 + T_1) T_1} + \frac{p_{1,4} p_{2,5} x_{1,4} x_{2,5}}{(-1 + T_1) (-1 + T_1 T_2)} +$$

$$\frac{p_{3,5} x_{3,4}}{T_1 (-1 + T_1 T_2)} - p_{1,5} p_{3,5} x_{1,4} x_{3,4} + \frac{p_{1,5} p_{3,4} x_{1,5} x_{3,4}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{p_{1,5} p_{3,5} x_{1,5} x_{3,4}}{-1 + T_1} -$$

$$\frac{T_2 p_{2,5} p_{3,5} x_{2,4} x_{3,4}}{T_1} - \frac{p_{2,5} p_{3,4} x_{2,5} x_{3,4}}{T_1 (-1 + T_1 T_2)} - \frac{p_{1,4} p_{3,5} x_{1,4} x_{3,5}}{(-1 + T_1) (-1 + T_1 T_2)} + \frac{T_2 p_{2,5} p_{3,5} x_{2,4} x_{3,5}}{T_1 (-1 + T_1 T_2)}$$

In[*]:= $\{p^*, x^*, \pi^*, \xi^*\} = \{\pi, \xi, p, x\}; (u_{-i-})^* := (u^*)_i;$

In[*]:= $Zip_{\{ \}}[\mathcal{E}_-] := \mathcal{E};$

$Zip_{\{\xi, \xi_{-}\}}[\mathcal{E}_-] := (Collect[\mathcal{E} // Zip_{\{\xi, \xi_{-}\}}, \xi^*] /. f_-. \xi^{d-} \Rightarrow (D[f, \{\xi^*, d\}])) /. \xi^* \rightarrow \theta$

In[*]:= $px2g[\mathcal{E}_-] := Module[\{ps, xs, Q\},$
 $ps = Union[Cases[\mathcal{E}, p_-, \infty]]; xs = Union[Cases[\mathcal{E}, x_-, \infty]];$
 $Q = Sum[p\theta^* x\theta^* g_{p\theta[[2]], x\theta[[2]], p\theta[[3]], x\theta[[3]]}, \{p\theta, ps\}, \{x\theta, xs\}];$
 $Expand[Zip_{ps \cup xs}[\mathcal{E} e^Q] /. g_{\alpha, \beta, i, j} \Rightarrow If[\alpha == \beta, g_{\alpha, i, j}, \theta]]$
 $]$

In[*]:= $px2g[p_{2,j}^2 x_{2,i} x_{2,j}]$

Out[*]=

$$2 g_{2,j,i} g_{2,j,j}$$

In[*]:= $R_1[1, i_-, j_-] = px2g[r_{1,rest}[1, i, j]]$

Out[*]=

$$-\frac{g_{1,j,j} g_{2,i,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{T_2 g_{1,j,i} g_{2,j,i}}{-1 + T_1 T_2} - \frac{g_{1,j,j} g_{2,j,i}}{(-1 + T_1) T_1} + \frac{g_{1,i,i} g_{2,j,j}}{(-1 + T_1) (-1 + T_1 T_2)} +$$

$$\frac{g_{1,j,j} g_{3,i,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} - \frac{g_{2,j,j} g_{3,i,i}}{T_1 (-1 + T_1 T_2)} + \frac{g_{3,j,i}}{T_1 (-1 + T_1 T_2)} - g_{1,j,i} g_{3,j,i} +$$

$$\frac{g_{1,j,j} g_{3,j,i}}{-1 + T_1} - \frac{T_2 g_{2,j,i} g_{3,j,i}}{T_1} - \frac{g_{1,i,i} g_{3,j,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \frac{T_2 g_{2,j,i} g_{3,j,j}}{T_1 (-1 + T_1 T_2)}$$

In[*]:= $R_1[-1, i_-, j_-] = \text{px2g}[r_{1,\text{rest}}[-1, i, j]]$

Out[*]=

$$\begin{aligned} & \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{2,i,i}}{T_1^2 (-1 + T_1 T_2)} + \frac{\mathfrak{g}_{1,j,j} \mathfrak{g}_{2,i,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{1,i,i} \mathfrak{g}_{2,j,i}}{(-1 + T_1) (-1 + T_1 T_2)} + \frac{\mathfrak{g}_{1,i,i} \mathfrak{g}_{2,j,i}}{(-1 + T_1) T_2 (-1 + T_1 T_2)} - \\ & \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{2,j,i}}{T_1^2 (-1 + T_1 T_2)} + \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{2,j,i}}{T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{2,j,i}}{T_1 T_2 (-1 + T_1 T_2)} + \frac{\mathfrak{g}_{1,j,j} \mathfrak{g}_{2,j,i}}{T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{1,i,i} \mathfrak{g}_{2,j,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \\ & \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{2,j,j}}{T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{3,i,i}}{T_1^2 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{1,j,j} \mathfrak{g}_{3,i,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \frac{\mathfrak{g}_{2,j,i} \mathfrak{g}_{3,i,i}}{T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{2,j,i} \mathfrak{g}_{3,i,i}}{T_1 T_2 (-1 + T_1 T_2)} + \\ & \frac{\mathfrak{g}_{2,j,j} \mathfrak{g}_{3,i,i}}{T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{3,j,i}}{T_1 (-1 + T_1 T_2)} + \frac{\mathfrak{g}_{1,i,i} \mathfrak{g}_{3,j,i}}{(-1 + T_1) T_1 T_2} - \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{3,j,i}}{T_1^2 T_2} + \frac{\mathfrak{g}_{2,i,i} \mathfrak{g}_{3,j,i}}{T_1^2 T_2} + \frac{\mathfrak{g}_{2,j,i} \mathfrak{g}_{3,j,i}}{T_1^2 T_2} - \\ & \frac{2 \mathfrak{g}_{2,j,i} \mathfrak{g}_{3,j,i}}{T_1^2 T_2} - \frac{\mathfrak{g}_{2,j,j} \mathfrak{g}_{3,j,i}}{T_1^2 T_2} + \frac{\mathfrak{g}_{1,i,i} \mathfrak{g}_{3,j,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{1,j,i} \mathfrak{g}_{3,j,j}}{T_1 (-1 + T_1 T_2)} - \frac{\mathfrak{g}_{2,j,i} \mathfrak{g}_{3,j,j}}{T_1 (-1 + T_1 T_2)} \end{aligned}$$

In[*]:= $\text{px2g}[r_{0,\text{pxx}}[1, i_0, j_0] r_{1,\text{pxx}}[1, i_1, j_1]]$

Out[*]=

$$\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1} - \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1} - \mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1} + \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1}$$

In[*]:= $\theta[\{1, i_0, j_0\}, \{1, i_1, j_1\}] = \text{px2g}[r_{0,\text{pxx}}[1, i_0, j_0] r_{1,\text{pxx}}[1, i_1, j_1]]$

$\theta[\{1, i_0, j_0\}, \{-1, i_1, j_1\}] = \text{px2g}[r_{0,\text{pxx}}[1, i_0, j_0] r_{1,\text{pxx}}[-1, i_1, j_1]]$

$\theta[\{-1, i_0, j_0\}, \{1, i_1, j_1\}] = \text{px2g}[r_{0,\text{pxx}}[-1, i_0, j_0] r_{1,\text{pxx}}[1, i_1, j_1]]$

$\theta[\{-1, i_0, j_0\}, \{-1, i_1, j_1\}] = \text{px2g}[r_{0,\text{pxx}}[-1, i_0, j_0] r_{1,\text{pxx}}[-1, i_1, j_1]]$

Out[*]=

$$\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1} - \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1} - \mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1} + \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1}$$

Out[*]=

$$- \frac{\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1} + \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^2} + \frac{\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1} - \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^2}$$

Out[*]=

$$- \frac{\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^2 T_2} + \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1 T_2} + \frac{\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^2 T_2} - \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1 T_2}$$

Out[*]=

$$\frac{\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^3 T_2} - \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,i_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^2 T_2} - \frac{\mathfrak{g}_{1,j_1,i_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^3 T_2} + \frac{\mathfrak{g}_{1,j_1,j_0} \mathfrak{g}_{2,j_1,i_0} \mathfrak{g}_{3,j_0,i_1}}{T_1^2 T_2}$$

In[*]:= $T_1[\varphi, k] = \text{px2g}[\gamma_1[\varphi, k]]$

Out[*]=

$$- \frac{\varphi \mathfrak{g}_{3,k,k}}{T_1 (-1 + T_1 T_2)}$$

```

In[*]:= T3 = T1 T2;
CF[ε_] := Factor@Together[ε];
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} >=> (A[[{i, j}, {i + 1, j + 1}]] += (
    -T^s T^s - 1
    0 -1
  ))];
  Δ = T^(-Total[φ] - Total[Cs[[All, 1]])/2) Det[A];
  G = Inverse[A]; gEval[ε_] := CF[ε /. g_{ν, α, β} >=> (G[[α, β]] /. T -> T_ν)];
  z = gEval[Sum[Sum[θ[Cs[[k1]], Cs[[k2]]], {k2, 1, n}], {k1, 1, n}];
  z += gEval[Sum[R1 @@ Cs[[k]], {k, 1, n}];
  z += gEval[Sum[T1[φ[[k]], k], {k, 1, 2^n}];
  {Δ, (Δ /. T -> T1) (Δ /. T -> T2) (Δ /. T -> T3) z} // CF
];

```

```

In[*]:= Timing[θ[Knot[3, 1]]]

```

 KnotTheory: Loading precomputed data in PD4Knots`.

Out[*]=

$$\left\{ \theta., \left\{ \frac{1 - T + T^2}{T}, \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \right. \right. \\
 \left. \left. (-1 + T_1 - T_1^2 + T_2 - T_1^2 T_2 + 2 T_1^3 T_2 - T_2^2 - T_1 T_2^2 + T_1^2 T_2^2 - 2 T_1^3 T_2^2 + 2 T_1 T_2^3 - 2 T_1^2 T_2^3 + 2 T_1^3 T_2^3) \right\} \right\}$$

```

In[*]:= Timing[θ[Knot[4, 1]]]

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Out[*]=

$$\left\{ \theta., \left\{ -\frac{1 - 3 T + T^2}{T}, -\frac{(1 - 3 T_1 + T_1^2) (1 + T_1 T_2) (1 - 3 T_2 + T_2^2)}{T_1^3 T_2^2} \right\} \right\}$$

In[*]:= **Timing**[Θ [Knot["K11n34"]]]

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[*]=

$$\left\{ 0.03125, \left\{ 1, \frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} \left(T_1^2 - 2 T_1^3 + T_1^4 - 2 T_1 T_2 + 2 T_1^2 T_2 + 2 T_1^5 T_2 - 2 T_1^6 T_2 + T_2^2 + 2 T_1 T_2^2 - 2 T_1^2 T_2^2 - 2 T_1^4 T_2^2 - 2 T_1^6 T_2^2 + 2 T_1^7 T_2^2 + T_1^8 T_2^2 - 2 T_2^3 + T_1^4 T_2^3 + T_1^5 T_2^3 - 2 T_1^9 T_2^3 + T_2^4 - 2 T_1^2 T_2^4 + T_1^3 T_2^4 + 2 T_1^4 T_2^4 + 2 T_1^6 T_2^4 + T_1^7 T_2^4 - 2 T_1^8 T_2^4 + T_1^{10} T_2^4 + 2 T_1 T_2^5 + T_1^3 T_2^5 - 4 T_1^5 T_2^5 - 4 T_1^6 T_2^5 + T_1^8 T_2^5 + 2 T_1^{10} T_2^5 - 2 T_1 T_2^6 - 2 T_1^2 T_2^6 + 2 T_1^4 T_2^6 - 4 T_1^5 T_2^6 + 12 T_1^6 T_2^6 - 4 T_1^7 T_2^6 + 2 T_1^8 T_2^6 - 2 T_1^{10} T_2^6 - 2 T_1^{11} T_2^6 + 2 T_1^2 T_2^7 + T_1^4 T_2^7 - 4 T_1^6 T_2^7 - 4 T_1^7 T_2^7 + T_1^9 T_2^7 + 2 T_1^{11} T_2^7 + T_1^2 T_2^8 - 2 T_1^4 T_2^8 + T_1^5 T_2^8 + 2 T_1^6 T_2^8 + 2 T_1^8 T_2^8 + T_1^9 T_2^8 - 2 T_1^{10} T_2^8 + T_1^{12} T_2^8 - 2 T_1^3 T_2^9 + T_1^7 T_2^9 + T_1^8 T_2^9 - 2 T_1^{12} T_2^9 + T_1 T_2^{10} + 2 T_1^5 T_2^{10} - 2 T_1^6 T_2^{10} - 2 T_1^8 T_2^{10} - 2 T_1^{10} T_2^{10} + 2 T_1^{11} T_2^{10} + T_1^{12} T_2^{10} - 2 T_1^6 T_2^{11} + 2 T_1^7 T_2^{11} + 2 T_1^{10} T_2^{11} - 2 T_1^{11} T_2^{11} + T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + T_1^{10} T_2^{12} \right) \right\} \right\}$$

In[*]:= **Timing**[Θ [Knot["K11n42"]]]

Out[*]=

$$\left\{ 0.03125, \left\{ 1, -\frac{1}{T_1^4 T_2^3 (-1 + T_1 T_2)} \left(T_1 + T_1^2 + T_2 - 2 T_1 T_2 - 2 T_1^2 T_2 - 2 T_1^3 T_2 + T_1^4 T_2 + T_2^2 - 2 T_1 T_2^2 + 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - 2 T_1^4 T_2^2 + T_1^5 T_2^2 - 2 T_1 T_2^3 + 2 T_1^2 T_2^3 + 2 T_1^4 T_2^3 - 2 T_1^5 T_2^3 + T_1 T_2^4 - 2 T_1^2 T_2^4 + 2 T_1^3 T_2^4 + 2 T_1^4 T_2^4 - 2 T_1^5 T_2^4 + T_1^6 T_2^4 + T_1^2 T_2^5 + T_1^3 T_2^5 - 2 T_1^3 T_2^5 - 2 T_1^4 T_2^5 - 2 T_1^5 T_2^5 + T_1^6 T_2^5 + T_1^4 T_2^6 + T_1^5 T_2^6 \right) \right\} \right\}$$

In[*]:= **PD**[**GST48**] = **PD**[**X**[1, 15, 2, 14], **X**[29, 2, 30, 3], **X**[40, 4, 41, 3], **X**[4, 44, 5, 43], **X**[5, 26, 6, 27], **X**[95, 7, 96, 6], **X**[7, 1, 8, 96], **X**[8, 14, 9, 13], **X**[28, 9, 29, 10], **X**[41, 11, 42, 10], **X**[11, 43, 12, 42], **X**[12, 27, 13, 28], **X**[15, 31, 16, 30], **X**[61, 16, 62, 17], **X**[72, 17, 73, 18], **X**[83, 18, 84, 19], **X**[34, 20, 35, 19], **X**[20, 89, 21, 90], **X**[92, 21, 93, 22], **X**[22, 79, 23, 80], **X**[23, 68, 24, 69], **X**[24, 57, 25, 58], **X**[56, 25, 57, 26], **X**[31, 63, 32, 62], **X**[32, 74, 33, 73], **X**[33, 85, 34, 84], **X**[35, 50, 36, 51], **X**[81, 37, 82, 36], **X**[70, 38, 71, 37], **X**[59, 39, 60, 38], **X**[54, 39, 55, 40], **X**[55, 45, 56, 44], **X**[45, 59, 46, 58], **X**[46, 70, 47, 69], **X**[47, 81, 48, 80], **X**[91, 49, 92, 48], **X**[49, 91, 50, 90], **X**[82, 52, 83, 51], **X**[71, 53, 72, 52], **X**[60, 54, 61, 53], **X**[74, 63, 75, 64], **X**[85, 64, 86, 65], **X**[65, 76, 66, 77], **X**[66, 87, 67, 88], **X**[94, 67, 95, 68], **X**[86, 75, 87, 76], **X**[77, 88, 78, 89], **X**[93, 78, 94, 79]]];

In[*]:= **AbsoluteTiming**[**gst48** = Θ [**GST48**]]

Out[*]=

$$\left\{ 13.3561, \left\{ -\frac{(-1 + 2 T - T^2 - T^3 + 2 T^4 - T^5 + T^8) (-1 + T^3 - 2 T^4 + T^5 + T^6 - 2 T^7 + T^8)}{T^8}, -\frac{1}{T_1^{21} T_2^{20} (-1 + T_1 T_2)} \left(T_1^5 - 3 T_1^6 + 4 T_1^7 - 2 T_1^8 - 2 T_1^9 + 4 T_1^{10} - 2 T_1^{11} - 2 T_1^{12} + 4 T_1^{13} - 3 T_1^{14} + T_1^{15} - 3 T_1^5 T_2 + 6 T_1^6 T_2 - 3 T_1^7 T_2 - 6 T_1^8 T_2 + 12 T_1^9 T_2 - 6 T_1^{10} T_2 - 6 T_1^{11} T_2 + 12 T_1^{12} T_2 - 6 T_1^{13} T_2 - 3 T_1^{14} T_2 + 6 T_1^{15} T_2 - 3 T_1^{16} T_2 - T_1^3 T_2^2 + \right) \right\} \right\}$$

$$\begin{aligned}
 & 3 T_1^4 T_2^2 - T_1^6 T_2^2 - 4 T_1^7 T_2^2 + 9 T_1^8 T_2^2 - 7 T_1^9 T_2^2 - 3 T_1^{10} T_2^2 + 8 T_1^{11} T_2^2 - 3 T_1^{12} T_2^2 - 7 T_1^{13} T_2^2 + 9 T_1^{14} T_2^2 - \\
 & 4 T_1^{15} T_2^2 - T_1^{16} T_2^2 + 3 T_1^{18} T_2^2 - T_1^{19} T_2^2 - T_1^2 T_2^3 + 6 T_1^3 T_2^3 - 10 T_1^4 T_2^3 + 3 T_1^5 T_2^3 + 2 T_1^6 T_2^3 - 3 T_1^7 T_2^3 + \\
 & 4 T_1^8 T_2^3 - 2 T_1^9 T_2^3 + 2 T_1^{10} T_2^3 - T_1^{11} T_2^3 - T_1^{12} T_2^3 + 2 T_1^{13} T_2^3 - 2 T_1^{14} T_2^3 + 4 T_1^{15} T_2^3 - 3 T_1^{16} T_2^3 + 2 T_1^{17} T_2^3 + \\
 & 3 T_1^{18} T_2^3 - 10 T_1^{19} T_2^3 + 6 T_1^{20} T_2^3 - T_1^{21} T_2^3 + 3 T_1^2 T_2^4 - 10 T_1^3 T_2^4 + 3 T_1^4 T_2^4 + 17 T_1^5 T_2^4 - 19 T_1^6 T_2^4 + \\
 & 10 T_1^7 T_2^4 - 7 T_1^8 T_2^4 + 6 T_1^9 T_2^4 - T_1^{10} T_2^4 - 18 T_1^{11} T_2^4 + 35 T_1^{12} T_2^4 - 18 T_1^{13} T_2^4 - T_1^{14} T_2^4 + 6 T_1^{15} T_2^4 - 7 T_1^{16} T_2^4 + \\
 & 10 T_1^{17} T_2^4 - 19 T_1^{18} T_2^4 + 17 T_1^{19} T_2^4 + 3 T_1^{20} T_2^4 - 10 T_1^{21} T_2^4 + 3 T_1^{22} T_2^4 + T_2^5 - 3 T_1 T_2^5 + 3 T_1^3 T_2^5 + 17 T_1^4 T_2^5 - \\
 & 38 T_1^5 T_2^5 + 22 T_1^6 T_2^5 + 7 T_1^7 T_2^5 - 11 T_1^8 T_2^5 - 6 T_1^9 T_2^5 + 14 T_1^{10} T_2^5 + 11 T_1^{11} T_2^5 - 31 T_1^{12} T_2^5 + 9 T_1^{13} T_2^5 + \\
 & T_1^{14} T_2^5 + 8 T_1^{16} T_2^5 - 15 T_1^{17} T_2^5 + 9 T_1^{18} T_2^5 + 16 T_1^{19} T_2^5 - 32 T_1^{20} T_2^5 + 15 T_1^{21} T_2^5 + 3 T_1^{22} T_2^5 - 3 T_1^{24} T_2^5 + \\
 & T_1^{25} T_2^5 - 3 T_2^6 + 6 T_1 T_2^6 - T_1^2 T_2^6 + 2 T_1^3 T_2^6 - 19 T_1^4 T_2^6 + 22 T_1^5 T_2^6 + 24 T_1^6 T_2^6 - 68 T_1^7 T_2^6 + 43 T_1^8 T_2^6 + \\
 & 9 T_1^9 T_2^6 - 29 T_1^{10} T_2^6 + 2 T_1^{11} T_2^6 - 12 T_1^{12} T_2^6 + 28 T_1^{13} T_2^6 - 42 T_1^{14} T_2^6 + 26 T_1^{15} T_2^6 - 29 T_1^{16} T_2^6 - T_1^{17} T_2^6 + \\
 & 45 T_1^{18} T_2^6 - 64 T_1^{19} T_2^6 + 24 T_1^{20} T_2^6 + 18 T_1^{21} T_2^6 - 17 T_1^{22} T_2^6 + 2 T_1^{23} T_2^6 - T_1^{24} T_2^6 + 6 T_1^{25} T_2^6 - 3 T_1^{26} T_2^6 + \\
 & 4 T_2^7 - 3 T_1 T_2^7 - 4 T_1^2 T_2^7 - 3 T_1^3 T_2^7 + 10 T_1^4 T_2^7 + 7 T_1^5 T_2^7 - 68 T_1^6 T_2^7 + 74 T_1^7 T_2^7 + 14 T_1^8 T_2^7 - 56 T_1^9 T_2^7 + \\
 & 14 T_1^{10} T_2^7 + 55 T_1^{11} T_2^7 - 23 T_1^{12} T_2^7 + 11 T_1^{13} T_2^7 + 51 T_1^{14} T_2^7 - 33 T_1^{15} T_2^7 + 41 T_1^{16} T_2^7 + 28 T_1^{17} T_2^7 - 60 T_1^{18} T_2^7 + \\
 & 16 T_1^{19} T_2^7 + 68 T_1^{20} T_2^7 - 62 T_1^{21} T_2^7 + 5 T_1^{22} T_2^7 + 10 T_1^{23} T_2^7 - 3 T_1^{24} T_2^7 - 4 T_1^{25} T_2^7 - 3 T_1^{26} T_2^7 + 4 T_1^{27} T_2^7 - \\
 & 2 T_2^8 - 6 T_1 T_2^8 + 9 T_1^2 T_2^8 + 4 T_1^3 T_2^8 - 7 T_1^4 T_2^8 - 11 T_1^5 T_2^8 + 43 T_1^6 T_2^8 + 14 T_1^7 T_2^8 - 123 T_1^8 T_2^8 + 133 T_1^9 T_2^8 - \\
 & 36 T_1^{10} T_2^8 - 89 T_1^{11} T_2^8 + 136 T_1^{12} T_2^8 - 127 T_1^{13} T_2^8 + 31 T_1^{14} T_2^8 - 31 T_1^{15} T_2^8 + 16 T_1^{16} T_2^8 - 33 T_1^{17} T_2^8 - \\
 & 28 T_1^{18} T_2^8 + 109 T_1^{19} T_2^8 - 115 T_1^{20} T_2^8 + 14 T_1^{21} T_2^8 + 51 T_1^{22} T_2^8 - 27 T_1^{23} T_2^8 + T_1^{24} T_2^8 + 4 T_1^{25} T_2^8 + 9 T_1^{26} T_2^8 - \\
 & 6 T_1^{27} T_2^8 - 2 T_1^{28} T_2^8 - 2 T_2^9 + 12 T_1 T_2^9 - 7 T_1^2 T_2^9 - 2 T_1^3 T_2^9 + 6 T_1^4 T_2^9 - 6 T_1^5 T_2^9 + 9 T_1^6 T_2^9 - 56 T_1^7 T_2^9 + \\
 & 133 T_1^8 T_2^9 - 149 T_1^9 T_2^9 - 10 T_1^{10} T_2^9 + 224 T_1^{11} T_2^9 - 314 T_1^{12} T_2^9 + 67 T_1^{13} T_2^9 + 111 T_1^{14} T_2^9 - 124 T_1^{15} T_2^9 + \\
 & 38 T_1^{16} T_2^9 - 49 T_1^{17} T_2^9 + 50 T_1^{18} T_2^9 - 38 T_1^{19} T_2^9 - 47 T_1^{20} T_2^9 + 95 T_1^{21} T_2^9 - 68 T_1^{22} T_2^9 + 8 T_1^{23} T_2^9 + 32 T_1^{24} T_2^9 - \\
 & 19 T_1^{25} T_2^9 - 2 T_1^{26} T_2^9 - 7 T_1^{27} T_2^9 + 12 T_1^{28} T_2^9 - 2 T_1^{29} T_2^9 + 4 T_2^{10} - 6 T_1 T_2^{10} - 3 T_1^2 T_2^{10} + 2 T_1^3 T_2^{10} - T_1^4 T_2^{10} + \\
 & 14 T_1^5 T_2^{10} - 29 T_1^6 T_2^{10} + 14 T_1^7 T_2^{10} - 36 T_1^8 T_2^{10} - 10 T_1^9 T_2^{10} + 240 T_1^{10} T_2^{10} - 314 T_1^{11} T_2^{10} + 74 T_1^{12} T_2^{10} + \\
 & 431 T_1^{13} T_2^{10} - 386 T_1^{14} T_2^{10} + 200 T_1^{15} T_2^{10} + 34 T_1^{16} T_2^{10} - 37 T_1^{17} T_2^{10} + 186 T_1^{18} T_2^{10} - 186 T_1^{19} T_2^{10} + \\
 & 136 T_1^{20} T_2^{10} - 22 T_1^{21} T_2^{10} - 12 T_1^{22} T_2^{10} + 46 T_1^{23} T_2^{10} - 93 T_1^{24} T_2^{10} + 30 T_1^{25} T_2^{10} + 11 T_1^{26} T_2^{10} + 2 T_1^{27} T_2^{10} - \\
 & 3 T_1^{28} T_2^{10} - 6 T_1^{29} T_2^{10} + 4 T_1^{30} T_2^{10} - 2 T_2^{11} - 6 T_1 T_2^{11} + 8 T_1^2 T_2^{11} - T_1^3 T_2^{11} - 18 T_1^4 T_2^{11} + 11 T_1^5 T_2^{11} + \\
 & 2 T_1^6 T_2^{11} + 55 T_1^7 T_2^{11} - 89 T_1^8 T_2^{11} + 224 T_1^9 T_2^{11} - 314 T_1^{10} T_2^{11} - 92 T_1^{11} T_2^{11} + 764 T_1^{12} T_2^{11} - 899 T_1^{13} T_2^{11} + \\
 & 273 T_1^{14} T_2^{11} + 176 T_1^{15} T_2^{11} - 382 T_1^{16} T_2^{11} + 391 T_1^{17} T_2^{11} - 420 T_1^{18} T_2^{11} + 75 T_1^{19} T_2^{11} + 212 T_1^{20} T_2^{11} - \\
 & 156 T_1^{21} T_2^{11} - 46 T_1^{22} T_2^{11} - 6 T_1^{23} T_2^{11} + 65 T_1^{24} T_2^{11} + 76 T_1^{25} T_2^{11} - 107 T_1^{26} T_2^{11} + 31 T_1^{27} T_2^{11} - T_1^{28} T_2^{11} + \\
 & 8 T_1^{29} T_2^{11} - 6 T_1^{30} T_2^{11} - 2 T_1^{31} T_2^{11} - 2 T_2^{12} + 12 T_1 T_2^{12} - 3 T_1^2 T_2^{12} - T_1^3 T_2^{12} + 35 T_1^4 T_2^{12} - 31 T_1^5 T_2^{12} - \\
 & 12 T_1^6 T_2^{12} - 23 T_1^7 T_2^{12} + 136 T_1^8 T_2^{12} - 314 T_1^9 T_2^{12} + 74 T_1^{10} T_2^{12} + 764 T_1^{11} T_2^{12} - 1304 T_1^{12} T_2^{12} + 293 T_1^{13} T_2^{12} + \\
 & 744 T_1^{14} T_2^{12} - 996 T_1^{15} T_2^{12} + 616 T_1^{16} T_2^{12} - 380 T_1^{17} T_2^{12} - 68 T_1^{18} T_2^{12} + 589 T_1^{19} T_2^{12} - 596 T_1^{20} T_2^{12} - \\
 & 72 T_1^{21} T_2^{12} + 294 T_1^{22} T_2^{12} + 38 T_1^{23} T_2^{12} - 64 T_1^{24} T_2^{12} - 123 T_1^{25} T_2^{12} + 60 T_1^{26} T_2^{12} + 93 T_1^{27} T_2^{12} - 69 T_1^{28} T_2^{12} - \\
 & T_1^{29} T_2^{12} - 3 T_1^{30} T_2^{12} + 12 T_1^{31} T_2^{12} - 2 T_1^{32} T_2^{12} + 4 T_2^{13} - 6 T_1 T_2^{13} - 7 T_1^2 T_2^{13} + 2 T_1^3 T_2^{13} - 18 T_1^4 T_2^{13} + \\
 & 9 T_1^5 T_2^{13} + 28 T_1^6 T_2^{13} + 11 T_1^7 T_2^{13} - 127 T_1^8 T_2^{13} + 67 T_1^9 T_2^{13} + 431 T_1^{10} T_2^{13} - 899 T_1^{11} T_2^{13} + 293 T_1^{12} T_2^{13} + \\
 & 1556 T_1^{13} T_2^{13} - 1724 T_1^{14} T_2^{13} + 887 T_1^{15} T_2^{13} + 223 T_1^{16} T_2^{13} - 480 T_1^{17} T_2^{13} + 998 T_1^{18} T_2^{13} - 905 T_1^{19} T_2^{13} + \\
 & 212 T_1^{20} T_2^{13} + 686 T_1^{21} T_2^{13} - 294 T_1^{22} T_2^{13} - 313 T_1^{23} T_2^{13} + 146 T_1^{24} T_2^{13} + 24 T_1^{25} T_2^{13} + 123 T_1^{26} T_2^{13} - \\
 & 238 T_1^{27} T_2^{13} + 65 T_1^{28} T_2^{13} + 45 T_1^{29} T_2^{13} + 2 T_1^{30} T_2^{13} - 7 T_1^{31} T_2^{13} - 6 T_1^{32} T_2^{13} + 4 T_1^{33} T_2^{13} - 3 T_2^{14} - 3 T_1 T_2^{14} + \\
 & 9 T_1^2 T_2^{14} - 2 T_1^3 T_2^{14} - 4 T_1^4 T_2^{14} + T_1^5 T_2^{14} - 42 T_1^6 T_2^{14} + 51 T_1^7 T_2^{14} + 31 T_1^8 T_2^{14} + 111 T_1^9 T_2^{14} - 386 T_1^{10} T_2^{14} + \\
 & 273 T_1^{11} T_2^{14} + 744 T_1^{12} T_2^{14} - 1724 T_1^{13} T_2^{14} + 705 T_1^{14} T_2^{14} + 482 T_1^{15} T_2^{14} - 1315 T_1^{16} T_2^{14} + 1061 T_1^{17} T_2^{14} - \\
 & 855 T_1^{18} T_2^{14} - 140 T_1^{19} T_2^{14} + 809 T_1^{20} T_2^{14} - 758 T_1^{21} T_2^{14} - 370 T_1^{22} T_2^{14} + 595 T_1^{23} T_2^{14} + 58 T_1^{24} T_2^{14} - \\
 & 229 T_1^{25} T_2^{14} + T_1^{26} T_2^{14} + 95 T_1^{27} T_2^{14} + 124 T_1^{28} T_2^{14} - 151 T_1^{29} T_2^{14} + 19 T_1^{30} T_2^{14} - 2 T_1^{31} T_2^{14} + 9 T_1^{32} T_2^{14} - \\
 & 3 T_1^{33} T_2^{14} - 3 T_1^{34} T_2^{14} + T_2^{15} + 6 T_1 T_2^{15} - 4 T_1^2 T_2^{15} + 4 T_1^3 T_2^{15} + 6 T_1^4 T_2^{15} + 26 T_1^5 T_2^{15} - 33 T_1^6 T_2^{15} - 31 T_1^7 T_2^{15} - \\
 & 124 T_1^8 T_2^{15} + 200 T_1^9 T_2^{15} + 176 T_1^{10} T_2^{15} - 996 T_1^{11} T_2^{15} + 887 T_1^{12} T_2^{15} + 482 T_1^{13} T_2^{15} + 482 T_1^{14} T_2^{15} - 1534 T_1^{15} T_2^{15} + \\
 & 1712 T_1^{16} T_2^{15} - 619 T_1^{17} T_2^{15} - 569 T_1^{18} T_2^{15} + 1420 T_1^{19} T_2^{15} - 914 T_1^{20} T_2^{15} - 229 T_1^{21} T_2^{15} + 992 T_1^{22} T_2^{15} - \\
 & 257 T_1^{23} T_2^{15} - 598 T_1^{24} T_2^{15} + 440 T_1^{25} T_2^{15} - 15 T_1^{26} T_2^{15} - 50 T_1^{27} T_2^{15} - 167 T_1^{28} T_2^{15} + 92 T_1^{29} T_2^{15} + 74 T_1^{30} T_2^{15} - \\
 & 49 T_1^{31} T_2^{15} + 4 T_1^{32} T_2^{15} - 4 T_1^{33} T_2^{15} + 6 T_1^{34} T_2^{15} + T_1^{35} T_2^{15} - 3 T_1 T_2^{16} - T_1^2 T_2^{16} - 3 T_1^3 T_2^{16} - 7 T_1^4 T_2^{16} +
 \end{aligned}$$

$$\begin{aligned}
 & 8 T_1^5 T_2^{16} - 29 T_1^6 T_2^{16} + 41 T_1^7 T_2^{16} + 16 T_1^8 T_2^{16} + 38 T_1^9 T_2^{16} + 34 T_1^{10} T_2^{16} - 382 T_1^{11} T_2^{16} + 616 T_1^{12} T_2^{16} + \\
 & 223 T_1^{13} T_2^{16} - 1315 T_1^{14} T_2^{16} + 1712 T_1^{15} T_2^{16} - 720 T_1^{16} T_2^{16} - 1180 T_1^{17} T_2^{16} + 2146 T_1^{18} T_2^{16} - 1310 T_1^{19} T_2^{16} - \\
 & 260 T_1^{20} T_2^{16} + 1108 T_1^{21} T_2^{16} - 545 T_1^{22} T_2^{16} - 555 T_1^{23} T_2^{16} + 792 T_1^{24} T_2^{16} - 94 T_1^{25} T_2^{16} - 350 T_1^{26} T_2^{16} + \\
 & 256 T_1^{27} T_2^{16} - 24 T_1^{28} T_2^{16} + 109 T_1^{29} T_2^{16} - 189 T_1^{30} T_2^{16} + 60 T_1^{31} T_2^{16} + 17 T_1^{32} T_2^{16} - 3 T_1^{33} T_2^{16} - T_1^{34} T_2^{16} - \\
 & 3 T_1^{35} T_2^{16} + 2 T_1^3 T_2^{17} + 10 T_1^4 T_2^{17} - 15 T_1^5 T_2^{17} - T_1^6 T_2^{17} + 28 T_1^7 T_2^{17} - 33 T_1^8 T_2^{17} - 49 T_1^9 T_2^{17} - 37 T_1^{10} T_2^{17} + \\
 & 391 T_1^{11} T_2^{17} - 380 T_1^{12} T_2^{17} - 480 T_1^{13} T_2^{17} + 1061 T_1^{14} T_2^{17} - 619 T_1^{15} T_2^{17} - 1180 T_1^{16} T_2^{17} + 2566 T_1^{17} T_2^{17} - \\
 & 1730 T_1^{18} T_2^{17} - 591 T_1^{19} T_2^{17} + 1520 T_1^{20} T_2^{17} - 933 T_1^{21} T_2^{17} - 265 T_1^{22} T_2^{17} + 476 T_1^{23} T_2^{17} + 123 T_1^{24} T_2^{17} - \\
 & 791 T_1^{25} T_2^{17} + 681 T_1^{26} T_2^{17} - 213 T_1^{27} T_2^{17} - 82 T_1^{28} T_2^{17} - 8 T_1^{29} T_2^{17} + 74 T_1^{30} T_2^{17} + 42 T_1^{31} T_2^{17} - 59 T_1^{32} T_2^{17} + \\
 & 10 T_1^{33} T_2^{17} + 2 T_1^{34} T_2^{17} + 3 T_1^2 T_2^{18} + 3 T_1^3 T_2^{18} - 19 T_1^4 T_2^{18} + 9 T_1^5 T_2^{18} + 45 T_1^6 T_2^{18} - 60 T_1^7 T_2^{18} - 28 T_1^8 T_2^{18} + \\
 & 50 T_1^9 T_2^{18} + 186 T_1^{10} T_2^{18} - 420 T_1^{11} T_2^{18} - 68 T_1^{12} T_2^{18} + 998 T_1^{13} T_2^{18} - 855 T_1^{14} T_2^{18} - 569 T_1^{15} T_2^{18} + \\
 & 2146 T_1^{16} T_2^{18} - 1730 T_1^{17} T_2^{18} - 492 T_1^{18} T_2^{18} + 2218 T_1^{19} T_2^{18} - 1372 T_1^{20} T_2^{18} - 146 T_1^{21} T_2^{18} + 878 T_1^{22} T_2^{18} - \\
 & 163 T_1^{23} T_2^{18} - 695 T_1^{24} T_2^{18} + 872 T_1^{25} T_2^{18} - 162 T_1^{26} T_2^{18} - 458 T_1^{27} T_2^{18} + 506 T_1^{28} T_2^{18} - 208 T_1^{29} T_2^{18} + \\
 & 44 T_1^{30} T_2^{18} - 100 T_1^{31} T_2^{18} + 79 T_1^{32} T_2^{18} - 19 T_1^{33} T_2^{18} - 5 T_1^{34} T_2^{18} + 3 T_1^{35} T_2^{18} + 3 T_1^{36} T_2^{18} - T_1^{37} T_2^{18} - \\
 & 10 T_1^3 T_2^{19} + 17 T_1^4 T_2^{19} + 16 T_1^5 T_2^{19} - 64 T_1^6 T_2^{19} + 16 T_1^7 T_2^{19} + 109 T_1^8 T_2^{19} - 38 T_1^9 T_2^{19} - 186 T_1^{10} T_2^{19} + \\
 & 75 T_1^{11} T_2^{19} + 589 T_1^{12} T_2^{19} - 905 T_1^{13} T_2^{19} - 140 T_1^{14} T_2^{19} + 1420 T_1^{15} T_2^{19} - 1310 T_1^{16} T_2^{19} - 591 T_1^{17} T_2^{19} + \\
 & 2218 T_1^{18} T_2^{19} - 2027 T_1^{19} T_2^{19} + 155 T_1^{20} T_2^{19} + 1033 T_1^{21} T_2^{19} - 840 T_1^{22} T_2^{19} - 49 T_1^{23} T_2^{19} + 464 T_1^{24} T_2^{19} + \\
 & 37 T_1^{25} T_2^{19} - 842 T_1^{26} T_2^{19} + 972 T_1^{27} T_2^{19} - 412 T_1^{28} T_2^{19} - 44 T_1^{29} T_2^{19} + 150 T_1^{30} T_2^{19} - 21 T_1^{31} T_2^{19} - \\
 & 10 T_1^{32} T_2^{19} - 42 T_1^{33} T_2^{19} + 50 T_1^{34} T_2^{19} - 13 T_1^{35} T_2^{19} - 10 T_1^{36} T_2^{19} - T_1^{37} T_2^{19} + 6 T_1^3 T_2^{20} + 3 T_1^4 T_2^{20} - \\
 & 32 T_1^5 T_2^{20} + 24 T_1^6 T_2^{20} + 68 T_1^7 T_2^{20} - 115 T_1^8 T_2^{20} - 47 T_1^9 T_2^{20} + 136 T_1^{10} T_2^{20} + 212 T_1^{11} T_2^{20} - 596 T_1^{12} T_2^{20} + \\
 & 212 T_1^{13} T_2^{20} + 809 T_1^{14} T_2^{20} - 914 T_1^{15} T_2^{20} - 260 T_1^{16} T_2^{20} + 1520 T_1^{17} T_2^{20} - 1372 T_1^{18} T_2^{20} + 155 T_1^{19} T_2^{20} + \\
 & 1056 T_1^{20} T_2^{20} - 1291 T_1^{21} T_2^{20} + 674 T_1^{22} T_2^{20} - 128 T_1^{23} T_2^{20} - 56 T_1^{24} T_2^{20} - 374 T_1^{25} T_2^{20} + 603 T_1^{26} T_2^{20} - \\
 & 180 T_1^{27} T_2^{20} - 504 T_1^{28} T_2^{20} + 592 T_1^{29} T_2^{20} - 340 T_1^{30} T_2^{20} + 71 T_1^{31} T_2^{20} - 39 T_1^{32} T_2^{20} + 100 T_1^{33} T_2^{20} - \\
 & 60 T_1^{34} T_2^{20} - 8 T_1^{35} T_2^{20} + 19 T_1^{36} T_2^{20} + 6 T_1^{37} T_2^{20} - T_1^3 T_2^{21} - 10 T_1^4 T_2^{21} + 15 T_1^5 T_2^{21} + 18 T_1^6 T_2^{21} - \\
 & 62 T_1^7 T_2^{21} + 14 T_1^8 T_2^{21} + 95 T_1^9 T_2^{21} - 22 T_1^{10} T_2^{21} - 156 T_1^{11} T_2^{21} - 72 T_1^{12} T_2^{21} + 686 T_1^{13} T_2^{21} - 758 T_1^{14} T_2^{21} - \\
 & 229 T_1^{15} T_2^{21} + 1108 T_1^{16} T_2^{21} - 933 T_1^{17} T_2^{21} - 146 T_1^{18} T_2^{21} + 1033 T_1^{19} T_2^{21} - 1291 T_1^{20} T_2^{21} + 891 T_1^{21} T_2^{21} - \\
 & 152 T_1^{22} T_2^{21} - 395 T_1^{23} T_2^{21} + 328 T_1^{24} T_2^{21} + 152 T_1^{25} T_2^{21} - 52 T_1^{26} T_2^{21} - 695 T_1^{27} T_2^{21} + 1069 T_1^{28} T_2^{21} - \\
 & 559 T_1^{29} T_2^{21} - 14 T_1^{30} T_2^{21} + 166 T_1^{31} T_2^{21} - 35 T_1^{32} T_2^{21} - 12 T_1^{33} T_2^{21} - 40 T_1^{34} T_2^{21} + 52 T_1^{35} T_2^{21} - \\
 & 15 T_1^{36} T_2^{21} - 10 T_1^{37} T_2^{21} - T_1^{38} T_2^{21} + 3 T_1^4 T_2^{22} + 3 T_1^5 T_2^{22} - 17 T_1^6 T_2^{22} + 5 T_1^7 T_2^{22} + 51 T_1^8 T_2^{22} - 68 T_1^9 T_2^{22} - \\
 & 12 T_1^{10} T_2^{22} - 46 T_1^{11} T_2^{22} + 294 T_1^{12} T_2^{22} - 294 T_1^{13} T_2^{22} - 370 T_1^{14} T_2^{22} + 992 T_1^{15} T_2^{22} - 545 T_1^{16} T_2^{22} - \\
 & 265 T_1^{17} T_2^{22} + 878 T_1^{18} T_2^{22} - 840 T_1^{19} T_2^{22} + 674 T_1^{20} T_2^{22} - 152 T_1^{21} T_2^{22} - 206 T_1^{22} T_2^{22} + 744 T_1^{23} T_2^{22} - \\
 & 390 T_1^{24} T_2^{22} + 141 T_1^{25} T_2^{22} - 385 T_1^{26} T_2^{22} + 866 T_1^{27} T_2^{22} - 464 T_1^{28} T_2^{22} - 332 T_1^{29} T_2^{22} + 614 T_1^{30} T_2^{22} - \\
 & 304 T_1^{31} T_2^{22} + 60 T_1^{32} T_2^{22} - 108 T_1^{33} T_2^{22} + 85 T_1^{34} T_2^{22} - 23 T_1^{35} T_2^{22} - 3 T_1^{36} T_2^{22} + 3 T_1^{37} T_2^{22} + 3 T_1^{38} T_2^{22} + \\
 & 2 T_1^6 T_2^{23} + 10 T_1^7 T_2^{23} - 27 T_1^8 T_2^{23} + 8 T_1^9 T_2^{23} + 46 T_1^{10} T_2^{23} - 6 T_1^{11} T_2^{23} + 38 T_1^{12} T_2^{23} - 313 T_1^{13} T_2^{23} + \\
 & 595 T_1^{14} T_2^{23} - 257 T_1^{15} T_2^{23} - 555 T_1^{16} T_2^{23} + 476 T_1^{17} T_2^{23} - 163 T_1^{18} T_2^{23} - 49 T_1^{19} T_2^{23} - 128 T_1^{20} T_2^{23} - \\
 & 395 T_1^{21} T_2^{23} + 744 T_1^{22} T_2^{23} - 1174 T_1^{23} T_2^{23} + 198 T_1^{24} T_2^{23} + 191 T_1^{25} T_2^{23} - 109 T_1^{26} T_2^{23} + 48 T_1^{27} T_2^{23} - \\
 & 668 T_1^{28} T_2^{23} + 885 T_1^{29} T_2^{23} - 489 T_1^{30} T_2^{23} + 5 T_1^{31} T_2^{23} + 19 T_1^{32} T_2^{23} + 92 T_1^{33} T_2^{23} + 51 T_1^{34} T_2^{23} - \\
 & 71 T_1^{35} T_2^{23} + 10 T_1^{36} T_2^{23} + 2 T_1^{37} T_2^{23} - 3 T_1^5 T_2^{24} - T_1^6 T_2^{24} - 3 T_1^7 T_2^{24} + T_1^8 T_2^{24} + 32 T_1^9 T_2^{24} - 93 T_1^{10} T_2^{24} + \\
 & 65 T_1^{11} T_2^{24} - 64 T_1^{12} T_2^{24} + 146 T_1^{13} T_2^{24} + 58 T_1^{14} T_2^{24} - 598 T_1^{15} T_2^{24} + 792 T_1^{16} T_2^{24} + 123 T_1^{17} T_2^{24} - \\
 & 695 T_1^{18} T_2^{24} + 464 T_1^{19} T_2^{24} - 56 T_1^{20} T_2^{24} + 328 T_1^{21} T_2^{24} - 390 T_1^{22} T_2^{24} + 198 T_1^{23} T_2^{24} + 404 T_1^{24} T_2^{24} - \\
 & 140 T_1^{25} T_2^{24} + 75 T_1^{26} T_2^{24} - 655 T_1^{27} T_2^{24} + 968 T_1^{28} T_2^{24} - 310 T_1^{29} T_2^{24} - 326 T_1^{30} T_2^{24} + 364 T_1^{31} T_2^{24} - \\
 & 104 T_1^{32} T_2^{24} + 133 T_1^{33} T_2^{24} - 253 T_1^{34} T_2^{24} + 84 T_1^{35} T_2^{24} + 25 T_1^{36} T_2^{24} - 3 T_1^{37} T_2^{24} - T_1^{38} T_2^{24} - 3 T_1^{39} T_2^{24} + \\
 & T_1^5 T_2^{25} + 6 T_1^6 T_2^{25} - 4 T_1^7 T_2^{25} + 4 T_1^8 T_2^{25} - 19 T_1^9 T_2^{25} + 30 T_1^{10} T_2^{25} + 76 T_1^{11} T_2^{25} - 123 T_1^{12} T_2^{25} + \\
 & 24 T_1^{13} T_2^{25} - 229 T_1^{14} T_2^{25} + 440 T_1^{15} T_2^{25} - 94 T_1^{16} T_2^{25} - 791 T_1^{17} T_2^{25} + 872 T_1^{18} T_2^{25} + 37 T_1^{19} T_2^{25} - \\
 & 374 T_1^{20} T_2^{25} + 152 T_1^{21} T_2^{25} + 141 T_1^{22} T_2^{25} + 191 T_1^{23} T_2^{25} - 140 T_1^{24} T_2^{25} + 246 T_1^{25} T_2^{25} - 674 T_1^{26} T_2^{25} + \\
 & 977 T_1^{27} T_2^{25} - 52 T_1^{28} T_2^{25} - 868 T_1^{29} T_2^{25} + 680 T_1^{30} T_2^{25} - 120 T_1^{31} T_2^{25} + 5 T_1^{32} T_2^{25} - 257 T_1^{33} T_2^{25} + \\
 & 142 T_1^{34} T_2^{25} + 104 T_1^{35} T_2^{25} - 74 T_1^{36} T_2^{25} + 4 T_1^{37} T_2^{25} - 4 T_1^{38} T_2^{25} + 6 T_1^{39} T_2^{25} + T_1^{40} T_2^{25} - 3 T_1^6 T_2^{26} -
 \end{aligned}$$

$$\begin{aligned}
 & 3 T_1^7 T_2^{26} + 9 T_1^8 T_2^{26} - 2 T_1^9 T_2^{26} + 11 T_1^{10} T_2^{26} - 107 T_1^{11} T_2^{26} + 60 T_1^{12} T_2^{26} + 123 T_1^{13} T_2^{26} + T_1^{14} T_2^{26} - \\
 & 15 T_1^{15} T_2^{26} - 350 T_1^{16} T_2^{26} + 681 T_1^{17} T_2^{26} - 162 T_1^{18} T_2^{26} - 842 T_1^{19} T_2^{26} + 603 T_1^{20} T_2^{26} - 52 T_1^{21} T_2^{26} - \\
 & 385 T_1^{22} T_2^{26} - 109 T_1^{23} T_2^{26} + 75 T_1^{24} T_2^{26} - 674 T_1^{25} T_2^{26} + 707 T_1^{26} T_2^{26} + 124 T_1^{27} T_2^{26} - 1276 T_1^{28} T_2^{26} + \\
 & 1003 T_1^{29} T_2^{26} + 94 T_1^{30} T_2^{26} - 355 T_1^{31} T_2^{26} - 29 T_1^{32} T_2^{26} + 167 T_1^{33} T_2^{26} + 226 T_1^{34} T_2^{26} - 259 T_1^{35} T_2^{26} + \\
 & 31 T_1^{36} T_2^{26} - 2 T_1^{37} T_2^{26} + 9 T_1^{38} T_2^{26} - 3 T_1^{39} T_2^{26} - 3 T_1^{40} T_2^{26} + 4 T_1^7 T_2^{27} - 6 T_1^8 T_2^{27} - 7 T_1^9 T_2^{27} + 2 T_1^{10} T_2^{27} + \\
 & 31 T_1^{11} T_2^{27} + 93 T_1^{12} T_2^{27} - 238 T_1^{13} T_2^{27} + 95 T_1^{14} T_2^{27} - 50 T_1^{15} T_2^{27} + 256 T_1^{16} T_2^{27} - 213 T_1^{17} T_2^{27} - \\
 & 458 T_1^{18} T_2^{27} + 972 T_1^{19} T_2^{27} - 180 T_1^{20} T_2^{27} - 695 T_1^{21} T_2^{27} + 866 T_1^{22} T_2^{27} + 48 T_1^{23} T_2^{27} - 655 T_1^{24} T_2^{27} + \\
 & 977 T_1^{25} T_2^{27} + 124 T_1^{26} T_2^{27} - 1524 T_1^{27} T_2^{27} + 1365 T_1^{28} T_2^{27} + 147 T_1^{29} T_2^{27} - 957 T_1^{30} T_2^{27} + 335 T_1^{31} T_2^{27} + \\
 & 101 T_1^{32} T_2^{27} + 207 T_1^{33} T_2^{27} - 504 T_1^{34} T_2^{27} + 149 T_1^{35} T_2^{27} + 94 T_1^{36} T_2^{27} + 2 T_1^{37} T_2^{27} - 7 T_1^{38} T_2^{27} - \\
 & 6 T_1^{39} T_2^{27} + 4 T_1^{40} T_2^{27} - 2 T_1^8 T_2^{28} + 12 T_1^9 T_2^{28} - 3 T_1^{10} T_2^{28} - T_1^{11} T_2^{28} - 69 T_1^{12} T_2^{28} + 65 T_1^{13} T_2^{28} + \\
 & 124 T_1^{14} T_2^{28} - 167 T_1^{15} T_2^{28} - 24 T_1^{16} T_2^{28} - 82 T_1^{17} T_2^{28} + 506 T_1^{18} T_2^{28} - 412 T_1^{19} T_2^{28} - 504 T_1^{20} T_2^{28} + \\
 & 1069 T_1^{21} T_2^{28} - 464 T_1^{22} T_2^{28} - 668 T_1^{23} T_2^{28} + 968 T_1^{24} T_2^{28} - 52 T_1^{25} T_2^{28} - 1276 T_1^{26} T_2^{28} + 1365 T_1^{27} T_2^{28} + \\
 & 204 T_1^{28} T_2^{28} - 1248 T_1^{29} T_2^{28} + 726 T_1^{30} T_2^{28} + 270 T_1^{31} T_2^{28} - 224 T_1^{32} T_2^{28} - 267 T_1^{33} T_2^{28} + 196 T_1^{34} T_2^{28} + \\
 & 189 T_1^{35} T_2^{28} - 173 T_1^{36} T_2^{28} - T_1^{37} T_2^{28} - 3 T_1^{38} T_2^{28} + 12 T_1^{39} T_2^{28} - 2 T_1^{40} T_2^{28} - 2 T_1^9 T_2^{29} - 6 T_1^{10} T_2^{29} + \\
 & 8 T_1^{11} T_2^{29} - T_1^{12} T_2^{29} + 45 T_1^{13} T_2^{29} - 151 T_1^{14} T_2^{29} + 92 T_1^{15} T_2^{29} + 109 T_1^{16} T_2^{29} - 8 T_1^{17} T_2^{29} - 208 T_1^{18} T_2^{29} - \\
 & 44 T_1^{19} T_2^{29} + 592 T_1^{20} T_2^{29} - 559 T_1^{21} T_2^{29} - 332 T_1^{22} T_2^{29} + 885 T_1^{23} T_2^{29} - 310 T_1^{24} T_2^{29} - 868 T_1^{25} T_2^{29} + \\
 & 1003 T_1^{26} T_2^{29} + 147 T_1^{27} T_2^{29} - 1248 T_1^{28} T_2^{29} + 896 T_1^{29} T_2^{29} + 114 T_1^{30} T_2^{29} - 478 T_1^{31} T_2^{29} + 75 T_1^{32} T_2^{29} + \\
 & 119 T_1^{33} T_2^{29} + 166 T_1^{34} T_2^{29} - 269 T_1^{35} T_2^{29} + 94 T_1^{36} T_2^{29} - T_1^{37} T_2^{29} + 8 T_1^{38} T_2^{29} - 6 T_1^{39} T_2^{29} - 2 T_1^{40} T_2^{29} + \\
 & 4 T_1^{10} T_2^{30} - 6 T_1^{11} T_2^{30} - 3 T_1^{12} T_2^{30} + 2 T_1^{13} T_2^{30} + 19 T_1^{14} T_2^{30} + 74 T_1^{15} T_2^{30} - 189 T_1^{16} T_2^{30} + 74 T_1^{17} T_2^{30} + \\
 & 44 T_1^{18} T_2^{30} + 150 T_1^{19} T_2^{30} - 340 T_1^{20} T_2^{30} - 14 T_1^{21} T_2^{30} + 614 T_1^{22} T_2^{30} - 489 T_1^{23} T_2^{30} - 326 T_1^{24} T_2^{30} + \\
 & 680 T_1^{25} T_2^{30} + 94 T_1^{26} T_2^{30} - 957 T_1^{27} T_2^{30} + 726 T_1^{28} T_2^{30} + 114 T_1^{29} T_2^{30} - 444 T_1^{30} T_2^{30} + 138 T_1^{31} T_2^{30} + \\
 & 68 T_1^{32} T_2^{30} + 106 T_1^{33} T_2^{30} - 253 T_1^{34} T_2^{30} + 90 T_1^{35} T_2^{30} + 31 T_1^{36} T_2^{30} + 2 T_1^{37} T_2^{30} - 3 T_1^{38} T_2^{30} - 6 T_1^{39} T_2^{30} + \\
 & 4 T_1^{40} T_2^{30} - 2 T_1^{11} T_2^{31} + 12 T_1^{12} T_2^{31} - 7 T_1^{13} T_2^{31} - 2 T_1^{14} T_2^{31} - 49 T_1^{15} T_2^{31} + 60 T_1^{16} T_2^{31} + 42 T_1^{17} T_2^{31} - \\
 & 100 T_1^{18} T_2^{31} - 21 T_1^{19} T_2^{31} + 71 T_1^{20} T_2^{31} + 166 T_1^{21} T_2^{31} - 304 T_1^{22} T_2^{31} + 5 T_1^{23} T_2^{31} + 364 T_1^{24} T_2^{31} - \\
 & 120 T_1^{25} T_2^{31} - 355 T_1^{26} T_2^{31} + 335 T_1^{27} T_2^{31} + 270 T_1^{28} T_2^{31} - 478 T_1^{29} T_2^{31} + 138 T_1^{30} T_2^{31} + 173 T_1^{31} T_2^{31} - \\
 & 59 T_1^{32} T_2^{31} - 112 T_1^{33} T_2^{31} + 41 T_1^{34} T_2^{31} + 98 T_1^{35} T_2^{31} - 74 T_1^{36} T_2^{31} - 2 T_1^{37} T_2^{31} - 7 T_1^{38} T_2^{31} + 12 T_1^{39} T_2^{31} - \\
 & 2 T_1^{40} T_2^{31} - 2 T_1^{12} T_2^{32} - 6 T_1^{13} T_2^{32} + 9 T_1^{14} T_2^{32} + 4 T_1^{15} T_2^{32} + 17 T_1^{16} T_2^{32} - 59 T_1^{17} T_2^{32} + 79 T_1^{18} T_2^{32} - \\
 & 10 T_1^{19} T_2^{32} - 39 T_1^{20} T_2^{32} - 35 T_1^{21} T_2^{32} + 60 T_1^{22} T_2^{32} + 19 T_1^{23} T_2^{32} - 104 T_1^{24} T_2^{32} + 5 T_1^{25} T_2^{32} - \\
 & 29 T_1^{26} T_2^{32} + 101 T_1^{27} T_2^{32} - 224 T_1^{28} T_2^{32} + 75 T_1^{29} T_2^{32} + 68 T_1^{30} T_2^{32} - 59 T_1^{31} T_2^{32} - 31 T_1^{32} T_2^{32} - \\
 & 10 T_1^{33} T_2^{32} + 87 T_1^{34} T_2^{32} - 75 T_1^{35} T_2^{32} + 25 T_1^{36} T_2^{32} + 4 T_1^{37} T_2^{32} + 9 T_1^{38} T_2^{32} - 6 T_1^{39} T_2^{32} - 2 T_1^{40} T_2^{32} + \\
 & 4 T_1^{13} T_2^{33} - 3 T_1^{14} T_2^{33} - 4 T_1^{15} T_2^{33} - 3 T_1^{16} T_2^{33} + 10 T_1^{17} T_2^{33} - 19 T_1^{18} T_2^{33} - 42 T_1^{19} T_2^{33} + 100 T_1^{20} T_2^{33} - \\
 & 12 T_1^{21} T_2^{33} - 108 T_1^{22} T_2^{33} + 92 T_1^{23} T_2^{33} + 133 T_1^{24} T_2^{33} - 257 T_1^{25} T_2^{33} + 167 T_1^{26} T_2^{33} + 207 T_1^{27} T_2^{33} - \\
 & 267 T_1^{28} T_2^{33} + 119 T_1^{29} T_2^{33} + 106 T_1^{30} T_2^{33} - 112 T_1^{31} T_2^{33} - 10 T_1^{32} T_2^{33} + 94 T_1^{33} T_2^{33} - 36 T_1^{34} T_2^{33} - \\
 & 21 T_1^{35} T_2^{33} + 10 T_1^{36} T_2^{33} - 3 T_1^{37} T_2^{33} - 4 T_1^{38} T_2^{33} - 3 T_1^{39} T_2^{33} + 4 T_1^{40} T_2^{33} - 3 T_1^{14} T_2^{34} + 6 T_1^{15} T_2^{34} - \\
 & T_1^{16} T_2^{34} + 2 T_1^{17} T_2^{34} - 5 T_1^{18} T_2^{34} + 50 T_1^{19} T_2^{34} - 60 T_1^{20} T_2^{34} - 40 T_1^{21} T_2^{34} + 85 T_1^{22} T_2^{34} + 51 T_1^{23} T_2^{34} - \\
 & 253 T_1^{24} T_2^{34} + 142 T_1^{25} T_2^{34} + 226 T_1^{26} T_2^{34} - 504 T_1^{27} T_2^{34} + 196 T_1^{28} T_2^{34} + 166 T_1^{29} T_2^{34} - 253 T_1^{30} T_2^{34} + \\
 & 41 T_1^{31} T_2^{34} + 87 T_1^{32} T_2^{34} - 36 T_1^{33} T_2^{34} - 60 T_1^{34} T_2^{34} + 46 T_1^{35} T_2^{34} - 3 T_1^{36} T_2^{34} + 2 T_1^{37} T_2^{34} - T_1^{38} T_2^{34} + \\
 & 6 T_1^{39} T_2^{34} - 3 T_1^{40} T_2^{34} + T_1^{15} T_2^{35} - 3 T_1^{16} T_2^{35} + 3 T_1^{18} T_2^{35} - 13 T_1^{19} T_2^{35} - 8 T_1^{20} T_2^{35} + 52 T_1^{21} T_2^{35} - \\
 & 23 T_1^{22} T_2^{35} - 71 T_1^{23} T_2^{35} + 84 T_1^{24} T_2^{35} + 104 T_1^{25} T_2^{35} - 259 T_1^{26} T_2^{35} + 149 T_1^{27} T_2^{35} + 189 T_1^{28} T_2^{35} - \\
 & 269 T_1^{29} T_2^{35} + 90 T_1^{30} T_2^{35} + 98 T_1^{31} T_2^{35} - 75 T_1^{32} T_2^{35} - 21 T_1^{33} T_2^{35} + 46 T_1^{34} T_2^{35} - 2 T_1^{35} T_2^{35} - 15 T_1^{36} T_2^{35} + \\
 & 3 T_1^{37} T_2^{35} - 3 T_1^{39} T_2^{35} + T_1^{40} T_2^{35} + 3 T_1^{18} T_2^{36} - 10 T_1^{19} T_2^{36} + 19 T_1^{20} T_2^{36} - 15 T_1^{21} T_2^{36} - 3 T_1^{22} T_2^{36} + \\
 & 10 T_1^{23} T_2^{36} + 25 T_1^{24} T_2^{36} - 74 T_1^{25} T_2^{36} + 31 T_1^{26} T_2^{36} + 94 T_1^{27} T_2^{36} - 173 T_1^{28} T_2^{36} + 94 T_1^{29} T_2^{36} + \\
 & 31 T_1^{30} T_2^{36} - 74 T_1^{31} T_2^{36} + 25 T_1^{32} T_2^{36} + 10 T_1^{33} T_2^{36} - 3 T_1^{34} T_2^{36} - 15 T_1^{35} T_2^{36} + 19 T_1^{36} T_2^{36} - 10 T_1^{37} T_2^{36} + \\
 & 3 T_1^{38} T_2^{36} - T_1^{39} T_2^{36} + 6 T_1^{20} T_2^{37} - 10 T_1^{21} T_2^{37} + 3 T_1^{22} T_2^{37} + 2 T_1^{23} T_2^{37} - 3 T_1^{24} T_2^{37} + 4 T_1^{25} T_2^{37} - \\
 & 2 T_1^{26} T_2^{37} + 2 T_1^{27} T_2^{37} - T_1^{28} T_2^{37} - T_1^{29} T_2^{37} + 2 T_1^{30} T_2^{37} - 2 T_1^{31} T_2^{37} + 4 T_1^{32} T_2^{37} - 3 T_1^{33} T_2^{37} + 2 T_1^{34} T_2^{37} + \\
 & 3 T_1^{35} T_2^{37} - 10 T_1^{36} T_2^{37} + 6 T_1^{37} T_2^{37} - T_1^{38} T_2^{37} - T_1^{21} T_2^{38} + 3 T_1^{22} T_2^{38} - T_1^{24} T_2^{38} - 4 T_1^{25} T_2^{38} + 9 T_1^{26} T_2^{38} -
 \end{aligned}$$

$$\left. \begin{aligned} &7 T_1^{27} T_2^{38} - 3 T_1^{28} T_2^{38} + 8 T_1^{29} T_2^{38} - 3 T_1^{30} T_2^{38} - 7 T_1^{31} T_2^{38} + 9 T_1^{32} T_2^{38} - 4 T_1^{33} T_2^{38} - T_1^{34} T_2^{38} + 3 T_1^{36} T_2^{38} - \\ &T_1^{37} T_2^{38} - 3 T_1^{24} T_2^{39} + 6 T_1^{25} T_2^{39} - 3 T_1^{26} T_2^{39} - 6 T_1^{27} T_2^{39} + 12 T_1^{28} T_2^{39} - 6 T_1^{29} T_2^{39} - 6 T_1^{30} T_2^{39} + \\ &12 T_1^{31} T_2^{39} - 6 T_1^{32} T_2^{39} - 3 T_1^{33} T_2^{39} + 6 T_1^{34} T_2^{39} - 3 T_1^{35} T_2^{39} + T_1^{25} T_2^{40} - 3 T_1^{26} T_2^{40} + 4 T_1^{27} T_2^{40} - \\ &2 T_1^{28} T_2^{40} - 2 T_1^{29} T_2^{40} + 4 T_1^{30} T_2^{40} - 2 T_1^{31} T_2^{40} - 2 T_1^{32} T_2^{40} + 4 T_1^{33} T_2^{40} - 3 T_1^{34} T_2^{40} + T_1^{35} T_2^{40} \end{aligned} \right\}$$

```
In[*]:= DuplicateFreeQ[θ /@ AllKnots[{3, 10}]]
```

Out[*]= True

```
In[*]:= DuplicateFreeQ[θ /@ AllKnots[{3, 12}]]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.
 KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

Out[*]= False

```
In[*]:= tab11 = Table[K → θ@K, {K, AllKnots[{3, 11}]}]
```

Out[*]=

$$\left\{ \text{Knot}[3, 1] \rightarrow \left\{ \frac{1-T+T^2}{T}, \frac{-1+T_1-T_1^2+T_2-T_1^2 T_2+2 T_1^3 T_2-T_1^2 T_2^2-T_1 T_2^2+T_1^2 T_2^2-2 T_1^3 T_2^2+2 T_1 T_2^2-2 T_1^2 T_2^2+2 T_1^3 T_2^2}{T_1^2 T_2 (-1+T_1 T_2)} \right\}, \right.$$

$$\text{Knot}[4, 1] \rightarrow \left\{ -\frac{1-3 T+T^2}{T}, -\frac{(1-3 T_1+T_1^2)(1+T_1 T_2)(1-3 T_2+T_2^2)}{T_1^3 T_2^2} \right\}, \text{Knot}[5, 1] \rightarrow \left\{ \frac{1-T+T^2-T^3+T^4}{T^2}, \frac{-1+\dots 67 \dots +4 T_1^7 T_2^2}{T_1^4 T_2^2 (-1+T_1 T_2)} \right\},$$

$$\dots 795 \dots, \text{Knot}[11, \text{NonAlternating}, 183] \rightarrow \left\{ \frac{\dots 1 \dots}{T^3}, -\dots 1 \dots \right\},$$

$$\text{Knot}[11, \text{NonAlternating}, 184] \rightarrow \left\{ \frac{(1-T+T^2)(2-7 T+11 T^2-7 T^3+2 T^4)}{T^3}, \frac{-33+149 T_1-332 T_1^2+\dots 177 \dots +148 T_1^9 T_2^2-67 T_1^{11} T_2^2+15 T_1^{12} T_2^2}{T_1^7 T_2^2 (-1+T_1 T_2)} \right\},$$

$$\text{Knot}[11, \text{NonAlternating}, 185] \rightarrow \left\{ -\frac{(1-3 T+T^2)(1-T+T^2)(2-3 T+2 T^2)}{T^3}, \right.$$

$$\left. -\frac{1}{T_1^2 T_2^2 (-1+T_1 T_2)} (-41+225 T_1-490 T_1^2+633 T_1^3-490 T_1^4+225 T_1^5-41 T_1^6+225 T_2-1054 T_1 T_2+1693 T_1^2 T_2-1311 T_1^3 T_2-101 T_1^4 T_2+923 T_1^5 T_2-768 T_1^6 T_2+\dots 139 \dots +5 T_1^5 T_2^2-64 T_1^6 T_2^2+263 T_1^7 T_2^2-541 T_1^8 T_2^2+669 T_1^9 T_2^2-507 T_1^{10} T_2^2+222 T_1^{11} T_2^2-39 T_1^{12} T_2^2+7 T_1^6 T_2^{12}-39 T_1^7 T_2^{12}+86 T_1^8 T_2^{12}-111 T_1^9 T_2^{12}+86 T_1^{10} T_2^{12}-39 T_1^{11} T_2^{12}+7 T_1^{12} T_2^{12}) \right\}$$

Full expression not available (original memory size: 36.2 MB)

```
In[*]:= Gather[tab11, Last[#1] == Last[#2] &]
```

Out[*]=

$$\left\{ \left\{ \text{Knot}[3, 1] \rightarrow \left\{ \frac{1-T+T^2}{T}, \frac{-1+T_1-T_1^2+T_2-T_1^2 T_2+2 T_1^3 T_2-T_1^2 T_2^2-T_1 T_2^2+T_1^2 T_2^2-2 T_1^3 T_2^2+2 T_1 T_2^2-2 T_1^2 T_2^2+2 T_1^3 T_2^2}{T_1^2 T_2 (-1+T_1 T_2)} \right\} \right\}, \right.$$

$$\left\{ \text{Knot}[4, 1] \rightarrow \left\{ -\frac{1-3 T+T^2}{T}, -\frac{(1-3 T_1+T_1^2)(1+T_1 T_2)(1-3 T_2+T_2^2)}{T_1^3 T_2^2} \right\} \right\}, \left\{ \text{Knot}[5, 1] \rightarrow \left\{ \frac{1-T+T^2-T^3+T^4}{T^2}, \frac{-1+\dots 67 \dots +4 T_1^7 T_2^2}{T_1^4 T_2^2 (-1+T_1 T_2)} \right\} \right\},$$

$$\dots 792 \dots, \left\{ \text{Knot}[11, \text{NonAlternating}, 183] \rightarrow \left\{ \frac{\dots 1 \dots}{T^3}, -\dots 1 \dots \right\} \right\},$$

$$\left\{ \text{Knot}[11, \text{NonAlternating}, 184] \rightarrow \left\{ \frac{(1-T+T^2)(2-7 T+11 T^2-7 T^3+2 T^4)}{T^3}, \frac{-33+149 T_1-332 T_1^2+415 T_1^3-332 T_1^4+\dots 174 \dots +148 T_1^9 T_2^2-67 T_1^{11} T_2^2+15 T_1^{12} T_2^2}{T_1^7 T_2^2 (-1+T_1 T_2)} \right\} \right\},$$

$$\left\{ \text{Knot}[11, \text{NonAlternating}, 185] \rightarrow \right.$$

$$\left\{ -\frac{(1-3 T+T^2)(1-T+T^2)(2-3 T+2 T^2)}{T^3}, -\frac{1}{T_1^2 T_2^2 (-1+T_1 T_2)} (-41+225 T_1-490 T_1^2+633 T_1^3-490 T_1^4+225 T_1^5-41 T_1^6+225 T_2-1054 T_1 T_2+1693 T_1^2 T_2-1311 T_1^3 T_2-101 T_1^4 T_2+923 T_1^5 T_2-768 T_1^6 T_2+\dots 142 \dots +263 T_1^7 T_2^2-541 T_1^8 T_2^2+669 T_1^9 T_2^2-507 T_1^{10} T_2^2+222 T_1^{11} T_2^2-39 T_1^{12} T_2^2+7 T_1^6 T_2^{12}-39 T_1^7 T_2^{12}+86 T_1^8 T_2^{12}-111 T_1^9 T_2^{12}+86 T_1^{10} T_2^{12}-39 T_1^{11} T_2^{12}+7 T_1^{12} T_2^{12}) \right\} \right\}$$

Stored in notebook: 36.2 MB Show more Show all Iconize

```
In[*]:= Select[Gather[tab11, Last[#1] == Last[#2] &], Length[#] > 1 &]
```

```
Out[*]=
```

$$\left\{ \left\{ \text{Knot}[11, \text{Alternating}, 44] \rightarrow \left\{ \frac{(1 - T + T^2)^2 (1 - 3T + 5T^2 - 3T^3 + T^4)}{T^4}, \frac{1}{T_1^9 T_2^8 (-1 + T_1 T_2)} (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) \right. \right. \right.$$

$$\left. \left. \left((1 - T_1 T_2 + T_1^2 T_2^2) (-4 + 16T_1 - 36T_1^2 + 44T_1^3 - 36T_1^4 + 16T_1^5 - 4T_1^6 + 16T_2 - 53T_1 T_2 + 100T_1^2 T_2 - 77T_1^3 T_2 + 23T_1^4 T_2 + 35T_1^5 T_2 - 28T_1^6 T_2 + 11T_1^7 T_2 - 36T_2^2 + 100T_1 T_2^2 - 161T_1^2 T_2^2 + 54T_1^3 T_2^2 + 39T_1^4 T_2^2 - 107T_1^5 T_2^2 + 23T_1^6 T_2^2 + 8T_1^7 T_2^2 - 13T_1^8 T_2^2 + 44T_2^3 - 77T_1 T_2^3 + 54T_1^2 T_2^3 + 189T_1^3 T_2^3 - 215T_1^4 T_2^3 + 153T_1^5 T_2^3 + 106T_1^6 T_2^3 - 107T_1^7 T_2^3 + 52T_1^8 T_2^3 - 36T_2^4 + 23T_1 T_2^4 + 39T_1^2 T_2^4 - 215T_1^3 T_2^4 - 23T_1^4 T_2^4 + 238T_1^5 T_2^4 - 478T_1^6 T_2^4 + 214T_1^7 T_2^4 - 4T_1^8 T_2^4 - 52T_1^9 T_2^4 + 13T_1^{10} T_2^4 + 16T_2^5 + 35T_1 T_2^5 - 107T_1^2 T_2^5 + 153T_1^3 T_2^5 + 238T_1^4 T_2^5 - 485T_1^5 T_2^5 + 547T_1^6 T_2^5 - 52T_1^7 T_2^5 - 194T_1^8 T_2^5 + 111T_1^9 T_2^5 - 8T_1^{10} T_2^5 - 11T_1^{11} T_2^5 - 4T_2^6 - 28T_1 T_2^6 + 23T_1^2 T_2^6 + 106T_1^3 T_2^6 - 478T_1^4 T_2^6 + 547T_1^5 T_2^6 - 192T_1^6 T_2^6 - 419T_1^7 T_2^6 + 498T_1^8 T_2^6 - 126T_1^9 T_2^6 - 23T_1^{10} T_2^6 + 28T_1^{11} T_2^6 + 4T_1^{12} T_2^6 + 11T_1 T_2^7 + 8T_1^2 T_2^7 - 107T_1^3 T_2^7 + 214T_1^4 T_2^7 - 52T_1^5 T_2^7 - 419T_1^6 T_2^7 + 613T_1^7 T_2^7 - 342T_1^8 T_2^7 - 133T_1^9 T_2^7 + 111T_1^{10} T_2^7 - 35T_1^{11} T_2^7 - 16T_1^{12} T_2^7 - 13T_1^2 T_2^8 + 52T_1^3 T_2^8 - 4T_1^4 T_2^8 - 194T_1^5 T_2^8 + 498T_1^6 T_2^8 - 342T_1^7 T_2^8 + 43T_1^8 T_2^8 + 235T_1^9 T_2^8 - 47T_1^{10} T_2^8 - 23T_1^{11} T_2^8 + 36T_1^{12} T_2^8 - 52T_1^4 T_2^9 + 111T_1^5 T_2^9 - 126T_1^6 T_2^9 - 133T_1^7 T_2^9 + 235T_1^8 T_2^9 - 209T_1^9 T_2^9 - 50T_1^{10} T_2^9 + 77T_1^{11} T_2^9 - 44T_1^{12} T_2^9 + 13T_1^4 T_2^{10} - 8T_1^5 T_2^{10} - 23T_1^6 T_2^{10} + 111T_1^7 T_2^{10} - 47T_1^8 T_2^{10} - 50T_1^9 T_2^{10} + 161T_1^{10} T_2^{10} - 100T_1^{11} T_2^{10} + 36T_1^{12} T_2^{10} - 11T_1^5 T_2^{11} + 28T_1^6 T_2^{11} - 35T_1^7 T_2^{11} - 23T_1^8 T_2^{11} + 77T_1^9 T_2^{11} - 100T_1^{10} T_2^{11} + 53T_1^{11} T_2^{11} - 16T_1^{12} T_2^{11} + 4T_1^6 T_2^{12} - 16T_1^7 T_2^{12} + 36T_1^8 T_2^{12} - 44T_1^9 T_2^{12} + 36T_1^{10} T_2^{12} - 16T_1^{11} T_2^{12} + 4T_1^{12} T_2^{12} \right\} \right\},$$

$$\text{Knot}[11, \text{Alternating}, 47] \rightarrow \left\{ \frac{(1 - T + T^2)^2 (1 - 3T + 5T^2 - 3T^3 + T^4)}{T^4}, \right.$$

$$\left. \frac{1}{T_1^9 T_2^8 (-1 + T_1 T_2)} (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) \right.$$

$$\left. \left(-4 + 16T_1 - 36T_1^2 + 44T_1^3 - 36T_1^4 + 16T_1^5 - 4T_1^6 + 16T_2 - 53T_1 T_2 + 100T_1^2 T_2 - 77T_1^3 T_2 + 23T_1^4 T_2 + 35T_1^5 T_2 - 28T_1^6 T_2 + 11T_1^7 T_2 - 36T_2^2 + 100T_1 T_2^2 - 161T_1^2 T_2^2 + 54T_1^3 T_2^2 + 39T_1^4 T_2^2 - 107T_1^5 T_2^2 + 23T_1^6 T_2^2 + 8T_1^7 T_2^2 - 13T_1^8 T_2^2 + 44T_2^3 - 77T_1 T_2^3 + 54T_1^2 T_2^3 + 189T_1^3 T_2^3 - 215T_1^4 T_2^3 + 153T_1^5 T_2^3 + 106T_1^6 T_2^3 - 107T_1^7 T_2^3 + 52T_1^8 T_2^3 - 36T_2^4 + 23T_1 T_2^4 + 39T_1^2 T_2^4 - 215T_1^3 T_2^4 - 23T_1^4 T_2^4 + 238T_1^5 T_2^4 - 478T_1^6 T_2^4 + 214T_1^7 T_2^4 - 4T_1^8 T_2^4 - 52T_1^9 T_2^4 + 13T_1^{10} T_2^4 + 16T_2^5 + 35T_1 T_2^5 - 107T_1^2 T_2^5 + 153T_1^3 T_2^5 + 238T_1^4 T_2^5 - 485T_1^5 T_2^5 + 547T_1^6 T_2^5 - 52T_1^7 T_2^5 - 194T_1^8 T_2^5 + 111T_1^9 T_2^5 - 8T_1^{10} T_2^5 - 11T_1^{11} T_2^5 - 4T_2^6 - 28T_1 T_2^6 + 23T_1^2 T_2^6 + 106T_1^3 T_2^6 - 478T_1^4 T_2^6 + 547T_1^5 T_2^6 - 192T_1^6 T_2^6 - 419T_1^7 T_2^6 + 498T_1^8 T_2^6 - 126T_1^9 T_2^6 - 23T_1^{10} T_2^6 + 28T_1^{11} T_2^6 + 4T_1^{12} T_2^6 + 11T_1 T_2^7 + 8T_1^2 T_2^7 - 107T_1^3 T_2^7 + 214T_1^4 T_2^7 - 52T_1^5 T_2^7 - 419T_1^6 T_2^7 + 613T_1^7 T_2^7 - 342T_1^8 T_2^7 - 133T_1^9 T_2^7 + 111T_1^{10} T_2^7 - 35T_1^{11} T_2^7 - 16T_1^{12} T_2^7 - 13T_1^2 T_2^8 + 52T_1^3 T_2^8 - 4T_1^4 T_2^8 - 194T_1^5 T_2^8 + 498T_1^6 T_2^8 - 342T_1^7 T_2^8 + 43T_1^8 T_2^8 + 235T_1^9 T_2^8 - 47T_1^{10} T_2^8 - 23T_1^{11} T_2^8 + 36T_1^{12} T_2^8 - 52T_1^4 T_2^9 + 111T_1^5 T_2^9 - 126T_1^6 T_2^9 - 133T_1^7 T_2^9 + 235T_1^8 T_2^9 - 209T_1^9 T_2^9 - 50T_1^{10} T_2^9 + 77T_1^{11} T_2^9 - 44T_1^{12} T_2^9 + 13T_1^4 T_2^{10} - 8T_1^5 T_2^{10} - 23T_1^6 T_2^{10} + 111T_1^7 T_2^{10} - 47T_1^8 T_2^{10} - 50T_1^9 T_2^{10} + 161T_1^{10} T_2^{10} - 100T_1^{11} T_2^{10} + 36T_1^{12} T_2^{10} - 11T_1^5 T_2^{11} + 28T_1^6 T_2^{11} - 35T_1^7 T_2^{11} - 23T_1^8 T_2^{11} + 77T_1^9 T_2^{11} - 100T_1^{10} T_2^{11} + 53T_1^{11} T_2^{11} - 16T_1^{12} T_2^{11} + 4T_1^6 T_2^{12} - 16T_1^7 T_2^{12} + 36T_1^8 T_2^{12} - 44T_1^9 T_2^{12} + 36T_1^{10} T_2^{12} - 16T_1^{11} T_2^{12} + 4T_1^{12} T_2^{12} \right\} \right\},$$

$$\left\{ \text{Knot}[11, \text{Alternating}, 57] \rightarrow \left\{ -\frac{(1 - T + T^2)^2 (1 - 3T + 3T^2 - 3T^3 + T^4)}{T^4}, \right. \right.$$

$$\begin{aligned}
 & - \frac{1}{T_1^9 T_2^8 (-1 + T_1 T_2)} (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) \\
 & \left(-3 + 12 T_1 - 21 T_1^2 + 27 T_1^3 - 21 T_1^4 + 12 T_1^5 - 3 T_1^6 + 12 T_2 - 41 T_1 T_2 + 56 T_1^2 T_2 - 59 T_1^3 T_2 + 21 T_1^4 T_2 + \right. \\
 & T_1^5 T_2 - 16 T_1^6 T_2 + 7 T_1^7 T_2 - 21 T_2^2 + 56 T_1 T_2^2 - 37 T_1^2 T_2^2 + 5 T_1^3 T_2^2 + 85 T_1^4 T_2^2 - 90 T_1^5 T_2^2 + 77 T_1^6 T_2^2 - \\
 & 20 T_1^7 T_2^2 - 2 T_1^8 T_2^2 + 27 T_2^3 - 59 T_1 T_2^3 + 5 T_1^2 T_2^3 + 39 T_1^3 T_2^3 - 152 T_1^4 T_2^3 + 108 T_1^5 T_2^3 - 52 T_1^6 T_2^3 - \\
 & 42 T_1^7 T_2^3 + 44 T_1^8 T_2^3 - 9 T_1^9 T_2^3 - 21 T_2^4 + 21 T_1 T_2^4 + 85 T_1^2 T_2^4 - 152 T_1^3 T_2^4 + 257 T_1^4 T_2^4 - 118 T_1^5 T_2^4 - \\
 & 16 T_1^6 T_2^4 + 109 T_1^7 T_2^4 - 52 T_1^8 T_2^4 - 28 T_1^9 T_2^4 + 16 T_1^{10} T_2^4 + 12 T_2^5 + T_1 T_2^5 - 90 T_1^2 T_2^5 + 108 T_1^3 T_2^5 - \\
 & 118 T_1^4 T_2^5 - 55 T_1^5 T_2^5 + 133 T_1^6 T_2^5 - 68 T_1^7 T_2^5 - 35 T_1^8 T_2^5 + 92 T_1^9 T_2^5 - 4 T_1^{10} T_2^5 - 15 T_1^{11} T_2^5 - 3 T_2^6 - \\
 & 16 T_1 T_2^6 + 77 T_1^2 T_2^6 - 52 T_1^3 T_2^6 - 16 T_1^4 T_2^6 + 133 T_1^5 T_2^6 - 156 T_1^6 T_2^6 - T_1^7 T_2^6 + 4 T_1^8 T_2^6 - 12 T_1^9 T_2^6 - \\
 & 93 T_1^{10} T_2^6 + 40 T_1^{11} T_2^6 + 5 T_1^{12} T_2^6 + 7 T_2^7 - 20 T_1 T_2^7 - 42 T_1^2 T_2^7 + 109 T_1^3 T_2^7 - 68 T_1^4 T_2^7 - T_1^5 T_2^7 + \\
 & 187 T_1^6 T_2^7 - 18 T_1^7 T_2^7 - 34 T_1^8 T_2^7 + 140 T_1^9 T_2^7 - 25 T_1^{10} T_2^7 - 20 T_1^{11} T_2^7 - 2 T_1^{12} T_2^7 - 2 T_2^8 + 44 T_1 T_2^8 - 52 T_1^2 T_2^8 - \\
 & 35 T_1^3 T_2^8 + 4 T_1^4 T_2^8 - 18 T_1^5 T_2^8 - 269 T_1^6 T_2^8 + 226 T_1^7 T_2^8 - 189 T_1^8 T_2^8 - 5 T_1^{10} T_2^8 + 35 T_1^{11} T_2^8 - 9 T_1^{12} T_2^8 - \\
 & 28 T_1^4 T_2^9 + 92 T_1^5 T_2^9 - 12 T_1^6 T_2^9 - 34 T_1^7 T_2^9 + 226 T_1^8 T_2^9 - 103 T_1^9 T_2^9 + 45 T_1^{10} T_2^9 + 75 T_1^{11} T_2^9 - 45 T_1^{12} T_2^9 + \\
 & 16 T_1^4 T_2^{10} - 4 T_1^5 T_2^{10} - 93 T_1^6 T_2^{10} + 140 T_1^7 T_2^{10} - 189 T_1^8 T_2^{10} + 45 T_1^9 T_2^{10} + 21 T_1^{10} T_2^{10} - 80 T_1^{11} T_2^{10} + \\
 & 35 T_1^{12} T_2^{10} - 15 T_1^5 T_2^{11} + 40 T_1^6 T_2^{11} - 25 T_1^7 T_2^{11} - 5 T_1^8 T_2^{11} + 75 T_1^9 T_2^{11} - 80 T_1^{10} T_2^{11} + 65 T_1^{11} T_2^{11} - \\
 & 20 T_1^{12} T_2^{11} + 5 T_1^6 T_2^{12} - 20 T_1^7 T_2^{12} + 35 T_1^8 T_2^{12} - 45 T_1^9 T_2^{12} + 35 T_1^{10} T_2^{12} - 20 T_1^{11} T_2^{12} + 5 T_1^{12} T_2^{12} \left. \right\},
 \end{aligned}$$

$$\text{Knot}[11, \text{Alternating}, 231] \rightarrow \left\{ - \frac{(1 - T + T^2)^2 (1 - 3T + 3T^2 - 3T^3 + T^4)}{T^4}, \right.$$

$$\begin{aligned}
 & - \frac{1}{T_1^9 T_2^8 (-1 + T_1 T_2)} (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) \\
 & \left(-3 + 12 T_1 - 21 T_1^2 + 27 T_1^3 - 21 T_1^4 + 12 T_1^5 - 3 T_1^6 + 12 T_2 - 41 T_1 T_2 + 56 T_1^2 T_2 - 59 T_1^3 T_2 + 21 T_1^4 T_2 + \right. \\
 & T_1^5 T_2 - 16 T_1^6 T_2 + 7 T_1^7 T_2 - 21 T_2^2 + 56 T_1 T_2^2 - 37 T_1^2 T_2^2 + 5 T_1^3 T_2^2 + 85 T_1^4 T_2^2 - 90 T_1^5 T_2^2 + 77 T_1^6 T_2^2 - \\
 & 20 T_1^7 T_2^2 - 2 T_1^8 T_2^2 + 27 T_2^3 - 59 T_1 T_2^3 + 5 T_1^2 T_2^3 + 39 T_1^3 T_2^3 - 152 T_1^4 T_2^3 + 108 T_1^5 T_2^3 - 52 T_1^6 T_2^3 - \\
 & 42 T_1^7 T_2^3 + 44 T_1^8 T_2^3 - 9 T_1^9 T_2^3 - 21 T_2^4 + 21 T_1 T_2^4 + 85 T_1^2 T_2^4 - 152 T_1^3 T_2^4 + 257 T_1^4 T_2^4 - 118 T_1^5 T_2^4 - \\
 & 16 T_1^6 T_2^4 + 109 T_1^7 T_2^4 - 52 T_1^8 T_2^4 - 28 T_1^9 T_2^4 + 16 T_1^{10} T_2^4 + 12 T_2^5 + T_1 T_2^5 - 90 T_1^2 T_2^5 + 108 T_1^3 T_2^5 - \\
 & 118 T_1^4 T_2^5 - 55 T_1^5 T_2^5 + 133 T_1^6 T_2^5 - 68 T_1^7 T_2^5 - 35 T_1^8 T_2^5 + 92 T_1^9 T_2^5 - 4 T_1^{10} T_2^5 - 15 T_1^{11} T_2^5 - 3 T_2^6 - \\
 & 16 T_1 T_2^6 + 77 T_1^2 T_2^6 - 52 T_1^3 T_2^6 - 16 T_1^4 T_2^6 + 133 T_1^5 T_2^6 - 156 T_1^6 T_2^6 - T_1^7 T_2^6 + 4 T_1^8 T_2^6 - 12 T_1^9 T_2^6 - \\
 & 93 T_1^{10} T_2^6 + 40 T_1^{11} T_2^6 + 5 T_1^{12} T_2^6 + 7 T_2^7 - 20 T_1 T_2^7 - 42 T_1^2 T_2^7 - 42 T_1^3 T_2^7 + 109 T_1^4 T_2^7 - 68 T_1^5 T_2^7 - T_1^6 T_2^7 + \\
 & 187 T_1^6 T_2^7 - 18 T_1^7 T_2^7 - 34 T_1^8 T_2^7 + 140 T_1^9 T_2^7 - 25 T_1^{10} T_2^7 - 20 T_1^{11} T_2^7 - 2 T_1^{12} T_2^7 - 2 T_2^8 + 44 T_1 T_2^8 - 52 T_1^2 T_2^8 - \\
 & 35 T_1^3 T_2^8 + 4 T_1^4 T_2^8 - 18 T_1^5 T_2^8 - 269 T_1^6 T_2^8 + 226 T_1^7 T_2^8 - 189 T_1^8 T_2^8 - 5 T_1^{10} T_2^8 + 35 T_1^{11} T_2^8 - 9 T_1^{12} T_2^8 - \\
 & 28 T_1^4 T_2^9 + 92 T_1^5 T_2^9 - 12 T_1^6 T_2^9 - 34 T_1^7 T_2^9 + 226 T_1^8 T_2^9 - 103 T_1^9 T_2^9 + 45 T_1^{10} T_2^9 + 75 T_1^{11} T_2^9 - 45 T_1^{12} T_2^9 + \\
 & 16 T_1^4 T_2^{10} - 4 T_1^5 T_2^{10} - 93 T_1^6 T_2^{10} + 140 T_1^7 T_2^{10} - 189 T_1^8 T_2^{10} + 45 T_1^9 T_2^{10} + 21 T_1^{10} T_2^{10} - 80 T_1^{11} T_2^{10} + \\
 & 35 T_1^{12} T_2^{10} - 15 T_1^5 T_2^{11} + 40 T_1^6 T_2^{11} - 25 T_1^7 T_2^{11} - 5 T_1^8 T_2^{11} + 75 T_1^9 T_2^{11} - 80 T_1^{10} T_2^{11} + 65 T_1^{11} T_2^{11} - \\
 & 20 T_1^{12} T_2^{11} + 5 T_1^6 T_2^{12} - 20 T_1^7 T_2^{12} + 35 T_1^8 T_2^{12} - 45 T_1^9 T_2^{12} + 35 T_1^{10} T_2^{12} - 20 T_1^{11} T_2^{12} + 5 T_1^{12} T_2^{12} \left. \right\},
 \end{aligned}$$

$$\left\{ \text{Knot}[11, \text{NonAlternating}, 73] \rightarrow \left\{ \frac{(1 - T + T^2)^2}{T^2}, \frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} \right. \right.$$

$$\begin{aligned}
 & 2 (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) \\
 & \left. \left(-1 + T_1 - T_1^2 + T_2 + 2 T_1^2 T_2 - T_2^2 + 2 T_1 T_2^2 - 6 T_1^2 T_2^2 + T_1^4 T_2^2 + 2 T_1^3 T_2^2 - T_1^4 T_2^2 + T_1^2 T_2^4 - T_1^3 T_2^4 + T_1^4 T_2^4 \right) \right\},
 \end{aligned}$$

$$\text{Knot}[11, \text{NonAlternating}, 74] \rightarrow$$

$$\left\{ \frac{(1 - T + T^2)^2}{T^2}, \right.$$

$$\frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} 2 (1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (1 - T_1 T_2 + T_1^2 T_2^2) (-1 + T_1 - T_1^2 + T_2 + 2 T_1^2 T_2 - T_2^2 + 2 T_1 T_2^2 - 6 T_1^2 T_2^2 + T_1^4 T_2^2 + 2 T_1^3 T_2^3 - T_1^4 T_2^3 + T_1^2 T_2^4 - T_1^3 T_2^4 + T_1^4 T_2^4) \}} \}}}$$

In[*]:= `tab12 = Table[K -> e@K, {K, AllKnots[{3, 12]}]}`

Out[*]=

{ Knot [3, 1] -> { $\frac{1-T+T^2}{T}$, $\frac{-1+T_1-T_1^2+T_2-T_1^2 T_2+2 T_1^3 T_2-T_2^2-T_1 T_2^2+T_1^2 T_2^2-2 T_1^3 T_2^2+2 T_1 T_2^3-2 T_1^2 T_2^3+2 T_1^3 T_2^3}{T_1^2 T_2 (-1+T_1 T_2)}$ },

Knot [4, 1] -> { $-\frac{1-3 T+T^2}{T}$, $-\frac{(1-3 T_1+T_1^2)(1+T_1 T_2)(1-3 T_2+T_2^2)}{T_1^3 T_2^2}$ }, Knot [5, 1] -> { $\frac{1-T+T^2-T^3+T^4}{T^2}$, $\frac{-1+\dots 67 \dots+4 T_1^4 T_2^2}{T_1^4 T_2^2 (-1+\dots 1 \dots)}$ },

$\dots 2971 \dots$, Knot [12, NonAlternating, 886] -> { $\dots 1 \dots$, $\dots 1 \dots$ },

Knot [12, NonAlternating, 887] -> { $\frac{1-6 T+16 T^2-25 T^3+29 T^4-25 T^5+16 T^6-6 T^7+T^8}{T^4}$, $\frac{-6+36 T_1-96 T_1^2+\dots 325 \dots+32 T_1^4 T_2^5-12 T_1^5 T_2^5+2 T_1^6 T_2^5}{T_1^5 T_2^5 (-1+T_1 T_2)}$ },

Knot [12, NonAlternating, 888] -> { $\frac{(1-T+T^2)^2(1+T-2 T^2+T^3-2 T^4+T^5+T^6)}{T^5}$, $\frac{1}{T_1^{11} T_2^{10} (-1+T_1 T_2)}$ },

$(1 - T_1 + T_1^2) (1 - T_2 + T_2^2) (\dots 1 \dots) (-10 + 20 T_1^2 - 40 T_1^3 + 50 T_1^4 - 40 T_1^5 + 20 T_1^6 - 10 T_1^8 - T_1 T_2 + 2 T_1^3 T_2 - 4 T_1^4 T_2 + 5 T_1^5 T_2 - 4 T_1^6 T_2 + 2 T_1^7 T_2 - T_1^9 T_2 + 20 T_2^2 - 24 T_1^2 T_2^2 + \dots 213 \dots + 3 T_1^5 T_2^{14} - 7 T_1^8 T_2^{14} + 11 T_1^9 T_2^{14} - 9 T_1^{10} T_2^{14} + 3 T_1^{11} T_2^{14} - T_1^{13} T_2^{14} + 2 T_1^{14} T_2^{14} + T_1^7 T_2^{15} - 2 T_1^9 T_2^{15} + 4 T_1^{10} T_2^{15} - 5 T_1^{11} T_2^{15} + 4 T_1^{12} T_2^{15} - 2 T_1^{13} T_2^{15} + T_1^{15} T_2^{15}) \}} \}}}$

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In[*]:= `dup12 = Map[First, Select[Gather[tab12, Last[#1] === Last[#2] &], Length[#] > 1 &], {2}]`

Out[*]=

- { Knot [10, 106], Knot [12, NonAlternating, 369] },
- { Knot [11, Alternating, 44], Knot [11, Alternating, 47] },
- { Knot [11, Alternating, 57], Knot [11, Alternating, 231] },
- { Knot [11, NonAlternating, 73], Knot [11, NonAlternating, 74] },
- { Knot [12, Alternating, 30], Knot [12, Alternating, 33] },
- { Knot [12, Alternating, 122], Knot [12, Alternating, 182] },
- { Knot [12, Alternating, 164], Knot [12, Alternating, 166] },
- { Knot [12, Alternating, 167], Knot [12, Alternating, 692] },
- { Knot [12, Alternating, 273], Knot [12, Alternating, 890] },
- { Knot [12, Alternating, 341], Knot [12, Alternating, 627] },
- { Knot [12, Alternating, 427], Knot [12, Alternating, 435], Knot [12, Alternating, 990] },
- { Knot [12, Alternating, 458], Knot [12, Alternating, 887] },
- { Knot [12, Alternating, 510], Knot [12, Alternating, 821] },
- { Knot [12, NonAlternating, 56], Knot [12, NonAlternating, 57] },
- { Knot [12, NonAlternating, 60], Knot [12, NonAlternating, 61] },
- { Knot [12, NonAlternating, 62], Knot [12, NonAlternating, 66] },
- { Knot [12, NonAlternating, 144], Knot [12, NonAlternating, 507] },
- { Knot [12, NonAlternating, 313], Knot [12, NonAlternating, 430] }

In[*]:= `Length /@ dup12`

Out[*]=

- { 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2 }

```

In[*]:= Total[(Length/@dup12) - 1]
Out[*]=
19

In[*]:= Length/@Select[
  Gather[tab12 /. {T1 -> 22 / 7, T2 -> 13 / 21}, Last[#1] === Last[#2] &], Length[#] > 1 &]
Out[*]=
{2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}

In[*]:= Put[tab12 /. {T1 -> T1, T2 -> T2}, "Data12.m"]

In[*]:= Table[K -> \theta[K],
  {K, {Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27], Knot[9, 41],
    Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42], Knot[10, 48],
    Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123], Knot[10, 129],
    Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]}}}
Out[*]=
{Knot[6, 1] -> {- ( -2 + T ) ( -1 + 2 T )
  / T
  - 1 / ( T1^3 T2^2 ( -1 + T1 T2 ) ) ( -7 + 17 T1 - 7 T1^2 + 17 T2 - 44 T1 T2 + 26 T1^2 T2 - 3 T1^3 T2 - 7 T2^2 + 26 T1 T2^2 - 24 T1^2 T2^2 -
  14 T1^3 T2^2 + 9 T1^4 T2^2 - 3 T1 T2^3 - 14 T1^2 T2^3 + 56 T1^3 T2^3 - 23 T1^4 T2^3 + 9 T1^2 T2^4 - 23 T1^3 T2^4 + 9 T1^4 T2^4 ) },
  Knot[8, 8] -> { ( 2 - 2 T + T^2 ) ( 1 - 2 T + 2 T^2 ) / T^2, 1 / ( T1^5 T2^4 ( -1 + T1 T2 ) )
  ( -17 + 51 T1 - 77 T1^2 + 51 T1^3 - 17 T1^4 + 51 T2 - 126 T1 T2 + 150 T1^2 T2 - 30 T1^3 T2 - 30 T1^4 T2 +
  27 T1^5 T2 - 77 T2^2 + 150 T1 T2^2 - 117 T1^2 T2^2 - 113 T1^3 T2^2 + 135 T1^4 T2^2 - 66 T1^5 T2^2 - 5 T1^6 T2^2 + 51 T2^3 -
  30 T1 T2^3 - 113 T1^2 T2^3 + 336 T1^3 T2^3 - 186 T1^4 T2^3 - 5 T1^5 T2^3 + 78 T1^6 T2^3 - 21 T1^7 T2^3 - 17 T2^4 - 30 T1 T2^4 +
  135 T1^2 T2^4 - 186 T1^3 T2^4 - 60 T1^4 T2^4 + 222 T1^5 T2^4 - 153 T1^6 T2^4 + 18 T1^7 T2^4 + 15 T1^8 T2^4 + 27 T1 T2^5 -
  66 T1^2 T2^5 - 5 T1^3 T2^5 + 222 T1^4 T2^5 - 300 T1^5 T2^5 + 103 T1^6 T2^5 + 42 T1^7 T2^5 - 45 T1^8 T2^5 - 5 T1^2 T2^6 + 78 T1^3 T2^6 -
  153 T1^4 T2^6 + 103 T1^5 T2^6 + 99 T1^6 T2^6 - 138 T1^7 T2^6 + 67 T1^8 T2^6 - 21 T1^3 T2^7 + 18 T1^4 T2^7 + 42 T1^5 T2^7 -
  138 T1^6 T2^7 + 114 T1^7 T2^7 - 45 T1^8 T2^7 + 15 T1^4 T2^8 - 45 T1^5 T2^8 + 67 T1^6 T2^8 - 45 T1^7 T2^8 + 15 T1^8 T2^8 ) },
  Knot[8, 9] -> { - ( -1 + T - 2 T^2 + T^3 ) ( -1 + 2 T - T^2 + T^3 ) / T^3,
  - 1 / ( T1^7 T2^6 ) ( -1 + T1 - 2 T1^2 + T1^3 ) ( -1 + 2 T1 - T1^2 + T1^3 ) ( 1 + T1 T2 ) ( -1 + T2 - 2 T2^2 + T2^3 )
  ( -1 + 2 T2 - T2^2 + T2^3 ) ( 3 - 6 T1 T2 + 8 T1^2 T2^2 - 6 T1^3 T2^3 + 3 T1^4 T2^4 ) },
  Knot[8, 20] -> { ( 1 - T + T^2 )^2 / T^2, 1 / ( T1^5 T2^4 ( -1 + T1 T2 ) )
  2 ( -1 + 2 T1 - 3 T1^2 + 2 T1^3 - T1^4 + 2 T2 - 3 T1 T2 + 4 T1^2 T2 -
  T1^3 T2 + T1^5 T2 - 3 T2^2 + 4 T1 T2^2 - 2 T1^2 T2^2 - 4 T1^3 T2^2 + 4 T1^4 T2^2 - 2 T1^5 T2^2 + 2 T2^3 - T1 T2^3 - 4 T1^2 T2^3 + 5 T1^3 T2^3 -
  T1^4 T2^3 - 4 T1^5 T2^3 + 2 T1^6 T2^3 - T1^7 T2^3 - T2^4 + 4 T1^2 T2^4 - T1^3 T2^4 + 3 T1^5 T2^4 + 2 T1^6 T2^4 + T1^8 T2^4 + T1 T2^5 - 2 T1^2 T2^5 -
  4 T1^3 T2^5 + 3 T1^4 T2^5 - 3 T1^5 T2^5 - 4 T1^6 T2^5 + T1^7 T2^5 - 2 T1^8 T2^5 + 2 T1^3 T2^6 + 2 T1^4 T2^6 - 4 T1^5 T2^6 + 8 T1^6 T2^6 - 4 T1^7 T2^6 +
  3 T1^8 T2^6 - T1^3 T2^7 + T1^5 T2^7 - 4 T1^6 T2^7 + 3 T1^7 T2^7 - 2 T1^8 T2^7 + T1^4 T2^8 - 2 T1^5 T2^8 + 3 T1^6 T2^8 - 2 T1^7 T2^8 + T1^8 T2^8 ) },

```

$$\text{Knot}[9, 27] \rightarrow \left\{ -\frac{(-1 + 2T - 3T^2 + T^3)(-1 + 3T - 2T^2 + T^3)}{T^3}, -\frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} \right.$$

$$\left. \begin{aligned} &(-3 + 15T_1 - 33T_1^2 + 45T_1^3 - 33T_1^4 + 15T_1^5 - 3T_1^6 + 15T_2 - 65T_1 T_2 + 115T_1^2 T_2 - 115T_1^3 T_2 + 15T_1^4 T_2 + \\ &35T_1^5 T_2 - 35T_1^6 T_2 + 10T_1^7 T_2 - 33T_2^2 + 115T_1 T_2^2 - 125T_1^2 T_2^2 + T_1^3 T_2^2 + 265T_1^4 T_2^2 - 219T_1^5 T_2^2 + \\ &95T_1^6 T_2^2 + 5T_1^7 T_2^2 - 11T_1^8 T_2^2 + 45T_2^3 - 115T_1 T_2^3 + T_1^2 T_2^3 + 268T_1^3 T_2^3 - 554T_1^4 T_2^3 + 156T_1^5 T_2^3 + \\ &108T_1^6 T_2^3 - 164T_1^7 T_2^3 + 55T_1^8 T_2^3 - 33T_2^4 + 15T_1 T_2^4 + 265T_1^2 T_2^4 - 554T_1^3 T_2^4 + 518T_1^4 T_2^4 + 342T_1^5 T_2^4 - \\ &542T_1^6 T_2^4 + 286T_1^7 T_2^4 - T_1^8 T_2^4 - 55T_1^9 T_2^4 + 11T_1^{10} T_2^4 + 15T_2^5 + 35T_1 T_2^5 - 219T_1^2 T_2^5 + 156T_1^3 T_2^5 + \\ &342T_1^4 T_2^5 - 1095T_1^5 T_2^5 + 705T_1^6 T_2^5 + 67T_1^7 T_2^5 - 294T_1^8 T_2^5 + 166T_1^9 T_2^5 - 5T_1^{10} T_2^5 - 10T_1^{11} T_2^5 - 3T_2^6 - \\ &35T_1 T_2^6 + 95T_1^2 T_2^6 + 108T_1^3 T_2^6 - 542T_1^4 T_2^6 + 705T_1^5 T_2^6 + 12T_1^6 T_2^6 - 745T_1^7 T_2^6 + 444T_1^8 T_2^6 - 92T_1^9 T_2^6 - \\ &97T_1^{10} T_2^6 + 35T_1^{11} T_2^6 + 3T_2^7 + 10T_1 T_2^7 + 5T_1^2 T_2^7 - 164T_1^3 T_2^7 + 286T_1^4 T_2^7 + 67T_1^5 T_2^7 - 745T_1^6 T_2^7 + \\ &1055T_1^7 T_2^7 - 208T_1^8 T_2^7 - 164T_1^9 T_2^7 + 221T_1^{10} T_2^7 - 35T_1^{11} T_2^7 - 15T_2^8 + 11T_1 T_2^8 + 55T_1^2 T_2^8 - T_1^3 T_2^8 - \\ &294T_1^4 T_2^8 + 444T_1^5 T_2^8 - 208T_1^6 T_2^8 - 616T_1^7 T_2^8 + 546T_1^8 T_2^8 - 267T_1^9 T_2^8 - 15T_1^{10} T_2^8 + 33T_1^{11} T_2^8 - \\ &55T_1^4 T_2^9 + 166T_1^5 T_2^9 - 92T_1^6 T_2^9 - 164T_1^7 T_2^9 + 546T_1^8 T_2^9 - 252T_1^9 T_2^9 + T_1^{10} T_2^9 + 115T_1^{11} T_2^9 - 45T_1^{12} T_2^9 + \\ &11T_1^4 T_2^{10} - 5T_1^5 T_2^{10} - 97T_1^6 T_2^{10} + 221T_1^7 T_2^{10} - 267T_1^8 T_2^{10} + T_1^9 T_2^{10} + 123T_1^{10} T_2^{10} - 115T_1^{11} T_2^{10} + \\ &33T_1^{12} T_2^{10} - 10T_1^5 T_2^{11} + 35T_1^6 T_2^{11} - 35T_1^7 T_2^{11} - 15T_1^8 T_2^{11} + 115T_1^9 T_2^{11} - 115T_1^{10} T_2^{11} + 65T_1^{11} T_2^{11} - \\ &15T_1^{12} T_2^{11} + 3T_1^6 T_2^{12} - 15T_1^7 T_2^{12} + 33T_1^8 T_2^{12} - 45T_1^9 T_2^{12} + 33T_1^{10} T_2^{12} - 15T_1^{11} T_2^{12} + 3T_1^{12} T_2^{12} \end{aligned} \right\},$$

$$\text{Knot}[9, 41] \rightarrow \left\{ \frac{(3 - 3T + T^2)(1 - 3T + 3T^2)}{T^2}, \frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} \right.$$

$$\left. \begin{aligned} &(-51 + 201T_1 - 315T_1^2 + 201T_1^3 - 51T_1^4 + 201T_2 - 698T_1 T_2 + 880T_1^2 T_2 - 236T_1^3 T_2 - \\ &158T_1^4 T_2 + 93T_1^5 T_2 - 315T_2^2 + 880T_1 T_2^2 - 519T_1^2 T_2^2 - 1035T_1^3 T_2^2 + 1305T_1^4 T_2^2 - 488T_1^5 T_2^2 + \\ &27T_1^6 T_2^2 + 201T_1^7 T_2^2 - 236T_1 T_2^3 - 1035T_1^2 T_2^3 + 2964T_1^3 T_2^3 - 2484T_1^4 T_2^3 + 333T_1^5 T_2^3 + 376T_1^6 T_2^3 - \\ &123T_1^7 T_2^3 - 51T_2^4 - 158T_1 T_2^4 + 1305T_1^2 T_2^4 - 2484T_1^3 T_2^4 + 1188T_1^4 T_2^4 + 1692T_1^5 T_2^4 - \\ &1467T_1^6 T_2^4 + 274T_1^7 T_2^4 + 57T_1^8 T_2^4 + 93T_1 T_2^5 - 488T_1^2 T_2^5 + 333T_1^3 T_2^5 + 1692T_1^4 T_2^5 - \\ &3756T_1^5 T_2^5 + 1701T_1^6 T_2^5 + 124T_1^7 T_2^5 - 231T_1^8 T_2^5 + 27T_1^9 T_2^5 + 376T_1^3 T_2^6 - 1467T_1^4 T_2^6 + \\ &1701T_1^5 T_2^6 + 357T_1^6 T_2^6 - 992T_1^7 T_2^6 + 369T_1^8 T_2^6 - 123T_1^3 T_2^7 + 274T_1^4 T_2^7 + 124T_1^5 T_2^7 - \\ &992T_1^6 T_2^7 + 814T_1^7 T_2^7 - 231T_1^8 T_2^7 + 57T_1^4 T_2^8 - 231T_1^5 T_2^8 + 369T_1^6 T_2^8 - 231T_1^7 T_2^8 + 57T_1^8 T_2^8 \end{aligned} \right\},$$

$$\text{Knot}[9, 46] \rightarrow \left\{ -\frac{(-2 + T)(-1 + 2T)}{T}, -\frac{1}{T_1^3 T_2^2 (-1 + T_1 T_2)} \right.$$

$$\left. \begin{aligned} &(-5 + 11T_1 - 5T_1^2 + 11T_2 - 32T_1 T_2 + 38T_1^2 T_2 - 9T_1^3 T_2 - 5T_2^2 + 38T_1 T_2^2 - 72T_1^2 T_2^2 - 2T_1^3 T_2^2 + \\ &11T_1^4 T_2^2 - 9T_1 T_2^3 - 2T_1^2 T_2^3 + 68T_1^3 T_2^3 - 29T_1^4 T_2^3 + 11T_1^2 T_2^4 - 29T_1^3 T_2^4 + 11T_1^4 T_2^4 \end{aligned} \right\},$$

$$\text{Knot}[10, 3] \rightarrow \left\{ -\frac{(-3 + 2T)(-2 + 3T)}{T}, -\frac{1}{T_1^3 T_2^2 (-1 + T_1 T_2)} \right.$$

$$\left. \begin{aligned} &(-171 + 367T_1 - 171T_1^2 + \\ &367T_2 - 888T_1 T_2 + 594T_1^2 T_2 - 101T_1^3 T_2 - 171T_2^2 + 594T_1 T_2^2 - 420T_1^2 T_2^2 - 342T_1^3 T_2^2 + \\ &261T_1^4 T_2^2 - 101T_1 T_2^3 - 342T_1^2 T_2^3 + 1140T_1^3 T_2^3 - 569T_1^4 T_2^3 + 261T_1^2 T_2^4 - 569T_1^3 T_2^4 + 261T_1^4 T_2^4 \end{aligned} \right\},$$

$$\text{Knot}[10, 22] \rightarrow \left\{ -\frac{(-2 + 2T - 2T^2 + T^3)(-1 + 2T - 2T^2 + 2T^3)}{T^3}, \right.$$

$$\left. -\frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} \right.$$

$$\left. \begin{aligned} &(-25 + 75T_1 - 125T_1^2 + 163T_1^3 - 125T_1^4 + 75T_1^5 - 25T_1^6 + 75T_2 - 174T_1 T_2 + \\ &222T_1^2 T_2 - 234T_1^3 T_2 + 42T_1^4 T_2 + 30T_1^5 T_2 - 78T_1^6 T_2 + 51T_1^7 T_2 - 125T_2^2 + 222T_1 T_2^2 - 211T_1^2 T_2^2 + \\ &189T_1^3 T_2^2 + 141T_1^4 T_2^2 - 91T_1^5 T_2^2 + 109T_1^6 T_2^2 - 18T_1^7 T_2^2 - 45T_1^8 T_2^2 + 163T_2^3 - 234T_1 T_2^3 + 189T_1^2 T_2^3 - \\ &204T_1^3 T_2^3 - 184T_1^4 T_2^3 - 52T_1^5 T_2^3 + 6T_1^6 T_2^3 - 111T_1^7 T_2^3 + 114T_1^8 T_2^3 + 7T_1^9 T_2^3 - 125T_2^4 + 42T_1 T_2^4 + \\ &141T_1^2 T_2^4 - 184T_1^3 T_2^4 + 557T_1^4 T_2^4 - 83T_1^5 T_2^4 + 105T_1^6 T_2^4 + 24T_1^7 T_2^4 + 5T_1^8 T_2^4 - 126T_1^9 T_2^4 + \end{aligned} \right.$$

$$\begin{aligned}
 & 35 T_1^{10} T_2^4 + 75 T_2^5 + 30 T_1 T_2^5 - 91 T_1^2 T_2^5 - 52 T_1^3 T_2^5 - 83 T_1^4 T_2^5 - 416 T_1^5 T_2^5 + 174 T_1^6 T_2^5 - 63 T_1^7 T_2^5 - \\
 & 32 T_1^8 T_2^5 + 129 T_1^9 T_2^5 + 30 T_1^{10} T_2^5 - 45 T_1^{11} T_2^5 - 25 T_2^6 - 78 T_1 T_2^6 + 109 T_1^2 T_2^6 + 6 T_1^3 T_2^6 + 105 T_1^4 T_2^6 + \\
 & 174 T_1^5 T_2^6 + 108 T_1^6 T_2^6 - 218 T_1^7 T_2^6 + 73 T_1^8 T_2^6 - 66 T_1^9 T_2^6 - 115 T_1^{10} T_2^6 + 66 T_1^{11} T_2^6 + 23 T_1^{12} T_2^6 + \\
 & 51 T_1 T_2^7 - 18 T_1^2 T_2^7 - 111 T_1^3 T_2^7 + 24 T_1^4 T_2^7 - 63 T_1^5 T_2^7 - 218 T_1^6 T_2^7 + 372 T_1^7 T_2^7 - 43 T_1^8 T_2^7 + 44 T_1^9 T_2^7 + \\
 & 109 T_1^{10} T_2^7 - 18 T_1^{11} T_2^7 - 69 T_1^{12} T_2^7 - 45 T_1^2 T_2^8 + 114 T_1^3 T_2^8 + 5 T_1^4 T_2^8 - 32 T_1^5 T_2^8 + 73 T_1^6 T_2^8 - 43 T_1^7 T_2^8 - \\
 & 379 T_1^8 T_2^8 + 176 T_1^9 T_2^8 - 131 T_1^{10} T_2^8 - 54 T_1^{11} T_2^8 + 115 T_1^{12} T_2^8 + 7 T_1^3 T_2^9 - 126 T_1^4 T_2^9 + 129 T_1^5 T_2^9 - \\
 & 66 T_1^6 T_2^9 + 44 T_1^7 T_2^9 + 176 T_1^8 T_2^9 + 144 T_1^9 T_2^9 - 171 T_1^{10} T_2^9 + 222 T_1^{11} T_2^9 - 149 T_1^{12} T_2^9 + 35 T_1^4 T_2^{10} + \\
 & 30 T_1^5 T_2^{10} - 115 T_1^6 T_2^{10} + 109 T_1^7 T_2^{10} - 131 T_1^8 T_2^{10} - 171 T_1^9 T_2^{10} + 205 T_1^{10} T_2^{10} - 210 T_1^{11} T_2^{10} + \\
 & 115 T_1^{12} T_2^{10} - 45 T_1^5 T_2^{11} + 66 T_1^6 T_2^{11} - 18 T_1^7 T_2^{11} - 54 T_1^8 T_2^{11} + 222 T_1^9 T_2^{11} - 210 T_1^{10} T_2^{11} + 162 T_1^{11} T_2^{11} - \\
 & 69 T_1^{12} T_2^{11} + 23 T_1^6 T_2^{12} - 69 T_1^7 T_2^{12} + 115 T_1^8 T_2^{12} - 149 T_1^9 T_2^{12} + 115 T_1^{10} T_2^{12} - 69 T_1^{11} T_2^{12} + 23 T_1^{12} T_2^{12} \Big\},
 \end{aligned}$$

$$\text{Knot}[10, 35] \rightarrow \left\{ \frac{(2 - 4 T + T^2)(1 - 4 T + 2 T^2)}{T^2}, \frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} \right.$$

$$\begin{aligned}
 & (-17 + 103 T_1 - 181 T_1^2 + 103 T_1^3 - 17 T_1^4 + 103 T_2 - 570 T_1 T_2 + 762 T_1^2 T_2 - 30 T_1^3 T_2 - 234 T_1^4 T_2 + \\
 & 55 T_1^5 T_2 - 181 T_2^2 + 762 T_1 T_2^2 + 112 T_1^2 T_2^2 - 2442 T_1^3 T_2^2 + 1708 T_1^4 T_2^2 - 246 T_1^5 T_2^2 - 13 T_1^6 T_2^2 + 103 T_2^3 - \\
 & 30 T_1 T_2^3 - 2442 T_1^2 T_2^3 + 5136 T_1^3 T_2^3 - 2172 T_1^4 T_2^3 - 426 T_1^5 T_2^3 + 330 T_1^6 T_2^3 - 41 T_1^7 T_2^3 - 17 T_2^4 - 234 T_1 T_2^4 + \\
 & 1708 T_1^2 T_2^4 - 2172 T_1^3 T_2^4 - 1392 T_1^4 T_2^4 + 3108 T_1^5 T_2^4 - 1412 T_1^6 T_2^4 + 150 T_1^7 T_2^4 + 15 T_1^8 T_2^4 + 55 T_1 T_2^5 - \\
 & 246 T_1^2 T_2^5 - 426 T_1^3 T_2^5 + 3108 T_1^4 T_2^5 - 4200 T_1^5 T_2^5 + 1590 T_1^6 T_2^5 + 114 T_1^7 T_2^5 - 89 T_1^8 T_2^5 - 13 T_1^2 T_2^6 + \\
 & 330 T_1^3 T_2^6 - 1412 T_1^4 T_2^6 + 1590 T_1^5 T_2^6 + 184 T_1^6 T_2^6 - 678 T_1^7 T_2^6 + 155 T_1^8 T_2^6 - 41 T_1^3 T_2^7 + 150 T_1^4 T_2^7 + \\
 & 114 T_1^5 T_2^7 - 678 T_1^6 T_2^7 + 486 T_1^7 T_2^7 - 89 T_1^8 T_2^7 + 15 T_1^4 T_2^8 - 89 T_1^5 T_2^8 + 155 T_1^6 T_2^8 - 89 T_1^7 T_2^8 + 15 T_1^8 T_2^8 \Big\},
 \end{aligned}$$

$$\text{Knot}[10, 42] \rightarrow \left\{ -\frac{(-1 + 3 T - 4 T^2 + T^3)(-1 + 4 T - 3 T^2 + T^3)}{T^3}, \right.$$

$$\begin{aligned}
 & -\frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} (-3 + 21 T_1 - 57 T_1^2 + 81 T_1^3 - 57 T_1^4 + 21 T_1^5 - 3 T_1^6 + 21 T_2 - 133 T_1 T_2 + 301 T_1^2 T_2 - \\
 & 301 T_1^3 T_2 + 21 T_1^4 T_2 + 119 T_1^5 T_2 - 77 T_1^6 T_2 + 14 T_1^7 T_2 - 57 T_2^2 + 301 T_1 T_2^2 - 422 T_1^2 T_2^2 - 166 T_1^3 T_2^2 + \\
 & 1164 T_1^4 T_2^2 - 926 T_1^5 T_2^2 + 262 T_1^6 T_2^2 + 35 T_1^7 T_2^2 - 19 T_1^8 T_2^2 + 81 T_2^3 - 301 T_1 T_2^3 - 166 T_1^2 T_2^3 + 1864 T_1^3 T_2^3 - \\
 & 3062 T_1^4 T_2^3 + 950 T_1^5 T_2^3 + 674 T_1^6 T_2^3 - 641 T_1^7 T_2^3 + 133 T_1^8 T_2^3 - 57 T_2^4 + 21 T_1 T_2^4 + 1164 T_1^2 T_2^4 - \\
 & 3062 T_1^3 T_2^4 + 2392 T_1^4 T_2^4 + 2552 T_1^5 T_2^4 - 3800 T_1^6 T_2^4 + 1690 T_1^7 T_2^4 - 38 T_1^8 T_2^4 - 133 T_1^9 T_2^4 + 19 T_1^{10} T_2^4 + \\
 & 21 T_2^5 + 119 T_1 T_2^5 - 926 T_1^2 T_2^5 + 950 T_1^3 T_2^5 + 2552 T_1^4 T_2^5 - 7805 T_1^5 T_2^5 + 5699 T_1^6 T_2^5 - 279 T_1^7 T_2^5 - \\
 & 1582 T_1^8 T_2^5 + 689 T_1^9 T_2^5 - 35 T_1^{10} T_2^5 - 14 T_1^{11} T_2^5 - 3 T_2^6 - 77 T_1 T_2^6 + 262 T_1^2 T_2^6 + 674 T_1^3 T_2^6 - 3800 T_1^4 T_2^6 + \\
 & 5699 T_1^5 T_2^6 - 552 T_1^6 T_2^6 - 5255 T_1^7 T_2^6 + 4018 T_1^8 T_2^6 - 818 T_1^9 T_2^6 - 274 T_1^{10} T_2^6 + 77 T_1^{11} T_2^6 + 3 T_1^{12} T_2^6 + \\
 & 14 T_1 T_2^7 + 35 T_1^2 T_2^7 - 641 T_1^3 T_2^7 + 1690 T_1^4 T_2^7 - 279 T_1^5 T_2^7 - 5255 T_1^6 T_2^7 + 8249 T_1^7 T_2^7 - 3110 T_1^8 T_2^7 - \\
 & 842 T_1^9 T_2^7 + 974 T_1^{10} T_2^7 - 119 T_1^{11} T_2^7 - 21 T_1^{12} T_2^7 - 19 T_2^8 + 133 T_1 T_2^8 - 38 T_1^2 T_2^8 - 1582 T_1^3 T_2^8 + \\
 & 4018 T_1^4 T_2^8 - 3110 T_1^5 T_2^8 - 2174 T_1^6 T_2^8 + 3170 T_1^7 T_2^8 - 1240 T_1^8 T_2^8 - 21 T_1^{11} T_2^8 + 57 T_1^{12} T_2^8 - 133 T_1^4 T_2^9 + \\
 & 689 T_1^5 T_2^9 - 818 T_1^6 T_2^9 - 842 T_1^7 T_2^9 + 3170 T_1^8 T_2^9 - 2008 T_1^9 T_2^9 + 214 T_1^{10} T_2^9 + 301 T_1^{11} T_2^9 - 81 T_1^{12} T_2^9 + \\
 & 19 T_1^4 T_2^{10} - 35 T_1^5 T_2^{10} - 274 T_1^6 T_2^{10} + 974 T_1^7 T_2^{10} - 1240 T_1^8 T_2^{10} + 214 T_1^9 T_2^{10} + 410 T_1^{10} T_2^{10} - 301 T_1^{11} T_2^{10} + \\
 & 57 T_1^{12} T_2^{10} - 14 T_1^5 T_2^{11} + 77 T_1^6 T_2^{11} - 119 T_1^7 T_2^{11} - 21 T_1^8 T_2^{11} + 301 T_1^9 T_2^{11} - 301 T_1^{10} T_2^{11} + 133 T_1^{11} T_2^{11} - \\
 & 21 T_1^{12} T_2^{11} + 3 T_1^6 T_2^{12} - 21 T_1^7 T_2^{12} + 57 T_1^8 T_2^{12} - 81 T_1^9 T_2^{12} + 57 T_1^{10} T_2^{12} - 21 T_1^{11} T_2^{12} + 3 T_1^{12} T_2^{12} \Big\},
 \end{aligned}$$

$$\text{Knot}[10, 48] \rightarrow \left\{ \frac{(1 - T + 2 T^2 - 2 T^3 + T^4)(1 - 2 T + 2 T^2 - T^3 + T^4)}{T^4}, \right.$$

$$\begin{aligned}
 & \frac{1}{T_1^9 T_2^8 (-1 + T_1 T_2)} \\
 & (-4 + 12 T_1 - 24 T_1^2 + 36 T_1^3 - 44 T_1^4 + 36 T_1^5 - 24 T_1^6 + 12 T_1^7 - 4 T_1^8 + 12 T_2 - 27 T_1 T_2 + 45 T_1^2 T_2 - \\
 & 54 T_1^3 T_2 + 51 T_1^4 T_2 - 9 T_1^5 T_2 - 9 T_1^6 T_2 + 18 T_1^7 T_2 - 15 T_1^8 T_2 + 9 T_1^9 T_2 - 24 T_2^2 + 45 T_1 T_2^2 - 74 T_1^2 T_2^2 +
 \end{aligned}$$

$$\begin{aligned}
 & 88 T_1^3 T_2^2 - 91 T_1^4 T_2^2 + 26 T_1^5 T_2^2 - 31 T_1^6 T_2^2 + 16 T_1^7 T_2^2 - 14 T_1^8 T_2^2 + 9 T_1^9 T_2^2 - 12 T_1^{10} T_2^2 + 36 T_1^3 - \\
 & 54 T_1 T_2^3 + 88 T_1^2 T_2^3 - 97 T_1^3 T_2^3 + 100 T_1^4 T_2^3 - 3 T_1^5 T_2^3 + 42 T_1^6 T_2^3 - 8 T_1^7 T_2^3 + 11 T_1^8 T_2^3 - 2 T_1^9 T_2^3 + \\
 & 9 T_1^{10} T_2^3 + 9 T_1^{11} T_2^3 - 44 T_2^4 + 51 T_1 T_2^4 - 91 T_1^2 T_2^4 + 100 T_1^3 T_2^4 - 115 T_1^4 T_2^4 - 83 T_1^5 T_2^4 + 6 T_1^7 T_2^4 + T_1^8 T_2^4 - \\
 & 26 T_1^9 T_2^4 + 11 T_1^{10} T_2^4 - 27 T_1^{11} T_2^4 + 36 T_2^5 - 9 T_1 T_2^5 + 26 T_1^2 T_2^5 - 3 T_1^3 T_2^5 + 110 T_1^5 T_2^5 + 44 T_1^6 T_2^5 + \\
 & 44 T_1^7 T_2^5 - 16 T_1^8 T_2^5 + 9 T_1^9 T_2^5 + 24 T_1^{10} T_2^5 - T_1^{11} T_2^5 + 27 T_1^{12} T_2^5 - 9 T_1^{13} T_2^5 - 24 T_2^6 - 9 T_1 T_2^6 - 31 T_1^2 T_2^6 + \\
 & 42 T_1^3 T_2^6 - 83 T_1^4 T_2^6 + 44 T_1^5 T_2^6 - 212 T_1^6 T_2^6 + 38 T_1^7 T_2^6 - 20 T_1^8 T_2^6 + 8 T_1^9 T_2^6 + T_1^{10} T_2^6 - 30 T_1^{11} T_2^6 - \\
 & 7 T_1^{12} T_2^6 - 9 T_1^{13} T_2^6 + 12 T_1^{14} T_2^6 + 12 T_2^7 + 18 T_1 T_2^7 + 16 T_1^2 T_2^7 - 8 T_1^3 T_2^7 + 6 T_1^4 T_2^7 + 44 T_1^5 T_2^7 + 38 T_1^6 T_2^7 + \\
 & 158 T_1^7 T_2^7 - 118 T_1^8 T_2^7 + 29 T_1^9 T_2^7 - 28 T_1^{10} T_2^7 + 9 T_1^{11} T_2^7 + 28 T_1^{12} T_2^7 - 2 T_1^{13} T_2^7 - 9 T_1^{14} T_2^7 - 9 T_1^{15} T_2^7 - \\
 & 4 T_2^8 - 15 T_1 T_2^8 - 14 T_1^2 T_2^8 + 11 T_1^3 T_2^8 + T_1^4 T_2^8 - 16 T_1^5 T_2^8 - 20 T_1^6 T_2^8 - 118 T_1^7 T_2^8 + 48 T_1^8 T_2^8 + 74 T_1^9 T_2^8 + \\
 & 10 T_1^{10} T_2^8 + 20 T_1^{11} T_2^8 - 19 T_1^{12} T_2^8 - 7 T_1^{13} T_2^8 + 16 T_1^{14} T_2^8 + 15 T_1^{15} T_2^8 + 4 T_1^{16} T_2^8 + 9 T_1 T_2^9 + 9 T_1^2 T_2^9 - \\
 & 2 T_1^3 T_2^9 - 26 T_1^4 T_2^9 + 9 T_1^5 T_2^9 + 8 T_1^6 T_2^9 + 29 T_1^7 T_2^9 + 74 T_1^8 T_2^9 - 202 T_1^9 T_2^9 + 20 T_1^{10} T_2^9 - 64 T_1^{11} T_2^9 + \\
 & 12 T_1^{12} T_2^9 + 10 T_1^{13} T_2^9 - 20 T_1^{14} T_2^9 - 18 T_1^{15} T_2^9 - 12 T_1^{16} T_2^9 - 12 T_2^{10} + 9 T_1 T_2^{10} + 11 T_1^2 T_2^{10} + \\
 & 24 T_1^5 T_2^{10} + T_1^6 T_2^{10} - 28 T_1^7 T_2^{10} + 10 T_1^8 T_2^{10} + 20 T_1^9 T_2^{10} + 202 T_1^{10} T_2^{10} - 64 T_1^{11} T_2^{10} + 85 T_1^{12} T_2^{10} - \\
 & 48 T_1^{13} T_2^{10} + 35 T_1^{14} T_2^{10} + 9 T_1^{15} T_2^{10} + 24 T_1^{16} T_2^{10} + 9 T_1^3 T_2^{11} - 27 T_1^4 T_2^{11} - T_1^5 T_2^{11} - 30 T_1^6 T_2^{11} + \\
 & 9 T_1^7 T_2^{11} + 20 T_1^8 T_2^{11} - 64 T_1^9 T_2^{11} - 64 T_1^{10} T_2^{11} - 106 T_1^{11} T_2^{11} + 18 T_1^{12} T_2^{11} - 3 T_1^{13} T_2^{11} - 28 T_1^{14} T_2^{11} + \\
 & 9 T_1^{15} T_2^{11} - 36 T_1^{16} T_2^{11} + 27 T_1^5 T_2^{12} - 7 T_1^6 T_2^{12} + 28 T_1^7 T_2^{12} - 19 T_1^8 T_2^{12} + 12 T_1^9 T_2^{12} + 85 T_1^{10} T_2^{12} + \\
 & 18 T_1^{11} T_2^{12} + 97 T_1^{12} T_2^{12} - 98 T_1^{13} T_2^{12} + 95 T_1^{14} T_2^{12} - 51 T_1^{15} T_2^{12} + 44 T_1^{16} T_2^{12} - 9 T_1^5 T_2^{13} - 9 T_1^6 T_2^{13} - \\
 & 2 T_1^7 T_2^{13} - 7 T_1^8 T_2^{13} + 10 T_1^9 T_2^{13} - 48 T_1^{10} T_2^{13} - 3 T_1^{11} T_2^{13} - 98 T_1^{12} T_2^{13} + 101 T_1^{13} T_2^{13} - 92 T_1^{14} T_2^{13} + \\
 & 54 T_1^{15} T_2^{13} - 36 T_1^{16} T_2^{13} + 12 T_1^6 T_2^{14} - 9 T_1^7 T_2^{14} + 16 T_1^8 T_2^{14} - 20 T_1^9 T_2^{14} + 35 T_1^{10} T_2^{14} - 28 T_1^{11} T_2^{14} + \\
 & 95 T_1^{12} T_2^{14} - 92 T_1^{13} T_2^{14} + 76 T_1^{14} T_2^{14} - 45 T_1^{15} T_2^{14} + 24 T_1^{16} T_2^{14} - 9 T_1^7 T_2^{15} + 15 T_1^8 T_2^{15} - 18 T_1^9 T_2^{15} + \\
 & 9 T_1^{10} T_2^{15} + 9 T_1^{11} T_2^{15} - 51 T_1^{12} T_2^{15} + 54 T_1^{13} T_2^{15} - 45 T_1^{14} T_2^{15} + 27 T_1^{15} T_2^{15} - 12 T_1^{16} T_2^{15} + 4 T_1^8 T_2^{16} - \\
 & 12 T_1^9 T_2^{16} + 24 T_1^{10} T_2^{16} - 36 T_1^{11} T_2^{16} + 44 T_1^{12} T_2^{16} - 36 T_1^{13} T_2^{16} + 24 T_1^{14} T_2^{16} - 12 T_1^{15} T_2^{16} + 4 T_1^{16} T_2^{16} \} ,
 \end{aligned}$$

$$\text{Knot}[10, 75] \rightarrow \left\{ - \frac{(-1 + 3 T - 4 T^2 + T^3) (-1 + 4 T - 3 T^2 + T^3)}{T^3}, \right.$$

$$\begin{aligned}
 & - \frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} \left(-3 + 21 T_1 - 57 T_1^2 + 81 T_1^3 - 57 T_1^4 + 21 T_1^5 - 3 T_1^6 + 21 T_2 - 133 T_1 T_2 + 301 T_1^2 T_2 - \right. \\
 & 301 T_1^3 T_2 + 21 T_1^4 T_2 + 119 T_1^5 T_2 - 77 T_1^6 T_2 + 14 T_1^7 T_2 - 57 T_2^2 + 301 T_1 T_2^2 - 414 T_1^2 T_2^2 - 198 T_1^3 T_2^2 + \\
 & 1218 T_1^4 T_2^2 - 958 T_1^5 T_2^2 + 270 T_1^6 T_2^2 + 35 T_1^7 T_2^2 - 19 T_1^8 T_2^2 + 81 T_2^3 - 301 T_1 T_2^3 - 198 T_1^2 T_2^3 + 1954 T_1^3 T_2^3 - \\
 & 3148 T_1^4 T_2^3 + 864 T_1^5 T_2^3 + 764 T_1^6 T_2^3 - 673 T_1^7 T_2^3 + 133 T_1^8 T_2^3 - 57 T_2^4 + 21 T_1 T_2^4 + 1218 T_1^2 T_2^4 - \\
 & 3148 T_1^3 T_2^4 + 2358 T_1^4 T_2^4 + 2906 T_1^5 T_2^4 - 3834 T_1^6 T_2^4 + 1604 T_1^7 T_2^4 + 16 T_1^8 T_2^4 - 133 T_1^9 T_2^4 + 19 T_1^{10} T_2^4 + \\
 & 21 T_2^5 + 119 T_1 T_2^5 - 958 T_1^2 T_2^5 + 864 T_1^3 T_2^5 + 2906 T_1^4 T_2^5 - 8283 T_1^5 T_2^5 + 5221 T_1^6 T_2^5 + 75 T_1^7 T_2^5 - \\
 & 1668 T_1^8 T_2^5 + 657 T_1^9 T_2^5 - 35 T_1^{10} T_2^5 - 14 T_1^{11} T_2^5 - 3 T_2^6 - 77 T_1 T_2^6 + 270 T_1^2 T_2^6 + 764 T_1^3 T_2^6 - 3834 T_1^4 T_2^6 + \\
 & 5221 T_1^5 T_2^6 + 900 T_1^6 T_2^6 - 5733 T_1^7 T_2^6 + 3984 T_1^8 T_2^6 - 728 T_1^9 T_2^6 - 266 T_1^{10} T_2^6 + 77 T_1^{11} T_2^6 + 3 T_1^{12} T_2^6 + \\
 & 14 T_1 T_2^7 + 35 T_1^2 T_2^7 - 673 T_1^3 T_2^7 + 1604 T_1^4 T_2^7 + 75 T_1^5 T_2^7 - 5733 T_1^6 T_2^7 + 7771 T_1^7 T_2^7 - 2756 T_1^8 T_2^7 - \\
 & 928 T_1^9 T_2^7 + 942 T_1^{10} T_2^7 - 119 T_1^{11} T_2^7 - 21 T_1^{12} T_2^7 - 19 T_2^8 + 133 T_1 T_2^8 + 16 T_1^2 T_2^8 - 1668 T_1^3 T_2^8 + \\
 & 3984 T_1^4 T_2^8 - 2756 T_1^5 T_2^8 - 2208 T_1^6 T_2^8 + 3084 T_1^7 T_2^8 - 1186 T_1^{10} T_2^8 - 21 T_1^{11} T_2^8 + 57 T_1^{12} T_2^8 - 133 T_1^4 T_2^9 + \\
 & 657 T_1^5 T_2^9 - 728 T_1^6 T_2^9 - 928 T_1^7 T_2^9 + 3084 T_1^8 T_2^9 - 1918 T_1^9 T_2^9 + 182 T_1^{10} T_2^9 + 301 T_1^{11} T_2^9 - 81 T_1^{12} T_2^9 + \\
 & 19 T_1^4 T_2^{10} - 35 T_1^5 T_2^{10} - 266 T_1^6 T_2^{10} + 942 T_1^7 T_2^{10} - 1186 T_1^8 T_2^{10} + 182 T_1^9 T_2^{10} + 418 T_1^{10} T_2^{10} - 301 T_1^{11} T_2^{10} + \\
 & 57 T_1^{12} T_2^{10} - 14 T_1^5 T_2^{11} + 77 T_1^6 T_2^{11} - 119 T_1^7 T_2^{11} - 21 T_1^8 T_2^{11} + 301 T_1^9 T_2^{11} - 301 T_1^{10} T_2^{11} + 133 T_1^{11} T_2^{11} - \\
 & 21 T_1^{12} T_2^{11} + 3 T_1^6 T_2^{12} - 21 T_1^7 T_2^{12} + 57 T_1^8 T_2^{12} - 81 T_1^9 T_2^{12} + 57 T_1^{10} T_2^{12} - 21 T_1^{11} T_2^{12} + 3 T_1^{12} T_2^{12} \} ,
 \end{aligned}$$

$$\text{Knot}[10, 87] \rightarrow \left\{ - \frac{(-2 + T) (-1 + 2 T) (1 - T + T^2)^2}{T^3}, \right.$$

$$\begin{aligned}
 & - \frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} \left(-25 + 112 T_1 - 224 T_1^2 + 287 T_1^3 - 224 T_1^4 + 112 T_1^5 - 25 T_1^6 + 112 T_2 - 426 T_1 T_2 + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 664 T_1^2 T_2 - 606 T_1^3 T_2 + 132 T_1^4 T_2 + 178 T_1^5 T_2 - 228 T_1^6 T_2 + 76 T_1^7 T_2 - 224 T_2^2 + 664 T_1 T_2^2 - 570 T_1^2 T_2^2 - \\
 & 95 T_1^3 T_2^2 + 1152 T_1^4 T_2^2 - 1103 T_1^5 T_2^2 + 582 T_1^6 T_2^2 + 16 T_1^7 T_2^2 - 80 T_1^8 T_2^2 + 287 T_2^3 - 606 T_1 T_2^3 - \\
 & 95 T_1^2 T_2^3 + 1138 T_1^3 T_2^3 - 2061 T_1^4 T_2^3 + 855 T_1^5 T_2^3 + 214 T_1^6 T_2^3 - 797 T_1^7 T_2^3 + 312 T_1^8 T_2^3 + 11 T_1^9 T_2^3 - \\
 & 224 T_2^4 + 132 T_1 T_2^4 + 1152 T_1^2 T_2^4 - 2061 T_1^3 T_2^4 + 2114 T_1^4 T_2^4 + 358 T_1^5 T_2^4 - 1378 T_1^6 T_2^4 + 1251 T_1^7 T_2^4 + \\
 & 18 T_1^8 T_2^4 - 336 T_1^9 T_2^4 + 64 T_1^{10} T_2^4 + 112 T_2^5 + 178 T_1 T_2^5 - 1103 T_1^2 T_2^5 + 855 T_1^3 T_2^5 + 358 T_1^4 T_2^5 - \\
 & 2758 T_1^5 T_2^5 + 2084 T_1^6 T_2^5 - 290 T_1^7 T_2^5 - 1269 T_1^8 T_2^5 + 823 T_1^9 T_2^5 + 16 T_1^{10} T_2^5 - 68 T_1^{11} T_2^5 - 25 T_2^6 - \\
 & 228 T_1 T_2^6 + 582 T_1^2 T_2^6 + 214 T_1^3 T_2^6 - 1378 T_1^4 T_2^6 + 2084 T_1^5 T_2^6 + 72 T_1^6 T_2^6 - 2020 T_1^7 T_2^6 + 1970 T_1^8 T_2^6 - \\
 & 338 T_1^9 T_2^6 - 606 T_1^{10} T_2^6 + 204 T_1^{11} T_2^6 + 23 T_2^7 + 76 T_1 T_2^7 + 16 T_1^2 T_2^7 - 797 T_1^3 T_2^7 + 1251 T_1^4 T_2^7 - \\
 & 290 T_1^5 T_2^7 - 2020 T_1^6 T_2^7 + 2822 T_1^7 T_2^7 - 938 T_1^8 T_2^7 - 873 T_1^9 T_2^7 + 1129 T_1^{10} T_2^7 - 146 T_1^{11} T_2^7 - 104 T_1^{12} T_2^7 - \\
 & 80 T_1^2 T_2^8 + 312 T_1^3 T_2^8 + 18 T_1^4 T_2^8 - 1269 T_1^5 T_2^8 + 1970 T_1^6 T_2^8 - 938 T_1^7 T_2^8 - 1522 T_1^8 T_2^8 + 2043 T_1^9 T_2^8 - \\
 & 1116 T_1^{10} T_2^8 - 156 T_1^{11} T_2^8 + 208 T_1^{12} T_2^8 + 11 T_2^9 + 336 T_1 T_2^9 - 336 T_1^2 T_2^9 + 823 T_1^3 T_2^9 - 338 T_1^4 T_2^9 - 873 T_1^5 T_2^9 + \\
 & 2043 T_1^6 T_2^9 - 1262 T_1^7 T_2^9 + 121 T_1^8 T_2^9 + 582 T_1^9 T_2^9 - 265 T_1^{10} T_2^9 + 64 T_1^{11} T_2^9 + 16 T_1^{12} T_2^9 - 606 T_1^6 T_2^{10} + \\
 & 1129 T_1^7 T_2^{10} - 1116 T_1^8 T_2^{10} + 121 T_1^9 T_2^{10} + 546 T_1^{10} T_2^{10} - 632 T_1^{11} T_2^{10} + 208 T_1^{12} T_2^{10} - 68 T_1^5 T_2^{11} + \\
 & 204 T_1^6 T_2^{11} - 146 T_1^7 T_2^{11} - 156 T_1^8 T_2^{11} + 582 T_1^9 T_2^{11} - 632 T_1^{10} T_2^{11} + 402 T_1^{11} T_2^{11} - 104 T_1^{12} T_2^{11} + \\
 & 23 T_1^6 T_2^{12} - 104 T_1^7 T_2^{12} + 208 T_1^8 T_2^{12} - 265 T_1^9 T_2^{12} + 208 T_1^{10} T_2^{12} - 104 T_1^{11} T_2^{12} + 23 T_1^{12} T_2^{12} \},
 \end{aligned}$$

$$\text{Knot [10, 99]} \rightarrow \left\{ \frac{(1 - T + T^2)^4}{T^4}, \frac{4 (1 - T_1 + T_1^2)^4 (1 + T_1 T_2) (1 - T_2 + T_2^2)^4 (1 - T_1 T_2 + T_1^2 T_2^2)^3}{T_1^9 T_2^8} \right\},$$

Knot [

10,

123] →

$$\left\{ \frac{(1 - 3 T + 3 T^2 - 3 T^3 + T^4)^2}{T^4}, \right.$$

$$\frac{1}{T_1^9 T_2^8} 2 (1 - 3 T_1 + 3 T_1^2 - 3 T_1^3 + T_1^4)^2 (1 + T_1 T_2) (2 - 3 T_1 T_2 + 2 T_1^2 T_2^2)$$

$$\left. (1 - 3 T_2 + 3 T_2^2 - 3 T_2^3 + T_2^4)^2 (1 - 3 T_1 T_2 + 3 T_1^2 T_2^2 - 3 T_1^3 T_2^3 + T_1^4 T_2^4) \right\},$$

$$\text{Knot [10, 129]} \rightarrow \left\{ \frac{(2 - 2 T + T^2) (1 - 2 T + 2 T^2)}{T^2}, \frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} \right.$$

$$\begin{aligned}
 & (-17 + 50 T_1 - 75 T_1^2 + 50 T_1^3 - 17 T_1^4 + 50 T_2 - 124 T_1 T_2 + 146 T_1^2 T_2 - 34 T_1^3 T_2 - 28 T_1^4 T_2 + \\
 & 26 T_1^5 T_2 - 75 T_2^2 + 146 T_1 T_2^2 - 77 T_1^2 T_2^2 - 151 T_1^3 T_2^2 + 175 T_1^4 T_2^2 - 70 T_1^5 T_2^2 - 3 T_1^6 T_2^2 + 50 T_2^3 - \\
 & 34 T_1 T_2^3 - 151 T_1^2 T_2^3 + 332 T_1^3 T_2^3 - 190 T_1^4 T_2^3 - 43 T_1^5 T_2^3 + 74 T_1^6 T_2^3 - 22 T_1^7 T_2^3 - 17 T_2^4 - 28 T_1 T_2^4 + \\
 & 175 T_1^2 T_2^4 - 190 T_1^3 T_2^4 - 12 T_1^4 T_2^4 + 218 T_1^5 T_2^4 - 113 T_1^6 T_2^4 + 20 T_1^7 T_2^4 + 15 T_1^8 T_2^4 + 26 T_1 T_2^5 - \\
 & 70 T_1^2 T_2^5 - 43 T_1^3 T_2^5 + 218 T_1^4 T_2^5 - 304 T_1^5 T_2^5 + 65 T_1^6 T_2^5 + 38 T_1^7 T_2^5 - 46 T_1^8 T_2^5 - 3 T_1^9 T_2^5 + 74 T_1^3 T_2^6 - \\
 & 113 T_1^4 T_2^6 + 65 T_1^5 T_2^6 + 139 T_1^6 T_2^6 - 142 T_1^7 T_2^6 + 69 T_1^8 T_2^6 - 22 T_1^9 T_2^6 + 20 T_1^4 T_2^7 + 38 T_1^5 T_2^7 - \\
 & 142 T_1^6 T_2^7 + 116 T_1^7 T_2^7 - 46 T_1^8 T_2^7 + 15 T_1^4 T_2^8 - 46 T_1^5 T_2^8 + 69 T_1^6 T_2^8 - 46 T_1^7 T_2^8 + 15 T_1^8 T_2^8 \},
 \end{aligned}$$

$$\text{Knot [10, 137]} \rightarrow \left\{ \frac{(1 - 3 T + T^2)^2}{T^2}, \frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} 2 (1 - 3 T_1 + T_1^2) (1 - 3 T_2 + T_2^2) \right.$$

$$\left. (1 - 3 T_1 T_2 + T_1^2 T_2^2) (-1 + 3 T_1 - T_1^2 + 3 T_2 - 8 T_1 T_2 + 4 T_1^2 T_2 - T_2^2 + 4 T_1 T_2^2 - 6 T_1^2 T_2^2 - 2 T_1^3 T_2^2 + T_1^4 T_2^2 - 2 T_1^2 T_2^3 + 10 T_1^3 T_2^3 - 3 T_1^4 T_2^3 + T_1^2 T_2^4 - 3 T_1^3 T_2^4 + T_1^4 T_2^4) \right\},$$

$$\text{Knot [10, 140]} \rightarrow \left\{ \frac{(1 - T + T^2)^2}{T^2}, \frac{1}{T_1^5 T_2^4 (-1 + T_1 T_2)} 2 (-1 + 2 T_1 - 3 T_1^2 + 2 T_1^3 - T_1^4 + 2 T_2 - \right.$$

$$\left. 3 T_1 T_2 + 4 T_1^2 T_2 - T_1^3 T_2 + T_1^5 T_2 - 3 T_2^2 + 4 T_1 T_2^2 + T_1^2 T_2^2 - 8 T_1^3 T_2^2 + 7 T_1^4 T_2^2 - 2 T_1^5 T_2^2 + 2 T_2^3 - T_1 T_2^3 - \right.$$

$$\left. \begin{aligned} & 8 T_1^2 T_2^3 + 6 T_1^3 T_2^3 - 8 T_1^5 T_2^3 + 2 T_1^6 T_2^3 - T_1^7 T_2^3 - T_2^4 + 7 T_1^2 T_2^4 + 4 T_1^5 T_2^4 + 5 T_1^6 T_2^4 + T_1^8 T_2^4 + T_1 T_2^5 - 2 T_1^2 T_2^5 - \\ & 8 T_1^3 T_2^5 + 4 T_1^4 T_2^5 - 2 T_1^5 T_2^5 - 8 T_1^6 T_2^5 + T_1^7 T_2^5 - 2 T_1^8 T_2^5 + 2 T_1^3 T_2^6 + 5 T_1^4 T_2^6 - 8 T_1^5 T_2^6 + 11 T_1^6 T_2^6 - 4 T_1^7 T_2^6 + \\ & 3 T_1^8 T_2^6 - T_1^3 T_2^7 + T_1^5 T_2^7 - 4 T_1^6 T_2^7 + 3 T_1^7 T_2^7 - 2 T_1^8 T_2^7 + T_1^4 T_2^8 - 2 T_1^5 T_2^8 + 3 T_1^6 T_2^8 - 2 T_1^7 T_2^8 + T_1^8 T_2^8 \end{aligned} \right\},$$

$$\text{Knot}[10, 153] \rightarrow \left\{ \frac{(1 - T + T^3)(1 - T^2 + T^3)}{T^3}, \frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} \right.$$

$$\left. \begin{aligned} & (-2 + 2 T_1 + 2 T_1^2 - 6 T_1^3 + 2 T_1^4 + 2 T_1^5 - 2 T_1^6 + 2 T_2 - 3 T_1 T_2 + 3 T_1^3 T_2 - T_1^4 T_2 - T_1^6 T_2 + T_1^7 T_2 + 2 T_2^2 - \\ & 2 T_1^2 T_2^2 + 6 T_1^3 T_2^2 - 2 T_1^5 T_2^2 + 2 T_1^6 T_2^2 + 2 T_1^7 T_2^2 - 6 T_2^3 + 3 T_1 T_2^3 + 6 T_1^2 T_2^3 - 14 T_1^3 T_2^3 - 2 T_1^4 T_2^3 + \\ & 2 T_1^5 T_2^3 + 4 T_1^6 T_2^3 - 5 T_1^7 T_2^3 - 5 T_1^8 T_2^3 + 3 T_1^9 T_2^3 + 2 T_2^4 - T_1 T_2^4 - 2 T_1^3 T_2^4 + 5 T_1^4 T_2^4 + 7 T_1^5 T_2^4 - 11 T_1^6 T_2^4 - \\ & 2 T_1^7 T_2^4 + 10 T_1^8 T_2^4 - 3 T_1^9 T_2^4 - 2 T_1^{10} T_2^4 + 2 T_2^5 - 2 T_1^2 T_2^5 + 2 T_1^3 T_2^5 + 7 T_1^4 T_2^5 - 5 T_1^5 T_2^5 - 7 T_1^6 T_2^5 + \\ & 18 T_1^7 T_2^5 - 10 T_1^8 T_2^5 - 3 T_1^9 T_2^5 + 8 T_1^{10} T_2^5 - 3 T_1^{11} T_2^5 - 2 T_2^6 - T_1 T_2^6 + 2 T_1^2 T_2^6 + 4 T_1^3 T_2^6 - 11 T_1^4 T_2^6 - \\ & 7 T_1^5 T_2^6 + 24 T_1^6 T_2^6 - 13 T_1^7 T_2^6 - 9 T_1^8 T_2^6 + 16 T_1^9 T_2^6 - 6 T_1^{10} T_2^6 - 3 T_1^{11} T_2^6 + 4 T_1^{12} T_2^6 + T_1 T_2^7 + 2 T_1^2 T_2^7 - \\ & 5 T_1^3 T_2^7 - 2 T_1^4 T_2^7 + 18 T_1^5 T_2^7 - 13 T_1^6 T_2^7 - 15 T_1^7 T_2^7 + 29 T_1^8 T_2^7 - 14 T_1^9 T_2^7 - 6 T_1^{10} T_2^7 + 10 T_1^{11} T_2^7 - \\ & 4 T_1^{12} T_2^7 - 5 T_1^3 T_2^8 + 10 T_1^4 T_2^8 - 10 T_1^5 T_2^8 - 9 T_1^6 T_2^8 + 29 T_1^7 T_2^8 - 25 T_1^8 T_2^8 - 10 T_1^9 T_2^8 + 20 T_1^{10} T_2^8 - \\ & 7 T_1^{11} T_2^8 - 4 T_1^{12} T_2^8 + 3 T_1^3 T_2^9 - 3 T_1^4 T_2^9 - 3 T_1^5 T_2^9 + 16 T_1^6 T_2^9 - 14 T_1^7 T_2^9 - 10 T_1^8 T_2^9 + 34 T_1^9 T_2^9 - \\ & 14 T_1^{10} T_2^9 - 11 T_1^{11} T_2^9 + 12 T_1^{12} T_2^9 - 2 T_2^{10} + 8 T_1 T_2^{10} - 6 T_1^2 T_2^{10} - 6 T_1^3 T_2^{10} + 20 T_1^4 T_2^{10} - 14 T_1^5 T_2^{10} - \\ & 2 T_1^{10} T_2^{10} + 10 T_1^{11} T_2^{10} - 4 T_1^{12} T_2^{10} - 3 T_1^5 T_2^{11} - 3 T_1^6 T_2^{11} + 10 T_1^7 T_2^{11} - 7 T_1^8 T_2^{11} - 11 T_1^9 T_2^{11} + 10 T_1^{10} T_2^{11} - \\ & T_1^{11} T_2^{11} - 4 T_1^{12} T_2^{11} + 4 T_1^6 T_2^{12} - 4 T_1^7 T_2^{12} - 4 T_1^8 T_2^{12} + 12 T_1^9 T_2^{12} - 4 T_1^{10} T_2^{12} - 4 T_1^{11} T_2^{12} + 4 T_1^{12} T_2^{12} \end{aligned} \right\},$$

$$\text{Knot}[10, 155] \rightarrow \left\{ -\frac{(-1 + T - 2 T^2 + T^3)(-1 + 2 T - T^2 + T^3)}{T^3}, \right.$$

$$\left. \begin{aligned} & -\frac{1}{T_1^7 T_2^6 (-1 + T_1 T_2)} (-3 + 9 T_1 - 15 T_1^2 + 21 T_1^3 - 15 T_1^4 + 9 T_1^5 - 3 T_1^6 + 9 T_2 - 21 T_1 T_2 + 27 T_1^2 T_2 - 33 T_1^3 T_2 + \\ & 3 T_1^4 T_2 + 3 T_1^5 T_2 - 9 T_1^6 T_2 + 6 T_1^7 T_2 - 15 T_2^2 + 27 T_1 T_2^2 - 28 T_1^2 T_2^2 + 38 T_1^3 T_2^2 + 16 T_1^4 T_2^2 - 2 T_1^5 T_2^2 + \\ & 12 T_1^6 T_2^2 - 3 T_1^7 T_2^2 - 5 T_1^8 T_2^2 + 21 T_2^3 - 33 T_1 T_2^3 + 38 T_1^2 T_2^3 - 64 T_1^3 T_2^3 - 24 T_1^4 T_2^3 - 12 T_1^5 T_2^3 - 16 T_1^6 T_2^3 - \\ & 7 T_1^7 T_2^3 + 15 T_1^8 T_2^3 - 15 T_2^4 + 3 T_1 T_2^4 + 16 T_1^2 T_2^4 - 24 T_1^3 T_2^4 + 15 T_1^4 T_2^4 - 53 T_1^5 T_2^4 + 63 T_1^6 T_2^4 + 4 T_1^7 T_2^4 - \\ & 10 T_1^8 T_2^4 - 15 T_1^9 T_2^4 + 5 T_1^{10} T_2^4 + 9 T_2^5 + 3 T_1 T_2^5 - 2 T_1^2 T_2^5 - 12 T_1^3 T_2^5 - 53 T_1^4 T_2^5 - 91 T_1^5 T_2^5 + 29 T_1^6 T_2^5 - \\ & 48 T_1^7 T_2^5 + 8 T_1^8 T_2^5 + 23 T_1^9 T_2^5 + 3 T_1^{10} T_2^5 - 6 T_1^{11} T_2^5 - 3 T_2^6 - 9 T_1 T_2^6 + 12 T_1^2 T_2^6 - 16 T_1^3 T_2^6 + 63 T_1^4 T_2^6 + \\ & 29 T_1^5 T_2^6 + 60 T_1^6 T_2^6 - 45 T_1^7 T_2^6 + 41 T_1^8 T_2^6 - 28 T_1^9 T_2^6 - 16 T_1^{10} T_2^6 + 9 T_1^{11} T_2^6 + 3 T_1^{12} T_2^6 + 6 T_1 T_2^7 - \\ & 3 T_1^2 T_2^7 - 7 T_1^3 T_2^7 + 4 T_1^4 T_2^7 - 48 T_1^5 T_2^7 - 45 T_1^6 T_2^7 + 75 T_1^7 T_2^7 - 43 T_1^8 T_2^7 + 24 T_1^9 T_2^7 + 18 T_1^{10} T_2^7 - 3 T_1^{11} T_2^7 - \\ & 9 T_1^{12} T_2^7 - 5 T_1^2 T_2^8 + 15 T_1^3 T_2^8 - 10 T_1^4 T_2^8 + 8 T_1^5 T_2^8 + 41 T_1^6 T_2^8 - 43 T_1^7 T_2^8 - 47 T_1^8 T_2^8 + 36 T_1^9 T_2^8 - \\ & 36 T_1^{10} T_2^8 - 3 T_1^{11} T_2^8 + 15 T_1^{12} T_2^8 - 15 T_1^4 T_2^9 + 23 T_1^5 T_2^9 - 28 T_1^6 T_2^9 + 24 T_1^7 T_2^9 + 36 T_1^8 T_2^9 + 20 T_1^9 T_2^9 - \\ & 22 T_1^{10} T_2^9 + 33 T_1^{11} T_2^9 - 21 T_1^{12} T_2^9 + 5 T_1^4 T_2^{10} + 3 T_1^5 T_2^{10} - 16 T_1^6 T_2^{10} + 18 T_1^7 T_2^{10} - 36 T_1^8 T_2^{10} - 22 T_1^9 T_2^{10} + \\ & 24 T_1^{10} T_2^{10} - 27 T_1^{11} T_2^{10} + 15 T_1^{12} T_2^{10} - 6 T_1^5 T_2^{11} + 9 T_1^6 T_2^{11} - 3 T_1^7 T_2^{11} - 3 T_1^8 T_2^{11} + 33 T_1^9 T_2^{11} - 27 T_1^{10} T_2^{11} + \\ & 21 T_1^{11} T_2^{11} - 9 T_1^{12} T_2^{11} + 3 T_1^6 T_2^{12} - 9 T_1^7 T_2^{12} + 15 T_1^8 T_2^{12} - 21 T_1^9 T_2^{12} + 15 T_1^{10} T_2^{12} - 9 T_1^{11} T_2^{12} + 3 T_1^{12} T_2^{12} \end{aligned} \right\}$$

In[*]:= `DunfieldKnots = ReadList["../People/Dunfield/nmd_random_knots"] /. k_Integer => k + 1;`

In[*]:= `(Crossings/@DunfieldKnots) == Range[3, 1000]`

Out[*]=

True

In[*]:= `DK[n_] := DunfieldKnots[[n - 2]]`

In[*]:= `Crossings[DK[576]]`

Out[*]=

576

```

In[*]:= AbsoluteTiming[ $\theta$ [DK[3]]]
Out[*]=
{0.0625038,
 $\left\{ \frac{1 - T + T^2}{T}, \frac{-2 + 2 T_1 - 2 T_1^2 + 2 T_2 - T_1 T_2 + T_1^2 T_2 + T_1^3 T_2 - 2 T_2^2 + T_1 T_2^2 - T_1^3 T_2^2 + T_1 T_2^3 - T_1^2 T_2^3 + T_1^3 T_2^3}{T_1^3 T_2^2 (-1 + T_1 T_2)} \right\}$ }

In[*]:= AbsoluteTiming[ $\theta$ [DK[30]]];]
Out[*]=
{4.88079, Null}

In[*]:= AbsoluteTiming[ $\theta$ [DK[60]]];]
Out[*]=
{44.9012, Null}

In[*]:= AbsoluteTiming[ $\theta$ [DK[90]]];]
Out[*]=
{314.553, Null}

In[*]:= AbsoluteTiming[ $\theta$ 120 =  $\theta$ [DK[120]]];]
Out[*]=
{761.428, Null}

In[*]:= Put[ $\theta$ 120, "Theta4DK120.m"]

In[*]:= AbsoluteTiming[ $\theta$ [DK[150]]];]
Out[*]=
{2357.39, Null}

(during the previous computation I biked home, so the AbsoluteTiming is too much)

In[*]:= AbsoluteTiming[ $\theta$ [DK[180]]];]
Out[*]=
{5391.24, Null}

In[*]:= AbsoluteTiming[ $\theta$ [DK[210]]];]
Out[*]=
{9613.68, Null}

In[*]:= AbsoluteTiming[ $\theta$ [DK[240]]];]
Out[*]=
{22462.4, Null}

In[*]:= AbsoluteTiming[ $\theta$ [DK[270]]];]

Mathematica crashed while trying the above computation.

In[*]:= AbsoluteTiming[ $\theta$ [DK[300]]];]

```