In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`];
<< Rot.m
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.

In[ ]:=
```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | π | g)__, ∞] ⋃ {x, p, ϵ}, ps, c},
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) ↦ CCF[c] (Times @@ vs^ps)]];
```

$In[\circ]:=$
```
{
    {r₀,pxx[1, i_, j_], r₀,pxx[-1, i_, j_]},
    {r₁,ppx[1, i_, j_], r₁,ppx[-1, i_, j_]},
    {r₁,rest[1, i_, j_], r₁,rest[-1, i_, j_]},
    γ₁[φ_, k_]
} = CF[{1, 1 - T₁ T₂, 1 - T₁ T₂, 1 - T₁ T₂} * Get["px-data.m"]]
```

$Out[\circ]=$

$$\left\{ \left\{ p_{3,j}\, x_{1,i}\, x_{2,i} - \frac{p_{3,j}\, x_{1,j}\, x_{2,i}}{T_1}, \; -\frac{p_{3,j}\, x_{1,i}\, x_{2,i}}{T_1^2\, T_2} + \frac{p_{3,j}\, x_{1,j}\, x_{2,i}}{T_1\, T_2} \right\}, \right.$$

$$\left\{ (1 - T_1\, T_2)\, p_{1,j}\, p_{2,i}\, x_{3,i} + (-1 + T_1\, T_2)\, p_{1,j}\, p_{2,j}\, x_{3,i}, \right.$$

$$\left. \frac{(-1 + T_1\, T_2)\, p_{1,j}\, p_{2,i}\, x_{3,i}}{T_1} - \frac{(-1 + T_1\, T_2)\, p_{1,j}\, p_{2,j}\, x_{3,i}}{T_1} \right\},$$

$$\left\{ -T_2\, p_{1,j}\, p_{2,j}\, x_{1,i}\, x_{2,i} + \frac{p_{1,j}\, p_{2,i}\, x_{1,j}\, x_{2,i}}{(-1 + T_1)\, T_1} + \frac{(-1 + T_1\, T_2)\, p_{1,j}\, p_{2,j}\, x_{1,j}\, x_{2,i}}{(-1 + T_1)\, T_1} - \frac{p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,j}}{-1 + T_1} - \right.$$

$$\frac{p_{3,j}\, x_{3,i}}{T_1} + (-1 + T_1\, T_2)\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,i} - \frac{p_{1,j}\, p_{3,i}\, x_{1,j}\, x_{3,i}}{(-1 + T_1)\, T_1} - \frac{(-1 + T_1\, T_2)\, p_{1,j}\, p_{3,j}\, x_{1,j}\, x_{3,i}}{-1 + T_1} +$$

$$\frac{T_2\, (-1 + T_1\, T_2)\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,i}}{T_1} + \frac{p_{2,j}\, p_{3,i}\, x_{2,j}\, x_{3,i}}{T_1} + \frac{p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{-1 + T_1} - \frac{T_2\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,j}}{T_1},$$

$$-\frac{p_{1,j}\, p_{2,i}\, x_{1,i}\, x_{2,i}}{T_1^2} + \frac{(-1 + T_2)\, p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,i}}{(-1 + T_1)\, T_2} - \frac{(-T_1 - T_2 + T_1\, T_2)\, p_{1,j}\, p_{2,j}\, x_{1,i}\, x_{2,i}}{T_1^2\, T_2} -$$

$$\frac{p_{1,j}\, p_{2,i}\, x_{1,j}\, x_{2,i}}{(-1 + T_1)\, T_1} - \frac{p_{1,j}\, p_{2,j}\, x_{1,j}\, x_{2,i}}{T_1} + \frac{p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,j}}{-1 + T_1} - \frac{p_{1,j}\, p_{2,j}\, x_{1,i}\, x_{2,j}}{T_1} + \frac{p_{3,j}\, x_{3,i}}{T_1} +$$

$$\frac{p_{1,j}\, p_{3,i}\, x_{1,i}\, x_{3,i}}{T_1^2} - \frac{(-1 + T_1\, T_2)\, p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,i}}{(-1 + T_1)\, T_1\, T_2} + \frac{(-1 + T_1\, T_2)\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,i}}{T_1^2\, T_2} +$$

$$\frac{p_{1,j}\, p_{3,i}\, x_{1,j}\, x_{3,i}}{(-1 + T_1)\, T_1} - \frac{(-1 + T_2)\, p_{2,j}\, p_{3,i}\, x_{2,i}\, x_{3,i}}{T_1\, T_2} - \frac{(-1 + T_1\, T_2)\, p_{2,i}\, p_{3,j}\, x_{2,i}\, x_{3,i}}{T_1^2\, T_2} +$$

$$\frac{(-1 + 2\, T_2)\, (-1 + T_1\, T_2)\, p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,i}}{T_1^2\, T_2^2} - \frac{p_{2,j}\, p_{3,i}\, x_{2,j}\, x_{3,i}}{T_1} + \frac{(-1 + T_1\, T_2)\, p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,i}}{T_1^2\, T_2} -$$

$$\left. \left. \frac{p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{-1 + T_1} + \frac{p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{T_1} + \frac{p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,j}}{T_1} \right\}, \; \frac{\varphi\, p_{3,k}\, x_{3,k}}{T_1} \right\}$$

$In[\circ]:=$ **r₁,rest[1, 4, 5]**

$Out[\circ]=$

$$-T_2\, p_{1,5}\, p_{2,5}\, x_{1,4}\, x_{2,4} + \frac{p_{1,5}\, p_{2,4}\, x_{1,5}\, x_{2,4}}{(-1 + T_1)\, T_1} + \frac{(-1 + T_1\, T_2)\, p_{1,5}\, p_{2,5}\, x_{1,5}\, x_{2,4}}{(-1 + T_1)\, T_1} - \frac{p_{1,4}\, p_{2,5}\, x_{1,4}\, x_{2,5}}{-1 + T_1} -$$

$$\frac{p_{3,5}\, x_{3,4}}{T_1} + (-1 + T_1\, T_2)\, p_{1,5}\, p_{3,5}\, x_{1,4}\, x_{3,4} - \frac{p_{1,5}\, p_{3,4}\, x_{1,5}\, x_{3,4}}{(-1 + T_1)\, T_1} - \frac{(-1 + T_1\, T_2)\, p_{1,5}\, p_{3,5}\, x_{1,5}\, x_{3,4}}{-1 + T_1} +$$

$$\frac{T_2\, (-1 + T_1\, T_2)\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,4}}{T_1} + \frac{p_{2,5}\, p_{3,4}\, x_{2,5}\, x_{3,4}}{T_1} + \frac{p_{1,4}\, p_{3,5}\, x_{1,4}\, x_{3,5}}{-1 + T_1} - \frac{T_2\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,5}}{T_1}$$

$In[\circ]:=$ **{p*, x*, π*, ξ*} = {π, ξ, p, x}; (u₋ᵢ₋₋)* := (u*)ᵢ;**

$In[\circ]:=$
```
Zip_{}[ℰ_] := ℰ;
Zip_{ζ_,ζs___}[ℰ_] := (Collect[ℰ // Zip_{ζs}, ζ] /. f_. ζ^d_. ⧴ (D[f, {ζ*, d}])) /. ζ* → 0
```

$In[\circ]:=$
```
px2g[ℰ_] := CF@Module[{ps, xs, Q},
    ps = Union[Cases[ℰ, p__, ∞]]; xs = Union[Cases[ℰ, x__, ∞]];
    Q = Sum[p0* x0* g_{p0[[2]],x0[[2]],p0[[3]],x0[[3]]}, {p0, ps}, {x0, xs}];
    Expand[Zip_{ps⋃xs}[ℰ e^Q] /. g_{α_,β_,i_,j_} ⧴ If[α === β, g_{α,i,j}, 0]]
]
```

$In[\circ]:=$ $\mathbf{px2g}\left[\, p_{2,j}^2\, x_{2,i}\, x_{2,j}\right]$

$Out[\circ]=$

$2\, g_{2,j,i}\, g_{2,j,j}$

$In[\circ]:=$ $R_1[\mathbf{1}, i\_, j\_] = \mathbf{px2g}[r_{1,\text{rest}}[\mathbf{1}, i, j]]$

$Out[\circ]=$

$$\frac{g_{1,j,j}\, g_{2,i,i}}{(-1 + T_1)\, T_1} - T_2\, g_{1,j,i}\, g_{2,j,i} + \frac{(-1 + T_1\, T_2)\, g_{1,j,j}\, g_{2,j,i}}{(-1 + T_1)\, T_1} -$$
$$\frac{g_{1,i,i}\, g_{2,j,j}}{-1 + T_1} - \frac{g_{1,j,j}\, g_{3,i,i}}{(-1 + T_1)\, T_1} + \frac{g_{2,j,j}\, g_{3,i,i}}{T_1} - \frac{g_{3,j,i}}{T_1} + (-1 + T_1\, T_2)\, g_{1,j,i}\, g_{3,j,i} -$$
$$\frac{(-1 + T_1\, T_2)\, g_{1,j,j}\, g_{3,j,i}}{-1 + T_1} + \frac{T_2\, (-1 + T_1\, T_2)\, g_{2,j,i}\, g_{3,j,i}}{T_1} + \frac{g_{1,i,i}\, g_{3,j,j}}{-1 + T_1} - \frac{T_2\, g_{2,j,i}\, g_{3,j,j}}{T_1}$$

$In[\circ]:=$ $R_1[-\mathbf{1}, i\_, j\_] = \mathbf{px2g}[r_{1,\text{rest}}[-\mathbf{1}, i, j]]$

$Out[\circ]=$

$$-\frac{g_{1,j,i}\, g_{2,i,i}}{T_1^2} - \frac{g_{1,j,j}\, g_{2,i,i}}{(-1 + T_1)\, T_1} + \frac{(-1 + T_2)\, g_{1,i,i}\, g_{2,j,i}}{(-1 + T_1)\, T_2} - \frac{(-T_1 - T_2 + T_1\, T_2)\, g_{1,j,i}\, g_{2,j,i}}{T_1^2\, T_2} -$$
$$\frac{g_{1,j,j}\, g_{2,j,i}}{T_1} + \frac{g_{1,i,i}\, g_{2,j,j}}{-1 + T_1} - \frac{g_{1,j,i}\, g_{2,j,j}}{T_1} + \frac{g_{1,j,i}\, g_{3,i,i}}{T_1^2} + \frac{g_{1,j,j}\, g_{3,i,i}}{(-1 + T_1)\, T_1} -$$
$$\frac{(-1 + T_2)\, g_{2,j,i}\, g_{3,i,i}}{T_1\, T_2} - \frac{g_{2,j,j}\, g_{3,i,i}}{T_1} + \frac{g_{3,j,i}}{T_1} - \frac{(-1 + T_1\, T_2)\, g_{1,i,i}\, g_{3,j,i}}{(-1 + T_1)\, T_1\, T_2} +$$
$$\frac{(-1 + T_1\, T_2)\, g_{1,j,i}\, g_{3,j,i}}{T_1^2\, T_2} - \frac{(-1 + T_1\, T_2)\, g_{2,i,i}\, g_{3,j,i}}{T_1^2\, T_2} + \frac{(-1 + 2\, T_2)\, (-1 + T_1\, T_2)\, g_{2,j,i}\, g_{3,j,i}}{T_1^2\, T_2^2} +$$
$$\frac{(-1 + T_1\, T_2)\, g_{2,j,j}\, g_{3,j,i}}{T_1^2\, T_2} - \frac{g_{1,i,i}\, g_{3,j,j}}{-1 + T_1} + \frac{g_{1,j,i}\, g_{3,j,j}}{T_1} + \frac{g_{2,j,i}\, g_{3,j,j}}{T_1}$$

$In[\circ]:=$ $\mathbf{px2g}[r_{0,\text{pxx}}[\mathbf{1}, \text{i0}, \text{j0}]\, r_{1,\text{ppx}}[\mathbf{1}, \text{i1}, \text{j1}]]$

$Out[\circ]=$

$$(1 - T_1\, T_2)\, g_{1,j1,i0}\, g_{2,i1,i0}\, g_{3,j0,i1} + \frac{(-1 + T_1\, T_2)\, g_{1,j1,j0}\, g_{2,i1,i0}\, g_{3,j0,i1}}{T_1} +$$
$$(-1 + T_1\, T_2)\, g_{1,j1,i0}\, g_{2,j1,i0}\, g_{3,j0,i1} - \frac{(-1 + T_1\, T_2)\, g_{1,j1,j0}\, g_{2,j1,i0}\, g_{3,j0,i1}}{T_1}$$

$In[\circ]:=$ $\theta[\{1, i0\_, j0\_\}, \{1, i1\_, j1\_\}] = \text{px2g}[r_{0,\text{pxx}}[1, i0, j0]\ r_{1,\text{ppx}}[1, i1, j1]]$

$\theta[\{1, i0\_, j0\_\}, \{-1, i1\_, j1\_\}] = \text{px2g}[r_{0,\text{pxx}}[1, i0, j0]\ r_{1,\text{ppx}}[-1, i1, j1]]$

$\theta[\{-1, i0\_, j0\_\}, \{1, i1\_, j1\_\}] = \text{px2g}[r_{0,\text{pxx}}[-1, i0, j0]\ r_{1,\text{ppx}}[1, i1, j1]]$

$\theta[\{-1, i0\_, j0\_\}, \{-1, i1\_, j1\_\}] = \text{px2g}[r_{0,\text{pxx}}[-1, i0, j0]\ r_{1,\text{ppx}}[-1, i1, j1]]$

$Out[\circ]=$

$$(1 - T_1 T_2)\ g_{1,j1,i0}\ g_{2,i1,i0}\ g_{3,j0,i1} + \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1} +$$
$$(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,j1,i0}\ g_{3,j0,i1} - \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1}$$

$Out[\circ]=$

$$\frac{(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1} - \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1^2} -$$
$$\frac{(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1} + \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1^2}$$

$Out[\circ]=$

$$\frac{(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1^2\ T_2} - \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1\ T_2} -$$
$$\frac{(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1^2\ T_2} + \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1\ T_2}$$

$Out[\circ]=$

$$-\frac{(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1^3\ T_2} + \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,i1,i0}\ g_{3,j0,i1}}{T_1^2\ T_2} +$$
$$\frac{(-1 + T_1 T_2)\ g_{1,j1,i0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1^3\ T_2} - \frac{(-1 + T_1 T_2)\ g_{1,j1,j0}\ g_{2,j1,i0}\ g_{3,j0,i1}}{T_1^2\ T_2}$$

$In[\circ]:=$ $\Gamma_1[\varphi\_, k\_] = \text{px2g}[\gamma_1[\varphi, k]]$

$Out[\circ]=$

$$\frac{\varphi\ g_{3,k,k}}{T_1}$$

```mathematica
In[◦]:=  T₃ = T₁ T₂;
         θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, z},
            {Cs, φ} = Rot[K]; n = Length[Cs];
            A = IdentityMatrix[2 n + 1];
            Cases[Cs, {s_, i_, j_} ⧴ (A⟦{i, j}, {i + 1, j + 1}⟧ += ( -Tˢ  Tˢ - 1 ))];
                                                                    (  0   -1   )
            Δ = T^(-Total[φ]-Total[Cs⟦All,1⟧])/2 Det[A];
            G = Inverse[A]; gEval[ℰ_] := CCF[ℰ /. g_{ν_,α_,β_} ⧴ (G⟦α, β⟧ /. T → T_ν)];
            z = gEval[∑ₖ₁₌₁ⁿ ∑ₖ₂₌₁ⁿ θ[Cs⟦k1⟧, Cs⟦k2⟧]];
            z += gEval[∑ₖ₌₁ⁿ R₁ @@ Cs⟦k⟧];
            z += gEval[∑ₖ₌₁²ⁿ Γ₁[φ⟦k⟧, k]];
            {Δ, (Δ /. T → T₁) (Δ /. T → T₂) (Δ /. T → T₃) z} // CCF
         ];
```

```mathematica
In[◦]:=  θ_{T1_,T2_}[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, z = 0},
            {Cs, φ} = Rot[K]; n = Length[Cs];
            temp = PrintTemporary["At work, n=", n];
            A = IdentityMatrix[2 n + 1];
            Cases[Cs, {s_, i_, j_} ⧴ (A⟦{i, j}, {i + 1, j + 1}⟧ += ( -Tˢ  Tˢ - 1 ))];
                                                                    (  0   -1   )
            Δ[0] := Δ[0] = T^(-Total[φ]-Total[Cs⟦All,1⟧])/2 Det[A];
            G[0] := G[0] = Inverse[A];
            {Δ[1], G[1]} = If[NumberQ@T1,
               {Det[A /. T → T1], Inverse[A /. T → T1]}, {Δ[0], G[0]} /. T → T1];
            temp = PrintTemporary@"Done with {Δ[1],G[1]}.";
            {Δ[2], G[2]} = If[NumberQ@T2,
               {Det[A /. T → T2], Inverse[A /. T → T2]}, {Δ[0], G[0]} /. T → T2];
            NotebookDelete[temp]; temp = PrintTemporary@"Done with {Δ[2],G[2]}.";
            {Δ[3], G[3]} = If[NumberQ[T1 T2],
               {Det[A /. T → T1 T2], Inverse[A /. T → T1 T2]}, {Δ[0], G[0]} /. T → T1 T2];
            NotebookDelete[temp]; temp = PrintTemporary@"Done with {Δ[3],G[3]}.";
            gEval[ℰ_] := CCF[ℰ //. {T₁ → T1, T₂ → T2, g_{ν_,α_,β_} ⧴ G[ν]⟦α, β⟧}];
            Do[z += gEval[θ[Cs⟦k1⟧, Cs⟦k2⟧]], {k1, n}, {k2, n}];
            Do[z += gEval[R₁ @@ Cs⟦k⟧], {k, n}];
            Do[z += gEval[Γ₁[φ⟦k⟧, k]], {k, 2 n}];
            NotebookDelete[temp];
            {Δ[1], Δ[2], Δ[3], Δ[1] × Δ[2] × Δ[3] z} // CCF
         ];
```

$In[\circ]:=$ **Timing[θ[Knot[3, 1]]]**

$Out[\circ]=$

$$\left\{0.015625, \left\{\frac{1 - T + T^2}{T}, -\frac{-1 + T_1 - T_1^2 + T_2 - T_1^2 T_2 + 2 T_1^3 T_2 - T_2^2 - T_1 T_2^2 + T_1^2 T_2^2 - 2 T_1^3 T_2^2 + 2 T_1 T_2^3 - 2 T_1^2 T_2^3 + 2 T_1^3 T_2^3}{T_1^2 T_2}\right\}\right\}$$

$In[\circ]:=$ **Timing[θ$_{T_1, T_2}$[Knot[3, 1]]]**

$Out[\circ]=$

$$\left\{0., \left\{\frac{1 - T_1 + T_1^2}{T_1}, \frac{1 - T_2 + T_2^2}{T_2}, \frac{1 - T_1 T_2 + T_1^2 T_2^2}{T_1 T_2}, \right.\right.$$
$$\left.\left. -\frac{-1 + T_1 - T_1^2 + T_2 - T_1^2 T_2 + 2 T_1^3 T_2 - T_2^2 - T_1 T_2^2 + T_1^2 T_2^2 - 2 T_1^3 T_2^2 + 2 T_1 T_2^3 - 2 T_1^2 T_2^3 + 2 T_1^3 T_2^3}{T_1^2 T_2}\right\}\right\}$$

$In[\circ]:=$ **Timing[θ[Knot[4, 1]]]**

$Out[\circ]=$

$$\left\{0., \left\{-\frac{1 - 3 T + T^2}{T}, \frac{\left(1 - 3 T_1 + T_1^2\right) \left(-1 + T_1 T_2\right) \left(1 + T_1 T_2\right) \left(1 - 3 T_2 + T_2^2\right)}{T_1^3 T_2^2}\right\}\right\}$$

$In[\circ]:=$ **Timing[θ[Knot["K11n34"]]]**

$Out[\circ]=$

$$\left\{0.03125, \right.$$
$$\left\{1, -\frac{1}{T_1^7 T_2^6} \left(T_1^2 - 2 T_1^3 + T_1^4 - 2 T_1 T_2 + 2 T_1^2 T_2 + 2 T_1^5 T_2 - 2 T_1^6 T_2 + T_2^2 + 2 T_1 T_2^2 - 2 T_1^2 T_2^2 - 2 T_1^4 T_2^2 - 2 T_1^6 T_2^2 + \right.$$
$$2 T_1^7 T_2^2 + T_1^8 T_2^2 - 2 T_2^3 + T_1^4 T_2^3 + T_1^5 T_2^3 - 2 T_1^9 T_2^3 + T_2^4 - 2 T_1^2 T_2^4 + T_1^3 T_2^4 + 2 T_1^4 T_2^4 + 2 T_1^6 T_2^4 + T_1^7 T_2^4 -$$
$$2 T_1^8 T_2^4 + T_1^{10} T_2^4 + 2 T_1 T_2^5 + T_1^3 T_2^5 - 4 T_1^5 T_2^5 - 4 T_1^6 T_2^5 + T_1^8 T_2^5 + 2 T_1^{10} T_2^5 - 2 T_1 T_2^6 - 2 T_1^2 T_2^6 +$$
$$2 T_1^4 T_2^6 - 4 T_1^5 T_2^6 + 12 T_1^6 T_2^6 - 4 T_1^7 T_2^6 + 2 T_1^8 T_2^6 - 2 T_1^{10} T_2^6 - 2 T_1^{11} T_2^6 + 2 T_1^2 T_2^7 + T_1^4 T_2^7 - 4 T_1^6 T_2^7 -$$
$$4 T_1^7 T_2^7 + T_1^9 T_2^7 + 2 T_1^{11} T_2^7 + T_1^2 T_2^8 - 2 T_1^4 T_2^8 + T_1^5 T_2^8 + 2 T_1^6 T_2^8 + 2 T_1^8 T_2^8 + T_1^9 T_2^8 - 2 T_1^{10} T_2^8 +$$
$$T_1^{12} T_2^8 - 2 T_1^3 T_2^9 + T_1^7 T_2^9 + T_1^8 T_2^9 - 2 T_1^{12} T_2^9 + T_1^4 T_2^{10} + 2 T_1^5 T_2^{10} - 2 T_1^6 T_2^{10} - 2 T_1^8 T_2^{10} - 2 T_1^{10} T_2^{10} +$$
$$\left.\left. 2 T_1^{11} T_2^{10} + T_1^{12} T_2^{10} - 2 T_1^6 T_2^{11} + 2 T_1^7 T_2^{11} + 2 T_1^{10} T_2^{11} - 2 T_1^{11} T_2^{11} + T_1^8 T_2^{12} - 2 T_1^9 T_2^{12} + T_1^{10} T_2^{12}\right)\right\}\right\}$$

$In[\circ]:=$ **Timing[θ[Knot["K11n42"]]]**

$Out[\circ]=$

$$\left\{0., \right.$$
$$\left\{1, \frac{1}{T_1^4 T_2^3} \left(T_1 + T_1^2 + T_2 - 2 T_1 T_2 - 2 T_1^2 T_2 - 2 T_1^3 T_2 + T_1^4 T_2 + T_2^2 - 2 T_1 T_2^2 + 2 T_1^2 T_2^2 + 2 T_1^3 T_2^2 - 2 T_1^4 T_2^2 + T_1^5 T_2^2 - \right.$$
$$2 T_1 T_2^3 + 2 T_1^2 T_2^3 + 2 T_1^4 T_2^3 - 2 T_1^5 T_2^3 + T_1 T_2^4 - 2 T_1^2 T_2^4 + 2 T_1^3 T_2^4 + 2 T_1^4 T_2^4 -$$
$$\left.\left. 2 T_1^5 T_2^4 + T_1^6 T_2^4 + T_1^2 T_2^5 - 2 T_1^3 T_2^5 - 2 T_1^4 T_2^5 - 2 T_1^5 T_2^5 + T_1^6 T_2^5 + T_1^4 T_2^6 + T_1^5 T_2^6\right)\right\}\right\}$$

```
In[ ]:= PD[GST48] = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
          X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
          X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
          X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
          X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
          X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
          X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
          X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
          X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
          X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
          X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
          X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];

In[ ]:= AbsoluteTiming[gst48 = θ[GST48]]
```

$Out[ ]=$

$$\left\{11.4869, \left\{-\frac{\left(-1 + 2\,T - T^2 - T^3 + 2\,T^4 - T^5 + T^8\right)\left(-1 + T^3 - 2\,T^4 + T^5 + T^6 - 2\,T^7 + T^8\right)}{T^8}\right., \right.$$

$\dfrac{1}{T_1^{21}\,T_2^{20}}\left(T_1^5 - 3\,T_1^6 + 4\,T_1^7 - 2\,T_1^8 - 2\,T_1^9 + 4\,T_1^{10} - 2\,T_1^{11} - 2\,T_1^{12} + 4\,T_1^{13} - 3\,T_1^{14} + T_1^{15} - 3\,T_1^5\,T_2 + 6\,T_1^6\,T_2 - 3\,T_1^7\,T_2 - \right.$

$6\,T_1^8\,T_2 + 12\,T_1^9\,T_2 - 6\,T_1^{10}\,T_2 - 6\,T_1^{11}\,T_2 + 12\,T_1^{12}\,T_2 - 6\,T_1^{13}\,T_2 - 3\,T_1^{14}\,T_2 + 6\,T_1^{15}\,T_2 - 3\,T_1^{16}\,T_2 - T_1^3\,T_2^2 +$

$3\,T_1^4\,T_2^2 - T_1^6\,T_2^2 - 4\,T_1^7\,T_2^2 + 9\,T_1^8\,T_2^2 - 7\,T_1^9\,T_2^2 - 3\,T_1^{10}\,T_2^2 + 8\,T_1^{11}\,T_2^2 - 3\,T_1^{12}\,T_2^2 - 7\,T_1^{13}\,T_2^2 + 9\,T_1^{14}\,T_2^2 -$

$4\,T_1^{15}\,T_2^2 - T_1^{16}\,T_2^2 + 3\,T_1^{18}\,T_2^2 - T_1^{19}\,T_2^2 - T_1^2\,T_2^3 + 6\,T_1^3\,T_2^3 - 10\,T_1^4\,T_2^3 + 3\,T_1^5\,T_2^3 + 2\,T_1^6\,T_2^3 - 3\,T_1^7\,T_2^3 + 4\,T_1^8\,T_2^3 -$

$2\,T_1^9\,T_2^3 + 2\,T_1^{10}\,T_2^3 - T_1^{11}\,T_2^3 - T_1^{12}\,T_2^3 + 2\,T_1^{13}\,T_2^3 - 2\,T_1^{14}\,T_2^3 + 4\,T_1^{15}\,T_2^3 - 3\,T_1^{16}\,T_2^3 + 2\,T_1^{17}\,T_2^3 + 3\,T_1^{18}\,T_2^3 -$

$10\,T_1^{19}\,T_2^3 + 6\,T_1^{20}\,T_2^3 - T_1^{21}\,T_2^3 + 3\,T_1^2\,T_2^4 - 10\,T_1^3\,T_2^4 + 3\,T_1^4\,T_2^4 + 17\,T_1^5\,T_2^4 - 19\,T_1^6\,T_2^4 + 10\,T_1^7\,T_2^4 - 7\,T_1^8\,T_2^4 +$

$6\,T_1^9\,T_2^4 - T_1^{10}\,T_2^4 - 18\,T_1^{11}\,T_2^4 + 35\,T_1^{12}\,T_2^4 - 18\,T_1^{13}\,T_2^4 - T_1^{14}\,T_2^4 + 6\,T_1^{15}\,T_2^4 - 7\,T_1^{16}\,T_2^4 + 10\,T_1^{17}\,T_2^4 -$

$19\,T_1^{18}\,T_2^4 + 17\,T_1^{19}\,T_2^4 + 3\,T_1^{20}\,T_2^4 - 10\,T_1^{21}\,T_2^4 + 3\,T_1^{22}\,T_2^4 + T_2^5 - 3\,T_1\,T_2^5 + 3\,T_1^3\,T_2^5 + 17\,T_1^4\,T_2^5 - 38\,T_1^5\,T_2^5 +$

$22\,T_1^6\,T_2^5 + 7\,T_1^7\,T_2^5 - 11\,T_1^8\,T_2^5 - 6\,T_1^9\,T_2^5 + 14\,T_1^{10}\,T_2^5 + 11\,T_1^{11}\,T_2^5 - 31\,T_1^{12}\,T_2^5 + 9\,T_1^{13}\,T_2^5 + T_1^{14}\,T_2^5 + 8\,T_1^{16}\,T_2^5 -$

$15\,T_1^{17}\,T_2^5 + 9\,T_1^{18}\,T_2^5 + 16\,T_1^{19}\,T_2^5 - 32\,T_1^{20}\,T_2^5 + 15\,T_1^{21}\,T_2^5 + 3\,T_1^{22}\,T_2^5 - 3\,T_1^{24}\,T_2^5 + T_1^{25}\,T_2^5 - 3\,T_2^6 + 6\,T_1\,T_2^6 -$

$T_1^2\,T_2^6 + 2\,T_1^3\,T_2^6 - 19\,T_1^4\,T_2^6 + 22\,T_1^5\,T_2^6 + 24\,T_1^6\,T_2^6 - 68\,T_1^7\,T_2^6 + 43\,T_1^8\,T_2^6 + 9\,T_1^9\,T_2^6 - 29\,T_1^{10}\,T_2^6 + 2\,T_1^{11}\,T_2^6 -$

$12\,T_1^{12}\,T_2^6 + 28\,T_1^{13}\,T_2^6 - 42\,T_1^{14}\,T_2^6 + 26\,T_1^{15}\,T_2^6 - 29\,T_1^{16}\,T_2^6 - T_1^{17}\,T_2^6 + 45\,T_1^{18}\,T_2^6 - 64\,T_1^{19}\,T_2^6 + 24\,T_1^{20}\,T_2^6 +$

$18\,T_1^{21}\,T_2^6 - 17\,T_1^{22}\,T_2^6 + 2\,T_1^{23}\,T_2^6 - T_1^{24}\,T_2^6 + 6\,T_1^{25}\,T_2^6 - 3\,T_1^{26}\,T_2^6 + 4\,T_2^7 - 3\,T_1\,T_2^7 - 4\,T_1^2\,T_2^7 - 3\,T_1^3\,T_2^7 +$

$10\,T_1^4\,T_2^7 + 7\,T_1^5\,T_2^7 - 68\,T_1^6\,T_2^7 + 74\,T_1^7\,T_2^7 + 14\,T_1^8\,T_2^7 - 56\,T_1^9\,T_2^7 + 14\,T_1^{10}\,T_2^7 + 55\,T_1^{11}\,T_2^7 - 23\,T_1^{12}\,T_2^7 +$

$11\,T_1^{13}\,T_2^7 + 51\,T_1^{14}\,T_2^7 - 33\,T_1^{15}\,T_2^7 + 41\,T_1^{16}\,T_2^7 + 28\,T_1^{17}\,T_2^7 - 60\,T_1^{18}\,T_2^7 + 16\,T_1^{19}\,T_2^7 + 68\,T_1^{20}\,T_2^7 -$

$62\,T_1^{21}\,T_2^7 + 5\,T_1^{22}\,T_2^7 + 10\,T_1^{23}\,T_2^7 - 3\,T_1^{24}\,T_2^7 - 4\,T_1^{25}\,T_2^7 - 3\,T_1^{26}\,T_2^7 + 4\,T_1^{27}\,T_2^7 - 2\,T_2^8 - 6\,T_1\,T_2^8 + 9\,T_1^2\,T_2^8 +$

$4\,T_1^3\,T_2^8 - 7\,T_1^4\,T_2^8 - 11\,T_1^5\,T_2^8 + 43\,T_1^6\,T_2^8 + 14\,T_1^7\,T_2^8 - 123\,T_1^8\,T_2^8 + 133\,T_1^9\,T_2^8 - 36\,T_1^{10}\,T_2^8 - 89\,T_1^{11}\,T_2^8 +$

$136\,T_1^{12}\,T_2^8 - 127\,T_1^{13}\,T_2^8 + 31\,T_1^{14}\,T_2^8 - 31\,T_1^{15}\,T_2^8 + 16\,T_1^{16}\,T_2^8 - 33\,T_1^{17}\,T_2^8 - 28\,T_1^{18}\,T_2^8 + 109\,T_1^{19}\,T_2^8 -$

$115\,T_1^{20}\,T_2^8 + 14\,T_1^{21}\,T_2^8 + 51\,T_1^{22}\,T_2^8 - 27\,T_1^{23}\,T_2^8 + T_1^{24}\,T_2^8 + 4\,T_1^{25}\,T_2^8 + 9\,T_1^{26}\,T_2^8 - 6\,T_1^{27}\,T_2^8 - 2\,T_1^{28}\,T_2^8 -$

$2\,T_2^9 + 12\,T_1\,T_2^9 - 7\,T_1^2\,T_2^9 - 2\,T_1^3\,T_2^9 + 6\,T_1^4\,T_2^9 - 6\,T_1^5\,T_2^9 + 9\,T_1^6\,T_2^9 - 56\,T_1^7\,T_2^9 + 133\,T_1^8\,T_2^9 - 149\,T_1^9\,T_2^9 -$

$10\,T_1^{10}\,T_2^9 + 224\,T_1^{11}\,T_2^9 - 314\,T_1^{12}\,T_2^9 + 67\,T_1^{13}\,T_2^9 + 111\,T_1^{14}\,T_2^9 - 124\,T_1^{15}\,T_2^9 + 38\,T_1^{16}\,T_2^9 - 49\,T_1^{17}\,T_2^9 +$

$50\,T_1^{18}\,T_2^9 - 38\,T_1^{19}\,T_2^9 - 47\,T_1^{20}\,T_2^9 + 95\,T_1^{21}\,T_2^9 - 68\,T_1^{22}\,T_2^9 + 8\,T_1^{23}\,T_2^9 + 32\,T_1^{24}\,T_2^9 - 19\,T_1^{25}\,T_2^9 - 2\,T_1^{26}\,T_2^9 -$

$7\,T_1^{27}\,T_2^9 + 12\,T_1^{28}\,T_2^9 - 2\,T_1^{29}\,T_2^9 + 4\,T_2^{10} - 6\,T_1\,T_2^{10} - 3\,T_1^2\,T_2^{10} + 2\,T_1^3\,T_2^{10} - T_1^4\,T_2^{10} + 14\,T_1^5\,T_2^{10} - 29\,T_1^6\,T_2^{10} +$

$14\,T_1^7\,T_2^{10} - 36\,T_1^8\,T_2^{10} - 10\,T_1^9\,T_2^{10} + 240\,T_1^{10}\,T_2^{10} - 314\,T_1^{11}\,T_2^{10} + 74\,T_1^{12}\,T_2^{10} + 431\,T_1^{13}\,T_2^{10} - 386\,T_1^{14}\,T_2^{10} +$

$200\,T_1^{15}\,T_2^{10} + 34\,T_1^{16}\,T_2^{10} - 37\,T_1^{17}\,T_2^{10} + 186\,T_1^{18}\,T_2^{10} - 186\,T_1^{19}\,T_2^{10} + 136\,T_1^{20}\,T_2^{10} - 22\,T_1^{21}\,T_2^{10} - 12\,T_1^{22}\,T_2^{10} +$

$46\,T_1^{23}\,T_2^{10} - 93\,T_1^{24}\,T_2^{10} + 30\,T_1^{25}\,T_2^{10} + 11\,T_1^{26}\,T_2^{10} + 2\,T_1^{27}\,T_2^{10} - 3\,T_1^{28}\,T_2^{10} - 6\,T_1^{29}\,T_2^{10} + 4\,T_1^{30}\,T_2^{10} - 2\,T_2^{11} -$

$6\,T_1\,T_2^{11} + 8\,T_1^2\,T_2^{11} - T_1^3\,T_2^{11} - 18\,T_1^4\,T_2^{11} + 11\,T_1^5\,T_2^{11} + 2\,T_1^6\,T_2^{11} + 55\,T_1^7\,T_2^{11} - 89\,T_1^8\,T_2^{11} + 224\,T_1^9\,T_2^{11} -$

$$
\begin{aligned}
&314\, T_1^{10} T_2^{11} - 92\, T_1^{11} T_2^{11} + 764\, T_1^{12} T_2^{11} - 899\, T_1^{13} T_2^{11} + 273\, T_1^{14} T_2^{11} + 176\, T_1^{15} T_2^{11} - 382\, T_1^{16} T_2^{11} + \\
&391\, T_1^{17} T_2^{11} - 420\, T_1^{18} T_2^{11} + 75\, T_1^{19} T_2^{11} + 212\, T_1^{20} T_2^{11} - 156\, T_1^{21} T_2^{11} - 46\, T_1^{22} T_2^{11} - 6\, T_1^{23} T_2^{11} + 65\, T_1^{24} T_2^{11} + \\
&76\, T_1^{25} T_2^{11} - 107\, T_1^{26} T_2^{11} + 31\, T_1^{27} T_2^{11} - T_1^{28} T_2^{11} + 8\, T_1^{29} T_2^{11} - 6\, T_1^{30} T_2^{11} - 2\, T_1^{31} T_2^{11} - 2\, T_2^{12} + 12\, T_1 T_2^{12} - \\
&3\, T_1^2 T_2^{12} - T_1^3 T_2^{12} + 35\, T_1^4 T_2^{12} - 31\, T_1^5 T_2^{12} - 12\, T_1^6 T_2^{12} - 23\, T_1^7 T_2^{12} + 136\, T_1^8 T_2^{12} - 314\, T_1^9 T_2^{12} + 74\, T_1^{10} T_2^{12} + \\
&764\, T_1^{11} T_2^{12} - 1304\, T_1^{12} T_2^{12} + 293\, T_1^{13} T_2^{12} + 744\, T_1^{14} T_2^{12} - 996\, T_1^{15} T_2^{12} + 616\, T_1^{16} T_2^{12} - 380\, T_1^{17} T_2^{12} - \\
&68\, T_1^{18} T_2^{12} + 589\, T_1^{19} T_2^{12} - 596\, T_1^{20} T_2^{12} - 72\, T_1^{21} T_2^{12} + 294\, T_1^{22} T_2^{12} + 38\, T_1^{23} T_2^{12} - 64\, T_1^{24} T_2^{12} - \\
&123\, T_1^{25} T_2^{12} + 60\, T_1^{26} T_2^{12} + 93\, T_1^{27} T_2^{12} - 69\, T_1^{28} T_2^{12} - T_1^{29} T_2^{12} - 3\, T_1^{30} T_2^{12} + 12\, T_1^{31} T_2^{12} - 2\, T_1^{32} T_2^{12} + 4\, T_2^{13} - \\
&6\, T_1 T_2^{13} - 7\, T_1^2 T_2^{13} + 2\, T_1^3 T_2^{13} - 18\, T_1^4 T_2^{13} + 9\, T_1^5 T_2^{13} + 28\, T_1^6 T_2^{13} + 11\, T_1^7 T_2^{13} - 127\, T_1^8 T_2^{13} + 67\, T_1^9 T_2^{13} + \\
&431\, T_1^{10} T_2^{13} - 899\, T_1^{11} T_2^{13} + 293\, T_1^{12} T_2^{13} + 1556\, T_1^{13} T_2^{13} - 1724\, T_1^{14} T_2^{13} + 887\, T_1^{15} T_2^{13} + 223\, T_1^{16} T_2^{13} - \\
&480\, T_1^{17} T_2^{13} + 998\, T_1^{18} T_2^{13} - 905\, T_1^{19} T_2^{13} + 212\, T_1^{20} T_2^{13} + 686\, T_1^{21} T_2^{13} - 294\, T_1^{22} T_2^{13} - 313\, T_1^{23} T_2^{13} + \\
&146\, T_1^{24} T_2^{13} + 24\, T_1^{25} T_2^{13} + 123\, T_1^{26} T_2^{13} - 238\, T_1^{27} T_2^{13} + 65\, T_1^{28} T_2^{13} + 45\, T_1^{29} T_2^{13} + 2\, T_1^{30} T_2^{13} - 7\, T_1^{31} T_2^{13} - \\
&6\, T_1^{32} T_2^{13} + 4\, T_1^{33} T_2^{13} - 3\, T_2^{14} - 3\, T_1 T_2^{14} + 9\, T_1^2 T_2^{14} - 2\, T_1^3 T_2^{14} - T_1^4 T_2^{14} + T_1^5 T_2^{14} - 42\, T_1^6 T_2^{14} + 51\, T_1^7 T_2^{14} + \\
&31\, T_1^8 T_2^{14} + 111\, T_1^9 T_2^{14} - 386\, T_1^{10} T_2^{14} + 273\, T_1^{11} T_2^{14} + 744\, T_1^{12} T_2^{14} - 1724\, T_1^{13} T_2^{14} + 705\, T_1^{14} T_2^{14} + \\
&482\, T_1^{15} T_2^{14} - 1315\, T_1^{16} T_2^{14} + 1061\, T_1^{17} T_2^{14} - 855\, T_1^{18} T_2^{14} - 140\, T_1^{19} T_2^{14} + 809\, T_1^{20} T_2^{14} - 758\, T_1^{21} T_2^{14} - \\
&370\, T_1^{22} T_2^{14} + 595\, T_1^{23} T_2^{14} + 58\, T_1^{24} T_2^{14} - 229\, T_1^{25} T_2^{14} + T_1^{26} T_2^{14} + 95\, T_1^{27} T_2^{14} + 124\, T_1^{28} T_2^{14} - 151\, T_1^{29} T_2^{14} + \\
&19\, T_1^{30} T_2^{14} - 2\, T_1^{31} T_2^{14} + 9\, T_1^{32} T_2^{14} - 3\, T_1^{33} T_2^{14} - 3\, T_1^{34} T_2^{14} + T_2^{15} + 6\, T_1 T_2^{15} - 4\, T_1^2 T_2^{15} + 4\, T_1^3 T_2^{15} + \\
&6\, T_1^4 T_2^{15} + 26\, T_1^6 T_2^{15} - 33\, T_1^7 T_2^{15} - 31\, T_1^8 T_2^{15} - 124\, T_1^9 T_2^{15} + 200\, T_1^{10} T_2^{15} + 176\, T_1^{11} T_2^{15} - 996\, T_1^{12} T_2^{15} + \\
&887\, T_1^{13} T_2^{15} + 482\, T_1^{14} T_2^{15} - 1534\, T_1^{15} T_2^{15} + 1712\, T_1^{16} T_2^{15} - 619\, T_1^{17} T_2^{15} - 569\, T_1^{18} T_2^{15} + 1420\, T_1^{19} T_2^{15} - \\
&914\, T_1^{20} T_2^{15} - 229\, T_1^{21} T_2^{15} + 992\, T_1^{22} T_2^{15} - 257\, T_1^{23} T_2^{15} - 598\, T_1^{24} T_2^{15} + 440\, T_1^{25} T_2^{15} - 15\, T_1^{26} T_2^{15} - \\
&50\, T_1^{27} T_2^{15} - 167\, T_1^{28} T_2^{15} + 92\, T_1^{29} T_2^{15} + 74\, T_1^{30} T_2^{15} - 49\, T_1^{31} T_2^{15} + 4\, T_1^{32} T_2^{15} - 4\, T_1^{33} T_2^{15} + 6\, T_1^{34} T_2^{15} + \\
&T_1^{35} T_2^{15} - 3\, T_1 T_2^{16} - T_1^2 T_2^{16} - 3\, T_1^3 T_2^{16} - 7\, T_1^4 T_2^{16} + 8\, T_1^5 T_2^{16} - 29\, T_1^6 T_2^{16} + 41\, T_1^7 T_2^{16} + 16\, T_1^8 T_2^{16} + \\
&38\, T_1^9 T_2^{16} + 34\, T_1^{10} T_2^{16} - 382\, T_1^{11} T_2^{16} + 616\, T_1^{12} T_2^{16} + 223\, T_1^{13} T_2^{16} - 1315\, T_1^{14} T_2^{16} + 1712\, T_1^{15} T_2^{16} - \\
&720\, T_1^{16} T_2^{16} - 1180\, T_1^{17} T_2^{16} + 2146\, T_1^{18} T_2^{16} - 1310\, T_1^{19} T_2^{16} - 260\, T_1^{20} T_2^{16} + 1108\, T_1^{21} T_2^{16} - 545\, T_1^{22} T_2^{16} - \\
&555\, T_1^{23} T_2^{16} + 792\, T_1^{24} T_2^{16} - 94\, T_1^{25} T_2^{16} - 350\, T_1^{26} T_2^{16} + 256\, T_1^{27} T_2^{16} - 24\, T_1^{28} T_2^{16} + 109\, T_1^{29} T_2^{16} - \\
&189\, T_1^{30} T_2^{16} + 60\, T_1^{31} T_2^{16} + 17\, T_1^{32} T_2^{16} - 3\, T_1^{33} T_2^{16} - T_1^{34} T_2^{16} - 3\, T_1^{35} T_2^{16} + 2\, T_1^3 T_2^{17} + 10\, T_1^4 T_2^{17} - \\
&15\, T_1^5 T_2^{17} - T_1^6 T_2^{17} + 28\, T_1^7 T_2^{17} - 33\, T_1^8 T_2^{17} - 49\, T_1^9 T_2^{17} - 37\, T_1^{10} T_2^{17} + 391\, T_1^{11} T_2^{17} - 380\, T_1^{12} T_2^{17} - \\
&480\, T_1^{13} T_2^{17} + 1061\, T_1^{14} T_2^{17} - 619\, T_1^{15} T_2^{17} - 1180\, T_1^{16} T_2^{17} + 2566\, T_1^{17} T_2^{17} - 1730\, T_1^{18} T_2^{17} - 591\, T_1^{19} T_2^{17} + \\
&1520\, T_1^{20} T_2^{17} - 933\, T_1^{21} T_2^{17} - 265\, T_1^{22} T_2^{17} + 476\, T_1^{23} T_2^{17} + 123\, T_1^{24} T_2^{17} - 791\, T_1^{25} T_2^{17} + 681\, T_1^{26} T_2^{17} - \\
&213\, T_1^{27} T_2^{17} - 82\, T_1^{28} T_2^{17} - 8\, T_1^{29} T_2^{17} + 74\, T_1^{30} T_2^{17} + 42\, T_1^{31} T_2^{17} - 59\, T_1^{32} T_2^{17} + 10\, T_1^{33} T_2^{17} + 2\, T_1^{34} T_2^{17} + \\
&3\, T_1^2 T_2^{18} + 3\, T_1^3 T_2^{18} - 19\, T_1^4 T_2^{18} + 9\, T_1^5 T_2^{18} + 45\, T_1^6 T_2^{18} - 60\, T_1^7 T_2^{18} - 28\, T_1^8 T_2^{18} + 50\, T_1^9 T_2^{18} + 186\, T_1^{10} T_2^{18} - \\
&420\, T_1^{11} T_2^{18} - 68\, T_1^{12} T_2^{18} + 998\, T_1^{13} T_2^{18} - 855\, T_1^{14} T_2^{18} - 569\, T_1^{15} T_2^{18} + 2146\, T_1^{16} T_2^{18} - 1730\, T_1^{17} T_2^{18} - \\
&492\, T_1^{18} T_2^{18} + 2218\, T_1^{19} T_2^{18} - 1372\, T_1^{20} T_2^{18} - 146\, T_1^{21} T_2^{18} + 878\, T_1^{22} T_2^{18} - 163\, T_1^{23} T_2^{18} - 695\, T_1^{24} T_2^{18} + \\
&872\, T_1^{25} T_2^{18} - 162\, T_1^{26} T_2^{18} - 458\, T_1^{27} T_2^{18} + 506\, T_1^{28} T_2^{18} - 208\, T_1^{29} T_2^{18} + 44\, T_1^{30} T_2^{18} - 100\, T_1^{31} T_2^{18} + \\
&79\, T_1^{32} T_2^{18} - 19\, T_1^{33} T_2^{18} - 5\, T_1^{34} T_2^{18} + 3\, T_1^{35} T_2^{18} + 3\, T_1^{36} T_2^{18} - T_1^2 T_2^{19} - 10\, T_1^3 T_2^{19} + 17\, T_1^4 T_2^{19} + 16\, T_1^5 T_2^{19} - \\
&64\, T_1^6 T_2^{19} + 16\, T_1^7 T_2^{19} + 109\, T_1^8 T_2^{19} - 38\, T_1^9 T_2^{19} - 186\, T_1^{10} T_2^{19} + 75\, T_1^{11} T_2^{19} + 589\, T_1^{12} T_2^{19} - 905\, T_1^{13} T_2^{19} - \\
&140\, T_1^{14} T_2^{19} + 1420\, T_1^{15} T_2^{19} - 1310\, T_1^{16} T_2^{19} - 591\, T_1^{17} T_2^{19} + 2218\, T_1^{18} T_2^{19} - 2027\, T_1^{19} T_2^{19} + 155\, T_1^{20} T_2^{19} + \\
&1033\, T_1^{21} T_2^{19} - 840\, T_1^{22} T_2^{19} - 49\, T_1^{23} T_2^{19} + 464\, T_1^{24} T_2^{19} + 37\, T_1^{25} T_2^{19} - 842\, T_1^{26} T_2^{19} + 972\, T_1^{27} T_2^{19} - \\
&412\, T_1^{28} T_2^{19} - 44\, T_1^{29} T_2^{19} + 150\, T_1^{30} T_2^{19} - 21\, T_1^{31} T_2^{19} - 10\, T_1^{32} T_2^{19} - 42\, T_1^{33} T_2^{19} + 50\, T_1^{34} T_2^{19} - 13\, T_1^{35} T_2^{19} - \\
&10\, T_1^{36} T_2^{19} - T_1^{37} T_2^{19} + 6\, T_1^3 T_2^{20} + 3\, T_1^4 T_2^{20} - 32\, T_1^5 T_2^{20} + 24\, T_1^6 T_2^{20} + 68\, T_1^7 T_2^{20} - 115\, T_1^8 T_2^{20} - 47\, T_1^9 T_2^{20} + \\
&136\, T_1^{10} T_2^{20} + 212\, T_1^{11} T_2^{20} - 596\, T_1^{12} T_2^{20} + 212\, T_1^{13} T_2^{20} + 809\, T_1^{14} T_2^{20} - 914\, T_1^{15} T_2^{20} - 260\, T_1^{16} T_2^{20} + \\
&1520\, T_1^{17} T_2^{20} - 1372\, T_1^{18} T_2^{20} + 155\, T_1^{19} T_2^{20} + 1056\, T_1^{20} T_2^{20} - 1291\, T_1^{21} T_2^{20} + 674\, T_1^{22} T_2^{20} - 128\, T_1^{23} T_2^{20} - \\
&56\, T_1^{24} T_2^{20} - 374\, T_1^{25} T_2^{20} + 603\, T_1^{26} T_2^{20} - 180\, T_1^{27} T_2^{20} - 504\, T_1^{28} T_2^{20} + 592\, T_1^{29} T_2^{20} - 340\, T_1^{30} T_2^{20} + \\
&71\, T_1^{31} T_2^{20} - 39\, T_1^{32} T_2^{20} + 100\, T_1^{33} T_2^{20} - 60\, T_1^{34} T_2^{20} - 8\, T_1^{35} T_2^{20} + 19\, T_1^{36} T_2^{20} + 6\, T_1^{37} T_2^{20} - T_1^3 T_2^{21} - \\
&10\, T_1^4 T_2^{21} + 15\, T_1^5 T_2^{21} + 18\, T_1^6 T_2^{21} - 62\, T_1^7 T_2^{21} + 14\, T_1^8 T_2^{21} + 95\, T_1^9 T_2^{21} - 22\, T_1^{10} T_2^{21} - 156\, T_1^{11} T_2^{21} - \\
&72\, T_1^{12} T_2^{21} + 686\, T_1^{13} T_2^{21} - 758\, T_1^{14} T_2^{21} - 229\, T_1^{15} T_2^{21} + 1108\, T_1^{16} T_2^{21} - 933\, T_1^{17} T_2^{21} - 146\, T_1^{18} T_2^{21} + \\
&1033\, T_1^{19} T_2^{21} - 1291\, T_1^{20} T_2^{21} + 891\, T_1^{21} T_2^{21} - 152\, T_1^{22} T_2^{21} - 395\, T_1^{23} T_2^{21} + 328\, T_1^{24} T_2^{21} + 152\, T_1^{25} T_2^{21} -
\end{aligned}
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\begin{aligned}
&52\,T_1^{26}\,T_2^{21} - 695\,T_1^{27}\,T_2^{21} + 1069\,T_1^{28}\,T_2^{21} - 559\,T_1^{29}\,T_2^{21} - 14\,T_1^{30}\,T_2^{21} + 166\,T_1^{31}\,T_2^{21} - 35\,T_1^{32}\,T_2^{21} - \\
&12\,T_1^{33}\,T_2^{21} - 40\,T_1^{34}\,T_2^{21} + 52\,T_1^{35}\,T_2^{21} - 15\,T_1^{36}\,T_2^{21} - 10\,T_1^{37}\,T_2^{21} - T_1^{38}\,T_2^{21} + 3\,T_1^{4}\,T_2^{22} + 3\,T_1^{5}\,T_2^{22} - \\
&17\,T_1^{6}\,T_2^{22} + 5\,T_1^{7}\,T_2^{22} + 51\,T_1^{8}\,T_2^{22} - 68\,T_1^{9}\,T_2^{22} - 12\,T_1^{10}\,T_2^{22} - 46\,T_1^{11}\,T_2^{22} + 294\,T_1^{12}\,T_2^{22} - 294\,T_1^{13}\,T_2^{22} - \\
&370\,T_1^{14}\,T_2^{22} + 992\,T_1^{15}\,T_2^{22} - 545\,T_1^{16}\,T_2^{22} - 265\,T_1^{17}\,T_2^{22} + 878\,T_1^{18}\,T_2^{22} - 840\,T_1^{19}\,T_2^{22} + 674\,T_1^{20}\,T_2^{22} - \\
&152\,T_1^{21}\,T_2^{22} - 206\,T_1^{22}\,T_2^{22} + 744\,T_1^{23}\,T_2^{22} - 390\,T_1^{24}\,T_2^{22} + 141\,T_1^{25}\,T_2^{22} - 385\,T_1^{26}\,T_2^{22} + 866\,T_1^{27}\,T_2^{22} - \\
&464\,T_1^{28}\,T_2^{22} - 332\,T_1^{29}\,T_2^{22} + 614\,T_1^{30}\,T_2^{22} - 304\,T_1^{31}\,T_2^{22} + 60\,T_1^{32}\,T_2^{22} - 108\,T_1^{33}\,T_2^{22} + 85\,T_1^{34}\,T_2^{22} - \\
&23\,T_1^{35}\,T_2^{22} - 3\,T_1^{36}\,T_2^{22} + 3\,T_1^{37}\,T_2^{22} + 3\,T_1^{38}\,T_2^{22} + 2\,T_1^{6}\,T_2^{23} + 10\,T_1^{7}\,T_2^{23} - 27\,T_1^{8}\,T_2^{23} + 8\,T_1^{9}\,T_2^{23} + 46\,T_1^{10}\,T_2^{23} - \\
&6\,T_1^{11}\,T_2^{23} + 38\,T_1^{12}\,T_2^{23} - 313\,T_1^{13}\,T_2^{23} + 595\,T_1^{14}\,T_2^{23} - 257\,T_1^{15}\,T_2^{23} - 555\,T_1^{16}\,T_2^{23} + 476\,T_1^{17}\,T_2^{23} - \\
&163\,T_1^{18}\,T_2^{23} - 49\,T_1^{19}\,T_2^{23} - 128\,T_1^{20}\,T_2^{23} - 395\,T_1^{21}\,T_2^{23} + 744\,T_1^{22}\,T_2^{23} - 1174\,T_1^{23}\,T_2^{23} + 198\,T_1^{24}\,T_2^{23} + \\
&191\,T_1^{25}\,T_2^{23} - 109\,T_1^{26}\,T_2^{23} + 48\,T_1^{27}\,T_2^{23} - 668\,T_1^{28}\,T_2^{23} + 885\,T_1^{29}\,T_2^{23} - 489\,T_1^{30}\,T_2^{23} + 5\,T_1^{31}\,T_2^{23} + \\
&19\,T_1^{32}\,T_2^{23} + 92\,T_1^{33}\,T_2^{23} + 51\,T_1^{34}\,T_2^{23} - 71\,T_1^{35}\,T_2^{23} + 10\,T_1^{36}\,T_2^{23} + 2\,T_1^{37}\,T_2^{23} - 3\,T_1^{5}\,T_2^{24} - T_1^{6}\,T_2^{24} - \\
&3\,T_1^{7}\,T_2^{24} + T_1^{8}\,T_2^{24} + 32\,T_1^{9}\,T_2^{24} - 93\,T_1^{10}\,T_2^{24} + 65\,T_1^{11}\,T_2^{24} - 64\,T_1^{12}\,T_2^{24} + 146\,T_1^{13}\,T_2^{24} + 58\,T_1^{14}\,T_2^{24} - \\
&598\,T_1^{15}\,T_2^{24} + 792\,T_1^{16}\,T_2^{24} + 123\,T_1^{17}\,T_2^{24} - 695\,T_1^{18}\,T_2^{24} + 464\,T_1^{19}\,T_2^{24} - 56\,T_1^{20}\,T_2^{24} + 328\,T_1^{21}\,T_2^{24} - \\
&390\,T_1^{22}\,T_2^{24} + 198\,T_1^{23}\,T_2^{24} + 404\,T_1^{24}\,T_2^{24} - 140\,T_1^{25}\,T_2^{24} + 75\,T_1^{26}\,T_2^{24} - 655\,T_1^{27}\,T_2^{24} + 968\,T_1^{28}\,T_2^{24} - \\
&310\,T_1^{29}\,T_2^{24} - 326\,T_1^{30}\,T_2^{24} + 364\,T_1^{31}\,T_2^{24} - 104\,T_1^{32}\,T_2^{24} + 133\,T_1^{33}\,T_2^{24} - 253\,T_1^{34}\,T_2^{24} + 84\,T_1^{35}\,T_2^{24} + \\
&25\,T_1^{36}\,T_2^{24} - 3\,T_1^{37}\,T_2^{24} - T_1^{38}\,T_2^{24} - 3\,T_1^{39}\,T_2^{24} + T_1^{5}\,T_2^{25} + 6\,T_1^{6}\,T_2^{25} - 4\,T_1^{7}\,T_2^{25} + 4\,T_1^{8}\,T_2^{25} - 19\,T_1^{9}\,T_2^{25} + \\
&30\,T_1^{10}\,T_2^{25} + 76\,T_1^{11}\,T_2^{25} - 123\,T_1^{12}\,T_2^{25} + 24\,T_1^{13}\,T_2^{25} - 229\,T_1^{14}\,T_2^{25} + 440\,T_1^{15}\,T_2^{25} - 94\,T_1^{16}\,T_2^{25} - \\
&791\,T_1^{17}\,T_2^{25} + 872\,T_1^{18}\,T_2^{25} + 37\,T_1^{19}\,T_2^{25} - 374\,T_1^{20}\,T_2^{25} + 152\,T_1^{21}\,T_2^{25} + 141\,T_1^{22}\,T_2^{25} + 191\,T_1^{23}\,T_2^{25} - \\
&140\,T_1^{24}\,T_2^{25} + 246\,T_1^{25}\,T_2^{25} - 674\,T_1^{26}\,T_2^{25} + 977\,T_1^{27}\,T_2^{25} - 52\,T_1^{28}\,T_2^{25} - 868\,T_1^{29}\,T_2^{25} + 680\,T_1^{30}\,T_2^{25} - \\
&120\,T_1^{31}\,T_2^{25} + 5\,T_1^{32}\,T_2^{25} - 257\,T_1^{33}\,T_2^{25} + 142\,T_1^{34}\,T_2^{25} + 104\,T_1^{35}\,T_2^{25} - 74\,T_1^{36}\,T_2^{25} + 4\,T_1^{37}\,T_2^{25} - 4\,T_1^{38}\,T_2^{25} + \\
&6\,T_1^{39}\,T_2^{25} + T_1^{40}\,T_2^{25} - 3\,T_1^{6}\,T_2^{26} - 3\,T_1^{7}\,T_2^{26} + 9\,T_1^{8}\,T_2^{26} - 2\,T_1^{9}\,T_2^{26} + 11\,T_1^{10}\,T_2^{26} - 107\,T_1^{11}\,T_2^{26} + 60\,T_1^{12}\,T_2^{26} + \\
&123\,T_1^{13}\,T_2^{26} + T_1^{14}\,T_2^{26} - 15\,T_1^{15}\,T_2^{26} - 350\,T_1^{16}\,T_2^{26} + 681\,T_1^{17}\,T_2^{26} - 162\,T_1^{18}\,T_2^{26} - 842\,T_1^{19}\,T_2^{26} + \\
&603\,T_1^{20}\,T_2^{26} - 52\,T_1^{21}\,T_2^{26} - 385\,T_1^{22}\,T_2^{26} - 109\,T_1^{23}\,T_2^{26} + 75\,T_1^{24}\,T_2^{26} - 674\,T_1^{25}\,T_2^{26} + 707\,T_1^{26}\,T_2^{26} + \\
&124\,T_1^{27}\,T_2^{26} - 1276\,T_1^{28}\,T_2^{26} + 1003\,T_1^{29}\,T_2^{26} + 94\,T_1^{30}\,T_2^{26} - 355\,T_1^{31}\,T_2^{26} - 29\,T_1^{32}\,T_2^{26} + 167\,T_1^{33}\,T_2^{26} + \\
&226\,T_1^{34}\,T_2^{26} - 259\,T_1^{35}\,T_2^{26} + 31\,T_1^{36}\,T_2^{26} - 2\,T_1^{37}\,T_2^{26} + 9\,T_1^{38}\,T_2^{26} - 3\,T_1^{39}\,T_2^{26} - 3\,T_1^{40}\,T_2^{26} + 4\,T_1^{7}\,T_2^{27} - \\
&6\,T_1^{8}\,T_2^{27} - 7\,T_1^{9}\,T_2^{27} + 2\,T_1^{10}\,T_2^{27} + 31\,T_1^{11}\,T_2^{27} + 93\,T_1^{12}\,T_2^{27} - 238\,T_1^{13}\,T_2^{27} + 95\,T_1^{14}\,T_2^{27} - 50\,T_1^{15}\,T_2^{27} + \\
&256\,T_1^{16}\,T_2^{27} - 213\,T_1^{17}\,T_2^{27} - 458\,T_1^{18}\,T_2^{27} + 972\,T_1^{19}\,T_2^{27} - 180\,T_1^{20}\,T_2^{27} - 695\,T_1^{21}\,T_2^{27} + 866\,T_1^{22}\,T_2^{27} + \\
&48\,T_1^{23}\,T_2^{27} - 655\,T_1^{24}\,T_2^{27} + 977\,T_1^{25}\,T_2^{27} + 124\,T_1^{26}\,T_2^{27} - 1524\,T_1^{27}\,T_2^{27} + 1365\,T_1^{28}\,T_2^{27} + 147\,T_1^{29}\,T_2^{27} - \\
&957\,T_1^{30}\,T_2^{27} + 335\,T_1^{31}\,T_2^{27} + 101\,T_1^{32}\,T_2^{27} + 207\,T_1^{33}\,T_2^{27} - 504\,T_1^{34}\,T_2^{27} + 149\,T_1^{35}\,T_2^{27} + 94\,T_1^{36}\,T_2^{27} + \\
&2\,T_1^{37}\,T_2^{27} - 7\,T_1^{38}\,T_2^{27} - 6\,T_1^{39}\,T_2^{27} + 4\,T_1^{40}\,T_2^{27} - 2\,T_1^{8}\,T_2^{28} + 12\,T_1^{9}\,T_2^{28} - 3\,T_1^{10}\,T_2^{28} - T_1^{11}\,T_2^{28} - 69\,T_1^{12}\,T_2^{28} + \\
&65\,T_1^{13}\,T_2^{28} + 124\,T_1^{14}\,T_2^{28} - 167\,T_1^{15}\,T_2^{28} - 24\,T_1^{16}\,T_2^{28} - 82\,T_1^{17}\,T_2^{28} + 506\,T_1^{18}\,T_2^{28} - 412\,T_1^{19}\,T_2^{28} - \\
&504\,T_1^{20}\,T_2^{28} + 1069\,T_1^{21}\,T_2^{28} - 464\,T_1^{22}\,T_2^{28} - 668\,T_1^{23}\,T_2^{28} + 968\,T_1^{24}\,T_2^{28} - 52\,T_1^{25}\,T_2^{28} - 1276\,T_1^{26}\,T_2^{28} + \\
&1365\,T_1^{27}\,T_2^{28} + 204\,T_1^{28}\,T_2^{28} - 1248\,T_1^{29}\,T_2^{28} + 726\,T_1^{30}\,T_2^{28} + 270\,T_1^{31}\,T_2^{28} - 224\,T_1^{32}\,T_2^{28} - 267\,T_1^{33}\,T_2^{28} + \\
&196\,T_1^{34}\,T_2^{28} + 189\,T_1^{35}\,T_2^{28} - 173\,T_1^{36}\,T_2^{28} - T_1^{37}\,T_2^{28} - 3\,T_1^{38}\,T_2^{28} + 12\,T_1^{39}\,T_2^{28} - 2\,T_1^{40}\,T_2^{28} - 2\,T_1^{9}\,T_2^{29} - \\
&6\,T_1^{10}\,T_2^{29} + 8\,T_1^{11}\,T_2^{29} - T_1^{12}\,T_2^{29} + 45\,T_1^{13}\,T_2^{29} - 151\,T_1^{14}\,T_2^{29} + 92\,T_1^{15}\,T_2^{29} + 109\,T_1^{16}\,T_2^{29} - 8\,T_1^{17}\,T_2^{29} - \\
&208\,T_1^{18}\,T_2^{29} - 44\,T_1^{19}\,T_2^{29} + 592\,T_1^{20}\,T_2^{29} - 559\,T_1^{21}\,T_2^{29} - 332\,T_1^{22}\,T_2^{29} + 885\,T_1^{23}\,T_2^{29} - 310\,T_1^{24}\,T_2^{29} - \\
&868\,T_1^{25}\,T_2^{29} + 1003\,T_1^{26}\,T_2^{29} + 147\,T_1^{27}\,T_2^{29} - 1248\,T_1^{28}\,T_2^{29} + 896\,T_1^{29}\,T_2^{29} + 114\,T_1^{30}\,T_2^{29} - 478\,T_1^{31}\,T_2^{29} + \\
&75\,T_1^{32}\,T_2^{29} + 119\,T_1^{33}\,T_2^{29} + 166\,T_1^{34}\,T_2^{29} - 269\,T_1^{35}\,T_2^{29} + 94\,T_1^{36}\,T_2^{29} - T_1^{37}\,T_2^{29} + 8\,T_1^{38}\,T_2^{29} - 6\,T_1^{39}\,T_2^{29} - \\
&2\,T_1^{40}\,T_2^{29} + 4\,T_1^{10}\,T_2^{30} - 6\,T_1^{11}\,T_2^{30} - 3\,T_1^{12}\,T_2^{30} + 2\,T_1^{13}\,T_2^{30} + 19\,T_1^{14}\,T_2^{30} + 74\,T_1^{15}\,T_2^{30} - 189\,T_1^{16}\,T_2^{30} + \\
&74\,T_1^{17}\,T_2^{30} + 44\,T_1^{18}\,T_2^{30} + 150\,T_1^{19}\,T_2^{30} - 340\,T_1^{20}\,T_2^{30} - 14\,T_1^{21}\,T_2^{30} + 614\,T_1^{22}\,T_2^{30} - 489\,T_1^{23}\,T_2^{30} - \\
&326\,T_1^{24}\,T_2^{30} + 680\,T_1^{25}\,T_2^{30} + 94\,T_1^{26}\,T_2^{30} - 957\,T_1^{27}\,T_2^{30} + 726\,T_1^{28}\,T_2^{30} + 114\,T_1^{29}\,T_2^{30} - 444\,T_1^{30}\,T_2^{30} + \\
&138\,T_1^{31}\,T_2^{30} + 68\,T_1^{32}\,T_2^{30} + 106\,T_1^{33}\,T_2^{30} - 253\,T_1^{34}\,T_2^{30} + 90\,T_1^{35}\,T_2^{30} + 31\,T_1^{36}\,T_2^{30} + 2\,T_1^{37}\,T_2^{30} - 3\,T_1^{38}\,T_2^{30} - \\
&6\,T_1^{39}\,T_2^{30} + 4\,T_1^{40}\,T_2^{30} - 2\,T_1^{11}\,T_2^{31} + 12\,T_1^{12}\,T_2^{31} - 7\,T_1^{13}\,T_2^{31} - 2\,T_1^{14}\,T_2^{31} - 49\,T_1^{15}\,T_2^{31} + 60\,T_1^{16}\,T_2^{31} + \\
&42\,T_1^{17}\,T_2^{31} - 100\,T_1^{18}\,T_2^{31} - 21\,T_1^{19}\,T_2^{31} + 71\,T_1^{20}\,T_2^{31} + 166\,T_1^{21}\,T_2^{31} - 304\,T_1^{22}\,T_2^{31} + 5\,T_1^{23}\,T_2^{31} + 364\,T_1^{24}\,T_2^{31} - \\
&120\,T_1^{25}\,T_2^{31} - 355\,T_1^{26}\,T_2^{31} + 335\,T_1^{27}\,T_2^{31} + 270\,T_1^{28}\,T_2^{31} - 478\,T_1^{29}\,T_2^{31} + 138\,T_1^{30}\,T_2^{31} + 173\,T_1^{31}\,T_2^{31} - \\
&59\,T_1^{32}\,T_2^{31} - 112\,T_1^{33}\,T_2^{31} + 41\,T_1^{34}\,T_2^{31} + 98\,T_1^{35}\,T_2^{31} - 74\,T_1^{36}\,T_2^{31} - 2\,T_1^{37}\,T_2^{31} - 7\,T_1^{38}\,T_2^{31} + 12\,T_1^{39}\,T_2^{31} -
\end{aligned}
$$

$$2 T_1^{40} T_2^{31} - 2 T_1^{12} T_2^{32} - 6 T_1^{13} T_2^{32} + 9 T_1^{14} T_2^{32} + 4 T_1^{15} T_2^{32} + 17 T_1^{16} T_2^{32} - 59 T_1^{17} T_2^{32} + 79 T_1^{18} T_2^{32} -$$
$$10 T_1^{19} T_2^{32} - 39 T_1^{20} T_2^{32} - 35 T_1^{21} T_2^{32} + 60 T_1^{22} T_2^{32} + 19 T_1^{23} T_2^{32} - 104 T_1^{24} T_2^{32} + 5 T_1^{25} T_2^{32} - 29 T_1^{26} T_2^{32} +$$
$$101 T_1^{27} T_2^{32} - 224 T_1^{28} T_2^{32} + 75 T_1^{29} T_2^{32} + 68 T_1^{30} T_2^{32} - 59 T_1^{31} T_2^{32} - 31 T_1^{32} T_2^{32} - 10 T_1^{33} T_2^{32} + 87 T_1^{34} T_2^{32} -$$
$$75 T_1^{35} T_2^{32} + 25 T_1^{36} T_2^{32} + 4 T_1^{37} T_2^{32} + 9 T_1^{38} T_2^{32} - 6 T_1^{39} T_2^{32} - 2 T_1^{40} T_2^{32} + 4 T_1^{13} T_2^{33} - 3 T_1^{14} T_2^{33} -$$
$$4 T_1^{15} T_2^{33} - 3 T_1^{16} T_2^{33} + 10 T_1^{17} T_2^{33} - 19 T_1^{18} T_2^{33} - 42 T_1^{19} T_2^{33} + 100 T_1^{20} T_2^{33} - 12 T_1^{21} T_2^{33} - 108 T_1^{22} T_2^{33} +$$
$$92 T_1^{23} T_2^{33} + 133 T_1^{24} T_2^{33} - 257 T_1^{25} T_2^{33} + 167 T_1^{26} T_2^{33} + 207 T_1^{27} T_2^{33} - 267 T_1^{28} T_2^{33} + 119 T_1^{29} T_2^{33} +$$
$$106 T_1^{30} T_2^{33} - 112 T_1^{31} T_2^{33} - 10 T_1^{32} T_2^{33} + 94 T_1^{33} T_2^{33} - 36 T_1^{34} T_2^{33} - 21 T_1^{35} T_2^{33} + 10 T_1^{36} T_2^{33} - 3 T_1^{37} T_2^{33} -$$
$$4 T_1^{38} T_2^{33} - 3 T_1^{39} T_2^{33} + 4 T_1^{40} T_2^{33} - 3 T_1^{14} T_2^{34} + 6 T_1^{15} T_2^{34} - T_1^{16} T_2^{34} + 2 T_1^{17} T_2^{34} - 5 T_1^{18} T_2^{34} + 50 T_1^{19} T_2^{34} -$$
$$60 T_1^{20} T_2^{34} - 40 T_1^{21} T_2^{34} + 85 T_1^{22} T_2^{34} + 51 T_1^{23} T_2^{34} - 253 T_1^{24} T_2^{34} + 142 T_1^{25} T_2^{34} + 226 T_1^{26} T_2^{34} -$$
$$504 T_1^{27} T_2^{34} + 196 T_1^{28} T_2^{34} + 166 T_1^{29} T_2^{34} - 253 T_1^{30} T_2^{34} + 41 T_1^{31} T_2^{34} + 87 T_1^{32} T_2^{34} - 36 T_1^{33} T_2^{34} -$$
$$60 T_1^{34} T_2^{34} + 46 T_1^{35} T_2^{34} - 3 T_1^{36} T_2^{34} + 2 T_1^{37} T_2^{34} - T_1^{38} T_2^{34} + 6 T_1^{39} T_2^{34} - 3 T_1^{40} T_2^{34} + T_1^{15} T_2^{35} - 3 T_1^{16} T_2^{35} +$$
$$3 T_1^{18} T_2^{35} - 13 T_1^{19} T_2^{35} - 8 T_1^{20} T_2^{35} + 52 T_1^{21} T_2^{35} - 23 T_1^{22} T_2^{35} - 71 T_1^{23} T_2^{35} + 84 T_1^{24} T_2^{35} + 104 T_1^{25} T_2^{35} -$$
$$259 T_1^{26} T_2^{35} + 149 T_1^{27} T_2^{35} + 189 T_1^{28} T_2^{35} - 269 T_1^{29} T_2^{35} + 90 T_1^{30} T_2^{35} + 98 T_1^{31} T_2^{35} - 75 T_1^{32} T_2^{35} -$$
$$21 T_1^{33} T_2^{35} + 46 T_1^{34} T_2^{35} - 2 T_1^{35} T_2^{35} - 15 T_1^{36} T_2^{35} + 3 T_1^{37} T_2^{35} - 3 T_1^{39} T_2^{35} + T_1^{40} T_2^{35} + 3 T_1^{18} T_2^{36} -$$
$$10 T_1^{19} T_2^{36} + 19 T_1^{20} T_2^{36} - 15 T_1^{21} T_2^{36} - 3 T_1^{22} T_2^{36} + 10 T_1^{23} T_2^{36} + 25 T_1^{24} T_2^{36} - 74 T_1^{25} T_2^{36} + 31 T_1^{26} T_2^{36} +$$
$$94 T_1^{27} T_2^{36} - 173 T_1^{28} T_2^{36} + 94 T_1^{29} T_2^{36} + 31 T_1^{30} T_2^{36} - 74 T_1^{31} T_2^{36} + 25 T_1^{32} T_2^{36} + 10 T_1^{33} T_2^{36} - 3 T_1^{34} T_2^{36} -$$
$$15 T_1^{35} T_2^{36} + 19 T_1^{36} T_2^{36} - 10 T_1^{37} T_2^{36} + 3 T_1^{38} T_2^{36} - T_1^{19} T_2^{37} + 6 T_1^{20} T_2^{37} - 10 T_1^{21} T_2^{37} + 3 T_1^{22} T_2^{37} +$$
$$2 T_1^{23} T_2^{37} - 3 T_1^{24} T_2^{37} + 4 T_1^{25} T_2^{37} - 2 T_1^{26} T_2^{37} + 2 T_1^{27} T_2^{37} - T_1^{28} T_2^{37} - T_1^{29} T_2^{37} + 2 T_1^{30} T_2^{37} - 2 T_1^{31} T_2^{37} +$$
$$4 T_1^{32} T_2^{37} - 3 T_1^{33} T_2^{37} + 2 T_1^{34} T_2^{37} + 3 T_1^{35} T_2^{37} - 10 T_1^{36} T_2^{37} + 6 T_1^{37} T_2^{37} - T_1^{38} T_2^{37} - T_1^{21} T_2^{38} + 3 T_1^{22} T_2^{38} -$$
$$T_1^{24} T_2^{38} - 4 T_1^{25} T_2^{38} + 9 T_1^{26} T_2^{38} - 7 T_1^{27} T_2^{38} - 3 T_1^{28} T_2^{38} + 8 T_1^{29} T_2^{38} - 3 T_1^{30} T_2^{38} - 7 T_1^{31} T_2^{38} + 9 T_1^{32} T_2^{38} -$$
$$4 T_1^{33} T_2^{38} - T_1^{34} T_2^{38} + 3 T_1^{36} T_2^{38} - T_1^{37} T_2^{38} - 3 T_1^{24} T_2^{39} + 6 T_1^{25} T_2^{39} - 3 T_1^{26} T_2^{39} - 6 T_1^{27} T_2^{39} + 12 T_1^{28} T_2^{39} -$$
$$6 T_1^{29} T_2^{39} - 6 T_1^{30} T_2^{39} + 12 T_1^{31} T_2^{39} - 6 T_1^{32} T_2^{39} - 3 T_1^{33} T_2^{39} + 6 T_1^{34} T_2^{39} - 3 T_1^{35} T_2^{39} + T_1^{25} T_2^{40} - 3 T_1^{26} T_2^{40} +$$
$$4 T_1^{27} T_2^{40} - 2 T_1^{28} T_2^{40} - 2 T_1^{29} T_2^{40} + 4 T_1^{30} T_2^{40} - 2 T_1^{31} T_2^{40} - 2 T_1^{32} T_2^{40} + 4 T_1^{33} T_2^{40} - 3 T_1^{34} T_2^{40} + T_1^{35} T_2^{40} \big) \big\} \big\}$$

In[ ]:= **AbsoluteTiming[Θ_{T_1,T_2}[GST48];]**

Out[ ]=

$\{47.8885, \text{Null}\}$

In[ ]:= **AbsoluteTiming[$\Theta_{22/7, 34/21}$[GST48]]**

Out[ ]=

$$\Big\{ 0.414346, \Big\{ -\frac{1\,422\,357\,287\,561\,349\,859\,889}{10\,190\,414\,377\,180\,576}, -\frac{486\,885\,265\,100\,293\,177\,259\,569}{15\,915\,006\,754\,796\,041\,036\,704},$$
$$-\frac{6\,215\,902\,990\,719\,340\,337\,664\,427\,997\,383\,765\,280\,900\,656\,009}{162\,180\,513\,646\,999\,558\,542\,864\,476\,199\,651\,861\,504},$$
$$27\,865\,447\,243\,020\,145\,692\,710\,782\,578\,866\,822\,243\,946\,130\,255\,307\,494\,889\,384\,165\,533\,451\,655\,811\,667 \,\big/$$
$$624\,690\,321\,276\,677\,875\,188\,699\,659 \,\big/$$
$$14\,859\,552\,982\,340\,980\,842\,664\,641\,721\,957\,485\,175\,777\,711\,994\,175\,132\,487\,335\,203\,798\,845\,978\,446 \,\big/$$
$$417\,306\,385\,907\,712 \Big\} \Big\}$$

In[ ]:= **DuplicateFreeQ[θ /@ AllKnots[{3, 10}]]**

Out[ ]=

True

In[ ]:= **DuplicateFreeQ[θ /@ AllKnots[{3, 12}]]**

    ⬚ KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

    ⬚ KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

Out[ ]=

    False

In[ ]:= **tab11 = Table[K → θ@K, {K, AllKnots[{3, 11}] }]**

Out[ ]=

$$\left\{ \text{Knot}[3, 1] \rightarrow \left\{ \frac{1-T+T^2}{T} , -\frac{-1+T_1-T_1^2+T_2-T_1^2 T_2+2 T_1^3 T_2-T_2^2-T_1 T_2^2+T_1^2 T_2^2-2 T_1^3 T_2^2+2 T_1 T_2^3-2 T_1^2 T_2^3+2 T_1^3 T_2^3}{T_1^2 T_2} \right\}, \right.$$

$$\text{Knot}[4, 1] \rightarrow \left\{ -\frac{1-3 T+T^2}{T} , \frac{(1-3 T_1+T_1^2)\,(-1+T_1 T_2)\,(1+T_1 T_2)\,(1-3 T_2+T_2^2)}{T_1^3 T_2^2} \right\}, \text{Knot}[5, 1] \rightarrow \left\{ \frac{1-T+T^2-T^3+T^4}{T^2} , -\frac{-1+\boxed{\cdots 67 \cdots}+4 T_1^7 T_2^7}{T_1^4 T_2^3} \right\},$$

$$\boxed{\cdots 795 \cdots}, \text{Knot}[11, \text{NonAlternating}, 183] \rightarrow \left\{ \frac{\boxed{\cdots 1 \cdots}}{T^3} , \boxed{\cdots 1 \cdots} \right\},$$

$$\text{Knot}[11, \text{NonAlternating}, 184] \rightarrow \left\{ \frac{(1-T+T^2)\,(2-7 T+11 T^2-7 T^3+2 T^4)}{T^3} , -\frac{-33+\boxed{\cdots 183 \cdots}+15 T_1^{12} T_2^{12}}{T_1^7 T_2^6} \right\},$$

$$\text{Knot}[11, \text{NonAlternating}, 185] \rightarrow \left\{ -\frac{(1-3 T+T^2)\,(1-T+T^2)\,(2-3 T+2 T^2)}{T^3} , \right.$$

$$\left. \frac{-41+225 T_1-490 T_1^2+633 T_1^3-490 T_1^4+225 T_1^5-41 T_1^6+225 T_2-1054 T_1 T_2+\boxed{\cdots 156 \cdots}+222 T_1^{11} T_2^{11}-39 T_1^{12} T_2^{11}+7 T_1^6 T_2^{12}-39 T_1^7 T_2^{12}+86 T_1^8 T_2^{12}-111 T_1^9 T_2^{12}+86 T_1^{10} T_2^{12}-39 T_1^{11} T_2^{12}+7 T_1^{12} T_2^{12}}{T_1^7 T_2^6} \right\} \right\}$$

Full expression not available (original memory size: 35.9 MB)   ⚙

In[ ]:= **Gather[tab11, Last[#1] === Last[#2] &]**

Out[ ]=

$$\left\{ \left\{ \text{Knot}[3, 1] \rightarrow \left\{ \frac{1-T+T^2}{T} , -\frac{-1+T_1-T_1^2+T_2-T_1^2 T_2+2 T_1^3 T_2-T_2^2-T_1 T_2^2+T_1^2 T_2^2-2 T_1^3 T_2^2+2 T_1 T_2^3-2 T_1^2 T_2^3+2 T_1^3 T_2^3}{T_1^2 T_2} \right\} \right\}, \right.$$

$$\left\{ \text{Knot}[4, 1] \rightarrow \left\{ -\frac{1-3 T+T^2}{T} , \frac{(1-3 T_1+T_1^2)\,(-1+T_1 T_2)\,(1+T_1 T_2)\,(1-3 T_2+T_2^2)}{T_1^3 T_2^2} \right\} \right\},$$

$$\left\{ \text{Knot}[5, 1] \rightarrow \left\{ \frac{1-T+T^2-T^3+T^4}{T^2} , -\frac{-1+\boxed{\cdots 67 \cdots}+\boxed{\cdots 1 \cdots}}{T_1^4 T_2^3} \right\} \right\}, \boxed{\cdots 792 \cdots}, \left\{ \text{Knot}[11, \text{NonAlternating}, 183] \rightarrow \left\{ \boxed{\cdots 1 \cdots} \right\} \right\},$$

$$\left\{ \text{Knot}[11, \text{NonAlternating}, 184] \rightarrow \left\{ \frac{(1-T+T^2)\,(2-7 T+11 T^2-7 T^3+2 T^4)}{T^3} , -\frac{-33+\boxed{\cdots 183 \cdots}+15 T_1^{12} T_2^{12}}{T_1^7 T_2^6} \right\} \right\},$$

$$\left\{ \text{Knot}[11, \text{NonAlternating}, 185] \rightarrow \left\{ -\frac{(1-3 T+T^2)\,(1-T+T^2)\,(2-3 T+2 T^2)}{T^3} , \right. \right.$$

$$\left. \left. \frac{-41+225 T_1-490 T_1^2+633 T_1^3-490 T_1^4+225 T_1^5-41 T_1^6+225 T_2-1054 T_1 T_2+\boxed{\cdots 156 \cdots}+222 T_1^{11} T_2^{11}-39 T_1^{12} T_2^{11}+7 T_1^6 T_2^{12}-39 T_1^7 T_2^{12}+86 T_1^8 T_2^{12}-111 T_1^9 T_2^{12}+86 T_1^{10} T_2^{12}-39 T_1^{11} T_2^{12}+7 T_1^{12} T_2^{12}}{T_1^7 T_2^6} \right\} \right\}$$

$$\left. \right\}$$

Full expression not available (original memory size: 36 MB)   ⚙

In[ ]:= **Select[Gather[tab11, Last[#1] === Last[#2] &], Length[#] > 1 &]**

Out[ ]=

$$\left\{ \left\{ \text{Knot}[11, \text{Alternating}, 44] \rightarrow \right. \right.$$

$$\left\{ \frac{(1-T+T^2)^2\,(1-3 T+5 T^2-3 T^3+T^4)}{T^4} , -\frac{1}{T_1^9 T_2^8}\,(1-T_1+T_1^2)\,(1-T_2+T_2^2)\,(1-T_1 T_2+T_1^2 T_2^2) \right.$$

$$\left(-4+16 T_1-36 T_1^2+44 T_1^3-36 T_1^4+16 T_1^5-4 T_1^6+16 T_2-53 T_1 T_2+100 T_1^2 T_2-77 T_1^3 T_2+ \right.$$

$$23 T_1^4 T_2+35 T_1^5 T_2-28 T_1^6 T_2+11 T_1^7 T_2-36 T_2^2+100 T_1 T_2^2-161 T_1^2 T_2^2+54 T_1^3 T_2^2+$$

$$39 T_1^4 T_2^2-107 T_1^5 T_2^2+23 T_1^6 T_2^2+8 T_1^7 T_2^2-13 T_1^8 T_2^2+44 T_2^3-77 T_1 T_2^3+54 T_1^2 T_2^3+189 T_1^3 T_2^3-$$

$$215 T_1^4 T_2^3+153 T_1^5 T_2^3+106 T_1^6 T_2^3-107 T_1^7 T_2^3+52 T_1^8 T_2^3-36 T_2^4+23 T_1 T_2^4+39 T_1^2 T_2^4-$$

$$215 T_1^3 T_2^4-23 T_1^4 T_2^4+238 T_1^5 T_2^4-478 T_1^6 T_2^4+214 T_1^7 T_2^4-4 T_1^8 T_2^4-52 T_1^9 T_2^4+13 T_1^{10} T_2^4+$$

$$16\,T_2^5 + 35\,T_1\,T_2^5 - 107\,T_1^2\,T_2^5 + 153\,T_1^3\,T_2^5 + 238\,T_1^4\,T_2^5 - 485\,T_1^5\,T_2^5 + 547\,T_1^6\,T_2^5 - 52\,T_1^7\,T_2^5 -$$
$$194\,T_1^8\,T_2^5 + 111\,T_1^9\,T_2^5 - 8\,T_1^{10}\,T_2^5 - 11\,T_1^{11}\,T_2^5 - 4\,T_2^6 - 28\,T_1\,T_2^6 + 23\,T_1^2\,T_2^6 + 106\,T_1^3\,T_2^6 - 478\,T_1^4\,T_2^6 +$$
$$547\,T_1^5\,T_2^6 - 192\,T_1^6\,T_2^6 - 419\,T_1^7\,T_2^6 + 498\,T_1^8\,T_2^6 - 126\,T_1^9\,T_2^6 - 23\,T_1^{10}\,T_2^6 + 28\,T_1^{11}\,T_2^6 + 4\,T_1^{12}\,T_2^6 +$$
$$11\,T_1\,T_2^7 + 8\,T_1^2\,T_2^7 - 107\,T_1^3\,T_2^7 + 214\,T_1^4\,T_2^7 - 52\,T_1^5\,T_2^7 - 419\,T_1^6\,T_2^7 + 613\,T_1^7\,T_2^7 - 342\,T_1^8\,T_2^7 -$$
$$133\,T_1^9\,T_2^7 + 111\,T_1^{10}\,T_2^7 - 35\,T_1^{11}\,T_2^7 - 16\,T_1^{12}\,T_2^7 - 13\,T_1^2\,T_2^8 + 52\,T_1^3\,T_2^8 - 4\,T_1^4\,T_2^8 - 194\,T_1^5\,T_2^8 +$$
$$498\,T_1^6\,T_2^8 - 342\,T_1^7\,T_2^8 + 43\,T_1^8\,T_2^8 + 235\,T_1^9\,T_2^8 - 47\,T_1^{10}\,T_2^8 - 23\,T_1^{11}\,T_2^8 + 36\,T_1^{12}\,T_2^8 - 52\,T_1^4\,T_2^9 +$$
$$111\,T_1^5\,T_2^9 - 126\,T_1^6\,T_2^9 - 133\,T_1^7\,T_2^9 + 235\,T_1^8\,T_2^9 - 209\,T_1^9\,T_2^9 - 50\,T_1^{10}\,T_2^9 + 77\,T_1^{11}\,T_2^9 - 44\,T_1^{12}\,T_2^9 +$$
$$13\,T_1^4\,T_2^{10} - 8\,T_1^5\,T_2^{10} - 23\,T_1^6\,T_2^{10} + 111\,T_1^7\,T_2^{10} - 47\,T_1^8\,T_2^{10} - 50\,T_1^9\,T_2^{10} + 161\,T_1^{10}\,T_2^{10} - 100\,T_1^{11}\,T_2^{10} +$$
$$36\,T_1^{12}\,T_2^{10} - 11\,T_1^5\,T_2^{11} + 28\,T_1^6\,T_2^{11} - 35\,T_1^7\,T_2^{11} - 23\,T_1^8\,T_2^{11} + 77\,T_1^9\,T_2^{11} - 100\,T_1^{10}\,T_2^{11} + 53\,T_1^{11}\,T_2^{11} -$$
$$16\,T_1^{12}\,T_2^{11} + 4\,T_1^6\,T_2^{12} - 16\,T_1^7\,T_2^{12} + 36\,T_1^8\,T_2^{12} - 44\,T_1^9\,T_2^{12} + 36\,T_1^{10}\,T_2^{12} - 16\,T_1^{11}\,T_2^{12} + 4\,T_1^{12}\,T_2^{12}) \Big\},$$

$$\text{Knot}[11, \text{Alternating}, 47] \to \left\{ \frac{\left(1 - T + T^2\right)^2 \left(1 - 3\,T + 5\,T^2 - 3\,T^3 + T^4\right)}{T^4}, \right.$$

$$-\frac{1}{T_1^9\,T_2^8}\,\left(1 - T_1 + T_1^2\right)\,\left(1 - T_2 + T_2^2\right)\,\left(1 - T_1\,T_2 + T_1^2\,T_2^2\right)$$

$$\left(-4 + 16\,T_1 - 36\,T_1^2 + 44\,T_1^3 - 36\,T_1^4 + 16\,T_1^5 - 4\,T_1^6 + 16\,T_2 - 53\,T_1\,T_2 + 100\,T_1^2\,T_2 - 77\,T_1^3\,T_2 + \right.$$
$$23\,T_1^4\,T_2 + 35\,T_1^5\,T_2 - 28\,T_1^6\,T_2 + 11\,T_1^7\,T_2 - 36\,T_2^2 + 100\,T_1\,T_2^2 - 161\,T_1^2\,T_2^2 + 54\,T_1^3\,T_2^2 +$$
$$39\,T_1^4\,T_2^2 - 107\,T_1^5\,T_2^2 + 23\,T_1^6\,T_2^2 + 8\,T_1^7\,T_2^2 - 13\,T_1^8\,T_2^2 + 44\,T_2^3 - 77\,T_1\,T_2^3 + 54\,T_1^2\,T_2^3 + 189\,T_1^3\,T_2^3 -$$
$$215\,T_1^4\,T_2^3 + 153\,T_1^5\,T_2^3 + 106\,T_1^6\,T_2^3 - 107\,T_1^7\,T_2^3 + 52\,T_1^8\,T_2^3 - 36\,T_2^4 + 23\,T_1\,T_2^4 + 39\,T_1^2\,T_2^4 -$$
$$215\,T_1^3\,T_2^4 - 23\,T_1^4\,T_2^4 + 238\,T_1^5\,T_2^4 - 478\,T_1^6\,T_2^4 + 214\,T_1^7\,T_2^4 - 4\,T_1^8\,T_2^4 - 52\,T_1^9\,T_2^4 + 13\,T_1^{10}\,T_2^4 +$$
$$16\,T_2^5 + 35\,T_1\,T_2^5 - 107\,T_1^2\,T_2^5 + 153\,T_1^3\,T_2^5 + 238\,T_1^4\,T_2^5 - 485\,T_1^5\,T_2^5 + 547\,T_1^6\,T_2^5 - 52\,T_1^7\,T_2^5 -$$
$$194\,T_1^8\,T_2^5 + 111\,T_1^9\,T_2^5 - 8\,T_1^{10}\,T_2^5 - 11\,T_1^{11}\,T_2^5 - 4\,T_2^6 - 28\,T_1\,T_2^6 + 23\,T_1^2\,T_2^6 + 106\,T_1^3\,T_2^6 - 478\,T_1^4\,T_2^6 +$$
$$547\,T_1^5\,T_2^6 - 192\,T_1^6\,T_2^6 - 419\,T_1^7\,T_2^6 + 498\,T_1^8\,T_2^6 - 126\,T_1^9\,T_2^6 - 23\,T_1^{10}\,T_2^6 + 28\,T_1^{11}\,T_2^6 + 4\,T_1^{12}\,T_2^6 +$$
$$11\,T_1\,T_2^7 + 8\,T_1^2\,T_2^7 - 107\,T_1^3\,T_2^7 + 214\,T_1^4\,T_2^7 - 52\,T_1^5\,T_2^7 - 419\,T_1^6\,T_2^7 + 613\,T_1^7\,T_2^7 - 342\,T_1^8\,T_2^7 -$$
$$133\,T_1^9\,T_2^7 + 111\,T_1^{10}\,T_2^7 - 35\,T_1^{11}\,T_2^7 - 16\,T_1^{12}\,T_2^7 - 13\,T_1^2\,T_2^8 + 52\,T_1^3\,T_2^8 - 4\,T_1^4\,T_2^8 - 194\,T_1^5\,T_2^8 +$$
$$498\,T_1^6\,T_2^8 - 342\,T_1^7\,T_2^8 + 43\,T_1^8\,T_2^8 + 235\,T_1^9\,T_2^8 - 47\,T_1^{10}\,T_2^8 - 23\,T_1^{11}\,T_2^8 + 36\,T_1^{12}\,T_2^8 - 52\,T_1^4\,T_2^9 +$$
$$111\,T_1^5\,T_2^9 - 126\,T_1^6\,T_2^9 - 133\,T_1^7\,T_2^9 + 235\,T_1^8\,T_2^9 - 209\,T_1^9\,T_2^9 - 50\,T_1^{10}\,T_2^9 + 77\,T_1^{11}\,T_2^9 - 44\,T_1^{12}\,T_2^9 +$$
$$13\,T_1^4\,T_2^{10} - 8\,T_1^5\,T_2^{10} - 23\,T_1^6\,T_2^{10} + 111\,T_1^7\,T_2^{10} - 47\,T_1^8\,T_2^{10} - 50\,T_1^9\,T_2^{10} + 161\,T_1^{10}\,T_2^{10} - 100\,T_1^{11}\,T_2^{10} +$$
$$36\,T_1^{12}\,T_2^{10} - 11\,T_1^5\,T_2^{11} + 28\,T_1^6\,T_2^{11} - 35\,T_1^7\,T_2^{11} - 23\,T_1^8\,T_2^{11} + 77\,T_1^9\,T_2^{11} - 100\,T_1^{10}\,T_2^{11} + 53\,T_1^{11}\,T_2^{11} -$$
$$\left.\left. 16\,T_1^{12}\,T_2^{11} + 4\,T_1^6\,T_2^{12} - 16\,T_1^7\,T_2^{12} + 36\,T_1^8\,T_2^{12} - 44\,T_1^9\,T_2^{12} + 36\,T_1^{10}\,T_2^{12} - 16\,T_1^{11}\,T_2^{12} + 4\,T_1^{12}\,T_2^{12}\right)\right\}\Big\},$$

$$\left\{\text{Knot}[11, \text{Alternating}, 57] \to \left\{ -\frac{\left(1 - T + T^2\right)^2 \left(1 - 3\,T + 3\,T^2 - 3\,T^3 + T^4\right)}{T^4}, \right.\right.$$

$$\frac{1}{T_1^9\,T_2^8}$$

$$\left(1 - T_1 + T_1^2\right)\,\left(1 - T_2 + T_2^2\right)\,\left(1 - T_1\,T_2 + T_1^2\,T_2^2\right)$$

$$\left(-3 + 12\,T_1 - 21\,T_1^2 + 27\,T_1^3 - 21\,T_1^4 + 12\,T_1^5 - 3\,T_1^6 + 12\,T_2 - 41\,T_1\,T_2 + 56\,T_1^2\,T_2 - 59\,T_1^3\,T_2 + 21\,T_1^4\,T_2 + \right.$$
$$T_1^5\,T_2 - 16\,T_1^6\,T_2 + 7\,T_1^7\,T_2 - 21\,T_2^2 + 56\,T_1\,T_2^2 - 37\,T_1^2\,T_2^2 + 5\,T_1^3\,T_2^2 + 85\,T_1^4\,T_2^2 - 90\,T_1^5\,T_2^2 + 77\,T_1^6\,T_2^2 -$$
$$20\,T_1^7\,T_2^2 - 2\,T_1^8\,T_2^2 + 27\,T_2^3 - 59\,T_1\,T_2^3 + 5\,T_1^2\,T_2^3 + 39\,T_1^3\,T_2^3 - 152\,T_1^4\,T_2^3 + 108\,T_1^5\,T_2^3 - 52\,T_1^6\,T_2^3 -$$
$$42\,T_1^7\,T_2^3 + 44\,T_1^8\,T_2^3 - 9\,T_1^9\,T_2^3 - 21\,T_2^4 + 21\,T_1\,T_2^4 + 85\,T_1^2\,T_2^4 - 152\,T_1^3\,T_2^4 + 257\,T_1^4\,T_2^4 - 118\,T_1^5\,T_2^4 -$$
$$16\,T_1^6\,T_2^4 + 109\,T_1^7\,T_2^4 - 52\,T_1^8\,T_2^4 - 28\,T_1^9\,T_2^4 + 16\,T_1^{10}\,T_2^4 + 12\,T_2^5 + T_1\,T_2^5 - 90\,T_1^2\,T_2^5 + 108\,T_1^3\,T_2^5 -$$
$$118\,T_1^4\,T_2^5 - 55\,T_1^5\,T_2^5 + 133\,T_1^6\,T_2^5 - 68\,T_1^7\,T_2^5 - 35\,T_1^8\,T_2^5 + 92\,T_1^9\,T_2^5 - 4\,T_1^{10}\,T_2^5 - 15\,T_1^{11}\,T_2^5 - 3\,T_2^6 -$$
$$16\,T_1\,T_2^6 + 77\,T_1^2\,T_2^6 - 52\,T_1^3\,T_2^6 - 16\,T_1^4\,T_2^6 + 133\,T_1^5\,T_2^6 - 156\,T_1^6\,T_2^6 - T_1^7\,T_2^6 + 4\,T_1^8\,T_2^6 - 12\,T_1^9\,T_2^6 -$$
$$93\,T_1^{10}\,T_2^6 + 40\,T_1^{11}\,T_2^6 + 5\,T_1^{12}\,T_2^6 + 7\,T_1\,T_2^7 - 20\,T_1^2\,T_2^7 - 42\,T_1^3\,T_2^7 + 109\,T_1^4\,T_2^7 - 68\,T_1^5\,T_2^7 - T_1^6\,T_2^7 +$$
$$187\,T_1^7\,T_2^7 - 18\,T_1^8\,T_2^7 - 34\,T_1^9\,T_2^7 + 140\,T_1^{10}\,T_2^7 - 25\,T_1^{11}\,T_2^7 - 20\,T_1^{12}\,T_2^7 - 2\,T_1^2\,T_2^8 + 44\,T_1^3\,T_2^8 - 52\,T_1^4\,T_2^8 -$$

$$35\,T_1^5\,T_2^8 + 4\,T_1^6\,T_2^8 - 18\,T_1^7\,T_2^8 - 269\,T_1^8\,T_2^8 + 226\,T_1^9\,T_2^8 - 189\,T_1^{10}\,T_2^8 - 5\,T_1^{11}\,T_2^8 + 35\,T_1^{12}\,T_2^8 - 9\,T_1^3\,T_2^9 -$$
$$28\,T_1^4\,T_2^9 + 92\,T_1^5\,T_2^9 - 12\,T_1^6\,T_2^9 - 34\,T_1^7\,T_2^9 + 226\,T_1^8\,T_2^9 - 103\,T_1^9\,T_2^9 + 45\,T_1^{10}\,T_2^9 + 75\,T_1^{11}\,T_2^9 - 45\,T_1^{12}\,T_2^9 +$$
$$16\,T_1^4\,T_2^{10} - 4\,T_1^5\,T_2^{10} - 93\,T_1^6\,T_2^{10} + 140\,T_1^7\,T_2^{10} - 189\,T_1^8\,T_2^{10} + 45\,T_1^9\,T_2^{10} + 21\,T_1^{10}\,T_2^{10} - 80\,T_1^{11}\,T_2^{10} +$$
$$35\,T_1^{12}\,T_2^{10} - 15\,T_1^5\,T_2^{11} + 40\,T_1^6\,T_2^{11} - 25\,T_1^7\,T_2^{11} - 5\,T_1^8\,T_2^{11} + 75\,T_1^9\,T_2^{11} - 80\,T_1^{10}\,T_2^{11} + 65\,T_1^{11}\,T_2^{11} -$$
$$20\,T_1^{12}\,T_2^{11} + 5\,T_1^6\,T_2^{12} - 20\,T_1^7\,T_2^{12} + 35\,T_1^8\,T_2^{12} - 45\,T_1^9\,T_2^{12} + 35\,T_1^{10}\,T_2^{12} - 20\,T_1^{11}\,T_2^{12} + 5\,T_1^{12}\,T_2^{12}\big)\big\},$$

$$\text{Knot}[11, \text{Alternating}, 231] \to \left\{ -\frac{\left(1 - T + T^2\right)^2\,\left(1 - 3\,T + 3\,T^2 - 3\,T^3 + T^4\right)}{T^4}, \right.$$

$$\frac{1}{T_1^9\,T_2^8}\,\left(1 - T_1 + T_1^2\right)\,\left(1 - T_2 + T_2^2\right)\,\left(1 - T_1\,T_2 + T_1^2\,T_2^2\right)$$

$$\left(-3 + 12\,T_1 - 21\,T_1^2 + 27\,T_1^3 - 21\,T_1^4 + 12\,T_1^5 - 3\,T_1^6 + 12\,T_2 - 41\,T_1\,T_2 + 56\,T_1^2\,T_2 - 59\,T_1^3\,T_2 + 21\,T_1^4\,T_2 + \right.$$
$$T_1^5\,T_2 - 16\,T_1^6\,T_2 + 7\,T_1^7\,T_2 - 21\,T_2^2 + 56\,T_1\,T_2^2 - 37\,T_1^2\,T_2^2 + 5\,T_1^3\,T_2^2 + 85\,T_1^4\,T_2^2 - 90\,T_1^5\,T_2^2 + 77\,T_1^6\,T_2^2 -$$
$$20\,T_1^7\,T_2^2 - 2\,T_1^8\,T_2^2 + 27\,T_2^3 - 59\,T_1\,T_2^3 + 5\,T_1^2\,T_2^3 + 39\,T_1^3\,T_2^3 - 152\,T_1^4\,T_2^3 + 108\,T_1^5\,T_2^3 - 52\,T_1^6\,T_2^3 -$$
$$42\,T_1^7\,T_2^3 + 44\,T_1^8\,T_2^3 - 9\,T_1^9\,T_2^3 - 21\,T_2^4 + 21\,T_1\,T_2^4 + 85\,T_1^2\,T_2^4 - 152\,T_1^3\,T_2^4 + 257\,T_1^4\,T_2^4 - 118\,T_1^5\,T_2^4 -$$
$$16\,T_1^6\,T_2^4 + 109\,T_1^7\,T_2^4 - 52\,T_1^8\,T_2^4 - 28\,T_1^9\,T_2^4 + 16\,T_1^{10}\,T_2^4 + 12\,T_2^5 + T_1\,T_2^5 - 90\,T_1^2\,T_2^5 + 108\,T_1^3\,T_2^5 -$$
$$118\,T_1^4\,T_2^5 - 55\,T_1^5\,T_2^5 + 133\,T_1^6\,T_2^5 - 68\,T_1^7\,T_2^5 - 35\,T_1^8\,T_2^5 + 92\,T_1^9\,T_2^5 - 4\,T_1^{10}\,T_2^5 - 15\,T_1^{11}\,T_2^5 - 3\,T_2^6 -$$
$$16\,T_1\,T_2^6 + 77\,T_1^2\,T_2^6 - 52\,T_1^3\,T_2^6 - 16\,T_1^4\,T_2^6 + 133\,T_1^5\,T_2^6 - 156\,T_1^6\,T_2^6 - T_1^7\,T_2^6 + 4\,T_1^8\,T_2^6 - 12\,T_1^9\,T_2^6 -$$
$$93\,T_1^{10}\,T_2^6 + 40\,T_1^{11}\,T_2^6 + 5\,T_1^{12}\,T_2^6 + 7\,T_1\,T_2^7 - 20\,T_1^2\,T_2^7 - 42\,T_1^3\,T_2^7 + 109\,T_1^4\,T_2^7 - 68\,T_1^5\,T_2^7 - T_1^6\,T_2^7 +$$
$$187\,T_1^7\,T_2^7 - 18\,T_1^8\,T_2^7 - 34\,T_1^9\,T_2^7 + 140\,T_1^{10}\,T_2^7 - 25\,T_1^{11}\,T_2^7 - 20\,T_1^{12}\,T_2^7 - 2\,T_1^2\,T_2^8 + 44\,T_1^3\,T_2^8 - 52\,T_1^4\,T_2^8 -$$
$$35\,T_1^5\,T_2^8 + 4\,T_1^6\,T_2^8 - 18\,T_1^7\,T_2^8 - 269\,T_1^8\,T_2^8 + 226\,T_1^9\,T_2^8 - 189\,T_1^{10}\,T_2^8 - 5\,T_1^{11}\,T_2^8 + 35\,T_1^{12}\,T_2^8 - 9\,T_1^3\,T_2^9 -$$
$$28\,T_1^4\,T_2^9 + 92\,T_1^5\,T_2^9 - 12\,T_1^6\,T_2^9 - 34\,T_1^7\,T_2^9 + 226\,T_1^8\,T_2^9 - 103\,T_1^9\,T_2^9 + 45\,T_1^{10}\,T_2^9 + 75\,T_1^{11}\,T_2^9 - 45\,T_1^{12}\,T_2^9 +$$
$$16\,T_1^4\,T_2^{10} - 4\,T_1^5\,T_2^{10} - 93\,T_1^6\,T_2^{10} + 140\,T_1^7\,T_2^{10} - 189\,T_1^8\,T_2^{10} + 45\,T_1^9\,T_2^{10} + 21\,T_1^{10}\,T_2^{10} - 80\,T_1^{11}\,T_2^{10} +$$
$$35\,T_1^{12}\,T_2^{10} - 15\,T_1^5\,T_2^{11} + 40\,T_1^6\,T_2^{11} - 25\,T_1^7\,T_2^{11} - 5\,T_1^8\,T_2^{11} + 75\,T_1^9\,T_2^{11} - 80\,T_1^{10}\,T_2^{11} + 65\,T_1^{11}\,T_2^{11} -$$
$$\left.\left.20\,T_1^{12}\,T_2^{11} + 5\,T_1^6\,T_2^{12} - 20\,T_1^7\,T_2^{12} + 35\,T_1^8\,T_2^{12} - 45\,T_1^9\,T_2^{12} + 35\,T_1^{10}\,T_2^{12} - 20\,T_1^{11}\,T_2^{12} + 5\,T_1^{12}\,T_2^{12}\right)\right\}\right\},$$

$$\left\{\text{Knot}[11, \text{NonAlternating}, 73] \to \left\{\frac{\left(1 - T + T^2\right)^2}{T^2}, \right.\right.$$

$$-\frac{1}{T_1^5\,T_2^4}\,2\,\left(1 - T_1 + T_1^2\right)\,\left(1 - T_2 + T_2^2\right)\,\left(1 - T_1\,T_2 + T_1^2\,T_2^2\right)$$

$$\left.\left(-1 + T_1 - T_1^2 + T_2 + 2\,T_1^2\,T_2 - T_2^2 + 2\,T_1\,T_2^2 - 6\,T_1^2\,T_2^2 + T_1^4\,T_2^2 + 2\,T_1^3\,T_2^3 - T_1^4\,T_2^3 + T_1^2\,T_2^4 - T_1^3\,T_2^4 + T_1^4\,T_2^4\right)\right\},$$

$$\text{Knot}[11, \text{NonAlternating}, 74] \to \left\{\frac{\left(1 - T + T^2\right)^2}{T^2}, \right.$$

$$-\frac{1}{T_1^5\,T_2^4}\,2\,\left(1 - T_1 + T_1^2\right)\,\left(1 - T_2 + T_2^2\right)\,\left(1 - T_1\,T_2 + T_1^2\,T_2^2\right)$$

$$\left.\left.\left.\left(-1 + T_1 - T_1^2 + T_2 + 2\,T_1^2\,T_2 - T_2^2 + 2\,T_1\,T_2^2 - 6\,T_1^2\,T_2^2 + T_1^4\,T_2^2 + 2\,T_1^3\,T_2^3 - T_1^4\,T_2^3 + T_1^2\,T_2^4 - T_1^3\,T_2^4 + T_1^4\,T_2^4\right)\right\}\right\}\right\}$$

In[ ]:= **tab12 = Table[K → Θ@K, {K, AllKnots[{3, 12}] }]**

```
In[ ]:= dup12 = Map[First, Select[Gather[tab12, Last[#1] === Last[#2] &], Length[#] > 1 &], {2}]
```

```
Out[ ]=
        {{Knot[10, 106], Knot[12, NonAlternating, 369]},
         {Knot[11, Alternating, 44], Knot[11, Alternating, 47]},
         {Knot[11, Alternating, 57], Knot[11, Alternating, 231]},
         {Knot[11, NonAlternating, 73], Knot[11, NonAlternating, 74]},
         {Knot[12, Alternating, 30], Knot[12, Alternating, 33]},
         {Knot[12, Alternating, 122], Knot[12, Alternating, 182]},
         {Knot[12, Alternating, 164], Knot[12, Alternating, 166]},
         {Knot[12, Alternating, 167], Knot[12, Alternating, 692]},
         {Knot[12, Alternating, 273], Knot[12, Alternating, 890]},
         {Knot[12, Alternating, 341], Knot[12, Alternating, 627]},
         {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]},
         {Knot[12, Alternating, 458], Knot[12, Alternating, 887]},
         {Knot[12, Alternating, 510], Knot[12, Alternating, 821]},
         {Knot[12, NonAlternating, 56], Knot[12, NonAlternating, 57]},
         {Knot[12, NonAlternating, 60], Knot[12, NonAlternating, 61]},
         {Knot[12, NonAlternating, 62], Knot[12, NonAlternating, 66]},
         {Knot[12, NonAlternating, 144], Knot[12, NonAlternating, 507]},
         {Knot[12, NonAlternating, 313], Knot[12, NonAlternating, 430]}}
```

```
In[ ]:= Length /@ dup12
```

```
Out[ ]=
        {2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}
```

```
In[ ]:= Total[(Length /@ dup12) - 1]
```

```
Out[ ]=
        19
```

```
In[ ]:= Length /@ Select[
          Gather[tab12 /. {T_1 → 22 / 7, T_2 → 13 / 21}, Last[#1] === Last[#2] &], Length[#] > 1 &]
```

```
Out[ ]=
        {2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}
```

```
In[ ]:= Put[tab12 /. {T_1 → T1, T_2 → T2}, "Data12.m"]
```

```
In[ ]:= Table[K → θ[K],
          {K, {Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27], Knot[9, 41],
            Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42], Knot[10, 48],
            Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123], Knot[10, 129],
            Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]}}]
```

$$
Out[ ]=
$$

$$
\Big\{ Knot[6, 1] \rightarrow
$$

$$
\Big\{ -\frac{(-2 + T)\ (-1 + 2\,T)}{T},\ \frac{1}{T_1^3\,T_2^2}\ \big(-7 + 17\,T_1 - 7\,T_1^2 + 17\,T_2 - 44\,T_1\,T_2 + 26\,T_1^2\,T_2 - 3\,T_1^3\,T_2 - 7\,T_2^2 + 26\,T_1\,T_2^2 -
$$

$$
24\,T_1^2\,T_2^2 - 14\,T_1^3\,T_2^2 + 9\,T_1^4\,T_2^2 - 3\,T_1\,T_2^3 - 14\,T_1^2\,T_2^3 + 56\,T_1^3\,T_2^3 - 23\,T_1^4\,T_2^3 + 9\,T_1^2\,T_2^4 - 23\,T_1^3\,T_2^4 + 9\,T_1^4\,T_2^4\big) \Big\},
$$

$\text{Knot}[8, 8] \to \left\{ \dfrac{\left(2 - 2\,T + T^2\right)\left(1 - 2\,T + 2\,T^2\right)}{T^2}, \right.$

$-\dfrac{1}{T_1^5\,T_2^4}\left(-17 + 51\,T_1 - 77\,T_1^2 + 51\,T_1^3 - 17\,T_1^4 + 51\,T_2 - 126\,T_1\,T_2 + 150\,T_1^2\,T_2 - 30\,T_1^3\,T_2 - 30\,T_1^4\,T_2 + \right.$

$\quad 27\,T_1^5\,T_2 - 77\,T_2^2 + 150\,T_1\,T_2^2 - 117\,T_1^2\,T_2^2 - 113\,T_1^3\,T_2^2 + 135\,T_1^4\,T_2^2 - 66\,T_1^5\,T_2^2 - 5\,T_1^6\,T_2^2 + 51\,T_2^3 -$

$\quad 30\,T_1\,T_2^3 - 113\,T_1^2\,T_2^3 + 336\,T_1^3\,T_2^3 - 186\,T_1^4\,T_2^3 - 5\,T_1^5\,T_2^3 + 78\,T_1^6\,T_2^3 - 21\,T_1^7\,T_2^3 - 17\,T_2^4 - 30\,T_1\,T_2^4 +$

$\quad 135\,T_1^2\,T_2^4 - 186\,T_1^3\,T_2^4 - 60\,T_1^4\,T_2^4 + 222\,T_1^5\,T_2^4 - 153\,T_1^6\,T_2^4 + 18\,T_1^7\,T_2^4 + 15\,T_1^8\,T_2^4 + 27\,T_1\,T_2^5 -$

$\quad 66\,T_1^2\,T_2^5 - 5\,T_1^3\,T_2^5 + 222\,T_1^4\,T_2^5 - 300\,T_1^5\,T_2^5 + 103\,T_1^6\,T_2^5 + 42\,T_1^7\,T_2^5 - 45\,T_1^8\,T_2^5 - 5\,T_1^2\,T_2^6 + 78\,T_1^3\,T_2^6 -$

$\quad 153\,T_1^4\,T_2^6 + 103\,T_1^5\,T_2^6 + 99\,T_1^6\,T_2^6 - 138\,T_1^7\,T_2^6 + 67\,T_1^8\,T_2^6 - 21\,T_1^3\,T_2^7 + 18\,T_1^4\,T_2^7 + 42\,T_1^5\,T_2^7 -$

$\quad \left. 138\,T_1^6\,T_2^7 + 114\,T_1^7\,T_2^7 - 45\,T_1^8\,T_2^7 + 15\,T_1^4\,T_2^8 - 45\,T_1^5\,T_2^8 + 67\,T_1^6\,T_2^8 - 45\,T_1^7\,T_2^8 + 15\,T_1^8\,T_2^8\right) \Big\},$

$\text{Knot}[8, 9] \to \left\{ -\dfrac{\left(-1 + T - 2\,T^2 + T^3\right)\left(-1 + 2\,T - T^2 + T^3\right)}{T^3}, \right.$

$\quad \dfrac{1}{T_1^7\,T_2^6}\left(-1 + T_1 - 2\,T_1^2 + T_1^3\right)\left(-1 + 2\,T_1 - T_1^2 + T_1^3\right)\left(-1 + T_1\,T_2\right)\left(1 + T_1\,T_2\right)$

$\quad\quad \left. \left(-1 + T_2 - 2\,T_2^2 + T_2^3\right)\left(-1 + 2\,T_2 - T_2^2 + T_2^3\right)\left(3 - 6\,T_1\,T_2 + 8\,T_1^2\,T_2^2 - 6\,T_1^3\,T_2^3 + 3\,T_1^4\,T_2^4\right)\right\},$

$\text{Knot}[8, 20] \to \left\{ \dfrac{\left(1 - T + T^2\right)^2}{T^2}, -\dfrac{1}{T_1^5\,T_2^4}\,2\,\left(-1 + 2\,T_1 - 3\,T_1^2 + 2\,T_1^3 - T_1^4 + 2\,T_2 - 3\,T_1\,T_2 + 4\,T_1^2\,T_2 - \right.\right.$

$\quad T_1^3\,T_2 + T_1^5\,T_2 - 3\,T_2^2 + 4\,T_1\,T_2^2 - 2\,T_1^2\,T_2^2 - 4\,T_1^3\,T_2^2 + 4\,T_1^4\,T_2^2 - 2\,T_1^5\,T_2^2 + 2\,T_2^3 - T_1\,T_2^3 - 4\,T_1^2\,T_2^3 + 5\,T_1^3\,T_2^3 -$

$\quad T_1^4\,T_2^3 - 4\,T_1^5\,T_2^3 + 2\,T_1^6\,T_2^3 - T_1^7\,T_2^3 - T_2^4 + 4\,T_1^2\,T_2^4 - T_1^3\,T_2^4 + 3\,T_1^5\,T_2^4 + 2\,T_1^6\,T_2^4 + T_1^8\,T_2^4 + T_1\,T_2^5 - 2\,T_1^2\,T_2^5 -$

$\quad 4\,T_1^3\,T_2^5 + 3\,T_1^4\,T_2^5 - 3\,T_1^5\,T_2^5 - 4\,T_1^6\,T_2^5 + T_1^7\,T_2^5 - 2\,T_1^8\,T_2^5 + 2\,T_1^3\,T_2^6 + 2\,T_1^4\,T_2^6 - 4\,T_1^5\,T_2^6 + 8\,T_1^6\,T_2^6 - 4\,T_1^7\,T_2^6 +$

$\quad \left.\left. 3\,T_1^8\,T_2^6 - T_1^3\,T_2^7 + T_1^5\,T_2^7 - 4\,T_1^6\,T_2^7 + 3\,T_1^7\,T_2^7 - 2\,T_1^8\,T_2^7 + T_1^4\,T_2^8 - 2\,T_1^5\,T_2^8 + 3\,T_1^6\,T_2^8 - 2\,T_1^7\,T_2^8 + T_1^8\,T_2^8\right)\right\},$

$\text{Knot}[9, 27] \to \left\{ -\dfrac{\left(-1 + 2\,T - 3\,T^2 + T^3\right)\left(-1 + 3\,T - 2\,T^2 + T^3\right)}{T^3}, \right.$

$\quad \dfrac{1}{T_1^7\,T_2^6}\left(-3 + 15\,T_1 - 33\,T_1^2 + 45\,T_1^3 - 33\,T_1^4 + 15\,T_1^5 - 3\,T_1^6 + 15\,T_2 - 65\,T_1\,T_2 + 115\,T_1^2\,T_2 - 115\,T_1^3\,T_2 + \right.$

$\quad 15\,T_1^4\,T_2 + 35\,T_1^5\,T_2 - 35\,T_1^6\,T_2 + 10\,T_1^7\,T_2 - 33\,T_2^2 + 115\,T_1\,T_2^2 - 125\,T_1^2\,T_2^2 + T_1^3\,T_2^2 + 265\,T_1^4\,T_2^2 - 219\,T_1^5\,T_2^2 +$

$\quad 95\,T_1^6\,T_2^2 + 5\,T_1^7\,T_2^2 - 11\,T_1^8\,T_2^2 + 45\,T_2^3 - 115\,T_1\,T_2^3 + T_1^2\,T_2^3 + 268\,T_1^3\,T_2^3 - 554\,T_1^4\,T_2^3 + 156\,T_1^5\,T_2^3 +$

$\quad 108\,T_1^6\,T_2^3 - 164\,T_1^7\,T_2^3 + 55\,T_1^8\,T_2^3 - 33\,T_2^4 + 15\,T_1\,T_2^4 + 265\,T_1^2\,T_2^4 - 554\,T_1^3\,T_2^4 + 518\,T_1^4\,T_2^4 + 342\,T_1^5\,T_2^4 -$

$\quad 542\,T_1^6\,T_2^4 + 286\,T_1^7\,T_2^4 - T_1^8\,T_2^4 - 55\,T_1^9\,T_2^4 + 11\,T_1^{10}\,T_2^4 + 15\,T_2^5 + 35\,T_1\,T_2^5 - 219\,T_1^2\,T_2^5 + 156\,T_1^3\,T_2^5 +$

$\quad 342\,T_1^4\,T_2^5 - 1095\,T_1^5\,T_2^5 + 705\,T_1^6\,T_2^5 + 67\,T_1^7\,T_2^5 - 294\,T_1^8\,T_2^5 + 166\,T_1^9\,T_2^5 - 5\,T_1^{10}\,T_2^5 - 10\,T_1^{11}\,T_2^5 - 3\,T_2^6 -$

$\quad 35\,T_1\,T_2^6 + 95\,T_1^2\,T_2^6 + 108\,T_1^3\,T_2^6 - 542\,T_1^4\,T_2^6 + 705\,T_1^5\,T_2^6 + 12\,T_1^6\,T_2^6 - 745\,T_1^7\,T_2^6 + 444\,T_1^8\,T_2^6 - 92\,T_1^9\,T_2^6 -$

$\quad 97\,T_1^{10}\,T_2^6 + 35\,T_1^{11}\,T_2^6 + 3\,T_1^{12}\,T_2^6 + 10\,T_1\,T_2^7 + 5\,T_1^2\,T_2^7 - 164\,T_1^3\,T_2^7 + 286\,T_1^4\,T_2^7 + 67\,T_1^5\,T_2^7 - 745\,T_1^6\,T_2^7 +$

$\quad 1055\,T_1^7\,T_2^7 - 208\,T_1^8\,T_2^7 - 164\,T_1^9\,T_2^7 + 221\,T_1^{10}\,T_2^7 - 35\,T_1^{11}\,T_2^7 - 15\,T_1^{12}\,T_2^7 - 11\,T_1^2\,T_2^8 + 55\,T_1^3\,T_2^8 - T_1^4\,T_2^8 -$

$\quad 294\,T_1^5\,T_2^8 + 444\,T_1^6\,T_2^8 - 208\,T_1^7\,T_2^8 - 616\,T_1^8\,T_2^8 + 546\,T_1^9\,T_2^8 - 267\,T_1^{10}\,T_2^8 - 15\,T_1^{11}\,T_2^8 + 33\,T_1^{12}\,T_2^8 -$

$\quad 55\,T_1^4\,T_2^9 + 166\,T_1^5\,T_2^9 - 92\,T_1^6\,T_2^9 - 164\,T_1^7\,T_2^9 + 546\,T_1^8\,T_2^9 - 252\,T_1^9\,T_2^9 + T_1^{10}\,T_2^9 + 115\,T_1^{11}\,T_2^9 - 45\,T_1^{12}\,T_2^9 +$

$\quad 11\,T_1^4\,T_2^{10} - 5\,T_1^5\,T_2^{10} - 97\,T_1^6\,T_2^{10} + 221\,T_1^7\,T_2^{10} - 267\,T_1^8\,T_2^{10} + T_1^9\,T_2^{10} + 123\,T_1^{10}\,T_2^{10} - 115\,T_1^{11}\,T_2^{10} +$

$\quad 33\,T_1^{12}\,T_2^{10} - 10\,T_1^5\,T_2^{11} + 35\,T_1^6\,T_2^{11} - 35\,T_1^7\,T_2^{11} - 15\,T_1^8\,T_2^{11} + 115\,T_1^9\,T_2^{11} - 115\,T_1^{10}\,T_2^{11} + 65\,T_1^{11}\,T_2^{11} -$

$\quad \left. 15\,T_1^{12}\,T_2^{11} + 3\,T_1^6\,T_2^{12} - 15\,T_1^7\,T_2^{12} + 33\,T_1^8\,T_2^{12} - 45\,T_1^9\,T_2^{12} + 33\,T_1^{10}\,T_2^{12} - 15\,T_1^{11}\,T_2^{12} + 3\,T_1^{12}\,T_2^{12}\right) \Big\},$

$\text{Knot}[9, 41] \to \left\{ \dfrac{\left(3 - 3\,T + T^2\right)\left(1 - 3\,T + 3\,T^2\right)}{T^2}, \right.$

$$-\frac{1}{T_1^5 T_2^4}\Big(-51 + 201\,T_1 - 315\,T_1^2 + 201\,T_1^3 - 51\,T_1^4 + 201\,T_2 - 698\,T_1\,T_2 + 880\,T_1^2\,T_2 - 236\,T_1^3\,T_2 -$$
$$158\,T_1^4\,T_2 + 93\,T_1^5\,T_2 - 315\,T_2^2 + 880\,T_1\,T_2^2 - 519\,T_1^2\,T_2^2 - 1035\,T_1^3\,T_2^2 + 1305\,T_1^4\,T_2^2 - 488\,T_1^5\,T_2^2 +$$
$$27\,T_1^6\,T_2^2 + 201\,T_2^3 - 236\,T_1\,T_2^3 - 1035\,T_1^2\,T_2^3 + 2964\,T_1^3\,T_2^3 - 2484\,T_1^4\,T_2^3 + 333\,T_1^5\,T_2^3 + 376\,T_1^6\,T_2^3 -$$
$$123\,T_1^7\,T_2^3 - 51\,T_2^4 - 158\,T_1\,T_2^4 + 1305\,T_1^2\,T_2^4 - 2484\,T_1^3\,T_2^4 + 1188\,T_1^4\,T_2^4 + 1692\,T_1^5\,T_2^4 -$$
$$1467\,T_1^6\,T_2^4 + 274\,T_1^7\,T_2^4 + 57\,T_1^8\,T_2^4 + 93\,T_1\,T_2^5 - 488\,T_1^2\,T_2^5 + 333\,T_1^3\,T_2^5 + 1692\,T_1^4\,T_2^5 -$$
$$3756\,T_1^5\,T_2^5 + 1701\,T_1^6\,T_2^5 + 124\,T_1^7\,T_2^5 - 231\,T_1^8\,T_2^5 + 27\,T_1^2\,T_2^6 + 376\,T_1^3\,T_2^6 - 1467\,T_1^4\,T_2^6 +$$
$$1701\,T_1^5\,T_2^6 + 357\,T_1^6\,T_2^6 - 992\,T_1^7\,T_2^6 + 369\,T_1^8\,T_2^6 - 123\,T_1^3\,T_2^7 + 274\,T_1^4\,T_2^7 + 124\,T_1^5\,T_2^7 -$$
$$992\,T_1^6\,T_2^7 + 814\,T_1^7\,T_2^7 - 231\,T_1^8\,T_2^7 + 57\,T_1^4\,T_2^8 - 231\,T_1^5\,T_2^8 + 369\,T_1^6\,T_2^8 - 231\,T_1^7\,T_2^8 + 57\,T_1^8\,T_2^8\Big)\Big\},$$

$$\text{Knot}[9, 46] \to \Big\{-\frac{(-2 + T)\,(-1 + 2\,T)}{T},\ \frac{1}{T_1^3 T_2^2}\Big(-5 + 11\,T_1 - 5\,T_1^2 + 11\,T_2 - 32\,T_1\,T_2 +$$
$$38\,T_1^2\,T_2 - 9\,T_1^3\,T_2 - 5\,T_2^2 + 38\,T_1\,T_2^2 - 72\,T_1^2\,T_2^2 - 2\,T_1^3\,T_2^2 + 11\,T_1^4\,T_2^2 -$$
$$9\,T_1\,T_2^3 - 2\,T_1^2\,T_2^3 + 68\,T_1^3\,T_2^3 - 29\,T_1^4\,T_2^3 + 11\,T_1^2\,T_2^4 - 29\,T_1^3\,T_2^4 + 11\,T_1^4\,T_2^4\Big)\Big\},$$

$$\text{Knot}[10, 3] \to \Big\{-\frac{(-3 + 2\,T)\,(-2 + 3\,T)}{T},\ \frac{1}{T_1^3 T_2^2}\Big(-171 + 367\,T_1 - 171\,T_1^2 + 367\,T_2 - 888\,T_1\,T_2 +$$
$$594\,T_1^2\,T_2 - 101\,T_1^3\,T_2 - 171\,T_2^2 + 594\,T_1\,T_2^2 - 420\,T_1^2\,T_2^2 - 342\,T_1^3\,T_2^2 + 261\,T_1^4\,T_2^2 -$$
$$101\,T_1\,T_2^3 - 342\,T_1^2\,T_2^3 + 1140\,T_1^3\,T_2^3 - 569\,T_1^4\,T_2^3 + 261\,T_1^2\,T_2^4 - 569\,T_1^3\,T_2^4 + 261\,T_1^4\,T_2^4\Big)\Big\},$$

$$\text{Knot}[10, 22] \to \Big\{-\frac{(-2 + 2\,T - 2\,T^2 + T^3)\,(-1 + 2\,T - 2\,T^2 + 2\,T^3)}{T^3},$$

$$\frac{1}{T_1^7 T_2^6}\Big(-25 + 75\,T_1 - 125\,T_1^2 + 163\,T_1^3 - 125\,T_1^4 + 75\,T_1^5 - 25\,T_1^6 + 75\,T_2 - 174\,T_1\,T_2 + 222\,T_1^2\,T_2 -$$
$$234\,T_1^3\,T_2 + 42\,T_1^4\,T_2 + 30\,T_1^5\,T_2 - 78\,T_1^6\,T_2 + 51\,T_1^7\,T_2 - 125\,T_2^2 + 222\,T_1\,T_2^2 - 211\,T_1^2\,T_2^2 + 189\,T_1^3\,T_2^2 +$$
$$141\,T_1^4\,T_2^2 - 91\,T_1^5\,T_2^2 + 109\,T_1^6\,T_2^2 - 18\,T_1^7\,T_2^2 - 45\,T_1^8\,T_2^2 + 163\,T_2^3 - 234\,T_1\,T_2^3 + 189\,T_1^2\,T_2^3 - 204\,T_1^3\,T_2^3 -$$
$$184\,T_1^4\,T_2^3 - 52\,T_1^5\,T_2^3 + 6\,T_1^6\,T_2^3 - 111\,T_1^7\,T_2^3 + 114\,T_1^8\,T_2^3 + 7\,T_1^9\,T_2^3 - 125\,T_2^4 + 42\,T_1\,T_2^4 + 141\,T_1^2\,T_2^4 -$$
$$184\,T_1^3\,T_2^4 + 557\,T_1^4\,T_2^4 - 83\,T_1^5\,T_2^4 + 105\,T_1^6\,T_2^4 + 24\,T_1^7\,T_2^4 + 5\,T_1^8\,T_2^4 - 126\,T_1^9\,T_2^4 + 35\,T_1^{10}\,T_2^4 + 75\,T_2^5 +$$
$$30\,T_1\,T_2^5 - 91\,T_1^2\,T_2^5 - 52\,T_1^3\,T_2^5 - 83\,T_1^4\,T_2^5 - 416\,T_1^5\,T_2^5 + 174\,T_1^6\,T_2^5 - 63\,T_1^7\,T_2^5 - 32\,T_1^8\,T_2^5 + 129\,T_1^9\,T_2^5 +$$
$$30\,T_1^{10}\,T_2^5 - 45\,T_1^{11}\,T_2^5 - 25\,T_2^6 - 78\,T_1\,T_2^6 + 109\,T_1^2\,T_2^6 + 6\,T_1^3\,T_2^6 + 105\,T_1^4\,T_2^6 + 174\,T_1^5\,T_2^6 + 108\,T_1^6\,T_2^6 -$$
$$218\,T_1^7\,T_2^6 + 73\,T_1^8\,T_2^6 - 66\,T_1^9\,T_2^6 - 115\,T_1^{10}\,T_2^6 + 66\,T_1^{11}\,T_2^6 + 23\,T_1^{12}\,T_2^6 + 51\,T_1\,T_2^7 - 18\,T_1^2\,T_2^7 -$$
$$111\,T_1^3\,T_2^7 + 24\,T_1^4\,T_2^7 - 63\,T_1^5\,T_2^7 - 218\,T_1^6\,T_2^7 + 372\,T_1^7\,T_2^7 - 43\,T_1^8\,T_2^7 + 44\,T_1^9\,T_2^7 + 109\,T_1^{10}\,T_2^7 -$$
$$18\,T_1^{11}\,T_2^7 - 69\,T_1^{12}\,T_2^7 - 45\,T_1^2\,T_2^8 + 114\,T_1^3\,T_2^8 + 5\,T_1^4\,T_2^8 - 32\,T_1^5\,T_2^8 + 73\,T_1^6\,T_2^8 - 43\,T_1^7\,T_2^8 - 379\,T_1^8\,T_2^8 +$$
$$176\,T_1^9\,T_2^8 - 131\,T_1^{10}\,T_2^8 - 54\,T_1^{11}\,T_2^8 + 115\,T_1^{12}\,T_2^8 + 7\,T_1^3\,T_2^9 - 126\,T_1^4\,T_2^9 + 129\,T_1^5\,T_2^9 - 66\,T_1^6\,T_2^9 +$$
$$44\,T_1^7\,T_2^9 + 176\,T_1^8\,T_2^9 + 144\,T_1^9\,T_2^9 - 171\,T_1^{10}\,T_2^9 + 222\,T_1^{11}\,T_2^9 - 149\,T_1^{12}\,T_2^9 + 35\,T_1^4\,T_2^{10} + 30\,T_1^5\,T_2^{10} -$$
$$115\,T_1^6\,T_2^{10} + 109\,T_1^7\,T_2^{10} - 131\,T_1^8\,T_2^{10} - 171\,T_1^9\,T_2^{10} + 205\,T_1^{10}\,T_2^{10} - 210\,T_1^{11}\,T_2^{10} + 115\,T_1^{12}\,T_2^{10} -$$
$$45\,T_1^5\,T_2^{11} + 66\,T_1^6\,T_2^{11} - 18\,T_1^7\,T_2^{11} - 54\,T_1^8\,T_2^{11} + 222\,T_1^9\,T_2^{11} - 210\,T_1^{10}\,T_2^{11} + 162\,T_1^{11}\,T_2^{11} - 69\,T_1^{12}\,T_2^{11} +$$
$$23\,T_1^6\,T_2^{12} - 69\,T_1^7\,T_2^{12} + 115\,T_1^8\,T_2^{12} - 149\,T_1^9\,T_2^{12} + 115\,T_1^{10}\,T_2^{12} - 69\,T_1^{11}\,T_2^{12} + 23\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10, 35] \to \Big\{\frac{(2 - 4\,T + T^2)\,(1 - 4\,T + 2\,T^2)}{T^2},$$

$$-\frac{1}{T_1^5 T_2^4}\Big(-17 + 103\,T_1 - 181\,T_1^2 + 103\,T_1^3 - 17\,T_1^4 + 103\,T_2 - 570\,T_1\,T_2 + 762\,T_1^2\,T_2 - 30\,T_1^3\,T_2 - 234\,T_1^4\,T_2 +$$
$$55\,T_1^5\,T_2 - 181\,T_2^2 + 762\,T_1\,T_2^2 + 112\,T_1^2\,T_2^2 - 2442\,T_1^3\,T_2^2 + 1708\,T_1^4\,T_2^2 - 246\,T_1^5\,T_2^2 - 13\,T_1^6\,T_2^2 + 103\,T_2^3 -$$
$$30\,T_1\,T_2^3 - 2442\,T_1^2\,T_2^3 + 5136\,T_1^3\,T_2^3 - 2172\,T_1^4\,T_2^3 - 426\,T_1^5\,T_2^3 + 330\,T_1^6\,T_2^3 - 41\,T_1^7\,T_2^3 - 17\,T_2^4 - 234\,T_1\,T_2^4 +$$
$$1708\,T_1^2\,T_2^4 - 2172\,T_1^3\,T_2^4 - 1392\,T_1^4\,T_2^4 + 3108\,T_1^5\,T_2^4 - 1412\,T_1^6\,T_2^4 + 150\,T_1^7\,T_2^4 + 15\,T_1^8\,T_2^4 + 55\,T_1\,T_2^5 -$$

$$246\,T_1^2\,T_2^5 - 426\,T_1^3\,T_2^5 + 3108\,T_1^4\,T_2^5 - 4200\,T_1^5\,T_2^5 + 1590\,T_1^6\,T_2^5 + 114\,T_1^7\,T_2^5 - 89\,T_1^8\,T_2^5 - 13\,T_1^2\,T_2^6 +$$
$$330\,T_1^3\,T_2^6 - 1412\,T_1^4\,T_2^6 + 1590\,T_1^5\,T_2^6 + 184\,T_1^6\,T_2^6 - 678\,T_1^7\,T_2^6 + 155\,T_1^8\,T_2^6 - 41\,T_1^3\,T_2^7 + 150\,T_1^4\,T_2^7 +$$
$$114\,T_1^5\,T_2^7 - 678\,T_1^6\,T_2^7 + 486\,T_1^7\,T_2^7 - 89\,T_1^8\,T_2^7 + 15\,T_1^4\,T_2^8 - 89\,T_1^5\,T_2^8 + 155\,T_1^6\,T_2^8 - 89\,T_1^7\,T_2^8 + 15\,T_1^8\,T_2^8) \Big\},$$

$$\text{Knot}[10, 42] \to \left\{ -\frac{\left(-1 + 3\,T - 4\,T^2 + T^3\right)\left(-1 + 4\,T - 3\,T^2 + T^3\right)}{T^3}, \right.$$

$$\frac{1}{T_1^7\,T_2^6}\left(-3 + 21\,T_1 - 57\,T_1^2 + 81\,T_1^3 - 57\,T_1^4 + 21\,T_1^5 - 3\,T_1^6 + 21\,T_2 - 133\,T_1\,T_2 + 301\,T_1^2\,T_2 - 301\,T_1^3\,T_2 + \right.$$
$$21\,T_1^4\,T_2 + 119\,T_1^5\,T_2 - 77\,T_1^6\,T_2 + 14\,T_1^7\,T_2 - 57\,T_2^2 + 301\,T_1\,T_2^2 - 422\,T_1^2\,T_2^2 - 166\,T_1^3\,T_2^2 + 1164\,T_1^4\,T_2^2 -$$
$$926\,T_1^5\,T_2^2 + 262\,T_1^6\,T_2^2 + 35\,T_1^7\,T_2^2 - 19\,T_1^8\,T_2^2 + 81\,T_2^3 - 301\,T_1\,T_2^3 - 166\,T_1^2\,T_2^3 + 1864\,T_1^3\,T_2^3 - 3062\,T_1^4\,T_2^3 +$$
$$950\,T_1^5\,T_2^3 + 674\,T_1^6\,T_2^3 - 641\,T_1^7\,T_2^3 + 133\,T_1^8\,T_2^3 - 57\,T_2^4 + 21\,T_1\,T_2^4 + 1164\,T_1^2\,T_2^4 - 3062\,T_1^3\,T_2^4 +$$
$$2392\,T_1^4\,T_2^4 + 2552\,T_1^5\,T_2^4 - 3800\,T_1^6\,T_2^4 + 1690\,T_1^7\,T_2^4 - 38\,T_1^8\,T_2^4 - 133\,T_1^9\,T_2^4 + 19\,T_1^{10}\,T_2^4 + 21\,T_2^5 +$$
$$119\,T_1\,T_2^5 - 926\,T_1^2\,T_2^5 + 950\,T_1^3\,T_2^5 + 2552\,T_1^4\,T_2^5 - 7805\,T_1^5\,T_2^5 + 5699\,T_1^6\,T_2^5 - 279\,T_1^7\,T_2^5 - 1582\,T_1^8\,T_2^5 +$$
$$689\,T_1^9\,T_2^5 - 35\,T_1^{10}\,T_2^5 - 14\,T_1^{11}\,T_2^5 - 3\,T_2^6 - 77\,T_1\,T_2^6 + 262\,T_1^2\,T_2^6 + 674\,T_1^3\,T_2^6 - 3800\,T_1^4\,T_2^6 + 5699\,T_1^5\,T_2^6 -$$
$$552\,T_1^6\,T_2^6 - 5255\,T_1^7\,T_2^6 + 4018\,T_1^8\,T_2^6 - 818\,T_1^9\,T_2^6 - 274\,T_1^{10}\,T_2^6 + 77\,T_1^{11}\,T_2^6 + 3\,T_1^{12}\,T_2^6 + 14\,T_1\,T_2^7 +$$
$$35\,T_1^2\,T_2^7 - 641\,T_1^3\,T_2^7 + 1690\,T_1^4\,T_2^7 - 279\,T_1^5\,T_2^7 - 5255\,T_1^6\,T_2^7 + 8249\,T_1^7\,T_2^7 - 3110\,T_1^8\,T_2^7 - 842\,T_1^9\,T_2^7 +$$
$$974\,T_1^{10}\,T_2^7 - 119\,T_1^{11}\,T_2^7 - 21\,T_1^{12}\,T_2^7 - 19\,T_1^2\,T_2^8 + 133\,T_1^3\,T_2^8 - 38\,T_1^4\,T_2^8 - 1582\,T_1^5\,T_2^8 + 4018\,T_1^6\,T_2^8 -$$
$$3110\,T_1^7\,T_2^8 - 2174\,T_1^8\,T_2^8 + 3170\,T_1^9\,T_2^8 - 1240\,T_1^{10}\,T_2^8 - 21\,T_1^{11}\,T_2^8 + 57\,T_1^{12}\,T_2^8 - 133\,T_1^4\,T_2^9 + 689\,T_1^5\,T_2^9 -$$
$$818\,T_1^6\,T_2^9 - 842\,T_1^7\,T_2^9 + 3170\,T_1^8\,T_2^9 - 2008\,T_1^9\,T_2^9 + 214\,T_1^{10}\,T_2^9 + 301\,T_1^{11}\,T_2^9 - 81\,T_1^{12}\,T_2^9 + 19\,T_1^4\,T_2^{10} -$$
$$35\,T_1^5\,T_2^{10} - 274\,T_1^6\,T_2^{10} + 974\,T_1^7\,T_2^{10} - 1240\,T_1^8\,T_2^{10} + 214\,T_1^9\,T_2^{10} + 410\,T_1^{10}\,T_2^{10} - 301\,T_1^{11}\,T_2^{10} +$$
$$57\,T_1^{12}\,T_2^{10} - 14\,T_1^5\,T_2^{11} + 77\,T_1^6\,T_2^{11} - 119\,T_1^7\,T_2^{11} - 21\,T_1^8\,T_2^{11} + 301\,T_1^9\,T_2^{11} - 301\,T_1^{10}\,T_2^{11} + 133\,T_1^{11}\,T_2^{11} -$$
$$\left. 21\,T_1^{12}\,T_2^{11} + 3\,T_1^6\,T_2^{12} - 21\,T_1^7\,T_2^{12} + 57\,T_1^8\,T_2^{12} - 81\,T_1^9\,T_2^{12} + 57\,T_1^{10}\,T_2^{12} - 21\,T_1^{11}\,T_2^{12} + 3\,T_1^{12}\,T_2^{12}\right) \Big\},$$

$$\text{Knot}[10, 48] \to \left\{ \frac{\left(1 - T + 2\,T^2 - 2\,T^3 + T^4\right)\left(1 - 2\,T + 2\,T^2 - T^3 + T^4\right)}{T^4}, \right.$$

$$-\frac{1}{T_1^9\,T_2^8}\left(-4 + 12\,T_1 - 24\,T_1^2 + 36\,T_1^3 - 44\,T_1^4 + 36\,T_1^5 - 24\,T_1^6 + 12\,T_1^7 - 4\,T_1^8 + 12\,T_2 - 27\,T_1\,T_2 + 45\,T_1^2\,T_2 - \right.$$
$$54\,T_1^3\,T_2 + 51\,T_1^4\,T_2 - 9\,T_1^5\,T_2 - 9\,T_1^6\,T_2 + 18\,T_1^7\,T_2 - 15\,T_1^8\,T_2 + 9\,T_1^9\,T_2 - 24\,T_2^2 + 45\,T_1\,T_2^2 - 74\,T_1^2\,T_2^2 +$$
$$88\,T_1^3\,T_2^2 - 91\,T_1^4\,T_2^2 + 26\,T_1^5\,T_2^2 - 31\,T_1^6\,T_2^2 + 16\,T_1^7\,T_2^2 - 14\,T_1^8\,T_2^2 + 9\,T_1^9\,T_2^2 - 12\,T_1^{10}\,T_2^2 + 36\,T_2^3 -$$
$$54\,T_1\,T_2^3 + 88\,T_1^2\,T_2^3 - 97\,T_1^3\,T_2^3 + 100\,T_1^4\,T_2^3 - 3\,T_1^5\,T_2^3 + 42\,T_1^6\,T_2^3 - 8\,T_1^7\,T_2^3 + 11\,T_1^8\,T_2^3 - 2\,T_1^9\,T_2^3 +$$
$$9\,T_1^{10}\,T_2^3 + 9\,T_1^{11}\,T_2^3 - 44\,T_2^4 + 51\,T_1\,T_2^4 - 91\,T_1^2\,T_2^4 + 100\,T_1^3\,T_2^4 - 115\,T_1^4\,T_2^4 - 83\,T_1^6\,T_2^4 + 6\,T_1^7\,T_2^4 + T_1^8\,T_2^4 -$$
$$26\,T_1^9\,T_2^4 + 11\,T_1^{10}\,T_2^4 - 27\,T_1^{11}\,T_2^4 + 36\,T_2^5 - 9\,T_1\,T_2^5 + 26\,T_1^2\,T_2^5 - 3\,T_1^3\,T_2^5 + 110\,T_1^5\,T_2^5 + 44\,T_1^6\,T_2^5 +$$
$$44\,T_1^7\,T_2^5 - 16\,T_1^8\,T_2^5 + 9\,T_1^9\,T_2^5 + 24\,T_1^{10}\,T_2^5 - T_1^{11}\,T_2^5 + 27\,T_1^{12}\,T_2^5 - 9\,T_1^{13}\,T_2^5 - 24\,T_2^6 - 9\,T_1\,T_2^6 - 31\,T_1^2\,T_2^6 +$$
$$42\,T_1^3\,T_2^6 - 83\,T_1^4\,T_2^6 + 44\,T_1^5\,T_2^6 - 212\,T_1^6\,T_2^6 + 38\,T_1^7\,T_2^6 - 20\,T_1^8\,T_2^6 + 8\,T_1^9\,T_2^6 + T_1^{10}\,T_2^6 - 30\,T_1^{11}\,T_2^6 -$$
$$7\,T_1^{12}\,T_2^6 - 9\,T_1^{13}\,T_2^6 + 12\,T_1^{14}\,T_2^6 + 12\,T_2^7 + 18\,T_1\,T_2^7 + 16\,T_1^2\,T_2^7 - 8\,T_1^3\,T_2^7 + 6\,T_1^4\,T_2^7 + 44\,T_1^5\,T_2^7 + 38\,T_1^6\,T_2^7 +$$
$$158\,T_1^7\,T_2^7 - 118\,T_1^8\,T_2^7 + 29\,T_1^9\,T_2^7 - 28\,T_1^{10}\,T_2^7 + 9\,T_1^{11}\,T_2^7 + 28\,T_1^{12}\,T_2^7 - 2\,T_1^{13}\,T_2^7 - 9\,T_1^{14}\,T_2^7 - 9\,T_1^{15}\,T_2^7 -$$
$$4\,T_2^8 - 15\,T_1\,T_2^8 - 14\,T_1^2\,T_2^8 + 11\,T_1^3\,T_2^8 + T_1^4\,T_2^8 - 16\,T_1^5\,T_2^8 - 20\,T_1^6\,T_2^8 - 118\,T_1^7\,T_2^8 + 48\,T_1^8\,T_2^8 + 74\,T_1^9\,T_2^8 +$$
$$10\,T_1^{10}\,T_2^8 + 20\,T_1^{11}\,T_2^8 - 19\,T_1^{12}\,T_2^8 - 7\,T_1^{13}\,T_2^8 + 16\,T_1^{14}\,T_2^8 + 15\,T_1^{15}\,T_2^8 + 4\,T_1^{16}\,T_2^8 + 9\,T_1\,T_2^9 + 9\,T_1^2\,T_2^9 -$$
$$2\,T_1^3\,T_2^9 - 26\,T_1^4\,T_2^9 + 9\,T_1^5\,T_2^9 + 8\,T_1^6\,T_2^9 + 29\,T_1^7\,T_2^9 + 74\,T_1^8\,T_2^9 - 202\,T_1^9\,T_2^9 + 20\,T_1^{10}\,T_2^9 - 64\,T_1^{11}\,T_2^9 +$$
$$12\,T_1^{12}\,T_2^9 + 10\,T_1^{13}\,T_2^9 - 20\,T_1^{14}\,T_2^9 - 18\,T_1^{15}\,T_2^9 - 12\,T_1^{16}\,T_2^9 - 12\,T_1^2\,T_2^{10} + 9\,T_1^3\,T_2^{10} + 11\,T_1^4\,T_2^{10} +$$
$$24\,T_1^5\,T_2^{10} + T_1^6\,T_2^{10} - 28\,T_1^7\,T_2^{10} + 10\,T_1^8\,T_2^{10} + 20\,T_1^9\,T_2^{10} + 202\,T_1^{10}\,T_2^{10} - 64\,T_1^{11}\,T_2^{10} + 85\,T_1^{12}\,T_2^{10} -$$
$$48\,T_1^{13}\,T_2^{10} + 35\,T_1^{14}\,T_2^{10} + 9\,T_1^{15}\,T_2^{10} + 24\,T_1^{16}\,T_2^{10} + 9\,T_1^3\,T_2^{11} - 27\,T_1^4\,T_2^{11} - T_1^5\,T_2^{11} - 30\,T_1^6\,T_2^{11} +$$
$$9\,T_1^7\,T_2^{11} + 20\,T_1^8\,T_2^{11} - 64\,T_1^9\,T_2^{11} - 64\,T_1^{10}\,T_2^{11} - 106\,T_1^{11}\,T_2^{11} + 18\,T_1^{12}\,T_2^{11} - 3\,T_1^{13}\,T_2^{11} - 28\,T_1^{14}\,T_2^{11} +$$
$$9\,T_1^{15}\,T_2^{11} - 36\,T_1^{16}\,T_2^{11} + 27\,T_1^5\,T_2^{12} - 7\,T_1^6\,T_2^{12} + 28\,T_1^7\,T_2^{12} - 19\,T_1^8\,T_2^{12} + 12\,T_1^9\,T_2^{12} + 85\,T_1^{10}\,T_2^{12} +$$
$$18\,T_1^{11}\,T_2^{12} + 97\,T_1^{12}\,T_2^{12} - 98\,T_1^{13}\,T_2^{12} + 95\,T_1^{14}\,T_2^{12} - 51\,T_1^{15}\,T_2^{12} + 44\,T_1^{16}\,T_2^{12} - 9\,T_1^5\,T_2^{13} - 9\,T_1^6\,T_2^{13} -$$
$$2\,T_1^7\,T_2^{13} - 7\,T_1^8\,T_2^{13} + 10\,T_1^9\,T_2^{13} - 48\,T_1^{10}\,T_2^{13} - 3\,T_1^{11}\,T_2^{13} - 98\,T_1^{12}\,T_2^{13} + 101\,T_1^{13}\,T_2^{13} - 92\,T_1^{14}\,T_2^{13} +$$

$$54 \, T_1^{15} \, T_2^{13} - 36 \, T_1^{16} \, T_2^{13} + 12 \, T_1^{6} \, T_2^{14} - 9 \, T_1^{7} \, T_2^{14} + 16 \, T_1^{8} \, T_2^{14} - 20 \, T_1^{9} \, T_2^{14} + 35 \, T_1^{10} \, T_2^{14} - 28 \, T_1^{11} \, T_2^{14} +$$
$$95 \, T_1^{12} \, T_2^{14} - 92 \, T_1^{13} \, T_2^{14} + 76 \, T_1^{14} \, T_2^{14} - 45 \, T_1^{15} \, T_2^{14} + 24 \, T_1^{16} \, T_2^{14} - 9 \, T_1^{7} \, T_2^{15} + 15 \, T_1^{8} \, T_2^{15} - 18 \, T_1^{9} \, T_2^{15} +$$
$$9 \, T_1^{10} \, T_2^{15} + 9 \, T_1^{11} \, T_2^{15} - 51 \, T_1^{12} \, T_2^{15} + 54 \, T_1^{13} \, T_2^{15} - 45 \, T_1^{14} \, T_2^{15} + 27 \, T_1^{15} \, T_2^{15} - 12 \, T_1^{16} \, T_2^{15} + 4 \, T_1^{8} \, T_2^{16} -$$
$$12 \, T_1^{9} \, T_2^{16} + 24 \, T_1^{10} \, T_2^{16} - 36 \, T_1^{11} \, T_2^{16} + 44 \, T_1^{12} \, T_2^{16} - 36 \, T_1^{13} \, T_2^{16} + 24 \, T_1^{14} \, T_2^{16} - 12 \, T_1^{15} \, T_2^{16} + 4 \, T_1^{16} \, T_2^{16}\Big) \Big\},$$

$$\text{Knot}[10, 75] \to \left\{ - \frac{\left(-1 + 3\,T - 4\,T^2 + T^3\right) \, \left(-1 + 4\,T - 3\,T^2 + T^3\right)}{T^3}, \right.$$

$$\frac{1}{T_1^7 \, T_2^6} \Big(-3 + 21 \, T_1 - 57 \, T_1^2 + 81 \, T_1^3 - 57 \, T_1^4 + 21 \, T_1^5 - 3 \, T_1^6 + 21 \, T_2 - 133 \, T_1 \, T_2 + 301 \, T_1^2 \, T_2 - 301 \, T_1^3 \, T_2 +$$
$$21 \, T_1^4 \, T_2 + 119 \, T_1^5 \, T_2 - 77 \, T_1^6 \, T_2 + 14 \, T_1^7 \, T_2 - 57 \, T_2^2 + 301 \, T_1 \, T_2^2 - 414 \, T_1^2 \, T_2^2 - 198 \, T_1^3 \, T_2^2 + 1218 \, T_1^4 \, T_2^2 -$$
$$958 \, T_1^5 \, T_2^2 + 270 \, T_1^6 \, T_2^2 + 35 \, T_1^7 \, T_2^2 - 19 \, T_1^8 \, T_2^2 + 81 \, T_2^3 - 301 \, T_1 \, T_2^3 - 198 \, T_1^2 \, T_2^3 + 1954 \, T_1^3 \, T_2^3 - 3148 \, T_1^4 \, T_2^3 +$$
$$864 \, T_1^5 \, T_2^3 + 764 \, T_1^6 \, T_2^3 - 673 \, T_1^7 \, T_2^3 + 133 \, T_1^8 \, T_2^3 - 57 \, T_2^4 + 21 \, T_1 \, T_2^4 + 1218 \, T_1^2 \, T_2^4 - 3148 \, T_1^3 \, T_2^4 +$$
$$2358 \, T_1^4 \, T_2^4 + 2906 \, T_1^5 \, T_2^4 - 3834 \, T_1^6 \, T_2^4 + 1604 \, T_1^7 \, T_2^4 + 16 \, T_1^8 \, T_2^4 - 133 \, T_1^9 \, T_2^4 + 19 \, T_1^{10} \, T_2^4 + 21 \, T_2^5 +$$
$$119 \, T_1 \, T_2^5 - 958 \, T_1^2 \, T_2^5 + 864 \, T_1^3 \, T_2^5 + 2906 \, T_1^4 \, T_2^5 - 8283 \, T_1^5 \, T_2^5 + 5221 \, T_1^6 \, T_2^5 + 75 \, T_1^7 \, T_2^5 - 1668 \, T_1^8 \, T_2^5 +$$
$$657 \, T_1^9 \, T_2^5 - 35 \, T_1^{10} \, T_2^5 - 14 \, T_1^{11} \, T_2^5 - 3 \, T_2^6 - 77 \, T_1 \, T_2^6 + 270 \, T_1^2 \, T_2^6 + 764 \, T_1^3 \, T_2^6 - 3834 \, T_1^4 \, T_2^6 + 5221 \, T_1^5 \, T_2^6 +$$
$$900 \, T_1^6 \, T_2^6 - 5733 \, T_1^7 \, T_2^6 + 3984 \, T_1^8 \, T_2^6 - 728 \, T_1^9 \, T_2^6 - 266 \, T_1^{10} \, T_2^6 + 77 \, T_1^{11} \, T_2^6 + 3 \, T_1^{12} \, T_2^6 + 14 \, T_1 \, T_2^7 +$$
$$35 \, T_1^2 \, T_2^7 - 673 \, T_1^3 \, T_2^7 + 1604 \, T_1^4 \, T_2^7 + 75 \, T_1^5 \, T_2^7 - 5733 \, T_1^6 \, T_2^7 + 7771 \, T_1^7 \, T_2^7 - 2756 \, T_1^8 \, T_2^7 - 928 \, T_1^9 \, T_2^7 +$$
$$942 \, T_1^{10} \, T_2^7 - 119 \, T_1^{11} \, T_2^7 - 21 \, T_1^{12} \, T_2^7 - 19 \, T_1^2 \, T_2^8 + 133 \, T_1^3 \, T_2^8 + 16 \, T_1^4 \, T_2^8 - 1668 \, T_1^5 \, T_2^8 + 3984 \, T_1^6 \, T_2^8 -$$
$$2756 \, T_1^7 \, T_2^8 - 2208 \, T_1^8 \, T_2^8 + 3084 \, T_1^9 \, T_2^8 - 1186 \, T_1^{10} \, T_2^8 - 21 \, T_1^{11} \, T_2^8 + 57 \, T_1^{12} \, T_2^8 - 133 \, T_1^4 \, T_2^9 + 657 \, T_1^5 \, T_2^9 -$$
$$728 \, T_1^6 \, T_2^9 - 928 \, T_1^7 \, T_2^9 + 3084 \, T_1^8 \, T_2^9 - 1918 \, T_1^9 \, T_2^9 + 182 \, T_1^{10} \, T_2^9 + 301 \, T_1^{11} \, T_2^9 - 81 \, T_1^{12} \, T_2^9 + 19 \, T_1^4 \, T_2^{10} -$$
$$35 \, T_1^5 \, T_2^{10} - 266 \, T_1^6 \, T_2^{10} + 942 \, T_1^7 \, T_2^{10} - 1186 \, T_1^8 \, T_2^{10} + 182 \, T_1^9 \, T_2^{10} + 418 \, T_1^{10} \, T_2^{10} - 301 \, T_1^{11} \, T_2^{10} +$$
$$57 \, T_1^{12} \, T_2^{10} - 14 \, T_1^5 \, T_2^{11} + 77 \, T_1^6 \, T_2^{11} - 119 \, T_1^7 \, T_2^{11} - 21 \, T_1^8 \, T_2^{11} + 301 \, T_1^9 \, T_2^{11} - 301 \, T_1^{10} \, T_2^{11} + 133 \, T_1^{11} \, T_2^{11} -$$
$$21 \, T_1^{12} \, T_2^{11} + 3 \, T_1^6 \, T_2^{12} - 21 \, T_1^7 \, T_2^{12} + 57 \, T_1^8 \, T_2^{12} - 81 \, T_1^9 \, T_2^{12} + 57 \, T_1^{10} \, T_2^{12} - 21 \, T_1^{11} \, T_2^{12} + 3 \, T_1^{12} \, T_2^{12}\Big) \Big\},$$

$$\text{Knot}[10, 87] \to \left\{ - \frac{\left(-2 + T\right) \, \left(-1 + 2\,T\right) \, \left(1 - T + T^2\right)^2}{T^3}, \right.$$

$$\frac{1}{T_1^7 \, T_2^6} \Big(-25 + 112 \, T_1 - 224 \, T_1^2 + 287 \, T_1^3 - 224 \, T_1^4 + 112 \, T_1^5 - 25 \, T_1^6 + 112 \, T_2 - 426 \, T_1 \, T_2 + 664 \, T_1^2 \, T_2 -$$
$$606 \, T_1^3 \, T_2 + 132 \, T_1^4 \, T_2 + 178 \, T_1^5 \, T_2 - 228 \, T_1^6 \, T_2 + 76 \, T_1^7 \, T_2 - 224 \, T_2^2 + 664 \, T_1 \, T_2^2 - 570 \, T_1^2 \, T_2^2 -$$
$$95 \, T_1^3 \, T_2^2 + 1152 \, T_1^4 \, T_2^2 - 1103 \, T_1^5 \, T_2^2 + 582 \, T_1^6 \, T_2^2 + 16 \, T_1^7 \, T_2^2 - 80 \, T_1^8 \, T_2^2 + 287 \, T_2^3 - 606 \, T_1 \, T_2^3 -$$
$$95 \, T_1^2 \, T_2^3 + 1138 \, T_1^3 \, T_2^3 - 2061 \, T_1^4 \, T_2^3 + 855 \, T_1^5 \, T_2^3 + 214 \, T_1^6 \, T_2^3 - 797 \, T_1^7 \, T_2^3 + 312 \, T_1^8 \, T_2^3 + 11 \, T_1^9 \, T_2^3 -$$
$$224 \, T_2^4 + 132 \, T_1 \, T_2^4 + 1152 \, T_1^2 \, T_2^4 - 2061 \, T_1^3 \, T_2^4 + 2114 \, T_1^4 \, T_2^4 + 358 \, T_1^5 \, T_2^4 - 1378 \, T_1^6 \, T_2^4 + 1251 \, T_1^7 \, T_2^4 +$$
$$18 \, T_1^8 \, T_2^4 - 336 \, T_1^9 \, T_2^4 + 64 \, T_1^{10} \, T_2^4 + 112 \, T_2^5 + 178 \, T_1 \, T_2^5 - 1103 \, T_1^2 \, T_2^5 + 855 \, T_1^3 \, T_2^5 + 358 \, T_1^4 \, T_2^5 -$$
$$2758 \, T_1^5 \, T_2^5 + 2084 \, T_1^6 \, T_2^5 - 290 \, T_1^7 \, T_2^5 - 1269 \, T_1^8 \, T_2^5 + 823 \, T_1^9 \, T_2^5 + 16 \, T_1^{10} \, T_2^5 - 68 \, T_1^{11} \, T_2^5 - 25 \, T_2^6 -$$
$$228 \, T_1 \, T_2^6 + 582 \, T_1^2 \, T_2^6 + 214 \, T_1^3 \, T_2^6 - 1378 \, T_1^4 \, T_2^6 + 2084 \, T_1^5 \, T_2^6 + 72 \, T_1^6 \, T_2^6 - 2020 \, T_1^7 \, T_2^6 + 1970 \, T_1^8 \, T_2^6 -$$
$$338 \, T_1^9 \, T_2^6 - 606 \, T_1^{10} \, T_2^6 + 204 \, T_1^{11} \, T_2^6 + 23 \, T_1^{12} \, T_2^6 + 76 \, T_1 \, T_2^7 + 16 \, T_1^2 \, T_2^7 - 797 \, T_1^3 \, T_2^7 + 1251 \, T_1^4 \, T_2^7 -$$
$$290 \, T_1^5 \, T_2^7 - 2020 \, T_1^6 \, T_2^7 + 2822 \, T_1^7 \, T_2^7 - 938 \, T_1^8 \, T_2^7 - 873 \, T_1^9 \, T_2^7 + 1129 \, T_1^{10} \, T_2^7 - 146 \, T_1^{11} \, T_2^7 - 104 \, T_1^{12} \, T_2^7 -$$
$$80 \, T_1^2 \, T_2^8 + 312 \, T_1^3 \, T_2^8 + 18 \, T_1^4 \, T_2^8 - 1269 \, T_1^5 \, T_2^8 + 1970 \, T_1^6 \, T_2^8 - 938 \, T_1^7 \, T_2^8 - 1522 \, T_1^8 \, T_2^8 + 2043 \, T_1^9 \, T_2^8 -$$
$$1116 \, T_1^{10} \, T_2^8 - 156 \, T_1^{11} \, T_2^8 + 208 \, T_1^{12} \, T_2^8 + 11 \, T_1^3 \, T_2^9 - 336 \, T_1^4 \, T_2^9 + 823 \, T_1^5 \, T_2^9 - 338 \, T_1^6 \, T_2^9 - 873 \, T_1^7 \, T_2^9 +$$
$$2043 \, T_1^8 \, T_2^9 - 1262 \, T_1^9 \, T_2^9 + 121 \, T_1^{10} \, T_2^9 + 582 \, T_1^{11} \, T_2^9 - 265 \, T_1^{12} \, T_2^9 + 64 \, T_1^4 \, T_2^{10} + 16 \, T_1^5 \, T_2^{10} - 606 \, T_1^6 \, T_2^{10} +$$
$$1129 \, T_1^7 \, T_2^{10} - 1116 \, T_1^8 \, T_2^{10} + 121 \, T_1^9 \, T_2^{10} + 546 \, T_1^{10} \, T_2^{10} - 632 \, T_1^{11} \, T_2^{10} + 208 \, T_1^{12} \, T_2^{10} - 68 \, T_1^5 \, T_2^{11} +$$
$$204 \, T_1^6 \, T_2^{11} - 146 \, T_1^7 \, T_2^{11} - 156 \, T_1^8 \, T_2^{11} + 582 \, T_1^9 \, T_2^{11} - 632 \, T_1^{10} \, T_2^{11} + 402 \, T_1^{11} \, T_2^{11} - 104 \, T_1^{12} \, T_2^{11} + 23 \, T_1^6 \, T_2^{12} -$$
$$104 \, T_1^7 \, T_2^{12} + 208 \, T_1^8 \, T_2^{12} - 265 \, T_1^9 \, T_2^{12} + 208 \, T_1^{10} \, T_2^{12} - 104 \, T_1^{11} \, T_2^{12} + 23 \, T_1^{12} \, T_2^{12}\Big) \Big\}, \text{Knot}[10, 99] \to$$

$$\left\{ \frac{\left(1 - T + T^2\right)^4}{T^4}, -\frac{4 \, \left(1 - T_1 + T_1^2\right)^4 \, \left(-1 + T_1 \, T_2\right) \, \left(1 + T_1 \, T_2\right) \, \left(1 - T_2 + T_2^2\right)^4 \, \left(1 - T_1 \, T_2 + T_1^2 \, T_2^2\right)^3}{T_1^9 \, T_2^8} \right\},$$

$\text{Knot}[10, 123] \rightarrow$

$\left\{ \dfrac{\left(1 - 3\,T + 3\,T^2 - 3\,T^3 + T^4\right)^2}{T^4} \right.,$

$-\dfrac{1}{T_1^9\,T_2^8}\, 2\,\left(1 - 3\,T_1 + 3\,T_1^2 - 3\,T_1^3 + T_1^4\right)^2\,\left(-1 + T_1\,T_2\right)\,\left(1 + T_1\,T_2\right)\,\left(2 - 3\,T_1\,T_2 + 2\,T_1^2\,T_2^2\right)$

$\left. \left(1 - 3\,T_2 + 3\,T_2^2 - 3\,T_2^3 + T_2^4\right)^2\,\left(1 - 3\,T_1\,T_2 + 3\,T_1^2\,T_2^2 - 3\,T_1^3\,T_2^3 + T_1^4\,T_2^4\right) \right\},$

$\text{Knot}[10, 129] \rightarrow \left\{ \dfrac{\left(2 - 2\,T + T^2\right)\,\left(1 - 2\,T + 2\,T^2\right)}{T^2} \right.,$

$-\dfrac{1}{T_1^5\,T_2^4}\,\left(-17 + 50\,T_1 - 75\,T_1^2 + 50\,T_1^3 - 17\,T_1^4 + 50\,T_2 - 124\,T_1\,T_2 + 146\,T_1^2\,T_2 - 34\,T_1^3\,T_2 - 28\,T_1^4\,T_2 + \right.$

$26\,T_1^5\,T_2 - 75\,T_2^2 + 146\,T_1\,T_2^2 - 77\,T_1^2\,T_2^2 - 151\,T_1^3\,T_2^2 + 175\,T_1^4\,T_2^2 - 70\,T_1^5\,T_2^2 - 3\,T_1^6\,T_2^2 + 50\,T_2^3 -$

$34\,T_1\,T_2^3 - 151\,T_1^2\,T_2^3 + 332\,T_1^3\,T_2^3 - 190\,T_1^4\,T_2^3 - 43\,T_1^5\,T_2^3 + 74\,T_1^6\,T_2^3 - 22\,T_1^7\,T_2^3 - 17\,T_2^4 - 28\,T_1\,T_2^4 +$

$175\,T_1^2\,T_2^4 - 190\,T_1^3\,T_2^4 - 12\,T_1^4\,T_2^4 + 218\,T_1^5\,T_2^4 - 113\,T_1^6\,T_2^4 + 20\,T_1^7\,T_2^4 + 15\,T_1^8\,T_2^4 + 26\,T_1\,T_2^5 -$

$70\,T_1^2\,T_2^5 - 43\,T_1^3\,T_2^5 + 218\,T_1^4\,T_2^5 - 304\,T_1^5\,T_2^5 + 65\,T_1^6\,T_2^5 + 38\,T_1^7\,T_2^5 - 46\,T_1^8\,T_2^5 - 3\,T_1^2\,T_2^6 + 74\,T_1^3\,T_2^6 -$

$113\,T_1^4\,T_2^6 + 65\,T_1^5\,T_2^6 + 139\,T_1^6\,T_2^6 - 142\,T_1^7\,T_2^6 + 69\,T_1^8\,T_2^6 - 22\,T_1^3\,T_2^7 + 20\,T_1^4\,T_2^7 + 38\,T_1^5\,T_2^7 -$

$\left. 142\,T_1^6\,T_2^7 + 116\,T_1^7\,T_2^7 - 46\,T_1^8\,T_2^7 + 15\,T_1^4\,T_2^8 - 46\,T_1^5\,T_2^8 + 69\,T_1^6\,T_2^8 - 46\,T_1^7\,T_2^8 + 15\,T_1^8\,T_2^8\right) \right\},$

$\text{Knot}[10, 137] \rightarrow \left\{ \dfrac{\left(1 - 3\,T + T^2\right)^2}{T^2} \right., \; -\dfrac{1}{T_1^5\,T_2^4}\, 2\,\left(1 - 3\,T_1 + T_1^2\right)\,\left(1 - 3\,T_2 + T_2^2\right)$

$\left(1 - 3\,T_1\,T_2 + T_1^2\,T_2^2\right)\,\left(-1 + 3\,T_1 - T_1^2 + 3\,T_2 - 8\,T_1\,T_2 + 4\,T_1^2\,T_2 - T_2^2 + 4\,T_1\,T_2^2 - \right.$

$\left. 6\,T_1^2\,T_2^2 - 2\,T_1^3\,T_2^2 + T_1^4\,T_2^2 - 2\,T_1^2\,T_2^3 + 10\,T_1^3\,T_2^3 - 3\,T_1^4\,T_2^3 + T_1^2\,T_2^4 - 3\,T_1^3\,T_2^4 + T_1^4\,T_2^4\right) \right\},$

$\text{Knot}[10, 140] \rightarrow \left\{ \dfrac{\left(1 - T + T^2\right)^2}{T^2} \right., \; -\dfrac{1}{T_1^5\,T_2^4}\, 2\,\left(-1 + 2\,T_1 - 3\,T_1^2 + 2\,T_1^3 - T_1^4 + 2\,T_2 - 3\,T_1\,T_2 + 4\,T_1^2\,T_2 - \right.$

$T_1^3\,T_2 + T_1^5\,T_2 - 3\,T_2^2 + 4\,T_1\,T_2^2 + T_1^2\,T_2^2 - 8\,T_1^3\,T_2^2 + 7\,T_1^4\,T_2^2 - 2\,T_1^5\,T_2^2 + 2\,T_2^3 - T_1\,T_2^3 - 8\,T_1^2\,T_2^3 + 6\,T_1^3\,T_2^3 -$

$8\,T_1^5\,T_2^3 + 2\,T_1^6\,T_2^3 - T_1^7\,T_2^3 - T_2^4 + 7\,T_1^2\,T_2^4 + 4\,T_1^5\,T_2^4 + 5\,T_1^6\,T_2^4 + T_1^8\,T_2^4 + T_1\,T_2^5 - 2\,T_1^2\,T_2^5 - 8\,T_1^3\,T_2^5 +$

$4\,T_1^4\,T_2^5 - 2\,T_1^5\,T_2^5 - 8\,T_1^6\,T_2^5 + T_1^7\,T_2^5 - 2\,T_1^8\,T_2^5 + 2\,T_1^3\,T_2^6 + 5\,T_1^4\,T_2^6 - 8\,T_1^5\,T_2^6 + 11\,T_1^6\,T_2^6 - 4\,T_1^7\,T_2^6 +$

$\left. 3\,T_1^8\,T_2^6 - T_1^3\,T_2^7 + T_1^5\,T_2^7 - 4\,T_1^6\,T_2^7 + 3\,T_1^7\,T_2^7 - 2\,T_1^8\,T_2^7 + T_1^4\,T_2^8 - 2\,T_1^5\,T_2^8 + 3\,T_1^6\,T_2^8 - 2\,T_1^7\,T_2^8 + T_1^8\,T_2^8\right) \right\},$

$\text{Knot}[10, 153] \rightarrow \left\{ \dfrac{\left(1 - T + T^3\right)\,\left(1 - T^2 + T^3\right)}{T^3} \right., \; -\dfrac{1}{T_1^7\,T_2^6}\,\left(-2 + 2\,T_1 + 2\,T_1^2 - 6\,T_1^3 + 2\,T_1^4 + 2\,T_1^5 - 2\,T_1^6 + \right.$

$2\,T_2 - 3\,T_1\,T_2 + 3\,T_1^3\,T_2 - T_1^4\,T_2 - T_1^6\,T_2 + T_1^7\,T_2 + 2\,T_2^2 - 2\,T_1^2\,T_2^2 + 6\,T_1^3\,T_2^2 - 2\,T_1^5\,T_2^2 + 2\,T_1^6\,T_2^2 + 2\,T_1^7\,T_2^2 -$

$6\,T_2^3 + 3\,T_1\,T_2^3 + 6\,T_1^2\,T_2^3 - 14\,T_1^3\,T_2^3 - 2\,T_1^4\,T_2^3 + 2\,T_1^5\,T_2^3 + 4\,T_1^6\,T_2^3 - 5\,T_1^7\,T_2^3 - 5\,T_1^8\,T_2^3 + 3\,T_1^9\,T_2^3 + 2\,T_2^4 -$

$T_1\,T_2^4 - 2\,T_1^3\,T_2^4 + 5\,T_1^4\,T_2^4 + 7\,T_1^5\,T_2^4 - 11\,T_1^6\,T_2^4 - 2\,T_1^7\,T_2^4 + 10\,T_1^8\,T_2^4 - 3\,T_1^9\,T_2^4 - 2\,T_1^{10}\,T_2^4 + 2\,T_2^5 - 2\,T_1^2\,T_2^5 +$

$2\,T_1^3\,T_2^5 + 7\,T_1^4\,T_2^5 - 5\,T_1^5\,T_2^5 - 7\,T_1^6\,T_2^5 + 18\,T_1^7\,T_2^5 - 10\,T_1^8\,T_2^5 - 3\,T_1^9\,T_2^5 + 8\,T_1^{10}\,T_2^5 - 3\,T_1^{11}\,T_2^5 - 2\,T_2^6 - T_1\,T_2^6 +$

$2\,T_1^2\,T_2^6 + 4\,T_1^3\,T_2^6 - 11\,T_1^4\,T_2^6 - 7\,T_1^5\,T_2^6 + 24\,T_1^6\,T_2^6 - 13\,T_1^7\,T_2^6 - 9\,T_1^8\,T_2^6 + 16\,T_1^9\,T_2^6 - 6\,T_1^{10}\,T_2^6 - 3\,T_1^{11}\,T_2^6 +$

$4\,T_1^{12}\,T_2^6 + T_1\,T_2^7 + 2\,T_1^2\,T_2^7 - 5\,T_1^3\,T_2^7 - 2\,T_1^4\,T_2^7 + 18\,T_1^5\,T_2^7 - 13\,T_1^6\,T_2^7 - 15\,T_1^7\,T_2^7 + 29\,T_1^8\,T_2^7 - 14\,T_1^9\,T_2^7 -$

$6\,T_1^{10}\,T_2^7 + 10\,T_1^{11}\,T_2^7 - 4\,T_1^{12}\,T_2^7 - 5\,T_1^3\,T_2^8 + 10\,T_1^4\,T_2^8 - 10\,T_1^5\,T_2^8 - 9\,T_1^6\,T_2^8 + 29\,T_1^7\,T_2^8 - 25\,T_1^8\,T_2^8 - 10\,T_1^9\,T_2^8 +$

$20\,T_1^{10}\,T_2^8 - 7\,T_1^{11}\,T_2^8 - 4\,T_1^{12}\,T_2^8 + 3\,T_1^3\,T_2^9 - 3\,T_1^4\,T_2^9 - 3\,T_1^5\,T_2^9 + 16\,T_1^6\,T_2^9 - 14\,T_1^7\,T_2^9 - 10\,T_1^8\,T_2^9 + 34\,T_1^9\,T_2^9 -$

$14\,T_1^{10}\,T_2^9 - 11\,T_1^{11}\,T_2^9 + 12\,T_1^{12}\,T_2^9 - 2\,T_1^4\,T_2^{10} + 8\,T_1^5\,T_2^{10} - 6\,T_1^6\,T_2^{10} - 6\,T_1^7\,T_2^{10} + 20\,T_1^8\,T_2^{10} - 14\,T_1^9\,T_2^{10} -$

$2\,T_1^{10}\,T_2^{10} + 10\,T_1^{11}\,T_2^{10} - 4\,T_1^{12}\,T_2^{10} - 3\,T_1^5\,T_2^{11} - 3\,T_1^6\,T_2^{11} + 10\,T_1^7\,T_2^{11} - 7\,T_1^8\,T_2^{11} - 11\,T_1^9\,T_2^{11} + 10\,T_1^{10}\,T_2^{11} -$

$\left. T_1^{11}\,T_2^{11} - 4\,T_1^{12}\,T_2^{11} + 4\,T_1^6\,T_2^{12} - 4\,T_1^7\,T_2^{12} - 4\,T_1^8\,T_2^{12} + 12\,T_1^9\,T_2^{12} - 4\,T_1^{10}\,T_2^{12} - 4\,T_1^{11}\,T_2^{12} + 4\,T_1^{12}\,T_2^{12}\right) \right\},$

$$\text{Knot}[10, 155] \rightarrow \left\{ -\frac{\left(-1 + T - 2\,T^2 + T^3\right)\left(-1 + 2\,T - T^2 + T^3\right)}{T^3} \right., $$

$$\frac{1}{T_1^7\,T_2^6}\,\left(-3 + 9\,T_1 - 15\,T_1^2 + 21\,T_1^3 - 15\,T_1^4 + 9\,T_1^5 - 3\,T_1^6 + 9\,T_2 - 21\,T_1\,T_2 + 27\,T_1^2\,T_2 - 33\,T_1^3\,T_2 + 3\,T_1^4\,T_2 + \right.$$

$$3\,T_1^5\,T_2 - 9\,T_1^6\,T_2 + 6\,T_1^7\,T_2 - 15\,T_2^2 + 27\,T_1\,T_2^2 - 28\,T_1^2\,T_2^2 + 38\,T_1^3\,T_2^2 + 16\,T_1^4\,T_2^2 - 2\,T_1^5\,T_2^2 + 12\,T_1^6\,T_2^2 -$$

$$3\,T_1^7\,T_2^2 - 5\,T_1^8\,T_2^2 + 21\,T_2^3 - 33\,T_1\,T_2^3 + 38\,T_1^2\,T_2^3 - 64\,T_1^3\,T_2^3 - 24\,T_1^4\,T_2^3 - 12\,T_1^5\,T_2^3 - 16\,T_1^6\,T_2^3 - 7\,T_1^7\,T_2^3 +$$

$$15\,T_1^8\,T_2^3 - 15\,T_2^4 + 3\,T_1\,T_2^4 + 16\,T_1^2\,T_2^4 - 24\,T_1^3\,T_2^4 + 151\,T_1^4\,T_2^4 - 53\,T_1^5\,T_2^4 + 63\,T_1^6\,T_2^4 + 4\,T_1^7\,T_2^4 - 10\,T_1^8\,T_2^4 -$$

$$15\,T_1^9\,T_2^4 + 5\,T_1^{10}\,T_2^4 + 9\,T_2^5 + 3\,T_1\,T_2^5 - 2\,T_1^2\,T_2^5 - 12\,T_1^3\,T_2^5 - 53\,T_1^4\,T_2^5 - 91\,T_1^5\,T_2^5 + 29\,T_1^6\,T_2^5 - 48\,T_1^7\,T_2^5 +$$

$$8\,T_1^8\,T_2^5 + 23\,T_1^9\,T_2^5 + 3\,T_1^{10}\,T_2^5 - 6\,T_1^{11}\,T_2^5 - 3\,T_2^6 - 9\,T_1\,T_2^6 + 12\,T_1^2\,T_2^6 - 16\,T_1^3\,T_2^6 + 63\,T_1^4\,T_2^6 + 29\,T_1^5\,T_2^6 +$$

$$60\,T_1^6\,T_2^6 - 45\,T_1^7\,T_2^6 + 41\,T_1^8\,T_2^6 - 28\,T_1^9\,T_2^6 - 16\,T_1^{10}\,T_2^6 + 9\,T_1^{11}\,T_2^6 + 3\,T_1^{12}\,T_2^6 + 6\,T_1\,T_2^7 - 3\,T_1^2\,T_2^7 - 7\,T_1^3\,T_2^7 +$$

$$4\,T_1^4\,T_2^7 - 48\,T_1^5\,T_2^7 - 45\,T_1^6\,T_2^7 + 75\,T_1^7\,T_2^7 - 43\,T_1^8\,T_2^7 + 24\,T_1^9\,T_2^7 + 18\,T_1^{10}\,T_2^7 - 3\,T_1^{11}\,T_2^7 - 9\,T_1^{12}\,T_2^7 -$$

$$5\,T_1^2\,T_2^8 + 15\,T_1^3\,T_2^8 - 10\,T_1^4\,T_2^8 + 8\,T_1^5\,T_2^8 + 41\,T_1^6\,T_2^8 - 43\,T_1^7\,T_2^8 - 47\,T_1^8\,T_2^8 + 36\,T_1^9\,T_2^8 - 36\,T_1^{10}\,T_2^8 -$$

$$3\,T_1^{11}\,T_2^8 + 15\,T_1^{12}\,T_2^8 - 15\,T_1^4\,T_2^9 + 23\,T_1^5\,T_2^9 - 28\,T_1^6\,T_2^9 + 24\,T_1^7\,T_2^9 + 36\,T_1^8\,T_2^9 + 20\,T_1^9\,T_2^9 - 22\,T_1^{10}\,T_2^9 +$$

$$33\,T_1^{11}\,T_2^9 - 21\,T_1^{12}\,T_2^9 + 5\,T_1^4\,T_2^{10} + 3\,T_1^5\,T_2^{10} - 16\,T_1^6\,T_2^{10} + 18\,T_1^7\,T_2^{10} - 36\,T_1^8\,T_2^{10} - 22\,T_1^9\,T_2^{10} + 24\,T_1^{10}\,T_2^{10} -$$

$$27\,T_1^{11}\,T_2^{10} + 15\,T_1^{12}\,T_2^{10} - 6\,T_1^5\,T_2^{11} + 9\,T_1^6\,T_2^{11} - 3\,T_1^7\,T_2^{11} - 3\,T_1^8\,T_2^{11} + 33\,T_1^9\,T_2^{11} - 27\,T_1^{10}\,T_2^{11} + 21\,T_1^{11}\,T_2^{11} -$$

$$\left.9\,T_1^{12}\,T_2^{11} + 3\,T_1^6\,T_2^{12} - 9\,T_1^7\,T_2^{12} + 15\,T_1^8\,T_2^{12} - 21\,T_1^9\,T_2^{12} + 15\,T_1^{10}\,T_2^{12} - 9\,T_1^{11}\,T_2^{12} + 3\,T_1^{12}\,T_2^{12}\right)\right\}\right\}$$

In[ ]:= `DunfieldKnots = ReadList["../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;`
`DK[n_] := DunfieldKnots[[n - 2]]`

In[ ]:= `Crossings[DK[576]]`

Out[ ]=

576

In[ ]:= `AbsoluteTiming[θ[DK[3]]]`

Out[ ]=

$$\left\{0.0110907, \right.$$

$$\left\{\frac{1 - T + T^2}{T}, -\frac{-2 + 2\,T_1 - 2\,T_1^2 + 2\,T_2 - T_1\,T_2 + T_1^2\,T_2 + T_1^3\,T_2 - 2\,T_2^2 + T_1\,T_2^2 - T_1^3\,T_2^2 + T_1\,T_2^3 - T_1^2\,T_2^3 + T_1^3\,T_2^3}{T_1^3\,T_2^2}\right\}\right\}$$

In[ ]:= `AbsoluteTiming[θ[DK[30]];]`

Out[ ]=

{2.91933, Null}

In[ ]:= `AbsoluteTiming[θ[DK[60]];]`

Out[ ]=

{27.4555, Null}

In[ ]:= `AbsoluteTiming[θ[DK[90]];]`

Out[ ]=

{227.389, Null}

In[ ]:= `AbsoluteTiming[θ120 = θ[DK[120]];]`

Out[ ]=

{761.428, Null}

$In[\bullet]:=$ **Put[θ120, "Theta4DK120.m"]**

$In[\bullet]:=$ **AbsoluteTiming[θ[DK[150]];]**

$Out[\bullet]=$
　　　{2357.39, Null}

(during the previous computation I biked home, so the AbsoluteTiming is too much)

$In[\bullet]:=$ **AbsoluteTiming[θ[DK[180]];]**

$Out[\bullet]=$
　　　{5391.24, Null}

$In[\bullet]:=$ **AbsoluteTiming[θ[DK[210]];]**

$Out[\bullet]=$
　　　{9613.68, Null}

$In[\bullet]:=$ **AbsoluteTiming[θ[DK[240]];]**

$Out[\bullet]=$
　　　{22462.4, Null}

$In[\bullet]:=$ **AbsoluteTiming[θ[DK[270]];]**

Mathematica crashed while trying the above computation.

$In[\bullet]:=$ **AbsoluteTiming[θ[DK[300]];]**

$In[\bullet]:=$ **Do[Echo@AbsoluteTiming[n → $\Theta_{22/7,34/21}$[DK[n]]], {n, 100, 1000, 100}]**

» $\Big\{1.61069, 100 → \Big\{$ $\dfrac{35\,388\,936\,522\,490\,931\,938\,908\,923\,343\,364\,558\,590\,414\,632\,463\,375\,508\,742\,089}{264\,554\,736\,545\,069\,605\,885\,631\,471\,128\,764\,401\,339\,301\,535\,744}$,

$\dfrac{525\,106\,180\,586\,933\,014\,293\,865\,927\,609\,379\,271\,742\,972\,076\,277\,257\,025\,413\,914\,338\,499}{37\,324\,734\,431\,368\,634\,257\,516\,595\,221\,111\,791\,096\,751\,570\,183\,668\,795\,296\,664\,772\,608}$,

$50\,463\,574\,955\,913\,231\,815\,385\,186\,261\,134\,862\,814\,456\,979\,779\,055\,953\,806\,229\,018\,368\,595\,827\,102\,502\,222\,063 -$
$117\,299\,430\,053\,887\,387\,799\,738\,329\,099\,644\,807\,147\,011\,110\,057\,363 /$
$78\,995\,482\,272\,843\,339\,527\,758\,555\,299\,340\,636\,345\,228\,530\,305\,737\,655\,210\,586\,135\,944\,082\,585\,735\,874\,960\,483 -$
$459\,404\,024\,632\,393\,880\,311\,552\,802\,816,$

$1\,454\,157\,786\,439\,833\,158\,073\,510\,296\,166\,664\,511\,893\,763\,273\,612\,593\,032\,508\,481\,123\,686\,040\,806\,094\,906\,156 -$
$614\,995\,898\,652\,264\,094\,554\,239\,129\,456\,588\,617\,926\,009\,448\,561\,362\,411\,131\,681\,477\,750\,921\,092\,323\,392\,446 -$
$495\,713\,256\,739\,833\,241\,088\,441\,458\,310\,488\,748\,504\,021\,676\,377\,213\,848\,231\,251\,594\,253\,709\,501\,604\,962\,486 -$
$232\,978\,740\,267\,199\,418\,705 /$
$6\,354\,394\,310\,390\,312\,822\,748\,925\,543\,501\,753\,140\,065\,543\,014\,212\,174\,242\,859\,988\,362\,148\,491\,816\,333\,172\,074 -$
$897\,312\,311\,833\,360\,632\,039\,282\,660\,548\,484\,996\,262\,406\,692\,880\,258\,968\,176\,579\,330\,558\,485\,056\,729\,052\,432 -$
$993\,117\,420\,466\,424\,511\,597\,633\,186\,035\,359\,746\,499\,955\,026\,301\,328\,379\,761\,576\,116\,224\Big\}\Big\}$

» {7.3406, 200 →

{– (72 941 025 249 230 622 091 769 886 034 332 903 937 878 867 275 035 495 850 289 152 467 601 139 729 946 680 –
691 983 449 444 238 470 173 260 899 434 879 455 547 646 677 /
79 780 391 006 864 379 747 986 053 920 193 038 680 545 693 079 622 955 011 027 668 359 182 291 645 896 903 –
218 461 275 510 571 008) ,
13 469 039 288 358 770 844 889 186 746 410 419 403 949 987 382 833 567 787 469 752 570 946 087 488 964 056 464 –
083 956 449 441 872 952 430 656 158 262 269 810 083 547 830 189 003 289 443 154 125 /
4 240 161 130 043 882 037 823 084 995 205 726 632 691 185 572 237 933 032 456 552 833 243 815 216 744 170 971 –
881 548 991 957 331 738 797 061 590 095 303 559 046 326 968 215 750 967 296,
3 058 236 953 956 402 226 943 593 388 603 713 021 071 954 699 338 326 371 450 792 000 285 430 803 814 324 110 –
911 806 690 348 020 780 088 584 382 124 603 092 971 693 299 841 778 094 187 288 377 810 035 496 408 283 188 –
130 224 093 352 681 965 580 164 395 682 496 054 504 489 551 954 332 992 465 733 972 977 594 735 369 459 115 –
633 590 163 189 798 671 672 600 349 071 866 872 120 468 309 375 /
7 389 876 778 587 670 278 409 931 856 936 212 530 694 800 372 408 625 530 583 166 986 417 139 021 654 981 203 –
589 910 511 227 601 136 991 125 732 955 086 827 137 765 975 954 473 403 792 833 419 463 344 119 138 486 741 –
874 061 457 114 480 552 952 530 491 222 541 669 872 799 328 574 041 719 777 250 405 019 238 495 420 416,
6 055 772 594 728 596 455 407 813 858 842 300 305 796 013 725 424 394 998 189 342 946 654 268 378 113 712 561 –
666 514 250 963 072 635 563 646 381 893 610 078 547 194 951 520 922 176 280 055 828 977 992 790 718 440 260 –
265 768 945 724 619 893 197 506 834 661 525 408 297 131 541 146 205 253 666 135 607 099 918 846 528 650 241 –
665 431 626 142 217 658 708 337 497 196 938 649 312 026 701 158 028 747 303 072 493 247 553 589 721 953 615 –
917 185 931 220 885 007 830 743 638 653 999 049 048 125 193 483 716 503 525 421 345 329 584 261 341 390 875 –
905 831 474 490 397 967 045 567 474 649 173 307 967 130 069 663 503 481 368 684 552 829 185 853 244 925 938 –
721 709 248 151 559 595 774 718 912 716 699 323 404 120 108 037 147 527 421 337 576 724 551 464 750 796 875 /
1 421 817 615 970 538 122 570 446 834 523 766 760 908 222 281 779 172 226 538 184 963 226 220 605 962 435 467 –
357 530 974 629 707 529 507 866 808 092 086 184 206 711 834 962 868 351 140 030 416 830 209 310 218 134 988 –
084 400 322 916 305 290 967 755 527 855 236 924 726 203 319 295 186 578 690 288 248 904 894 482 820 217 219 –
936 617 834 694 117 100 777 085 133 489 362 587 426 540 579 190 556 123 633 779 730 340 813 256 893 167 951 –
316 991 548 797 991 645 638 090 254 721 915 664 804 963 927 625 281 349 644 642 357 197 558 649 709 884 596 –
639 231 084 685 491 565 277 101 735 550 434 012 716 913 649 081 518 440 855 540 999 887 069 493 854 208}}

» {75.9612, 300 →
{54 300 428 014 802 247 763 147 703 343 836 297 447 025 108 824 684 772 425 762 525 822 095 039 545 899 375 981 953 473 178 602 586 048 430 534 584 880 163 873 723 541 762 115 735 883 067 341 959 560 581 371 283 178 656 972 648 408 925 263 946 669 /
6 741 838 682 197 306 940 008 962 116 848 220 280 436 936 971 437 572 995 472 014 771 688 913 708 639 211 514 814 195 885 491 758 038 709 972 366 558 512 006 372 340 250 849 089 814 593 530 683 936 627 298 651 512 766 464,

1 084 128 382 249 743 436 824 663 986 171 685 150 273 646 351 713 912 937 150 171 700 202 730 323 922 010 700 294 161 035 743 289 238 368 194 879 507 950 682 627 574 784 328 439 797 605 967 434 628 113 238 619 877 448 933 104 349 915 804 145 167 106 117 098 828 582 214 168 974 179 /
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