Pensieve header: Finding the A2 $d=1$ invariant using undetermined coefficients.

Searching for $Q + pxx + \epsilon(ppx + 1 + px + ppxx)$ solutions.

Initialization

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`; << Rot.m];
<< FormalGaussianIntegration.m;
i_⁺ := i + 1;
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

In[•]:=
```
Features[Knot[8, 17]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=
```
Features[18,
 C6[-1] C14[-1] X1,7[1] X3,9[-1] X5,13[-1] X8,16[1] X10,4[-1] X12,18[1] X15,2[-1] X17,11[1]]
```

In[•]:=
```
T3 = T1 T2;
S = {x__, p__};
q[s_, i_, j_] := Sum[
    xv,i (pv,i⁺ - pv,i) + xv,j (pv,j⁺ - pv,j) + (Tv^s - 1) xv,i (pv,i⁺ - pv,j⁺),
    {v, 3}];
ℒ[Xi_,j_[s_]] :=
  T3^s 𝔼[q[s, i, j] + B^-1 r0[s, i, j] + ϵ B r1[s, i, j] + ϵ r42[s, i, j] + O[ϵ]^2];
(*γ1[φ_,k_]:=φ(3/2-x1,kp1,k-x2,kp2,k-x3,kp3,k);*)
ℒ[Ck_[0]] := 𝔼[Sum[xv,k (pv,k⁺ - pv,k), {v, 3}] + O[ϵ]^2];
ℒ[Ck_[φ_]] :=
  T3^φ 𝔼[Sum[xv,k (pv,k⁺ - pv,k), {v, 3}] + B^-1 γ0[φ, k] + ϵ B γ1[φ, k] + ϵ γ42[φ, k] + O[ϵ]^2];
psi_ := Sequence[p1,i, p2,i, p3,i];
xsi_ := Sequence[x1,i, x2,i, x3,i];
vsi_ := Sequence[psi, xsi];
ℱ[is___] := 𝔼[Sum[πv,i pv,i, {i, {is}}, {v, 3}]];
ℒ[K_] := (2 π)^-Features[K][[1]] CF[ℒ /@ Features[K][[2]]];
vs[K_] := Union @@ Table[{vsi}, {i, Features[K][[1]]}]
```

In[•]:=
```
vsi
```

Out[•]=
```
Sequence[p1,i, p2,i, p3,i, x1,i, x2,i, x3,i]
```

The Various Terms (r_0)

The pxx Terms (r_0)

```
In[•]:= κ = 0;
        r₀[1, i_, j_] := Evaluate[Sum[
             a_{++κ} p_{3,k3} x_{1,k1} x_{2,k2},
             {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
            ]];
        r₀[1, i, j]
```

Out[•]=

$$a_1\, p_{3,i}\, x_{1,i}\, x_{2,i} + a_2\, p_{3,j}\, x_{1,i}\, x_{2,i} + a_5\, p_{3,i}\, x_{1,j}\, x_{2,i} + a_6\, p_{3,j}\, x_{1,j}\, x_{2,i} + a_3\, p_{3,i}\, x_{1,i}\, x_{2,j} + a_4\, p_{3,j}\, x_{1,i}\, x_{2,j} + a_7\, p_{3,i}\, x_{1,j}\, x_{2,j} + a_8\, p_{3,j}\, x_{1,j}\, x_{2,j}$$

```
In[•]:= κ = 0;
        r₀[-1, i_, j_] := Evaluate[Sum[
             d_{++κ} p_{3,k3} x_{1,k1} x_{2,k2},
             {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
            ]];
        r₀[-1, i, j]
```

Out[•]=

$$d_1\, p_{3,i}\, x_{1,i}\, x_{2,i} + d_2\, p_{3,j}\, x_{1,i}\, x_{2,i} + d_5\, p_{3,i}\, x_{1,j}\, x_{2,i} + d_6\, p_{3,j}\, x_{1,j}\, x_{2,i} + d_3\, p_{3,i}\, x_{1,i}\, x_{2,j} + d_4\, p_{3,j}\, x_{1,i}\, x_{2,j} + d_7\, p_{3,i}\, x_{1,j}\, x_{2,j} + d_8\, p_{3,j}\, x_{1,j}\, x_{2,j}$$

The ppx Terms (r_1)

```
In[•]:= κ = 0;
        r₁[1, i_, j_] := Evaluate[Sum[
             b_{++κ} x_{3,k3} p_{1,k1} p_{2,k2},
             {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
            ]];
        r₁[1, i, j]
```

Out[•]=

$$b_1\, p_{1,i}\, p_{2,i}\, x_{3,i} + b_5\, p_{1,j}\, p_{2,i}\, x_{3,i} + b_3\, p_{1,i}\, p_{2,j}\, x_{3,i} + b_7\, p_{1,j}\, p_{2,j}\, x_{3,i} + b_2\, p_{1,i}\, p_{2,i}\, x_{3,j} + b_6\, p_{1,j}\, p_{2,i}\, x_{3,j} + b_4\, p_{1,i}\, p_{2,j}\, x_{3,j} + b_8\, p_{1,j}\, p_{2,j}\, x_{3,j}$$

```
In[•]:= κ = 0;
        r₁[-1, i_, j_] := Evaluate[Sum[
             e_{++κ} x_{3,k3} p_{1,k1} p_{2,k2},
             {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
            ]];
        r₁[-1, i, j]
```

Out[•]=

$$e_1\, p_{1,i}\, p_{2,i}\, x_{3,i} + e_5\, p_{1,j}\, p_{2,i}\, x_{3,i} + e_3\, p_{1,i}\, p_{2,j}\, x_{3,i} + e_7\, p_{1,j}\, p_{2,j}\, x_{3,i} + e_2\, p_{1,i}\, p_{2,i}\, x_{3,j} + e_6\, p_{1,j}\, p_{2,i}\, x_{3,j} + e_4\, p_{1,i}\, p_{2,j}\, x_{3,j} + e_8\, p_{1,j}\, p_{2,j}\, x_{3,j}$$

The ppxx Terms (r_{42})

```
In[•]:= κ = 0;
Short[r42[1, i_, j_] = Evaluate[Plus[
     Sum[
      c++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
      {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1 + 1, 3}
     ],
     Sum[
      c++κ xν,k1 pν,k2,
      {k1, {i, j}}, {k2, {i, j}}, {ν, 3}
     ],
     c++κ
    ]]]
```

```
Out[•]//Short=
c61 + c49 p1,i x1,i + <<58>> + c48 p2,j p3,j x2,j x3,j
```

```
In[•]:= κ = 0;
Short[r42[-1, i_, j_] = Evaluate[Plus[
     Sum[
      f++κ xν1,k1 pν1,k2 xν2,k3 pν2,k4,
      {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {ν1, 2}, {ν2, ν1 + 1, 3}
     ],
     Sum[
      f++κ xν,k1 pν,k2,
      {k1, {i, j}}, {k2, {i, j}}, {ν, 3}
     ],
     f++κ
    ]]]
```

```
Out[•]//Short=
f61 + f49 p1,i x1,i + <<58>> + f48 p2,j p3,j x2,j x3,j
```

The γ Terms (γ_0, γ_1, γ_{42})

```
In[•]:= κ = 0;
γ0[1, k_] := Evaluate[g++κ p3,k x1,k x2,k];
γ1[1, k_] := Evaluate[g++κ x3,k p1,k p2,k];
γ42[1, k_] := Evaluate[Plus[
     Sum[g++κ xν,k pν,k, {ν, 3}],
     Sum[g++κ xν1,k pν1,k xν2,k pν2,k, {ν1, 2}, {ν2, ν1 + 1, 3}]
    ]];
{γ0[1, k], γ0[1, k], γ42[1, k]}
```

```
Out[•]=
{g1 p3,k x1,k x2,k, g1 p3,k x1,k x2,k,
 g3 p1,k x1,k + g4 p2,k x2,k + g6 p1,k p2,k x1,k x2,k + g5 p3,k x3,k + g7 p1,k p3,k x1,k x3,k + g8 p2,k p3,k x2,k x3,k}
```

```
In[•]:= κ = 0;
        γ0[-1, k_] := Evaluate[h++κ p3,k x1,k x2,k];
        γ1[-1, k_] := Evaluate[h++κ x3,k p1,k p2,k];
        γ42[-1, k_] := Evaluate[Plus[
             Sum[h++κ xν,k pν,k, {ν, 3}],
             Sum[h++κ xν1,k pν1,k xν2,k pν2,k, {ν1, 2}, {ν2, ν1 + 1, 3}]
            ]];
        {γ0[-1, k], γ0[-1, k], γ42[-1, k]}
```

```
Out[•]=
        {h1 p3,k x1,k x2,k, h1 p3,k x1,k x2,k,
         h3 p1,k x1,k + h4 p2,k x2,k + h6 p1,k p2,k x1,k x2,k + h5 p3,k x3,k + h7 p1,k p3,k x1,k x3,k + h8 p2,k p3,k x2,k x3,k}
```

Reidemeister 3b

```
In[•]:= Timing[{LeftR3b} =
          Cases[∫ℱ[i, j, k] × ℒ /@ (Xi,j[1] Xi⁺,k[1] Xj⁺,k⁺[1]) 𝕕{vsi, vsj, vsk, vsi⁺, vsj⁺, vsk⁺},
           𝔼[ℰ_] :↦ ℰ, ∞]]
```

```
Out[•]=
        {2.28125, {∈Series[
            T1^2 p1,2+i π1,i - (-1 + T1) T1 p1,2+j π1,i + (1 - T1) p1,2+k π1,i + T1 p1,2+j π1,j + (1 - T1) p1,2+k π1,j + p1,2+k π1,k + ··· 44 ··· +
             T1^2 T2^2 p3,2+i π3,i - T1 T2 (-1 + T1 T2) p3,2+j π3,i + (1 - T1 T2) p3,2+k π3,i + T1 T2 p3,2+j π3,j + (1 - T1 T2) p3,2+k π3,j + p3,2+k π3,k,
            3 (a1 b1 + a2 b2 + a3 b3 + a4 b4 + a5 b5 + a6 b6 + a7 b7 + a8 b8 + c1 + ··· 9 ··· + c47 + c48 + c49 + c50 + c51 + c58 + c59 + c60 + c61) +
             ··· 406 ··· + ··· 1 ···]}}
```

Full expression not available (original memory size: 3.6 MB)

```
In[•]:= Timing[{RightR3b} =
          Cases[∫ℱ[i, j, k] × ℒ /@ (Xj,k[1] Xi,k⁺[1] Xi⁺,j⁺[1]) 𝕕{vsi, vsj, vsk, vsi⁺, vsj⁺, vsk⁺},
           𝔼[ℰ_] :↦ ℰ, ∞];]
```

```
Out[•]=
        {2.40625, Null}
```

In[•]:=
```
Short[eqn = CF[LeftR3b[[1]] - RightR3b[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r0[1, i, j], a_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :↦ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[•]//Short=

$$\frac{T_1\,T_2\,(\text{<<1>>})\ \text{<<1>>}\ \pi_{1,i}\,\pi_{2,i}}{B} - \frac{\text{<<1>>}}{B} + \text{<<32>>} + \frac{a_7\ \text{<<5>>}\ \pi_{\text{<<1>>}}}{B}$$

Out[•]=

$$\{p_{3,2+i},\ p_{3,2+j},\ p_{3,2+k},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{1,k},\ \pi_{2,i},\ \pi_{2,j},\ \pi_{2,k}\}$$

Out[•]=

$$\{a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8\}$$

Out[•]//Short=

$$\left\{-\frac{a_3\,T_1^2\,T_2^2}{B} + \frac{a_3\,T_1^2\,T_2^3}{B} == 0,\ \frac{a_3\,T_1^2\,T_2}{B} - \frac{a_3\,T_1^2\,T_2^2}{B} == 0,\right.$$
$$\left.\text{<<22>>},\ -\frac{a_7}{B} - \frac{a_8}{B} + \frac{a_7\,T_2}{B} + \frac{a_8\,T_2}{B} + \frac{a_7\,T_1\,T_2}{B} - \frac{a_7\,T_1\,T_2^2}{B} == 0\right\}$$

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\left\{\left\{a_1 \to 0,\ a_3 \to 0,\ a_5 \to 0,\ a_6 \to -\frac{a_2}{T_1} - \frac{a_4\,T_2}{T_1},\ a_7 \to 0,\ a_8 \to 0\right\}\right\}$$

In[•]:=
```
sol /. (v_ → val_) :↦ (v = CF[val]);
r0[1, i, j]
```

Out[•]=

$$a_2\,p_{3,j}\,x_{1,i}\,x_{2,i} - \frac{(a_2 + a_4\,T_2)\,p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1} + a_4\,p_{3,j}\,x_{1,i}\,x_{2,j}$$

In[•]:=
```
Short[eqn = CF[Coefficient[
     LeftR3b[[2]] - RightR3b[[2]] /. v : (π | p)__ :> μ v,
     μ^3
    ]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r1[1, i, j], b_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[•]//Short=
$$B\,b_1\,(-1+T_1)\,T_1\,T_2^2\,p_{1,2+j}\,p_{2,2+i}\,\pi_{3,i} - B\,b_1\,(-1+T_1)\,T_1\,T_2^2\,p_{1,2+k}\,p_{2,2+i}\,\pi_{3,i} + \text{<<31>>} + B\,(-1+T_1)\,T_1\,(-b_2-b_4+b_2\,T_2)\,p_{1,2+i}\,p_{2,2+k}\,\pi_{3,k} - B\,(-1+T_1)\,T_1\,(-b_2-b_4+b_2\,T_2)\,p_{1,2+j}\,p_{2,2+k}\,\pi_{3,k}$$

Out[•]=
$$\{p_{1,2+i},\ p_{1,2+j},\ p_{1,2+k},\ p_{2,2+i},\ p_{2,2+j},\ p_{2,2+k},\ \pi_{3,i},\ \pi_{3,j},\ \pi_{3,k}\}$$

Out[•]=
$$\{b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6,\ b_7,\ b_8\}$$

Out[•]//Short=
$$\{-B\,b_2\,T_1^2\,T_2^2 + B\,b_2\,T_1^3\,T_2^3 == 0,\ B\,b_2\,T_1\,T_2 - B\,b_2\,T_1^2\,T_2^2 == 0,\ \text{<<1>>} == 0,\ \text{<<19>>},\ \text{<<1>>} == 0,\ B\,b_1 + \text{<<47>>} == 0,\ -B\,b_6\,T_1 - B\,b_8\,T_1 - B\,b_4\,T_2 - B\,b_8\,T_2 + B\,b_2\,T_1\,T_2 + \text{<<11>>} + B\,b_2\,T_1^2\,T_2^2 == 0\}$$

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=
$$\{\{b_1 \to 0,\ b_2 \to 0,\ b_4 \to 0,\ b_6 \to 0,\ b_7 \to -b_3 - b_5,\ b_8 \to 0\}\}$$

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val]);
r1[1, i, j]
```

Out[•]=
$$b_5\,p_{1,j}\,p_{2,i}\,x_{3,i} + b_3\,p_{1,i}\,p_{2,j}\,x_{3,i} + (-b_3-b_5)\,p_{1,j}\,p_{2,j}\,x_{3,i}$$

In[•]:=
```
Short[eqn = CF[LeftR3b[[2]] - RightR3b[[2]]], 5]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r42[1, i, j], c_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :↦ (c == 0), 3]
Short[{sol} = Solve[eqns, vars]]
```

Out[•]//Short=
$$-\left((c_{25}+c_{26}+c_{34}+c_{35}+c_{55})\,(-1+T_1)\,T_1^2\,p_{1,2+j}\,\pi_{1,i}\right) - (-1+T_1)\left(c_1+c_2+c_{10}+c_{11}+c_{13}+c_{14}+c_{22}+c_{23}+c_{49}+c_{52}+\text{<<5>>}+c_{38}\,T_1+c_{46}\,T_1+c_{47}\,T_1+c_{55}\,T_1+c_{58}\,T_1-c_{25}\,T_1^2-c_{26}\,T_1^2-c_{34}\,T_1^2-c_{35}\,T_1^2-c_{55}\,T_1^2\right)p_{1,\text{<<1>>}}\,\pi_{1,i}+\text{<<374>>}+\text{<<1>>}-(-1+T_2)\,T_2\,(-c_{33}-c_{36}+c_{33}\,T_1\,T_2)\,p_{2,2+j}\,p_{3,2+k}\,\pi_{2,k}\,\pi_{3,k}$$

Out[•]=
$$\{p_{1,2+i},\ p_{1,2+j},\ p_{1,2+k},\ p_{2,2+i},\ p_{2,2+j},\ p_{2,2+k},\ p_{3,2+i},\ p_{3,2+j},\ p_{3,2+k},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{1,k},\ \pi_{2,i},\ \pi_{2,j},\ \pi_{2,k},\ \pi_{3,i},\ \pi_{3,j},\ \pi_{3,k}\}$$

Out[•]=
$$\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}, c_{19}, c_{20}, c_{21}, c_{22}, c_{23}, c_{24}, c_{25}, c_{26}, c_{27}, c_{28}, c_{29}, c_{30}, c_{31}, c_{32}, c_{33}, c_{34}, c_{35}, c_{36}, c_{37}, c_{38}, c_{39}, c_{40}, c_{41}, c_{42}, c_{43}, c_{44}, c_{45}, c_{46}, c_{47}, c_{48}, c_{49}, c_{50}, c_{51}, c_{52}, c_{53}, c_{54}, c_{55}, c_{56}, c_{57}, c_{58}, c_{59}, c_{60}, c_{61}\}$$

Out[•]//Short=
$$\left\{-c_7\,T_1^2\,T_2^2+c_7\,T_1^2\,T_2^3==0,\ \text{<<250>>},\ c_8\,T_1\,T_2+c_9\,T_1\,T_2+c_{44}\,T_1\,T_2+c_{45}\,T_1\,T_2+c_{57}\,T_1\,T_2-c_8\,T_1^2\,T_2^2-c_9\,T_1^2\,T_2^2-c_{44}\,T_1^2\,T_2^2-c_{45}\,T_1^2\,T_2^2-c_{57}\,T_1^2\,T_2^2==0\right\}$$

Solve: Equations may not give solutions for all "solve" variables.

Out[•]//Short=
$$\left\{\left\{c_1\to 0,\ \text{<<46>>},\ c_{60}\to -\frac{c_{51}}{T_1\,T_2}-\frac{c_{\text{<<2>>}}}{\text{<<1>>}\ \text{<<1>>}\ \text{<<1>>}}-\frac{\text{<<8>>}+\text{<<1>>}}{T_1^2\ \text{<<1>>}\ (-1+\text{<<1>>})}\right\}\right\}$$

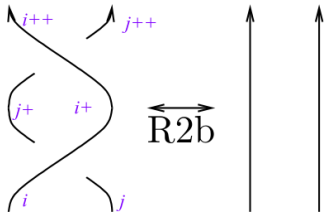
In[•]:=
```
sol /. (v_ → val_) :↦ (v = CF[val]);
```

```
In[•]:= Short[CF[r42[1, i, j]], 20]
```

Out[•]//Short=

$$c_{61} + c_{49}\, p_{1,i}\, x_{1,i} + c_{52}\, p_{1,j}\, x_{1,i} - \frac{(c_{49} + c_{52})\, p_{1,j}\, x_{1,j}}{T_1} + c_{50}\, p_{2,i}\, x_{2,i} +$$
$$c_{53}\, p_{2,j}\, x_{2,i} + c_{13}\, p_{1,j}\, p_{2,i}\, x_{1,i}\, x_{2,i} + c_4\, p_{1,i}\, p_{2,j}\, x_{1,i}\, x_{2,i} + \text{<<25>>} +$$
$$\frac{1}{(-1+T_1)\,(-1+T_2)}\left(-a_4\, b_5 - c_{15} + c_{15}\, T_1 + a_4\, b_3\, T_2 + a_4\, b_5\, T_2 - a_4\, b_3\, T_1\, T_2 + c_{15}\, T_1\, T_2 - c_{15}\, T_1^2\, T_2\right)$$
$$p_{2,j}\, p_{3,j}\, x_{2,j}\, x_{3,i} - \frac{1}{T_1^2\, T_2\,(-1+T_1\, T_2)}$$
$$\left(-a_2\, b_5 - a_2\, b_3\, T_1 + a_4\, b_3\, T_1 - c_{51}\, T_1 - c_{54}\, T_1 - a_4\, b_5\, T_2 - a_4\, b_3\, T_1\, T_2 + a_4\, b_5\, T_1\, T_2 + c_{51}\, T_1^2\, T_2 + c_{54}\, T_1^2\, T_2\right)$$
$$p_{3,j}\, x_{3,j} - \frac{(a_2\, b_5 - a_4\, b_5 - c_5 + c_{14} + c_5\, T_1 + a_4\, b_5\, T_2 - c_{14}\, T_1\, T_2)\, p_{1,i}\, p_{3,j}\, x_{1,i}\, x_{3,j}}{(-1+T_1)\,(-1+T_1\, T_2)} -$$
$$\left((-a_2\, b_3 + c_5 + a_2\, b_3\, T_1 - a_4\, b_3\, T_1 - c_5\, T_1 - a_4\, b_3\, T_2 - c_5\, T_2 + 2\, a_4\, b_3\, T_1\, T_2 + c_5\, T_1\, T_2)\, p_{1,j}\, p_{3,j}\, x_{1,i}\, x_{3,j}\right) /$$
$$((-1+T_2)\,(-1+T_1\, T_2)) -$$
$$\left(\left(a_2\, b_3 - c_6\, T_1 + c_{15}\, T_1 + a_4\, b_3\, T_2 - a_4\, b_3\, T_1\, T_2 + c_6\, T_1\, T_2 - c_{15}\, T_1^2\, T_2\right)\, p_{2,i}\, p_{3,j}\, x_{2,i}\, x_{3,j}\right) /$$
$$(T_1\,(-1+T_2)\,(-1+T_1\, T_2)) -$$
$$\left(\left(c_6\, T_1 - c_6\, T_1^2 + a_2\, b_5\, T_2 - a_2\, b_5\, T_1\, T_2 + a_4\, b_5\, T_1\, T_2 - c_6\, T_1\, T_2 + c_6\, T_1^2\, T_2 + a_4\, b_5\, T_2^2 - 2\, a_4\, b_5\, T_1\, T_2^2\right)\right.$$
$$\left.p_{2,j}\, p_{3,j}\, x_{2,i}\, x_{3,j}\right) / ((-1+T_1)\, T_1\,(-1+T_1\, T_2))$$

Reidemeister 2b



$\overset{\text{R2b}}{\longleftrightarrow}$

Territory: $(x \mid p)_{(i\,|\,j)^+}$, $x_{i\,|\,j}$.

```
In[•]:= Timing[ Short[LeftR2b = (∫ℱ[i, j] × ℒ /@ (Xi,j[1] Xi+,j+[-1]) dⅆ{vsi, vsj, vsi+, vsj+}) ⟦1⟧
        ]]
```

Out[•]=

$$\{0.265625,\ \in\text{Series}[p_{1,2+i}\, \pi_{1,i} + p_{1,2+j}\, \pi_{1,j} + \text{<<12>>} + p_{3,2+j}\, \pi_{3,j},\ \text{<<1>>}]\}$$

```
In[•]:= RightR2b = ∈Series[p1,2+i π1,i + p1,2+j π1,j + p2,2+i π2,i + p2,2+j π2,j + p3,2+i π3,i + p3,2+j π3,j, 0]
```

Out[•]=

$$\in\text{Series}[p_{1,2+i}\, \pi_{1,i} + p_{1,2+j}\, \pi_{1,j} + p_{2,2+i}\, \pi_{2,i} + p_{2,2+j}\, \pi_{2,j} + p_{3,2+i}\, \pi_{3,i} + p_{3,2+j}\, \pi_{3,j},\ 0]$$

In[•]:=
```
Short[eqn = CF[LeftR2b[[1]] - RightR2b[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[r0[-1, i, j], d_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[•]//Short=

$$\frac{(d_7 + \text{<<11>>} + d_7\,T_1\,T_2)\ \text{<<1>>}\ \text{<<1>>}\ \text{<<1>>}\ \pi_{\text{<<1>>}}}{B\,T_1\,T_2} + \text{<<7>>} + \frac{\text{<<1>>}}{B\ \text{<<1>>}\ T_2}$$

Out[•]=

$$\{p_{3,2+i},\ p_{3,2+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j}\}$$

Out[•]=

$$\{d_1,\ d_2,\ d_3,\ d_4,\ d_5,\ d_6,\ d_7,\ d_8\}$$

Out[•]//Short=

$$\left\{\frac{d_1}{B} - \frac{d_3}{B} - \frac{d_5}{B} + \frac{d_7}{B} + \frac{d_5}{B\,T_1} - \frac{d_7}{B\,T_1} + \frac{d_3}{B\,T_2} - \frac{d_7}{B\,T_2} + \frac{d_7}{B\,T_1\,T_2} == 0,\right.$$
$$\left.\frac{d_3}{B\,T_2} - \frac{d_7}{B\,T_2} + \frac{d_7}{B\,T_1\,T_2} == 0,\ \text{<<5>>},\ \frac{d_7}{B} + \frac{d_8}{B} - \frac{d_7}{B\,T_1\,T_2} == 0\right\}$$

Out[•]=

$$\left\{\left\{d_1 \to 0,\ d_2 \to -\frac{a_2 - a_4\,T_1 + a_4\,T_2}{T_1^2\,T_2},\ d_3 \to 0,\ d_4 \to -\frac{a_4}{T_1},\ d_5 \to 0,\ d_6 \to -\frac{-a_2 - a_4\,T_2}{T_1\,T_2},\ d_7 \to 0,\ d_8 \to 0\right\}\right\}$$

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val]);
r0[-1, i, j]
```

Out[•]=

$$\frac{(-a_2 + a_4\,T_1 - a_4\,T_2)\ p_{3,j}\ x_{1,i}\ x_{2,i}}{T_1^2\,T_2} + \frac{(a_2 + a_4\,T_2)\ p_{3,j}\ x_{1,j}\ x_{2,i}}{T_1\,T_2} - \frac{a_4\ p_{3,j}\ x_{1,i}\ x_{2,j}}{T_1}$$

```
In[•]:= Short[eqn = CF[LeftR2b[[2]] - RightR2b[[2]]]]
        cvs = Union@Cases[eqn, p__ | π__, ∞]
        vars = Union@Cases[r1[-1, i, j] + r42[-1, i, j], e_ | f_, ∞]
        Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
        Short[{sol} = Solve[eqns, vars]]
```

Out[•]//Short=

$$\text{<<85>>} + \frac{\text{<<1>>}}{\text{<<1>>}} + \frac{\text{<<1>>}}{T_1\ \text{<<1>>}}$$

Out[•]=

$$\{p_{1,2+i},\ p_{1,2+j},\ p_{2,2+i},\ p_{2,2+j},\ p_{3,2+i},\ p_{3,2+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j},\ \pi_{3,i},\ \pi_{3,j}\}$$

Out[•]=

$$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10},$$
$$f_{11}, f_{12}, f_{13}, f_{14}, f_{15}, f_{16}, f_{17}, f_{18}, f_{19}, f_{20}, f_{21}, f_{22}, f_{23}, f_{24}, f_{25}, f_{26}, f_{27},$$
$$f_{28}, f_{29}, f_{30}, f_{31}, f_{32}, f_{33}, f_{34}, f_{35}, f_{36}, f_{37}, f_{38}, f_{39}, f_{40}, f_{41}, f_{42}, f_{43}, f_{44},$$
$$f_{45}, f_{46}, f_{47}, f_{48}, f_{49}, f_{50}, f_{51}, f_{52}, f_{53}, f_{54}, f_{55}, f_{56}, f_{57}, f_{58}, f_{59}, f_{60}, f_{61}\}$$

Out[•]//Short=

$$\left\{f_1 - f_7 - f_{25} + f_{31} + \frac{f_{25}}{T_1} - \frac{f_{31}}{T_1} + \frac{f_7}{T_2} - \frac{f_{31}}{T_2} + \frac{f_{31}}{T_1\,T_2} == 0,\ \frac{f_7}{T_2} - \frac{\text{<<1>>}}{\text{<<1>>}} + \frac{f_{\text{<<2>>}}}{\text{<<1>>}\ \text{<<1>>}} == 0,\right.$$
$$\left.\text{<<66>>},\ \frac{2\,a_4\,b_3}{(1-T_1)\,(1-T_2)} + \frac{2\,c_{49}}{(1-T_1)\,(1-T_2)} + \text{<<216>>} + \frac{f_{61}\,T_1\,T_2}{(1-T_1)\,(1-T_2)} == 0\right\}$$

Out[•]//Short=

$$\{\{e_1 \to 0,\ e_2 \to 0,\ \text{<<66>>},\ f_{61} \to -c_{61}\}\}$$

```
In[•]:= sol /. (v_ → val_) :> (v = CF[val]);
```

```
In[•]:= r1[-1, i, j]
        Short[CF[r42[-1, i, j]], 5]
```

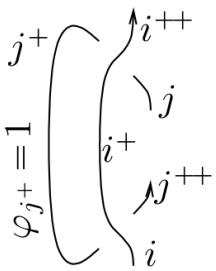
Out[•]=

$$-\frac{b_5\,p_{1,j}\,p_{2,i}\,x_{3,i}}{T_1} - \frac{b_3\,p_{1,i}\,p_{2,j}\,x_{3,i}}{T_2} + \frac{(b_3\,T_1 + b_5\,T_2)\,p_{1,j}\,p_{2,j}\,x_{3,i}}{T_1\,T_2}$$

Out[•]//Short=

$$-c_{61} - c_{49}\,p_{1,i}\,x_{1,i} + \frac{\left(-c_{49} - c_{52} + c_{49}\,T_1^2\right)\,p_{1,j}\,x_{1,i}}{T_1^2} + \text{<<41>>} + \frac{\text{<<1>>}}{\text{<<1>>}} + \frac{\text{<<1>>}}{T_1\,(-1+\text{<<1>>})\,(-1+T_1\,T_2)} +$$
$$\left(\left(a_2\,b_3 - c_6\,T_1 + c_{15}\,T_1 + a_4\,b_3\,T_2 - a_4\,b_3\,T_1\,T_2 + c_6\,T_1\,T_2 - c_{15}\,T_1^2\,T_2\right)\,p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,j}\right) /$$
$$\left(T_1\,(-1+T_2)\,(-1+T_1\,T_2)\right) + \frac{(\text{<<1>>})\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{(-1+T_1)\,T_1\,T_2\,(-1+T_1\,T_2)}$$

Reidemeister 2c



In[•]:=
```
Timing[ Short[{LeftR2c} = Cases[
    ∫ℱ[i, j] × ℒ /@ (X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}}, 𝔼[ℰ_] :↦ ℰ]
 ]]
```

Out[•]=
```
{0.15625, {∈Series[p_{1,2+i} π_{1,i} + p_{1,3+j} π_{1,j} + <<9>> + p_{3,3+j} π_{3,j}, g1 g2 + <<41>> + <<1>>]}}
```

In[•]:=
```
Timing[ Short[{RightR2c} =
   Cases[∫ℱ[i, j] × ℒ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}},
    𝔼[ℰ_] :↦ ℰ]
 ]]
```

Out[•]=
```
{0.015625, {∈Series[p_{1,2+i} π_{1,i} + p_{1,3+j} π_{1,j} + <<4>> + p_{3,3+j} π_{3,j}, g1 g2 + <<12>> + <<1>>]}}
```

In[•]:=
```
Short[eqn = CF[LeftR2c[[1]] - RightR2c[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ0[1, k], g_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :↦ (c == 0), 3]
{sol} = Solve[eqns, vars]
```

Out[•]//Short=
$$\frac{g_1\ (-1+T_1)\ \ll\!1\!\gg\ \ll\!1\!\gg\ \ll\!1\!\gg\ \pi_{1\ \ll 1\gg\ \ll 1\gg}\ \pi_{2,i}}{B\ T_1\ T_2} - \frac{\ll\!1\!\gg}{B\ \ll\!1\!\gg\ \ll\!1\!\gg} - \frac{g_1\ \ll\!3\!\gg\ \ll\!1\!\gg}{B\ T_1}$$

Out[•]=
$$\{p_{3,3+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j}\}$$

Out[•]=
$$\{g_1\}$$

Out[•]//Short=
$$\left\{\frac{g_1}{B} - \frac{g_1}{B\ T_1} - \frac{g_1}{B\ T_2} + \frac{g_1}{B\ T_1\ T_2} == 0,\ -\frac{g_1}{B} + \frac{g_1}{B\ T_1} == 0,\ -\frac{g_1}{B} + \frac{g_1}{B\ T_2} == 0\right\}$$

Out[•]=
$$\{\{g_1 \to 0\}\}$$

```
In[•]:= sol /. (v_ → val_) :↦ (v = CF[val]);
        γ0[1, k]
Out[•]=
        0

In[•]:= Short[eqn = CF[LeftR2c⟦2⟧ - RightR2c⟦2⟧]]
        cvs = Union@Cases[eqn, p__ | π__, ∞]
        vars = Union@Cases[γ1[1, k] + γ42[1, k], g_, ∞]
        Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :↦ (c == 0), 3]
        Short[{sol} = Solve[eqns, vars]]
Out[•]//Short=
        -((g3 + g6 + g7) (-1 + <<1>>) <<1>> π1,i)/T1 - <<1>>/<<1>> + <<18>>
Out[•]=
        {p1,3+j, p2,3+j, p3,3+j, π1,i, π1,j, π2,i, π2,j, π3,i, π3,j}
Out[•]=
        {g2, g3, g4, g5, g6, g7, g8}
Out[•]//Short=
        {g6 - g6/T1 - g6/T2 + (a2 g2)/(T1 T2) + g6/(T1 T2) == 0, -g6 + (a4 g2)/T1 + g6/T1 == 0, <<1>> == 0, <<7>>, -g8 + <<1>> == 0,
         -g4 - g6 - g8 + g4/T2 + g6/T2 + g8/T2 == 0, (2 a4 b3)/((1 - T1) (1 - T2)) + (a4 b3)/((1 - T1) T1^2 (1 - T2)) + <<50>> == 0}
Out[•]//Short=
        {{g2 → 0, g3 → 0, g4 → 0, g5 → -<<1>>/<<1>>, g6 → 0, g7 → 0, g8 → 0}}

In[•]:= sol /. (v_ → val_) :↦ (v = CF[val]);

In[•]:= γ1[1, k]
        Short[CF[γ42[1, k]], 5]
Out[•]=
        0
Out[•]//Short=
        ((-b3 + b5 + b3 T1 - b5 T2) (-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2) p3,k x3,k)/((-1 + T1) T1 (-1 + T2) (-1 + T1 T2))
```

$C_k[1]$ and $C_k[-1]$ are inverses

```
In[•]:= Timing[ Short[{LeftCC} = Cases[{∫ℱ[k] × ℒ /@ (Ck[1] Ck+1[-1]) 𝕕{vsk, vsk+}}, 𝔼[ℰ_] :↦ ℰ]
        ]]
Out[•]=
        {0.015625, {∈Series[p1,2+k π1,k + p2,2+k π2,k + <<1>>/B + p3,2+k π3,k, <<1>>/<<1>> + <<6>> + <<1>>]}}
```

```
In[•]:= Timing[ Short[ {RightCC} = Cases[ {∫ℱ[k] × ℒ /@ (C_k[0] C_{k+1}[0]) 𝕕{vs_k, vs_{k⁺}}}, 𝔼[ℰ_] :> ℰ ] ]]
```

```
Out[•]=
{0., {∈Series[p_{1,2+k} π_{1,k} + p_{2,2+k} π_{2,k} + p_{3,2+k} π_{3,k}, 0]}}
```

```
In[•]:= Short[eqn = CF[LeftCC[[1]] - RightCC[[1]]]]
cvs = Union@Cases[eqn, p__ | π__, ∞]
vars = Union@Cases[γ_0[-1, k], h_, ∞]
Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
{sol} = Solve[eqns, vars]
```

```
Out[•]//Short=
(h_1 p_3 <<1>> 2 <<1>> <<1>> π_<<1>> π_{2,k}) / B
```

```
Out[•]=
{p_{3,2+k}, π_{1,k}, π_{2,k}}
```

```
Out[•]=
{h_1}
```

```
Out[•]//Short=
{h_1/B == 0}
```

```
Out[•]=
{{h_1 → 0}}
```

```
In[•]:= sol /. (v_ → val_) :> (v = CF[val]);
γ_0[-1, k]
```

```
Out[•]=
0
```

```
In[•]:= Short[eqn = CF[LeftCC⟦2⟧ - RightCC⟦2⟧]]
        cvs = Union@Cases[eqn, p__ | π__, ∞]
        vars = Union@Cases[γ1[-1, k] + γ42[-1, k], h_, ∞]
        Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :↦ (c == 0), 3]
        Short[{sol} = Solve[eqns, vars]]
```

Out[•]//Short=

<<1>>

Out[•]=

$\{p_{1,2+k},\ p_{2,2+k},\ p_{3,2+k},\ \pi_{1,k},\ \pi_{2,k},\ \pi_{3,k}\}$

Out[•]=

$\{h_2,\ h_3,\ h_4,\ h_5,\ h_6,\ h_7,\ h_8\}$

Out[•]//Short=

$$\left\{h_6 == 0,\ B\,h_2 == 0,\ h_7 == 0,\ h_3 + h_6 + h_7 == 0,\ h_8 == 0,\ h_4 + h_6 + h_8 == 0,\ \frac{2\,a_2\,b_3}{(1-T_1)\,(1-T_2)\,(1-T_1\,T_2)} - \frac{a_4\,b_3}{(1-T_1)\,(1-\text{<<1>>})\,(1-T_1\,T_2)} - \frac{\text{<<1>>}\ \text{<<1>>}\ \text{<<1>>}}{\text{<<1>>}} + \text{<<48>>} == 0,\ \text{<<1>>} == 0\right\}$$

Out[•]//Short=

$$\left\{\left\{h_2 \to 0,\ h_3 \to 0,\ h_4 \to 0,\ h_5 \to -\frac{\text{<<1>>}}{\text{<<1>>}},\ h_6 \to 0,\ h_7 \to 0,\ h_8 \to 0\right\}\right\}$$

```
In[•]:= sol /. (v_ → val_) :↦ (v = CF[val]);
```

```
In[•]:= γ1[-1, k]
        Short[CF[γ42[-1, k]], 5]
```

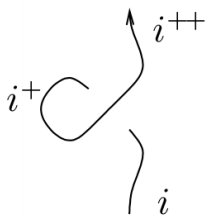
Out[•]=

0

Out[•]//Short=

$$-\frac{(-b_3 + b_5 + b_3\,T_1 - b_5\,T_2)\ (-a_2 + a_2\,T_1 - a_4\,T_1 - a_4\,T_2 + 2\,a_4\,T_1\,T_2)\ p_{3,k}\,x_{3,k}}{(-1+T_1)\,T_1\,(-1+T_2)\,(-1+T_1\,T_2)}$$

Invariance Under R1l



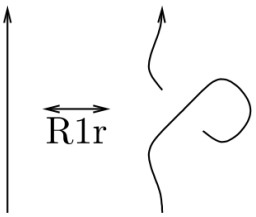

```
In[•]:= {LeftR1l} = Cases[{∫ℱ[i] × ℒ /@ (X_{i+2,i}[1] C_{i+1}[1]) 𝕕{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] :↦ ℰ, ∞]
```

Out[•]=

$\{\in \text{Series}[p_{1,3+i}\,\pi_{1,i} + p_{2,3+i}\,\pi_{2,i} + p_{3,3+i}\,\pi_{3,i},\ c_{61}]\}$

In[•]:=
```
c61 = 0
```
Out[•]=
```
0
```

Invariance Under R1r



In[•]:=
```
{LeftR1r} = Cases[{∫ℱ[i] × ℒ/@(X_{i,i+2}[1] C_{i+1}[-1]) 𝕕{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] :> ℰ, ∞]
```
Out[•]=
$$\left\{\in\text{Series}\left[p_{1,3+i}\,\pi_{1,i}+p_{2,3+i}\,\pi_{2,i}+p_{3,3+i}\,\pi_{3,i},\ \frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\left(a_2\,b_5+a_2\,b_3\,T_1-a_4\,b_3\,T_1+c_{51}\,T_1+c_{54}\,T_1+c_{50}\,T_1^2+c_{53}\,T_1^2+a_4\,b_5\,T_2+a_4\,b_3\,T_1\,T_2-a_4\,b_5\,T_1\,T_2+c_{49}\,T_1\,T_2+c_{52}\,T_1\,T_2-c_{49}\,T_1^2\,T_2-c_{50}\,T_1^2\,T_2-2\,c_{51}\,T_1^2\,T_2-c_{54}\,T_1^2\,T_2-c_{50}\,T_1^3\,T_2-c_{53}\,T_1^3\,T_2-c_{49}\,T_1^2\,T_2^2-c_{52}\,T_1^2\,T_2^2+c_{49}\,T_1^3\,T_2^2+c_{50}\,T_1^3\,T_2^2+c_{51}\,T_1^3\,T_2^2\right)\right]\right\}$$

In[•]:=
```
{RightR1r} = Cases[{∫ℱ[i] × ℒ/@(C_i[0] C_{i+1}[0] C_{i+2}[0]) 𝕕{vs_i, vs_{i⁺}, vs_{i+2}}}, 𝔼[ℰ_] :> ℰ, ∞]
```
Out[•]=
$$\{\in\text{Series}[p_{1,3+i}\,\pi_{1,i}+p_{2,3+i}\,\pi_{2,i}+p_{3,3+i}\,\pi_{3,i},\ 0]\}$$

In[•]:=
```
LeftR1r[[1]] == RightR1r[[1]]
```
Out[•]=
```
True
```

```
In[•]:= Short[eqn = CF[LeftR1r[[2]] - RightR1r[[2]]]]
        cvs = Union@Cases[eqn, p__ | π__, ∞]
        vars = Union@Cases[eqn, (c | d | e | f | g | h)_, ∞]
        Short[eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0), 3]
        {sol} = Solve[eqns, vars]
```

Out[•]//Short=

$$\frac{a_2\, b_5 + \text{<<31>>} + c_{51}\, T_1^3\, T_2^2}{T_1^2\, T_2\, (-1 + T_1\, T_2)}$$

Out[•]=

$\{\}$

Out[•]=

$\{c_{49}, c_{50}, c_{51}, c_{52}, c_{53}, c_{54}\}$

Out[•]//Short=

$$\left\{\frac{a_2\, b_5 + a_2\, b_3\, T_1 - a_4\, b_3\, T_1 + \text{<<26>>} + c_{49}\, T_1^3\, T_2^2 + c_{50}\, T_1^3\, T_2^2 + c_{51}\, T_1^3\, T_2^2}{T_1^2\, T_2\, (-1 + T_1\, T_2)} == 0\right\}$$

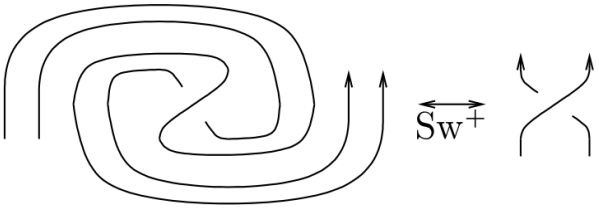
Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\left\{\left\{c_{54} \to -c_{53}\, T_1 - c_{52}\, T_2 - c_{51}\, (1 - T_1\, T_2) - c_{50}\, (T_1 - T_1\, T_2) - c_{49}\, (T_2 - T_1\, T_2) - \frac{-a_2\, b_5 - a_2\, b_3\, T_1 + a_4\, b_3\, T_1 - a_4\, b_5\, T_2 - a_4\, b_3\, T_1\, T_2 + a_4\, b_5\, T_1\, T_2}{T_1\, (-1 + T_1\, T_2)}\right\}\right\}$$

```
In[•]:= sol /. (v_ → val_) :> (v = CF[val]);
```

Invariance Under Sw




```
In[•]:= Timing[ Short[{LeftSw} = Cases[
          {∫ℱ[i, j] × ℒ /@ (X_{i+1,j+1}[1] C_i[-1] C_j[-1] C_{i+2}[1] C_{j+2}[1]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}}},
          𝔼[ℰ_] :> ℰ, ∞]
        ]]
```

Out[•]=

$$\left\{0.109375, \left\{\in\text{Series}\left[T_1\, p_{1,2+i}\, \pi_{1,i} + \text{<<20>>} + p_{3,3+i}\, x_{3,2+i}, \frac{\text{<<1>>}}{\text{<<1>>}} + \text{<<48>>} + \frac{\text{<<1>>}}{\text{<<1>>}}\right]\right\}\right\}$$

```
In[•]:= Timing[ Short[{RightSw} = Cases[
          {∫ℱ[i, j] × ℒ /@ (X_{i+1,j+1}[1] C_i[0] C_j[0] C_{i+2}[0] C_{j+2}[0]) 𝕕{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}, vs_{j+2}}},
          𝔼[ℰ_] :↦ ℰ, ∞]
        ]]
```

```
Out[•]=
{0.125, {∈Series[T1 p1,2+i π1,i + <<20>> + p3,3+i x3,2+i, <<1>>/<<1>> + <<48>> + <<1>>/<<1>>]}}
```

```
In[•]:= LeftSw[[1]] == RightSw[[1]]
```

```
Out[•]=
True
```

```
In[•]:= Short[eqn = CF[LeftSw[[2]] - RightSw[[2]]]]
        cvs = Union@Cases[eqn, p__ | π__, ∞]
        vars = Union@Cases[eqn, (c | d | e | f | g | h)_, ∞]
```

```
Out[•]//Short=
-((-b3 + b5 + b3 <<1>> - b5 T2) (<<1>>))/((-1 + T1) T1 (<<1>>) (-1 + T1 T2)) - <<1>>/<<1>> + <<1>>/<<1>>
```

```
Out[•]=
{p3,2+i, π3,i}
```

```
Out[•]=
{}
```

```
In[•]:= Factor[eqn]
```

```
Out[•]=
-(((-b3 + b5 + b3 T1 - b5 T2) (-a2 + a2 T1 - a4 T1 - a4 T2 + 2 a4 T1 T2)
      (1 + T1 T2 p3,2+i π3,i - p3,2+i x3,2+i)) / ((-1 + T1) T1 (-1 + T2) (-1 + T1 T2)))
```