

Pensieve header: Finding the A_2 $\mathcal{S}d=1$ invariant using undetermined coefficients.

Initialization

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];  
Once[<< KnotTheory` ; << Rot.m];  
<< FormalGaussianIntegration.m;
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[*]:= Features[Knot[8, 17]]
```

```
Out[*]=
```

Features[18,

$C_6[-1]$ $C_{14}[-1]$ $X_{1,7}[1]$ $X_{3,9}[-1]$ $X_{5,13}[-1]$ $X_{8,16}[1]$ $X_{10,4}[-1]$ $X_{12,18}[1]$ $X_{15,2}[-1]$ $X_{17,11}[1]$]

Producing generic perturbations

```

In[*]:=
mons0 = MonomialList [
  p3 x1 x2 /.
  { (v : p | x) _α_ => v_{α,i} + v_{α,j} }
] /. c_Integer * mon_ => mon;
mons1 = MonomialList [
  1 + p1 x1 + p2 x2 + p3 x3 + p1 p1 x1 x1 +
  p2 p2 x2 x2 + p1 p2 x1 x2 + p1 p3 x1 x3 + p2 p3 x2 x3 + p3 p3 x3 x3 + p1 p2 x3 /.
  { (v : p | x) _α_ => v_{α,i} + v_{α,j} }
] /. c_Integer * mon_ => mon;
k = 0;
r1[1, i_, j_] := Evaluate[Sum[c_{++k} mon, {mon, mons0}] + ε Sum[c_{++k} mon, {mon, mons1}]];
r1[-1, i_, j_] := Evaluate[Sum[c_{++k} mon, {mon, mons0}] + ε Sum[c_{++k} mon, {mon, mons1}]];
r1[-1, 4, 7]

```

Out[*]=

$$\begin{aligned}
& c_{105} p_{3,4} x_{1,4} x_{2,4} + c_{109} p_{3,7} x_{1,4} x_{2,4} + c_{107} p_{3,4} x_{1,7} x_{2,4} + c_{111} p_{3,7} x_{1,7} x_{2,4} + \\
& c_{106} p_{3,4} x_{1,4} x_{2,7} + c_{110} p_{3,7} x_{1,4} x_{2,7} + c_{108} p_{3,4} x_{1,7} x_{2,7} + c_{112} p_{3,7} x_{1,7} x_{2,7} + \\
& \in \left(c_{208} + c_{127} p_{1,4} x_{1,4} + c_{152} p_{1,7} x_{1,4} + c_{113} p_{1,4}^2 x_{1,4}^2 + c_{116} p_{1,4} p_{1,7} x_{1,4}^2 + \right. \\
& c_{141} p_{1,7}^2 x_{1,4}^2 + c_{136} p_{1,4} x_{1,7} + c_{161} p_{1,7} x_{1,7} + c_{114} p_{1,4}^2 x_{1,4} x_{1,7} + c_{117} p_{1,4} p_{1,7} x_{1,4} x_{1,7} + \\
& c_{142} p_{1,7}^2 x_{1,4} x_{1,7} + c_{115} p_{1,4}^2 x_{1,7}^2 + c_{118} p_{1,4} p_{1,7} x_{1,7}^2 + c_{143} p_{1,7}^2 x_{1,7}^2 + c_{176} p_{2,4} x_{2,4} + \\
& c_{189} p_{2,7} x_{2,4} + c_{119} p_{1,4} p_{2,4} x_{1,4} x_{2,4} + c_{144} p_{1,7} p_{2,4} x_{1,4} x_{2,4} + c_{121} p_{1,4} p_{2,7} x_{1,4} x_{2,4} + \\
& c_{146} p_{1,7} p_{2,7} x_{1,4} x_{2,4} + c_{128} p_{1,4} p_{2,4} x_{1,7} x_{2,4} + c_{153} p_{1,7} p_{2,4} x_{1,7} x_{2,4} + c_{130} p_{1,4} p_{2,7} x_{1,7} x_{2,4} + \\
& c_{155} p_{1,7} p_{2,7} x_{1,7} x_{2,4} + c_{166} p_{2,4}^2 x_{2,4}^2 + c_{169} p_{2,4} p_{2,7} x_{2,4}^2 + c_{182} p_{2,7}^2 x_{2,4}^2 + c_{181} p_{2,4} x_{2,7} + \\
& c_{194} p_{2,7} x_{2,7} + c_{120} p_{1,4} p_{2,4} x_{1,4} x_{2,7} + c_{145} p_{1,7} p_{2,4} x_{1,4} x_{2,7} + c_{122} p_{1,4} p_{2,7} x_{1,4} x_{2,7} + \\
& c_{147} p_{1,7} p_{2,7} x_{1,4} x_{2,7} + c_{129} p_{1,4} p_{2,4} x_{1,7} x_{2,7} + c_{154} p_{1,7} p_{2,4} x_{1,7} x_{2,7} + c_{131} p_{1,4} p_{2,7} x_{1,7} x_{2,7} + \\
& c_{156} p_{1,7} p_{2,7} x_{1,7} x_{2,7} + c_{167} p_{2,4}^2 x_{2,4} x_{2,7} + c_{170} p_{2,4} p_{2,7} x_{2,4} x_{2,7} + c_{183} p_{2,7}^2 x_{2,4} x_{2,7} + \\
& c_{168} p_{2,4}^2 x_{2,7}^2 + c_{171} p_{2,4} p_{2,7} x_{2,7}^2 + c_{184} p_{2,7}^2 x_{2,7}^2 + c_{137} p_{1,4} p_{2,4} x_{3,4} + c_{162} p_{1,7} p_{2,4} x_{3,4} + \\
& c_{139} p_{1,4} p_{2,7} x_{3,4} + c_{164} p_{1,7} p_{2,7} x_{3,4} + c_{201} p_{3,4} x_{3,4} + c_{202} p_{3,7} x_{3,4} + c_{123} p_{1,4} p_{3,4} x_{1,4} x_{3,4} + \\
& c_{148} p_{1,7} p_{3,4} x_{1,4} x_{3,4} + c_{124} p_{1,4} p_{3,7} x_{1,4} x_{3,4} + c_{149} p_{1,7} p_{3,7} x_{1,4} x_{3,4} + c_{132} p_{1,4} p_{3,4} x_{1,7} x_{3,4} + \\
& c_{157} p_{1,7} p_{3,4} x_{1,7} x_{3,4} + c_{133} p_{1,4} p_{3,7} x_{1,7} x_{3,4} + c_{158} p_{1,7} p_{3,7} x_{1,7} x_{3,4} + c_{172} p_{2,4} p_{3,4} x_{2,4} x_{3,4} + \\
& c_{185} p_{2,7} p_{3,4} x_{2,4} x_{3,4} + c_{173} p_{2,4} p_{3,7} x_{2,4} x_{3,4} + c_{186} p_{2,7} p_{3,7} x_{2,4} x_{3,4} + c_{177} p_{2,4} p_{3,4} x_{2,7} x_{3,4} + \\
& c_{190} p_{2,7} p_{3,4} x_{2,7} x_{3,4} + c_{178} p_{2,4} p_{3,7} x_{2,7} x_{3,4} + c_{191} p_{2,7} p_{3,7} x_{2,7} x_{3,4} + c_{195} p_{3,4}^2 x_{3,4}^2 + \\
& c_{196} p_{3,4} p_{3,7} x_{3,4}^2 + c_{197} p_{3,7}^2 x_{3,4}^2 + c_{138} p_{1,4} p_{2,4} x_{3,7} + c_{163} p_{1,7} p_{2,4} x_{3,7} + \\
& c_{140} p_{1,4} p_{2,7} x_{3,7} + c_{165} p_{1,7} p_{2,7} x_{3,7} + c_{206} p_{3,4} x_{3,7} + c_{207} p_{3,7} x_{3,7} + c_{125} p_{1,4} p_{3,4} x_{1,4} x_{3,7} + \\
& c_{150} p_{1,7} p_{3,4} x_{1,4} x_{3,7} + c_{126} p_{1,4} p_{3,7} x_{1,4} x_{3,7} + c_{151} p_{1,7} p_{3,7} x_{1,4} x_{3,7} + c_{134} p_{1,4} p_{3,4} x_{1,7} x_{3,7} + \\
& c_{159} p_{1,7} p_{3,4} x_{1,7} x_{3,7} + c_{135} p_{1,4} p_{3,7} x_{1,7} x_{3,7} + c_{160} p_{1,7} p_{3,7} x_{1,7} x_{3,7} + c_{174} p_{2,4} p_{3,4} x_{2,4} x_{3,7} + \\
& c_{187} p_{2,7} p_{3,4} x_{2,4} x_{3,7} + c_{175} p_{2,4} p_{3,7} x_{2,4} x_{3,7} + c_{188} p_{2,7} p_{3,7} x_{2,4} x_{3,7} + c_{179} p_{2,4} p_{3,4} x_{2,7} x_{3,7} + \\
& c_{192} p_{2,7} p_{3,4} x_{2,7} x_{3,7} + c_{180} p_{2,4} p_{3,7} x_{2,7} x_{3,7} + c_{193} p_{2,7} p_{3,7} x_{2,7} x_{3,7} + c_{198} p_{3,4}^2 x_{3,4} x_{3,7} + \\
& c_{199} p_{3,4} p_{3,7} x_{3,4} x_{3,7} + c_{200} p_{3,7}^2 x_{3,4} x_{3,7} + c_{203} p_{3,4}^2 x_{3,7}^2 + c_{204} p_{3,4} p_{3,7} x_{3,7}^2 + c_{205} p_{3,7}^2 x_{3,7}^2 \left. \right)
\end{aligned}$$

The A2 Integrand

Adopted from pensieve://Talks//Oaxaca-2210/Rho.nb.

```
In[*]:=
T3 = T1 T2;
S = {x_, p_};
q[s_, i_, j_] :=
  Sum[x $\alpha$ , i (p $\alpha$ , i - p $\alpha$ , i+1) + x $\alpha$ , j (p $\alpha$ , j - p $\alpha$ , j+1) + x $\alpha$ , i ((1 - T $\alpha^s$ ) p $\alpha$ , i+1 + (T $\alpha^s$  - 1) p $\alpha$ , j+1), { $\alpha$ , 3}];
 $\gamma_1[\varphi_, k_] := \epsilon \varphi (3/2 - x_{1,k} p_{1,k} - x_{2,k} p_{2,k} - x_{3,k} p_{3,k})$ ;
 $\mathcal{L}[X_{i,j}[s_]] := T_3^s \mathbb{E}[-q[s, i, j] + r_1[s, i, j] + O[\epsilon^1]]$ ;
 $\mathcal{L}[C_k[\varphi_]] :=$ 
  T $_3^\varphi \mathbb{E}[-x_{1,k} (p_{1,k} - p_{1,k+1}) - x_{2,k} (p_{2,k} - p_{2,k+1}) - x_{3,k} (p_{3,k} - p_{3,k+1}) + \gamma_1[\varphi, k] + O[\epsilon^1]]$ ;
 $\mathcal{L}[K_] := (2\pi)^{-\text{Features}[K][[1]]} \text{CF}[\mathcal{L} / @ \text{Features}[K][[2]]]$ ;
vs $_i := \text{Sequence}[p_{1,i}, x_{1,i}, p_{2,i}, x_{2,i}, p_{3,i}, x_{3,i}]$ ;
vs[K_] := Union @@ Table[{vs $_i$ }, {i, Features[K][[1]]}]
```

```
In[*]:= Features[Knot[3, 1]]
```

```
Out[*]=
Features[7, C $_4$ [-1] X $_{2,6}$ [-1] X $_{5,1}$ [-1] X $_{7,3}$ [-1]]
```

```
In[*]:= Short[ $\mathcal{L}$ [Knot[3, 1]], 10]
```

```
Out[*]//Short=

$$\frac{1}{128 \pi^7 T_1^4 T_2^4} \mathbb{E} \left[ \epsilon \text{Series} \left[ -p_{1,1} x_{1,1} + p_{1,2} x_{1,1} - p_{1,2} x_{1,2} + \frac{p_{1,3} x_{1,2}}{T_1} + \frac{(-1 + T_1) p_{1,7} x_{1,2}}{T_1} - p_{1,3} x_{1,3} + p_{1,4} x_{1,3} - \right. \right.$$


$$p_{1,4} x_{1,4} + p_{1,5} x_{1,4} + \frac{(-1 + T_1) p_{1,2} x_{1,5}}{T_1} - p_{1,5} x_{1,5} + \frac{p_{1,6} x_{1,5}}{T_1} - p_{1,6} x_{1,6} + p_{1,7} x_{1,6} +$$


$$\frac{(-1 + T_1) p_{1,4} x_{1,7}}{T_1} - p_{1,7} x_{1,7} + \frac{p_{1,8} x_{1,7}}{T_1} - p_{2,1} x_{2,1} + p_{2,2} x_{2,1} + c_{112} p_{3,1} x_{1,1} x_{2,1} + \ll 41 \gg +$$


$$c_{107} p_{3,7} x_{1,3} x_{2,7} + c_{109} p_{3,3} x_{1,7} x_{2,7} + c_{105} p_{3,7} x_{1,7} x_{2,7} - p_{3,1} x_{3,1} + p_{3,2} x_{3,1} - p_{3,2} x_{3,2} + \frac{p_{3,3} x_{3,2}}{T_1 T_2} +$$


$$\frac{(-1 + T_1 T_2) p_{3,7} x_{3,2}}{T_1 T_2} - p_{3,3} x_{3,3} + p_{3,4} x_{3,3} - p_{3,4} x_{3,4} + p_{3,5} x_{3,4} + \frac{(-1 + T_1 T_2) p_{3,2} x_{3,5}}{T_1 T_2} -$$

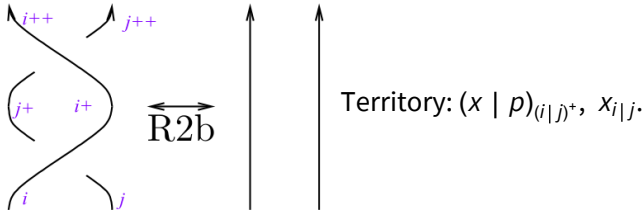

$$p_{3,5} x_{3,5} + \frac{p_{3,6} x_{3,5}}{T_1 T_2} - p_{3,6} x_{3,6} + p_{3,7} x_{3,6} + \frac{(-1 + T_1 T_2) p_{3,4} x_{3,7}}{T_1 T_2} - p_{3,7} x_{3,7} + \frac{p_{3,8} x_{3,7}}{T_1 T_2}, \ll 1 \gg \left. \right]$$

```

```
In[*]:= vs[Knot[3, 1]]
```

```
Out[*]=
{p $_{1,1}$ , p $_{1,2}$ , p $_{1,3}$ , p $_{1,4}$ , p $_{1,5}$ , p $_{1,6}$ , p $_{1,7}$ , p $_{2,1}$ , p $_{2,2}$ , p $_{2,3}$ , p $_{2,4}$ , p $_{2,5}$ , p $_{2,6}$ ,
p $_{2,7}$ , p $_{3,1}$ , p $_{3,2}$ , p $_{3,3}$ , p $_{3,4}$ , p $_{3,5}$ , p $_{3,6}$ , p $_{3,7}$ , x $_{1,1}$ , x $_{1,2}$ , x $_{1,3}$ , x $_{1,4}$ , x $_{1,5}$ , x $_{1,6}$ , x $_{1,7}$ ,
x $_{2,1}$ , x $_{2,2}$ , x $_{2,3}$ , x $_{2,4}$ , x $_{2,5}$ , x $_{2,6}$ , x $_{2,7}$ , x $_{3,1}$ , x $_{3,2}$ , x $_{3,3}$ , x $_{3,4}$ , x $_{3,5}$ , x $_{3,6}$ , x $_{3,7}$ }
```

Invariance Under Reidemeister 2b



In[*]:= {lhs} = Cases [

$$\int \mathbb{E} [\text{Sum}[\pi_{\alpha,i} p_{\alpha,i} + \pi_{\alpha,j} p_{\alpha,j}, \{\alpha, 3\}]] \mathcal{L} / @ (X_{i,j} [1] X_{i+1,j+1} [-1]) \mathfrak{d} \{\mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_{i+1}, \mathbf{vs}_{j+1}\},$$

$$\mathbf{eSeries}[\mathcal{E}_] \Rightarrow \mathcal{E}, \infty]$$

Out[*]=

$$\left\{ p_{1,2+i} \pi_{1,i} + p_{1,2+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + \right.$$

$$\frac{1}{T_1 T_2} (c_{108} + c_{106} T_1 - c_{108} T_1 + c_{107} T_2 - c_{108} T_2 + c_1 T_1 T_2 + c_{105} T_1 T_2 - c_{106} T_1 T_2 - c_{107} T_1 T_2 + c_{108} T_1 T_2)$$

$$p_{3,2+i} \pi_{1,i} \pi_{2,i} + \frac{1}{T_1 T_2}$$

$$(-c_{108} - c_{106} T_1 + c_{108} T_1 - c_{107} T_2 + c_{108} T_2 + c_5 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2 + c_{112} T_1 T_2 +$$

$$c_{106} T_1^2 T_2 - c_{108} T_1^2 T_2 + c_{110} T_1^2 T_2 - c_{112} T_1^2 T_2 + c_{107} T_1 T_2^2 - c_{108} T_1 T_2^2 + c_{111} T_1 T_2^2 - c_{112} T_1 T_2^2 +$$

$$c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 - c_{107} T_1^2 T_2^2 + c_{108} T_1^2 T_2^2 + c_{109} T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + c_{112} T_1^2 T_2^2)$$

$$p_{3,2+j} \pi_{1,i} \pi_{2,i} + \frac{(c_{108} + c_{107} T_2 - c_{108} T_2 + c_3 T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i}}{T_1 T_2} +$$

$$\frac{1}{T_1 T_2} (-c_{108} - c_{107} T_2 + c_{108} T_2 + c_7 T_1 T_2 + c_{108} T_1 T_2 + c_{112} T_1 T_2 + c_{107} T_1 T_2^2 -$$

$$c_{108} T_1 T_2^2 + c_{111} T_1 T_2^2 - c_{112} T_1 T_2^2) p_{3,2+j} \pi_{1,j} \pi_{2,i} +$$

$$p_{2,2+j} \pi_{2,j} + \frac{(c_{108} + c_{106} T_1 - c_{108} T_1 + c_2 T_1 T_2) p_{3,2+i} \pi_{1,i} \pi_{2,j}}{T_1 T_2} +$$

$$\frac{1}{T_1 T_2} (-c_{108} - c_{106} T_1 + c_{108} T_1 + c_6 T_1 T_2 + c_{108} T_1 T_2 + c_{112} T_1 T_2 +$$

$$c_{106} T_1^2 T_2 - c_{108} T_1^2 T_2 + c_{110} T_1^2 T_2 - c_{112} T_1^2 T_2) p_{3,2+j} \pi_{1,i} \pi_{2,j} +$$

$$\frac{(c_{108} + c_4 T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,j}}{T_1 T_2} + \frac{(-c_{108} + c_8 T_1 T_2 + c_{108} T_1 T_2 + c_{112} T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,j}}{T_1 T_2} +$$

$$p_{3,2+i} \pi_{3,i} + p_{3,2+j} \pi_{3,j} \left. \right\}$$

In[*]:= {rhs} = Cases [$\int \mathbb{E} [\text{Sum}[\pi_{\alpha,i} p_{\alpha,i} + \pi_{\alpha,j} p_{\alpha,j}, \{\alpha, 3\}]]$

$$\mathcal{L} / @ (C_i [0] C_{i+1} [0] C_j [0] C_{j+1} [0]) \mathfrak{d} \{\mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_{i+1}, \mathbf{vs}_{j+1}\}, \mathbf{eSeries}[\mathcal{E}_] \Rightarrow \mathcal{E}, \infty]$$

Out[*]=

$$\{ p_{1,2+i} \pi_{1,i} + p_{1,2+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + p_{2,2+j} \pi_{2,j} + p_{3,2+i} \pi_{3,i} + p_{3,2+j} \pi_{3,j} \}$$

In[*]:= **eqn** = **CF**[**lhs** - **rhs**]

Out[*]=

$$\frac{1}{T_1 T_2} (c_{108} + c_{106} T_1 - c_{108} T_1 + c_{107} T_2 - c_{108} T_2 + c_1 T_1 T_2 + c_{105} T_1 T_2 - c_{106} T_1 T_2 - c_{107} T_1 T_2 + c_{108} T_1 T_2) \\ p_{3,2+i} \pi_{1,i} \pi_{2,i} + \\ \frac{1}{T_1 T_2} (-c_{108} - c_{106} T_1 + c_{108} T_1 - c_{107} T_2 + c_{108} T_2 + c_5 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2 + c_{112} T_1 T_2 + \\ c_{106} T_1^2 T_2 - c_{108} T_1^2 T_2 + c_{110} T_1^2 T_2 - c_{112} T_1^2 T_2 + c_{107} T_1 T_2^2 - c_{108} T_1 T_2^2 + c_{111} T_1 T_2^2 - c_{112} T_1 T_2^2 + \\ c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 - c_{107} T_1^2 T_2^2 + c_{108} T_1^2 T_2^2 + c_{109} T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + c_{112} T_1^2 T_2^2) \\ p_{3,2+j} \pi_{1,i} \pi_{2,i} + \frac{(c_{108} + c_{107} T_2 - c_{108} T_2 + c_3 T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i}}{T_1 T_2} + \\ \frac{1}{T_1 T_2} (-c_{108} - c_{107} T_2 + c_{108} T_2 + c_7 T_1 T_2 + c_{108} T_1 T_2 + c_{112} T_1 T_2 + \\ c_{107} T_1 T_2^2 - c_{108} T_1 T_2^2 + c_{111} T_1 T_2^2 - c_{112} T_1 T_2^2) p_{3,2+j} \pi_{1,j} \pi_{2,i} + \\ \frac{(c_{108} + c_{106} T_1 - c_{108} T_1 + c_2 T_1 T_2) p_{3,2+i} \pi_{1,i} \pi_{2,j}}{T_1 T_2} + \frac{1}{T_1 T_2} (-c_{108} - c_{106} T_1 + c_{108} T_1 + c_6 T_1 T_2 + \\ c_{108} T_1 T_2 + c_{112} T_1 T_2 + c_{106} T_1^2 T_2 - c_{108} T_1^2 T_2 + c_{110} T_1^2 T_2 - c_{112} T_1^2 T_2) p_{3,2+j} \pi_{1,i} \pi_{2,j} + \\ \frac{(c_{108} + c_4 T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,j}}{T_1 T_2} + \frac{(-c_{108} + c_8 T_1 T_2 + c_{108} T_1 T_2 + c_{112} T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,j}}{T_1 T_2}$$

In[*]:= **cvs** = **Union@Cases**[**eqn**, **p_** | **$\pi_{$** , **∞**]

Out[*]=

{**p**_{3,2+i}, **p**_{3,2+j}, **π** _{1,i}, **π** _{1,j}, **π** _{2,i}, **π** _{2,j}}

In[*]:= **eqns** = **CoefficientRules**[**eqn**, **cvs**] /. (**_** → **c_**) ⇒ (**c** == **θ**)

Out[*]=

$$\left\{ \begin{aligned} c_1 + c_{105} - c_{106} - c_{107} + c_{108} + \frac{c_{107}}{T_1} - \frac{c_{108}}{T_1} + \frac{c_{106}}{T_2} - \frac{c_{108}}{T_2} + \frac{c_{108}}{T_1 T_2} &= \theta, \\ c_2 + \frac{c_{106}}{T_2} - \frac{c_{108}}{T_2} + \frac{c_{108}}{T_1 T_2} &= \theta, \quad c_3 + \frac{c_{107}}{T_1} - \frac{c_{108}}{T_1} + \frac{c_{108}}{T_1 T_2} &= \theta, \quad c_4 + \frac{c_{108}}{T_1 T_2} &= \theta, \\ c_5 - c_{105} + c_{106} + c_{107} + c_{112} - \frac{c_{107}}{T_1} + \frac{c_{108}}{T_1} + c_{106} T_1 - c_{108} T_1 + c_{110} T_1 - \\ c_{112} T_1 - \frac{c_{106}}{T_2} + \frac{c_{108}}{T_2} - \frac{c_{108}}{T_1 T_2} + c_{107} T_2 - c_{108} T_2 + c_{111} T_2 - c_{112} T_2 + c_{105} T_1 T_2 - \\ c_{106} T_1 T_2 - c_{107} T_1 T_2 + c_{108} T_1 T_2 + c_{109} T_1 T_2 - c_{110} T_1 T_2 - c_{111} T_1 T_2 + c_{112} T_1 T_2 &= \theta, \\ c_6 + c_{108} + c_{112} + c_{106} T_1 - c_{108} T_1 + c_{110} T_1 - c_{112} T_1 - \frac{c_{106}}{T_2} + \frac{c_{108}}{T_2} - \frac{c_{108}}{T_1 T_2} &= \theta, \\ c_7 + c_{108} + c_{112} - \frac{c_{107}}{T_1} + \frac{c_{108}}{T_1} - \frac{c_{108}}{T_1 T_2} + c_{107} T_2 - c_{108} T_2 + c_{111} T_2 - c_{112} T_2 &= \theta, \quad c_8 + c_{108} + c_{112} - \frac{c_{108}}{T_1 T_2} &= \theta \end{aligned} \right\}$$

In[*]:= **vars** = **Union@Cases**[**eqn**, **c_**, **∞**]

Out[*]=

{**c**₁, **c**₂, **c**₃, **c**₄, **c**₅, **c**₆, **c**₇, **c**₈, **c**₁₀₅, **c**₁₀₆, **c**₁₀₇, **c**₁₀₈, **c**₁₀₉, **c**₁₁₀, **c**₁₁₁, **c**₁₁₂}

In[*]:= **{sol} = Solve[eqns, vars]**

Solve: Equations may not give solutions for all "solve" variables.

Out[*]=

$$\left\{ \left\{ \begin{aligned} c_2 &\rightarrow -\frac{c_1}{-1+T_2} - \frac{c_{105}}{-1+T_2} - \frac{c_{107}(1-T_1)}{T_1(-1+T_2)}, & c_3 &\rightarrow -\frac{c_1}{-1+T_1} - \frac{c_{105}}{-1+T_1} - \frac{c_{106}(1-T_2)}{(-1+T_1)T_2}, \\ c_4 &\rightarrow \frac{c_1}{(-1+T_1)(-1+T_2)} + \frac{c_{105}}{(-1+T_1)(-1+T_2)} - \frac{c_{107}}{T_1(-1+T_2)} - \frac{c_{106}}{(-1+T_1)T_2}, \\ c_6 &\rightarrow -c_{112}(1-T_1) - c_{110}T_1 - \frac{c_1(-1+T_1T_2)}{-1+T_2} - \frac{c_{105}(-1+T_1T_2)}{-1+T_2} - \frac{c_{107}(-1+T_1+T_1T_2-T_1^2T_2)}{T_1(-1+T_2)}, \\ c_7 &\rightarrow -c_{112}(1-T_2) - c_{111}T_2 - \frac{c_1(-1+T_1T_2)}{-1+T_1} - \frac{c_{105}(-1+T_1T_2)}{-1+T_1} - \frac{c_{106}(-1+T_2+T_1T_2-T_1T_2^2)}{(-1+T_1)T_2}, \\ c_8 &\rightarrow -c_{112} - \frac{c_1(1-T_1T_2)}{(-1+T_1)(-1+T_2)} - \frac{c_{105}(1-T_1T_2)}{(-1+T_1)(-1+T_2)} - \frac{c_{107}(-1+T_1T_2)}{T_1(-1+T_2)} - \frac{c_{106}(-1+T_1T_2)}{(-1+T_1)T_2}, \\ c_{108} &\rightarrow \frac{c_{106}T_1}{-1+T_1} + \frac{c_{107}T_2}{-1+T_2} - \frac{c_1T_1T_2}{(-1+T_1)(-1+T_2)} - \frac{c_{105}T_1T_2}{(-1+T_1)(-1+T_2)}, \\ c_{109} &\rightarrow -\frac{c_{111}(1-T_1)}{T_1} - \frac{c_5}{T_1T_2} - \frac{c_{110}(1-T_2)}{T_2} - \frac{c_1(1-T_1T_2)}{T_1T_2} - \frac{c_{112}(1-T_1-T_2+T_1T_2)}{T_1T_2} \end{aligned} \right\} \right\}$$

In[*]:= **sol /. (v_ -> val_) -> (v = CF[val])**

Out[*]=

$$\left\{ -\frac{c_{107} + c_1T_1 + c_{105}T_1 - c_{107}T_1}{T_1(-1+T_2)}, -\frac{c_{106} + c_1T_2 + c_{105}T_2 - c_{106}T_2}{(-1+T_1)T_2}, \frac{c_{106}T_1 + c_{107}T_2 + c_1T_1T_2 + c_{105}T_1T_2 - c_{106}T_1T_2 - c_{107}T_1T_2}{(-1+T_1)T_1(-1+T_2)T_2}, \frac{1}{T_1(-1+T_2)} \left(c_{107} + c_1T_1 + c_{105}T_1 - c_{107}T_1 + c_{112}T_1 + c_{110}T_1^2 - c_{112}T_1^2 - c_{107}T_1T_2 - c_{112}T_1T_2 - c_1T_1^2T_2 - c_{105}T_1^2T_2 + c_{107}T_1^2T_2 - c_{110}T_1^2T_2 + c_{112}T_1^2T_2 \right), \frac{1}{(-1+T_1)T_2} \left(c_{106} + c_1T_2 + c_{105}T_2 - c_{106}T_2 + c_{112}T_2 - c_{106}T_1T_2 - c_{112}T_1T_2 + c_{111}T_2^2 - c_{112}T_2^2 - c_1T_1T_2^2 - c_{105}T_1T_2^2 + c_{106}T_1T_2^2 - c_{111}T_1T_2^2 + c_{112}T_1T_2^2 \right), -\left(\left(c_{106}T_1 + c_{107}T_2 + c_1T_1T_2 + c_{105}T_1T_2 - c_{106}T_1T_2 - c_{107}T_1T_2 + c_{112}T_1T_2 - c_{106}T_1^2T_2 - c_{112}T_1^2T_2 - c_{107}T_1T_2^2 - c_{112}T_1T_2^2 - c_1T_1^2T_2^2 - c_{105}T_1^2T_2^2 + c_{106}T_1^2T_2^2 + c_{107}T_1^2T_2^2 + c_{112}T_1^2T_2^2 \right) / \left((-1+T_1)T_1(-1+T_2)T_2 \right), \frac{-c_{106}T_1 - c_{107}T_2 - c_1T_1T_2 - c_{105}T_1T_2 + c_{106}T_1T_2 + c_{107}T_1T_2}{(-1+T_1)(-1+T_2)}, \frac{1}{T_1T_2} \left(-c_1 - c_5 - c_{112} - c_{110}T_1 + c_{112}T_1 - c_{111}T_2 + c_{112}T_2 + c_1T_1T_2 + c_{110}T_1T_2 + c_{111}T_1T_2 - c_{112}T_1T_2 \right) \right\}$$

Invariance Under Reidemeister 3b

In[*]:= **Clear[k]**

$$In[] := \{lhs\} = \text{Cases} \left[\int \mathbb{E}[\text{Sum}[\pi_{\alpha,i} p_{\alpha,i} + \pi_{\alpha,j} p_{\alpha,j} + \pi_{\alpha,k} p_{\alpha,k}, \{\alpha, 3\}]] \mathcal{L} / @ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1]) \right. \\ \left. \text{d} \{vs_i, vs_j, vs_k, vs_{i+1}, vs_{j+1}, vs_{k+1}\}, \text{eSeries}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty \right]$$

Out[] =

$$\left\{ \begin{aligned} & T_1^2 p_{1,2+i} \pi_{1,i} - (-1 + T_1) T_1 p_{1,2+j} \pi_{1,i} + (1 - T_1) p_{1,2+k} \pi_{1,i} + T_1 p_{1,2+j} \pi_{1,j} + \\ & (1 - T_1) p_{1,2+k} \pi_{1,j} + p_{1,2+k} \pi_{1,k} + T_2^2 p_{2,2+i} \pi_{2,i} - (-1 + T_2) T_2 p_{2,2+j} \pi_{2,i} + \\ & (1 - T_2) p_{2,2+k} \pi_{2,i} + 2 c_1 T_1^2 T_2^2 p_{3,2+i} \pi_{1,i} \pi_{2,i} + (c_{106} T_1^2 + 2 c_1 T_1 T_2 + c_5 T_1 T_2 + c_{105} T_1^2 T_2 - \\ & c_{106} T_1^2 T_2 + c_{107} T_2^2 + c_{105} T_1 T_2^2 - c_{107} T_1 T_2^2 - c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2) p_{3,2+j} \pi_{1,i} \pi_{2,i} + \\ & (2 c_1 + 2 c_5 - c_1 T_1 - c_5 T_1 - c_{112} T_1 + c_{112} T_1^2 - c_1 T_2 - c_5 T_2 - c_{112} T_2 + c_5 T_1 T_2 - c_{110} T_1 T_2 - \\ & c_{111} T_1 T_2 + 3 c_{112} T_1 T_2 + c_1 T_1^2 T_2 + c_{110} T_1^2 T_2 + c_{111} T_1^2 T_2 - 2 c_{112} T_1^2 T_2 + c_{112} T_2^2 + c_1 T_1 T_2^2 + \\ & c_{110} T_1 T_2^2 + c_{111} T_1 T_2^2 - 2 c_{112} T_1 T_2^2 - 2 c_1 T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + c_{112} T_1^2 T_2^2) \\ & p_{3,2+k} \pi_{1,i} \pi_{2,i} + \frac{T_1^2 T_2 (-c_{106} - c_1 T_2 - c_{105} T_2 + c_{106} T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i}}{-1 + T_1} + \\ & T_2 (c_1 T_1 - c_{112} T_1 + c_{107} T_2 + c_{105} T_1 T_2 - c_{107} T_1 T_2 - c_{111} T_1 T_2 + c_{112} T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,i} + \\ & \frac{1}{-1 + T_1} (-c_1 - c_5 + c_{112} + c_1 T_1 + c_5 T_1 - c_{106} T_1 - c_{112} T_1 + c_1 T_2 + c_5 T_2 + c_{111} T_2 - c_1 T_1 T_2 - \\ & c_5 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{110} T_1 T_2 - c_{111} T_1 T_2 - 2 c_{112} T_1 T_2 - c_1 T_1^2 T_2 + c_{106} T_1^2 T_2 - \\ & c_{110} T_1^2 T_2 + 2 c_{112} T_1^2 T_2 - c_{112} T_2^2 - c_1 T_1 T_2^2 - c_{110} T_1 T_2^2 - c_{111} T_1 T_2^2 + 3 c_{112} T_1 T_2^2 + \\ & 2 c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 + c_{110} T_1^2 T_2^2 + c_{111} T_1^2 T_2^2 - 2 c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,j} \pi_{2,i} + \\ & \frac{T_1 T_2 (-c_{106} - c_1 T_2 - c_{105} T_2 + c_{106} T_2) p_{3,2+i} \pi_{1,k} \pi_{2,i}}{-1 + T_1} + \\ & \frac{(-c_{106} T_1 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 - c_{107} T_2^2 + c_{107} T_1 T_2^2) p_{3,2+j} \pi_{1,k} \pi_{2,i}}{-1 + T_1} + \\ & (-c_{112} - c_{111} T_2 + c_{112} T_2^2) p_{3,2+k} \pi_{1,k} \pi_{2,i} + \\ & T_2 p_{2,2+j} \pi_{2,j} + (1 - T_2) p_{2,2+k} \pi_{2,j} + \\ & \frac{T_1 (-c_{107} - c_1 T_1 - c_{105} T_1 + c_{107} T_1) T_2^2 p_{3,2+i} \pi_{1,i} \pi_{2,j}}{-1 + T_2} - \\ & T_1 (-c_{106} T_1 - c_1 T_2 + c_{112} T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{110} T_1 T_2 - c_{112} T_1 T_2) p_{3,2+j} \pi_{1,i} \pi_{2,j} + \\ & \frac{1}{-1 + T_2} (-c_1 - c_5 + c_{112} + c_1 T_1 + c_5 T_1 + c_{110} T_1 - c_{112} T_1^2 + c_1 T_2 + c_5 T_2 - c_{107} T_2 - c_{112} T_2 - c_1 T_1 T_2 - \\ & c_5 T_1 T_2 - c_{105} T_1 T_2 + c_{107} T_1 T_2 - c_{110} T_1 T_2 + c_{111} T_1 T_2 - 2 c_{112} T_1 T_2 - c_1 T_1^2 T_2 - c_{110} T_1^2 T_2 - \\ & c_{111} T_1^2 T_2 + 3 c_{112} T_1^2 T_2 - c_1 T_1 T_2^2 + c_{107} T_1 T_2^2 - c_{111} T_1 T_2^2 + 2 c_{112} T_1 T_2^2 + 2 c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - \\ & c_{107} T_1^2 T_2^2 + c_{110} T_1^2 T_2^2 + c_{111} T_1^2 T_2^2 - 2 c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,i} \pi_{2,j} - \frac{1}{(-1 + T_1) (-1 + T_2)} \\ & T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,j} + \\ & \frac{1}{(-1 + T_1) (-1 + T_2)} (c_1 - c_{112}) T_1 T_2 p_{3,2+j} \pi_{1,j} \pi_{2,j} - \\ & (-c_1 - c_5 + c_{112} + c_1 T_1 + c_5 T_1 - c_{106} T_1 - c_{112} T_1 + c_1 T_2 + c_5 T_2 - c_{107} T_2 - c_{112} T_2 - c_1 T_1 T_2 - c_5 T_1 T_2 - \\ & c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2 - c_1 T_1^2 T_2 + c_{106} T_1^2 T_2 + c_{112} T_1^2 T_2 - c_1 T_1 T_2^2 + c_{107} T_1 T_2^2 + \\ & c_{112} T_1 T_2^2 + 2 c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 - c_{107} T_1^2 T_2^2 - c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,j} \pi_{2,j} + \end{aligned} \right.$$

$$\begin{aligned}
 & \frac{T_1 (-C_{106} - C_1 T_2 - C_{105} T_2 + C_{106} T_2) p_{3,2+j} \pi_{1,k} \pi_{2,j}}{-1 + T_1} + (-C_{112} - C_{111} T_2 + C_{112} T_2) p_{3,2+k} \pi_{1,k} \pi_{2,j} + \\
 & p_{2,2+k} \pi_{2,k} + \frac{T_1 (-C_{107} - C_1 T_1 - C_{105} T_1 + C_{107} T_1) T_2 p_{3,2+i} \pi_{1,i} \pi_{2,k}}{-1 + T_2} + \\
 & \frac{(-C_{106} T_1^2 - C_{107} T_2 - C_1 T_1 T_2 - C_{105} T_1 T_2 + C_{107} T_1 T_2 + C_{106} T_1^2 T_2) p_{3,2+j} \pi_{1,i} \pi_{2,k}}{-1 + T_2} + \\
 & \frac{(-C_{112} - C_{110} T_1 + C_{112} T_1^2) p_{3,2+k} \pi_{1,i} \pi_{2,k} + (-C_{107} - C_1 T_1 - C_{105} T_1 + C_{107} T_1) T_2 p_{3,2+j} \pi_{1,j} \pi_{2,k}}{-1 + T_2} + \\
 & \frac{(-C_{112} - C_{110} T_1 + C_{112} T_1) p_{3,2+k} \pi_{1,j} \pi_{2,k} - (-C_{106} T_1 - C_{107} T_2 - C_1 T_1 T_2 - C_{105} T_1 T_2 + C_{106} T_1 T_2 + C_{107} T_1 T_2) p_{3,2+i} \pi_{1,k} \pi_{2,k}}{(-1 + T_1) (-1 + T_2)} - \\
 & \frac{(-C_{106} T_1 - C_{107} T_2 - C_1 T_1 T_2 - C_{105} T_1 T_2 + C_{106} T_1 T_2 + C_{107} T_1 T_2) p_{3,2+j} \pi_{1,k} \pi_{2,k}}{(-1 + T_1) (-1 + T_2)} - \\
 & \left. \begin{aligned}
 & 2 C_{112} p_{3,2+k} \pi_{1,k} \pi_{2,k} + T_1^2 T_2^2 p_{3,2+i} \pi_{3,i} - T_1 T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{3,i} + \\
 & (1 - T_1 T_2) p_{3,2+k} \pi_{3,i} + T_1 T_2 p_{3,2+j} \pi_{3,j} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,j} + p_{3,2+k} \pi_{3,k} \}
 \end{aligned} \right\}
 \end{aligned}$$

In[*]:= {rhs} = Cases [∫ E [Sum [π_{α,i} p_{α,i} + π_{α,j} p_{α,j} + π_{α,k} p_{α,k}, {α, 3}]] L /@ (X_{j,k}[1] X_{i,k+1}[1] X_{i+1,j+1}[1])
d[{vs_i, vs_j, vs_k, vs_{i+1}, vs_{j+1}, vs_{k+1}}, eSeries [ε₋] → ε, ∞]

Out[*]=

$$\begin{aligned}
 & \left\{ T_1^2 p_{1,2+i} \pi_{1,i} - (-1 + T_1) T_1 p_{1,2+j} \pi_{1,i} + (1 - T_1) p_{1,2+k} \pi_{1,i} + T_1 p_{1,2+j} \pi_{1,j} + (1 - T_1) p_{1,2+k} \pi_{1,j} + \right. \\
 & p_{1,2+k} \pi_{1,k} + T_2^2 p_{2,2+i} \pi_{2,i} - (-1 + T_2) T_2 p_{2,2+j} \pi_{2,i} + (1 - T_2) p_{2,2+k} \pi_{2,i} + 2 C_1 T_1^2 T_2^2 p_{3,2+i} \pi_{1,i} \pi_{2,i} - \\
 & T_1 T_2 (-2 C_1 - C_5 + 2 C_1 T_1 T_2) p_{3,2+j} \pi_{1,i} \pi_{2,i} + (C_1 + C_5 - C_1 T_1 T_2) p_{3,2+k} \pi_{1,i} \pi_{2,i} - \\
 & \left. \frac{(-2 + T_1) T_1^2 T_2 (-C_{106} - C_1 T_2 - C_{105} T_2 + C_{106} T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i}}{-1 + T_1} + T_1 \right. \\
 & \left. \frac{(C_{106} + C_1 T_2 + C_{105} T_2 - C_{106} T_2 - C_{112} T_2 - C_{106} T_1 T_2 - C_{111} T_2^2 + C_{112} T_2^2 - C_1 T_1 T_2^2 - C_{105} T_1 T_2^2 + C_{106} T_1 T_2^2) \right. \\
 & p_{3,2+j} \pi_{1,j} \pi_{2,i} - (-1 + T_1) (-C_{112} - C_{111} T_2 + C_{112} T_2) p_{3,2+k} \pi_{1,j} \pi_{2,i} + \\
 & \left. \frac{T_1^2 T_2 (-C_{106} - C_1 T_2 - C_{105} T_2 + C_{106} T_2) p_{3,2+i} \pi_{1,k} \pi_{2,i}}{-1 + T_1} - \right. \\
 & \frac{T_1 (-C_{106} - C_1 T_2 - C_{105} T_2 + C_{106} T_2) (-1 + T_1 T_2) p_{3,2+j} \pi_{1,k} \pi_{2,i}}{-1 + T_1} + \\
 & (-C_{112} - C_{111} T_2 + C_{112} T_2) p_{3,2+k} \pi_{1,k} \pi_{2,i} + T_2 p_{2,2+j} \pi_{2,j} + (1 - T_2) p_{2,2+k} \pi_{2,j} - \\
 & \left. \frac{T_1 (-C_{107} - C_1 T_1 - C_{105} T_1 + C_{107} T_1) (-2 + T_2) T_2^2 p_{3,2+i} \pi_{1,i} \pi_{2,j}}{-1 + T_2} + T_2 \right. \\
 & \left. \frac{(C_{107} + C_1 T_1 + C_{105} T_1 - C_{107} T_1 - C_{112} T_1 - C_{110} T_1^2 + C_{112} T_1^2 - C_{107} T_1 T_2 - C_1 T_1^2 T_2 - C_{105} T_1^2 T_2 + C_{107} T_1^2 T_2) \right. \\
 & p_{3,2+j} \pi_{1,i} \pi_{2,j} - (-C_{112} - C_{110} T_1 + C_{112} T_1) (-1 + T_2) p_{3,2+k} \pi_{1,i} \pi_{2,j} - \\
 & (T_1 T_2 (2 - T_1 - T_2 + T_1 T_2) (-C_{106} T_1 - C_{107} T_2 - C_1 T_1 T_2 - C_{105} T_1 T_2 + C_{106} T_1 T_2 + C_{107} T_1 T_2) p_{3,2+i} \pi_{1,j} \\
 & \pi_{2,j}) / ((-1 + T_1) (-1 + T_2)) + (C_{106} T_1 + C_{107} T_2 + 2 C_1 T_1 T_2 + C_{105} T_1 T_2 - C_{106} T_1 T_2 - C_{107} T_1 T_2 - \\
 & C_{112} T_1 T_2 - C_{106} T_1^2 T_2 - C_{107} T_1 T_2^2 - C_1 T_1^2 T_2^2 - C_{105} T_1^2 T_2^2 + C_{106} T_1^2 T_2^2 + C_{107} T_1^2 T_2^2) p_{3,2+j} \pi_{1,j} \pi_{2,j} + \\
 & \left. (C_1 + C_5 - C_{112} + C_{112} T_1 + C_{112} T_2 - C_1 T_1 T_2 - C_{112} T_1 T_2) p_{3,2+k} \pi_{1,j} \pi_{2,j} + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{-1 + T_1} T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,k} \pi_{2,j} - \\
& \frac{1}{-1 + T_1} (2 c_{106} T_1 + c_{107} T_2 + 2 c_1 T_1 T_2 + 2 c_{105} T_1 T_2 - 2 c_{106} T_1 T_2 - c_{107} T_1 T_2 - \\
& \quad c_{106} T_1^2 T_2 - c_{107} T_1 T_2^2 - c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{106} T_1^2 T_2^2 + c_{107} T_1^2 T_2^2) p_{3,2+j} \pi_{1,k} \pi_{2,j} + \\
& (-2 c_{112} - c_{111} T_2 + 2 c_{112} T_2) p_{3,2+k} \pi_{1,k} \pi_{2,j} + p_{2,2+k} \pi_{2,k} + \\
& \frac{T_1 (-c_{107} - c_1 T_1 - c_{105} T_1 + c_{107} T_1) T_2^2 p_{3,2+i} \pi_{1,i} \pi_{2,k}}{-1 + T_2} - \\
& \frac{(-c_{107} - c_1 T_1 - c_{105} T_1 + c_{107} T_1) T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{1,i} \pi_{2,k}}{-1 + T_2} + \\
& (-c_{112} - c_{110} T_1 + c_{112} T_1) p_{3,2+k} \pi_{1,i} \pi_{2,k} + \\
& \frac{1}{-1 + T_2} T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,k} - \\
& \frac{1}{-1 + T_2} (c_{106} T_1 + 2 c_{107} T_2 + 2 c_1 T_1 T_2 + 2 c_{105} T_1 T_2 - c_{106} T_1 T_2 - 2 c_{107} T_1 T_2 - \\
& \quad c_{106} T_1^2 T_2 - c_{107} T_1 T_2^2 - c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{106} T_1^2 T_2^2 + c_{107} T_1^2 T_2^2) p_{3,2+j} \pi_{1,j} \pi_{2,k} + \\
& (-2 c_{112} - c_{110} T_1 + 2 c_{112} T_1) p_{3,2+k} \pi_{1,j} \pi_{2,k} - \frac{1}{(-1 + T_1) (-1 + T_2)} \\
& \frac{T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,k} \pi_{2,k} +}{(-1 + T_1) (-1 + T_2)} \\
& \left. \begin{aligned}
& p_{3,2+j} \pi_{1,k} \pi_{2,k} - 2 c_{112} p_{3,2+k} \pi_{1,k} \pi_{2,k} + T_1^2 T_2^2 p_{3,2+i} \pi_{3,i} - T_1 T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{3,i} + \\
& (1 - T_1 T_2) p_{3,2+k} \pi_{3,i} + T_1 T_2 p_{3,2+j} \pi_{3,j} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,j} + p_{3,2+k} \pi_{3,k} \}
\end{aligned} \right\}
\end{aligned}$$

In[*]:= eqn = CF[lhs - rhs]

Out[*]=

$$\begin{aligned}
& (c_{106} T_1^2 + c_{105} T_1^2 T_2 - c_{106} T_1^2 T_2 + c_{107} T_2^2 + c_{105} T_1 T_2^2 - c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2) p_{3,2+j} \pi_{1,i} \pi_{2,i} + \\
& (c_1 + c_5 - c_1 T_1 - c_5 T_1 - c_{112} T_1 + c_{112} T_1^2 - c_1 T_2 - c_5 T_2 - c_{112} T_2 + c_1 T_1 T_2 + c_5 T_1 T_2 - c_{110} T_1 T_2 - \\
& \quad c_{111} T_1 T_2 + 3 c_{112} T_1 T_2 + c_1 T_1^2 T_2 + c_{110} T_1^2 T_2 + c_{111} T_1^2 T_2 - 2 c_{112} T_1^2 T_2 + c_{112} T_2^2 + c_1 T_1 T_2^2 + \\
& \quad c_{110} T_1 T_2^2 + c_{111} T_1 T_2^2 - 2 c_{112} T_1 T_2^2 - 2 c_1 T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,i} \pi_{2,i} + \\
& T_1^2 T_2 (-c_{106} - c_1 T_2 - c_{105} T_2 + c_{106} T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i} + (-c_{106} T_1 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + \\
& \quad c_{106} T_1^2 T_2 + c_{107} T_2^2 + c_{105} T_1 T_2^2 - c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2) p_{3,2+j} \pi_{1,j} \pi_{2,i} - \\
& \frac{1}{-1 + T_1} (c_1 + c_5 - c_1 T_1 - c_5 T_1 + c_{106} T_1 - c_{112} T_1 + c_{112} T_1^2 - c_1 T_2 - c_5 T_2 - c_{112} T_2 + c_1 T_1 T_2 + \\
& \quad c_5 T_1 T_2 + c_{105} T_1 T_2 - c_{106} T_1 T_2 - c_{110} T_1 T_2 - c_{111} T_1 T_2 + 4 c_{112} T_1 T_2 + c_1 T_1^2 T_2 - c_{106} T_1^2 T_2 + \\
& \quad c_{110} T_1^2 T_2 + c_{111} T_1^2 T_2 - 3 c_{112} T_1^2 T_2 + c_{112} T_2^2 + c_1 T_1 T_2^2 + c_{110} T_1 T_2^2 + c_{111} T_1 T_2^2 - 3 c_{112} T_1 T_2^2 - \\
& \quad 2 c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{106} T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + 2 c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,j} \pi_{2,i} - \\
& T_1 T_2 (-c_{106} - c_1 T_2 - c_{105} T_2 + c_{106} T_2) p_{3,2+i} \pi_{1,k} \pi_{2,i} + \\
& \frac{T_2 (-c_{106} T_1^2 - c_{107} T_2 + c_{107} T_1 T_2 - c_1 T_1^2 T_2 - c_{105} T_1^2 T_2 + c_{106} T_1^2 T_2) p_{3,2+j} \pi_{1,k} \pi_{2,i}}{-1 + T_1} + \\
& c_{112} (-1 + T_2) T_2 p_{3,2+k} \pi_{1,k} \pi_{2,i} + \\
& T_1 (-c_{107} - c_1 T_1 - c_{105} T_1 + c_{107} T_1) T_2^2 p_{3,2+i} \pi_{1,i} \pi_{2,j} +
\end{aligned}$$

$$\begin{aligned}
 & (c_{106} T_1^2 - c_{107} T_2 - c_{105} T_1 T_2 + c_{107} T_1 T_2 + c_{105} T_1^2 T_2 - \\
 & \quad c_{106} T_1^2 T_2 + c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - c_{107} T_1^2 T_2^2) p_{3,2+j} \pi_{1,i} \pi_{2,j} - \\
 & \frac{1}{-1 + T_2} (c_1 + c_5 - c_1 T_1 - c_5 T_1 - c_{112} T_1 + c_{112} T_1^2 - c_1 T_2 - c_5 T_2 + c_{107} T_2 - c_{112} T_2 + c_1 T_1 T_2 + \\
 & \quad c_5 T_1 T_2 + c_{105} T_1 T_2 - c_{107} T_1 T_2 - c_{110} T_1 T_2 - c_{111} T_1 T_2 + 4 c_{112} T_1 T_2 + c_1 T_1^2 T_2 + c_{110} T_1^2 T_2 + \\
 & \quad c_{111} T_1^2 T_2 - 3 c_{112} T_1^2 T_2 + c_{112} T_2^2 + c_1 T_1 T_2^2 - c_{107} T_1 T_2^2 + c_{110} T_1 T_2^2 + c_{111} T_1 T_2^2 - 3 c_{112} T_1 T_2^2 - \\
 & \quad 2 c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{107} T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + 2 c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,i} \pi_{2,j} + \\
 & T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,j} - \\
 & (-1 + T_1 T_2) (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,j} - \\
 & \frac{1}{(-1 + T_1) (-1 + T_2)} (-c_{106} T_1 + c_{112} T_1 - c_{112} T_1^2 - c_{107} T_2 + c_{112} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + \\
 & \quad c_{106} T_1 T_2 + c_{107} T_1 T_2 - 4 c_{112} T_1 T_2 + c_{106} T_1^2 T_2 + 3 c_{112} T_1^2 T_2 - c_{112} T_2^2 + c_{107} T_1 T_2^2 + \\
 & \quad 3 c_{112} T_1 T_2^2 + c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 - c_{107} T_1^2 T_2^2 - 2 c_{112} T_1^2 T_2^2) p_{3,2+k} \pi_{1,j} \pi_{2,j} - \\
 & \frac{1}{-1 + T_1} T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,k} \pi_{2,j} + \\
 & \frac{1}{-1 + T_1} (-1 + T_1 T_2) (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+j} \pi_{1,k} \pi_{2,j} - \\
 & c_{112} (-1 + T_2) p_{3,2+k} \pi_{1,k} \pi_{2,j} - \\
 & T_1 (-c_{107} - c_1 T_1 - c_{105} T_1 + c_{107} T_1) T_2 p_{3,2+i} \pi_{1,i} \pi_{2,k} + \\
 & \frac{T_1 (-c_{106} T_1 + c_{106} T_1 T_2 - c_{107} T_2^2 - c_1 T_1 T_2^2 - c_{105} T_1 T_2^2 + c_{107} T_1 T_2^2) p_{3,2+j} \pi_{1,i} \pi_{2,k}}{-1 + T_2} + \\
 & c_{112} (-1 + T_1) T_1 p_{3,2+k} \pi_{1,i} \pi_{2,k} - \\
 & \frac{1}{-1 + T_2} T_1 T_2 (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,k} + \\
 & \frac{1}{-1 + T_2} (-1 + T_1 T_2) (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,k} - \\
 & c_{112} (-1 + T_1) p_{3,2+k} \pi_{1,j} \pi_{2,k} + \frac{1}{(-1 + T_1) (-1 + T_2)} (-1 + T_1 T_2) \\
 & \quad (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+i} \pi_{1,k} \pi_{2,k} - \frac{1}{(-1 + T_1) (-1 + T_2)} \\
 & \quad (-1 + T_1 T_2) (-c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{107} T_1 T_2) p_{3,2+j} \pi_{1,k} \pi_{2,k}
 \end{aligned}$$

In[]:= **cv**s = **Union@Cases**[**eqn**, **p**__ | **π**__, ∞]

Out[]:=

{**p**_{3,2+i}, **p**_{3,2+j}, **p**_{3,2+k}, **π**_{1,i}, **π**_{1,j}, **π**_{1,k}, **π**_{2,i}, **π**_{2,j}, **π**_{2,k}}

In[]:= **eqns** = **CoefficientRules**[**eqn**, **cv**s] /. (**_** → **c**_) ⇒ (**c** == **0**)

Out[]:=

$$\begin{aligned}
 & \left\{ -c_{107} T_1 T_2^2 - c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{107} T_1^2 T_2^2 = 0, \right. \\
 & \quad c_{107} T_1 T_2 + c_1 T_1^2 T_2 + c_{105} T_1^2 T_2 - c_{107} T_1^2 T_2 = 0, \quad -c_{106} T_1^2 T_2 - c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{106} T_1^2 T_2^2 = 0, \\
 & \quad -c_{106} T_1^2 T_2 - c_{107} T_1 T_2^2 - c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 + c_{106} T_1^2 T_2^2 + c_{107} T_1^2 T_2^2 = 0, \\
 & \quad \left. -\frac{c_{106} T_1^2 T_2}{1 - T_2} - \frac{c_{107} T_1 T_2^2}{1 - T_2} - \frac{c_1 T_1^2 T_2^2}{1 - T_2} - \frac{c_{105} T_1^2 T_2^2}{1 - T_2} + \frac{c_{106} T_1^2 T_2^2}{1 - T_2} + \frac{c_{107} T_1^2 T_2^2}{1 - T_2} = 0, \right.
 \end{aligned}$$

$$\begin{aligned}
 & c_{106} T_1 T_2 + c_1 T_1 T_2^2 + c_{105} T_1 T_2^2 - c_{106} T_1 T_2^2 = \theta, \\
 & - \frac{c_{106} T_1^2 T_2}{1 - T_1} - \frac{c_{107} T_1 T_2^2}{1 - T_1} - \frac{c_1 T_1^2 T_2^2}{1 - T_1} - \frac{c_{105} T_1^2 T_2^2}{1 - T_1} + \frac{c_{106} T_1^2 T_2^2}{1 - T_1} + \frac{c_{107} T_1^2 T_2^2}{1 - T_1} = \theta, \\
 & \frac{c_{106} T_1}{(1 - T_1)(1 - T_2)} + \frac{c_{107} T_2}{(1 - T_1)(1 - T_2)} + \frac{c_1 T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{c_{105} T_1 T_2}{(1 - T_1)(1 - T_2)} - \\
 & \frac{c_{106} T_1 T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{107} T_1 T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{106} T_1^2 T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{107} T_1 T_2^2}{(1 - T_1)(1 - T_2)} - \\
 & \frac{c_1 T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} - \frac{c_{105} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} + \frac{c_{106} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} + \frac{c_{107} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} = \theta, \\
 & c_{106} T_1^2 + c_{105} T_1^2 T_2 - c_{106} T_1^2 T_2 + c_{107} T_2^2 + c_{105} T_1 T_2^2 - c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 - c_{105} T_1^2 T_2^2 = \theta, \\
 & c_{106} T_1^2 - c_{107} T_2 - c_{105} T_1 T_2 + c_{107} T_1 T_2 + c_{105} T_1^2 T_2 - c_{106} T_1^2 T_2 + c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - \\
 & c_{107} T_1^2 T_2^2 = \theta, \frac{c_{106} T_1^2}{1 - T_2} - \frac{c_{106} T_1^2 T_2}{1 - T_2} + \frac{c_{107} T_1 T_2^2}{1 - T_2} + \frac{c_1 T_1^2 T_2^2}{1 - T_2} + \frac{c_{105} T_1^2 T_2^2}{1 - T_2} - \frac{c_{107} T_1^2 T_2^2}{1 - T_2} = \theta, \\
 & - c_{106} T_1 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + c_{106} T_1^2 T_2 + c_{107} T_2^2 + c_{105} T_1 T_2^2 - c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 + \\
 & c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 = \theta, - c_{106} T_1 - c_{107} T_2 - c_1 T_1 T_2 - c_{105} T_1 T_2 + c_{106} T_1 T_2 + \\
 & c_{107} T_1 T_2 + c_{106} T_1^2 T_2 + c_{107} T_1 T_2^2 + c_1 T_1^2 T_2^2 + c_{105} T_1^2 T_2^2 - c_{106} T_1^2 T_2^2 - c_{107} T_1^2 T_2^2 = \theta, \\
 & - \frac{c_{106} T_1}{1 - T_2} - \frac{c_{107} T_2}{1 - T_2} - \frac{c_1 T_1 T_2}{1 - T_2} - \frac{c_{105} T_1 T_2}{1 - T_2} + \frac{c_{106} T_1 T_2}{1 - T_2} + \frac{c_{107} T_1 T_2}{1 - T_2} + \frac{c_{106} T_1^2 T_2}{1 - T_2} + \\
 & \frac{c_{107} T_1 T_2^2}{1 - T_2} + \frac{c_1 T_1^2 T_2^2}{1 - T_2} + \frac{c_{105} T_1^2 T_2^2}{1 - T_2} - \frac{c_{106} T_1^2 T_2^2}{1 - T_2} - \frac{c_{107} T_1^2 T_2^2}{1 - T_2} = \theta, \\
 & \frac{c_{106} T_1^2 T_2}{1 - T_1} + \frac{c_{107} T_2^2}{1 - T_1} - \frac{c_{107} T_1 T_2^2}{1 - T_1} + \frac{c_1 T_1^2 T_2^2}{1 - T_1} + \frac{c_{105} T_1^2 T_2^2}{1 - T_1} - \frac{c_{106} T_1^2 T_2^2}{1 - T_1} = \theta, \\
 & - \frac{c_{106} T_1}{1 - T_1} - \frac{c_{107} T_2}{1 - T_1} - \frac{c_1 T_1 T_2}{1 - T_1} - \frac{c_{105} T_1 T_2}{1 - T_1} + \frac{c_{106} T_1 T_2}{1 - T_1} + \frac{c_{107} T_1 T_2}{1 - T_1} + \\
 & \frac{c_{106} T_1^2 T_2}{1 - T_1} + \frac{c_{107} T_1 T_2^2}{1 - T_1} + \frac{c_1 T_1^2 T_2^2}{1 - T_1} + \frac{c_{105} T_1^2 T_2^2}{1 - T_1} - \frac{c_{106} T_1^2 T_2^2}{1 - T_1} - \frac{c_{107} T_1^2 T_2^2}{1 - T_1} = \theta, \\
 & - \frac{c_{106} T_1}{(1 - T_1)(1 - T_2)} - \frac{c_{107} T_2}{(1 - T_1)(1 - T_2)} - \frac{c_1 T_1 T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{105} T_1 T_2}{(1 - T_1)(1 - T_2)} + \\
 & \frac{c_{106} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{c_{107} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{c_{106} T_1^2 T_2}{(1 - T_1)(1 - T_2)} + \frac{c_{107} T_1 T_2^2}{(1 - T_1)(1 - T_2)} + \\
 & \frac{c_1 T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} + \frac{c_{105} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} - \frac{c_{106} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} - \frac{c_{107} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} = \theta, \\
 & c_1 + c_5 - c_1 T_1 - c_5 T_1 - c_{112} T_1 + c_{112} T_1^2 - c_1 T_2 - c_5 T_2 - c_{112} T_2 + c_1 T_1 T_2 + c_5 T_1 T_2 - c_{110} T_1 T_2 - \\
 & c_{111} T_1 T_2 + 3 c_{112} T_1 T_2 + c_1 T_1^2 T_2 + c_{110} T_1^2 T_2 + c_{111} T_1^2 T_2 - 2 c_{112} T_1^2 T_2 + c_{112} T_2^2 + c_1 T_1 T_2^2 + \\
 & c_{110} T_1 T_2^2 + c_{111} T_1 T_2^2 - 2 c_{112} T_1 T_2^2 - 2 c_1 T_1^2 T_2^2 - c_{110} T_1^2 T_2^2 - c_{111} T_1^2 T_2^2 + c_{112} T_1^2 T_2^2 = \theta, \\
 & \frac{c_1}{1 - T_2} + \frac{c_5}{1 - T_2} - \frac{c_1 T_1}{1 - T_2} - \frac{c_5 T_1}{1 - T_2} - \frac{c_{112} T_1}{1 - T_2} + \frac{c_{112} T_1^2}{1 - T_2} - \frac{c_1 T_2}{1 - T_2} - \frac{c_5 T_2}{1 - T_2} + \frac{c_{107} T_2}{1 - T_2} - \frac{c_{112} T_2}{1 - T_2} + \frac{c_1 T_1 T_2}{1 - T_2} + \\
 & \frac{c_5 T_1 T_2}{1 - T_2} + \frac{c_{105} T_1 T_2}{1 - T_2} - \frac{c_{107} T_1 T_2}{1 - T_2} - \frac{c_{110} T_1 T_2}{1 - T_2} - \frac{c_{111} T_1 T_2}{1 - T_2} + \frac{4 c_{112} T_1 T_2}{1 - T_2} + \frac{c_1 T_1^2 T_2}{1 - T_2} + \frac{c_{110} T_1^2 T_2}{1 - T_2} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{c_{111} T_1^2 T_2}{1 - T_2} - \frac{3 c_{112} T_1^2 T_2}{1 - T_2} + \frac{c_{112} T_2^2}{1 - T_2} + \frac{c_1 T_1 T_2^2}{1 - T_2} - \frac{c_{107} T_1 T_2^2}{1 - T_2} + \frac{c_{110} T_1 T_2^2}{1 - T_2} + \frac{c_{111} T_1 T_2^2}{1 - T_2} - \frac{3 c_{112} T_1 T_2^2}{1 - T_2} - \\
 & \frac{2 c_1 T_1^2 T_2^2}{1 - T_2} - \frac{c_{105} T_1^2 T_2^2}{1 - T_2} + \frac{c_{107} T_1^2 T_2^2}{1 - T_2} - \frac{c_{110} T_1^2 T_2^2}{1 - T_2} - \frac{c_{111} T_1^2 T_2^2}{1 - T_2} + \frac{2 c_{112} T_1^2 T_2^2}{1 - T_2} = 0, \quad -c_{112} T_1 + c_{112} T_1^2 = 0, \\
 & \frac{c_1}{1 - T_1} + \frac{c_5}{1 - T_1} - \frac{c_1 T_1}{1 - T_1} - \frac{c_5 T_1}{1 - T_1} + \frac{c_{106} T_1}{1 - T_1} - \frac{c_{112} T_1}{1 - T_1} + \frac{c_{112} T_1^2}{1 - T_1} - \frac{c_1 T_2}{1 - T_1} - \frac{c_5 T_2}{1 - T_1} - \frac{c_{112} T_2}{1 - T_1} + \\
 & \frac{c_1 T_1 T_2}{1 - T_1} + \frac{c_5 T_1 T_2}{1 - T_1} + \frac{c_{105} T_1 T_2}{1 - T_1} - \frac{c_{106} T_1 T_2}{1 - T_1} - \frac{c_{110} T_1 T_2}{1 - T_1} - \frac{c_{111} T_1 T_2}{1 - T_1} + \frac{4 c_{112} T_1 T_2}{1 - T_1} + \frac{c_1 T_1^2 T_2}{1 - T_1} - \\
 & \frac{c_{106} T_1^2 T_2}{1 - T_1} + \frac{c_{110} T_1^2 T_2}{1 - T_1} + \frac{c_{111} T_1^2 T_2}{1 - T_1} - \frac{3 c_{112} T_1^2 T_2}{1 - T_1} + \frac{c_{112} T_2^2}{1 - T_1} + \frac{c_1 T_1 T_2^2}{1 - T_1} + \frac{c_{110} T_1 T_2^2}{1 - T_1} + \frac{c_{111} T_1 T_2^2}{1 - T_1} - \\
 & \frac{3 c_{112} T_1 T_2^2}{1 - T_1} - \frac{2 c_1 T_1^2 T_2^2}{1 - T_1} - \frac{c_{105} T_1^2 T_2^2}{1 - T_1} + \frac{c_{106} T_1^2 T_2^2}{1 - T_1} - \frac{c_{110} T_1^2 T_2^2}{1 - T_1} - \frac{c_{111} T_1^2 T_2^2}{1 - T_1} + \frac{2 c_{112} T_1^2 T_2^2}{1 - T_1} = 0, \\
 & \frac{c_{106} T_1}{(1 - T_1)(1 - T_2)} - \frac{c_{112} T_1}{(1 - T_1)(1 - T_2)} + \frac{c_{112} T_1^2}{(1 - T_1)(1 - T_2)} + \frac{c_{107} T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{112} T_2}{(1 - T_1)(1 - T_2)} + \\
 & \frac{c_1 T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{c_{105} T_1 T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{106} T_1 T_2}{(1 - T_1)(1 - T_2)} - \frac{c_{107} T_1 T_2}{(1 - T_1)(1 - T_2)} + \frac{4 c_{112} T_1 T_2}{(1 - T_1)(1 - T_2)} - \\
 & \frac{c_{106} T_1^2 T_2}{(1 - T_1)(1 - T_2)} - \frac{3 c_{112} T_1^2 T_2}{(1 - T_1)(1 - T_2)} + \frac{c_{112} T_2^2}{(1 - T_1)(1 - T_2)} - \frac{c_{107} T_1 T_2^2}{(1 - T_1)(1 - T_2)} - \frac{3 c_{112} T_1 T_2^2}{(1 - T_1)(1 - T_2)} - \\
 & \frac{c_1 T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} - \frac{c_{105} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} + \frac{c_{106} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} + \frac{c_{107} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} + \frac{2 c_{112} T_1^2 T_2^2}{(1 - T_1)(1 - T_2)} = 0, \\
 & \left. \begin{aligned} c_{112} - c_{112} T_1 = 0, \quad -c_{112} T_2 + c_{112} T_2^2 = 0, \quad c_{112} - c_{112} T_2 = 0 \end{aligned} \right\}
 \end{aligned}$$

In[*]:= vars = Union@Cases[eqn, c_, ∞]

Out[*]=

{c₁, c₅, c₁₀₅, c₁₀₆, c₁₀₇, c₁₁₀, c₁₁₁, c₁₁₂}

In[*]:= {sol} = Solve[eqns, vars]

 Solve: Equations may not give solutions for all "solve" variables.

Out[*]=

{ {c₁ → 0, c₁₀₅ → 0, c₁₀₆ → 0, c₁₀₇ → 0, c₁₁₁ → -c₁₁₀ + $\frac{c_5}{T_1 T_2}$, c₁₁₂ → 0 } }

In[*]:= sol /. (v_ → val_) => (v = CF[val])

Out[*]=

{ 0, 0, 0, 0, - $\frac{c_5 + c_{110} T_1 T_2}{T_1 T_2}$, 0 }

In[*]:= vars = Union@Cases[eqn, c_, ∞]

Out[*]=

{c₅, c₁₁₀}

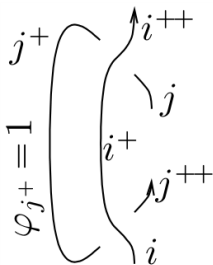
In[*]:= Factor@Cases[$\mathcal{L}[X_{i,j}[1]]$, eSeries[\mathcal{E}_-] \Rightarrow Coefficient[\mathcal{E} , c_5], ∞]

Out[*]=
$$\left\{ \frac{p_{3,j} (T_1 x_{1,i} - x_{1,j}) x_{2,i}}{T_1} \right\}$$

In[*]:= Factor@Cases[$\mathcal{L}[X_{i,j}[1]]$, eSeries[\mathcal{E}_-] \Rightarrow Coefficient[\mathcal{E} , c_{110}], ∞]

Out[*]=
$$\{-p_{3,j} (-T_2 x_{1,j} x_{2,i} + T_1 x_{1,i} x_{2,j})\}$$

Invariance Under R2c

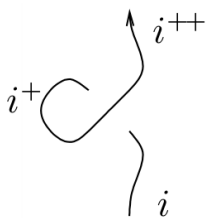


In[*]:= lhs = $\int \mathbb{E}[\pi_i p_i + \pi_j p_j] \mathcal{L} / @ (X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1])$
 $\mathbb{d}\{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{j+2}, p_{j+2}\}$
 rhs = $\int \mathbb{E}[\pi_i p_i + \pi_j p_j] \mathcal{L} / @ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0])$
 $\mathbb{d}\{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{j+2}, p_{j+2}\};$
lhs == rhs

Out[*]=
$$-32 i \pi^5 \sqrt{T} \mathbb{E} \left[\text{eSeries} \left[p_{2+i} \pi_i + p_{3+j} \pi_j, -\frac{1}{2} - p_{3+j} \pi_j, \frac{1}{2} p_{3+j} \pi_j \right] \right]$$

Out[*]= True

Invariance Under R1l

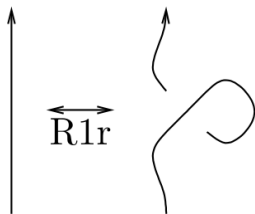


$$\begin{aligned}
 \text{In[*]:= lhs} &= \int \mathbb{E}[\pi_i p_i] \mathcal{L} / @ (\mathbf{X}_{i+2,i} [1] \mathbf{C}_{i+1} [1]) \mathcal{d}\{\mathbf{x}_i, \mathbf{p}_i, \mathbf{x}_{i+1}, \mathbf{p}_{i+1}, \mathbf{x}_{i+2}, \mathbf{p}_{i+2}\} \\
 \text{rhs} &= \int \mathbb{E}[\pi_i p_i] \mathcal{L} / @ (\mathbf{C}_i [0] \mathbf{C}_{i+1} [0] \mathbf{C}_{i+2} [0]) \mathcal{d}\{\mathbf{x}_i, \mathbf{p}_i, \mathbf{x}_{i+1}, \mathbf{p}_{i+1}, \mathbf{x}_{i+2}, \mathbf{p}_{i+2}\}; \\
 \text{lhs} &== \text{rhs}
 \end{aligned}$$

Out[*]=
 $-8 i \pi^3 \mathbb{E}[\text{Series}[p_{3+i} \pi_i, 0, 0]]$

Out[*]=
 True

Invariance Under R1r

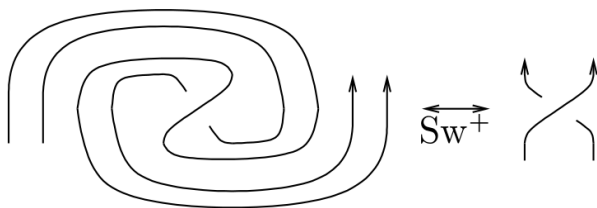


$$\begin{aligned}
 \text{In[*]:= lhs} &= \int \mathbb{E}[\pi_i p_i] \mathcal{L} / @ (\mathbf{X}_{i,i+2} [1] \mathbf{C}_{i+1} [-1]) \mathcal{d}\{\mathbf{x}_i, \mathbf{p}_i, \mathbf{x}_{i+1}, \mathbf{p}_{i+1}, \mathbf{x}_{i+2}, \mathbf{p}_{i+2}\} \\
 \text{rhs} &= \int \mathbb{E}[\pi_i p_i] \mathcal{L} / @ (\mathbf{C}_i [0] \mathbf{C}_{i+1} [0] \mathbf{C}_{i+2} [0]) \mathcal{d}\{\mathbf{x}_i, \mathbf{p}_i, \mathbf{x}_{i+1}, \mathbf{p}_{i+1}, \mathbf{x}_{i+2}, \mathbf{p}_{i+2}\}; \\
 \text{lhs} &== \text{rhs}
 \end{aligned}$$

Out[*]=
 $-8 i \pi^3 \mathbb{E}[\text{Series}[p_{3+i} \pi_i, 0, 0]]$

Out[*]=
 True

Invariance Under Sw



```

In[*]:= lhs = ∫ E[πi pi + πj pj] ℒ /@ (Xi+1,j+1[1] Ci[-1] Cj[-1] Ci+2[1] Cj+2[1])
      d{Xi, Xj, pi, pj, Xi+1, Xj+1, pi+1, pj+1, Xi+2, pi+2, Xj+2, pj+2}
rhs = ∫ E[πi pi + πj pj] ℒ /@ (Xi+1,j+1[1] Ci[0] Cj[0] Ci+2[0] Cj+2[0])
      d{Xi, Xj, pi, pj, Xi+1, Xj+1, pi+1, pj+1, Xi+2, pi+2, Xj+2, pj+2};
lhs == rhs

```

Out[*]=

$$\begin{aligned}
& 64 \pi^6 \sqrt{T} \mathbb{E} \left[\in \text{Series} \left[T p_{3+i} \pi_i + p_{3+j} (\pi_i - T \pi_i + \pi_j), \right. \right. \\
& \quad - \frac{1}{2} + \frac{1}{2} T p_{3+i} p_{3+j} \pi_i (-\pi_i + T \pi_i - 2 \pi_j) - \frac{1}{2} T p_{3+j}^2 \pi_i (-\pi_i + T \pi_i - 2 \pi_j) + p_{3+j} (T \pi_i - \pi_j), \\
& \quad \frac{1}{4} T p_{3+j}^2 \pi_i (-3 \pi_i + 5 T \pi_i - 10 \pi_j) - \frac{1}{4} T p_{3+i} p_{3+j} \pi_i (-\pi_i + 3 T \pi_i - 6 \pi_j) - \\
& \quad \frac{1}{6} T^2 p_{3+i}^2 p_{3+j} \pi_i^2 (-\pi_i + T \pi_i - 3 \pi_j) + \frac{1}{2} p_{3+j} (-T \pi_i + \pi_j) + \\
& \quad \left. \left. \frac{1}{6} T p_{3+i} p_{3+j}^2 \pi_i (\pi_i^2 - 5 T \pi_i^2 + 4 T^2 \pi_i^2 + 3 \pi_i \pi_j - 12 T \pi_i \pi_j + 3 \pi_j^2) - \right. \right. \\
& \quad \left. \left. \frac{1}{6} T p_{3+j}^3 \pi_i (\pi_i^2 - 4 T \pi_i^2 + 3 T^2 \pi_i^2 + 3 \pi_i \pi_j - 9 T \pi_i \pi_j + 3 \pi_j^2) \right] \right]
\end{aligned}$$

Out[*]=

True