Pensieve header: Proof of invariance of ρ_1 using integration techniques.

Initialization

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`; << Rot.m];
<< NilpotentIntegration.m;
$π = Normal[# + O[ϵ]^2] &;
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
Loading Rot.m from http://drorbn.net/icbs24 to compute rotation numbers.
```

The ρ_1 Integrand

Adopted from pensieve://Projects/APAI/PerturbedGaussianIntegration.nb.

```
In[•]:= q[s_, i_, j_] := x_i ((1 - T^s) p_{i+1} + (T^s - 1) p_{j+1});
r_1[s_, i_, j_] :=
  s (-1 + 2 p_i x_i - 2 p_j x_i + (T^s - 1) p_i p_j x_i^2 + (1 - T^s) p_j^2 x_i^2 - 2 p_i p_j x_i x_j + 2 p_j^2 x_i x_j) / 2;
γ_1[φ_, k_] := ϵ φ (1 / 2 - x_k p_k);
ρ1i[s_, i_, j_] := T^{s/2} 𝔼[-q[s, i, j] + ϵ r_1[s, i, j]];
ρ1i[φ_, k_] := T^{φ/2} 𝔼[-x_k (p_k - p_{k+1}) + γ_1[φ, k]];
ρ1i[End, k_] := 𝔼[-x_k p_k];
ρ1i[K_] := Module[{Cs, φ, n, c, k, ℰ},
   {Cs, φ} = Rot[K]; n = Length[Cs];
   ℰ = ρ1i[End, 2 n + 1];
   Do[ℰ *= ρ1i @@ c, {c, Cs}];
   Do[ℰ *= ρ1i[φ⟦k⟧, k], {k, 2 n}];
   CF@ℰ
  ];
ρ1vs[K_] := Union @@ Table[{x_i, p_i}, {i, 2 Crossings[K] + 1}]
```

In[∘]:= **ρ1i[Knot[3, 1]]**

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=

$$\frac{1}{T^2}\,\mathbb{E}\Big[\epsilon - p_1 x_1 + p_2 x_1 - p_2 x_2 - \epsilon\, p_2 x_2 + \frac{p_3 x_2}{T} + \epsilon\, p_5 x_2 + \frac{(-1+T)\, p_6 x_2}{T} + \frac{(-1+T)\,\epsilon\, p_2 p_5 x_2^2}{2\,T} - \frac{(-1+T)\,\epsilon\, p_5^2 x_2^2}{2\,T} - p_3 x_3 + p_4 x_3 + \epsilon\, p_1 x_4 + \frac{(-1+T)\, p_2 x_4}{T} - p_4 x_4 + \frac{p_5 x_4}{T} - \epsilon\, p_1^2 x_1 x_4 + \epsilon\, p_1 p_4 x_1 x_4 - \frac{(-1+T)\,\epsilon\, p_1^2 x_4^2}{2\,T} + \frac{(-1+T)\,\epsilon\, p_1 p_4 x_4^2}{2\,T} - p_5 x_5 + p_6 x_5 + \epsilon\, p_2 p_5 x_2 x_5 - \epsilon\, p_5^2 x_2 x_5 + \epsilon\, p_3 x_6 + \frac{(-1+T)\, p_4 x_6}{T} - p_6 x_6 - \epsilon\, p_6 x_6 + \frac{p_7 x_6}{T} - \epsilon\, p_3^2 x_3 x_6 + \epsilon\, p_3 p_6 x_3 x_6 - \frac{(-1+T)\,\epsilon\, p_3^2 x_6^2}{2\,T} + \frac{(-1+T)\,\epsilon\, p_3 p_6 x_6^2}{2\,T} - p_7 x_7\Big]$$

In[∘]:= **ρ1vs[Knot[3, 1]]**

Out[∘]=

$$\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$$

In[∘]:= **K = Knot[5, 2]; ρ1i[K]**

Out[∘]=

$$\frac{1}{T^3}\,\mathbb{E}\Big[2\,\epsilon - p_1 x_1 + p_2 x_1 - p_2 x_2 - \epsilon\, p_2 x_2 + \frac{p_3 x_2}{T} + \epsilon\, p_7 x_2 + \frac{(-1+T)\, p_8 x_2}{T} + \frac{(-1+T)\,\epsilon\, p_2 p_7 x_2^2}{2\,T} - \frac{(-1+T)\,\epsilon\, p_7^2 x_2^2}{2\,T} - p_3 x_3 + p_4 x_3 + \epsilon\, p_1 x_4 + \frac{(-1+T)\, p_2 x_4}{T} - p_4 x_4 + \frac{p_5 x_4}{T} - \epsilon\, p_1^2 x_1 x_4 + \epsilon\, p_1 p_4 x_1 x_4 - \frac{(-1+T)\,\epsilon\, p_1^2 x_4^2}{2\,T} + \frac{(-1+T)\,\epsilon\, p_1 p_4 x_4^2}{2\,T} - p_5 x_5 + p_6 x_5 - p_6 x_6 - \epsilon\, p_6 x_6 + \frac{p_7 x_6}{T} + \epsilon\, p_9 x_6 + \frac{(-1+T)\, p_{10} x_6}{T} + \frac{(-1+T)\,\epsilon\, p_6 p_9 x_6^2}{2\,T} - \frac{(-1+T)\,\epsilon\, p_9^2 x_6^2}{2\,T} - p_7 x_7 + p_8 x_7 + \epsilon\, p_2 p_7 x_2 x_7 - \epsilon\, p_7^2 x_2 x_7 + \epsilon\, p_3 x_8 + \frac{(-1+T)\, p_4 x_8}{T} - p_8 x_8 - \epsilon\, p_8 x_8 + \frac{p_9 x_8}{T} - \epsilon\, p_3^2 x_3 x_8 + \epsilon\, p_3 p_8 x_3 x_8 - \frac{(-1+T)\,\epsilon\, p_3^2 x_8^2}{2\,T} + \frac{(-1+T)\,\epsilon\, p_3 p_8 x_8^2}{2\,T} - p_9 x_9 - \epsilon\, p_9 x_9 + p_{10} x_9 + \epsilon\, p_6 p_9 x_6 x_9 - \epsilon\, p_9^2 x_6 x_9 + \epsilon\, p_5 x_{10} + \frac{(-1+T)\, p_6 x_{10}}{T} - p_{10} x_{10} + \frac{p_{11} x_{10}}{T} - \epsilon\, p_5^2 x_5 x_{10} + \epsilon\, p_5 p_{10} x_5 x_{10} - \frac{(-1+T)\,\epsilon\, p_5^2 x_{10}^2}{2\,T} + \frac{(-1+T)\,\epsilon\, p_5 p_{10} x_{10}^2}{2\,T} - p_{11} x_{11}\Big]$$

In[∘]:= **K = Knot[5, 2]; $\int$ρ1i[K] d(ρ1vs@K)**

Out[∘]=

$$-\frac{i\, T\, \mathbb{E}\Big[\frac{(-1+T)^2\,(5-4\,T+5\,T^2)\,\epsilon}{(2-3\,T+2\,T^2)^2}\Big]}{2048\,\pi^{11}\,(2-3\,T+2\,T^2)}$$

In[∘]:=
```
K = Knot[8, 19]; ∫ρ1i[K] ⅆ(ρ1vs@K)
```

Out[∘]=

$$-\frac{\mathbb{i}\,T^3\,\mathbb{E}\left[-\frac{(-1+T)^2\,(1+T^4)\,(3+4\,T^3+3\,T^6)\,\epsilon}{(1-T+T^2)^2\,(1-T^2+T^4)^2}\right]}{131072\,\pi^{17}\,(1-T+T^3-T^5+T^6)}$$

Concatenating edges

In[∘]:=
```
lhs = ∫(𝔼[π_i p_i] ρ1i[φ1, i] ρ1i[φ2, i + 1]) ⅆ{x_i, p_i, x_{i+1}, p_{i+1}}
rhs = ∫(𝔼[π_i p_i] ρ1i[φ1 + φ2, i]) ⅆ{x_i, p_i}
```

Out[∘]=

$$\frac{T^{\frac{\varphi 1}{2}+\frac{\varphi 2}{2}}\,\mathbb{E}\left[\frac{1}{2}\,\epsilon\,(-\varphi 1-\varphi 2)+p_{2+i}\,\pi_i-\epsilon\,(\varphi 1+\varphi 2)\,p_{2+i}\,\pi_i\right]}{4\,\pi^2}$$

Out[∘]=

$$-\frac{\mathbb{i}\,T^{\frac{\varphi 1}{2}+\frac{\varphi 2}{2}}\,\mathbb{E}\left[\frac{1}{2}\,\epsilon\,(-\varphi 1-\varphi 2)+p_{1+i}\,\pi_i-\epsilon\,(\varphi 1+\varphi 2)\,p_{1+i}\,\pi_i\right]}{2\,\pi}$$

Invariance Under Reidemeister 3b

In[∘]:=

```
lhs =
  ∫(𝔼[π_i p_i + π_j p_j + π_k p_k] ρ1i[1, i, j] ρ1i[1, i + 1, k] ρ1i[1, j + 1, k + 1] ρ1i[0, i] ρ1i[0, j]
      ρ1i[0, k] ρ1i[0, i + 1] ρ1i[0, j + 1] ρ1i[0, k + 1])
    ⅆ{x_i, x_j, x_k, p_i, p_j, p_k, x_{i+1}, x_{j+1}, x_{k+1}, p_{i+1}, p_{j+1}, p_{k+1}}
rhs = ∫(𝔼[π_i p_i + π_j p_j + π_k p_k] ρ1i[1, j, k] ρ1i[1, i, k + 1] ρ1i[1, i + 1, j + 1]
       ρ1i[0, i] ρ1i[0, j] ρ1i[0, k] ρ1i[0, i + 1] ρ1i[0, j + 1] ρ1i[0, k + 1])
     ⅆ{x_i, x_j, x_k, p_i, p_j, p_k, x_{i+1}, x_{j+1}, x_{k+1}, p_{i+1}, p_{j+1}, p_{k+1}};
lhs == rhs
```

Out[∘]=

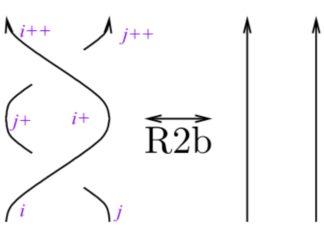
$$\frac{1}{64\,\pi^6}$$

$$T^{3/2}\,\mathbb{E}\Big[-\frac{3\,\epsilon}{2}+T^2\,p_{2+i}\,\pi_i+\frac{1}{2}\,T^3\,\epsilon\,p_{2+i}\,p_{2+j}\,\pi_i\,(-\pi_i+T\,\pi_i-2\,\pi_j)-\frac{1}{2}\,T^3\,\epsilon\,p_{2+j}^2\,\pi_i\,(-\pi_i+T\,\pi_i-2\,\pi_j)+$$
$$T\,\epsilon\,p_{2+j}\,(T\,\pi_i-\pi_j)-T\,p_{2+j}\,(-\pi_i+T\,\pi_i-\pi_j)-\frac{1}{2}\,T\,\epsilon\,p_{2+k}^2\,(\pi_i+\pi_j)\,(-\pi_i+T\,\pi_i-\pi_j+T\,\pi_j-2\,\pi_k)+$$
$$\frac{1}{2}\,T^2\,\epsilon\,p_{2+i}\,p_{2+k}\,\pi_i\,(-\pi_i+T\,\pi_i-2\,\pi_j+2\,T\,\pi_j-2\,\pi_k)+$$
$$\epsilon\,p_{2+k}\,(T\,\pi_i-\pi_j+2\,T\,\pi_j-2\,\pi_k)+p_{2+k}\,(\pi_i-T\,\pi_i+\pi_j-T\,\pi_j+\pi_k)-\frac{1}{2}\,T\,\epsilon\,p_{2+j}\,p_{2+k}$$
$$\left(\pi_i^2-2\,T\,\pi_i^2+T^2\,\pi_i^2+2\,\pi_i\,\pi_j-4\,T\,\pi_i\,\pi_j+2\,T^2\,\pi_i\,\pi_j+\pi_j^2-T\,\pi_j^2+2\,\pi_i\,\pi_k-2\,T\,\pi_i\,\pi_k+2\,\pi_j\,\pi_k\right)\Big]$$

Out[∘]=

True

Invariance Under Reidemeister 2b



Territory: $(x \mid p)_{(i|j)^+},\ x_{i|j}$.

In[∘]:=
```
lhs = ∫(𝔼[π_i p_i + π_j p_j] ρ1i[1, i, j] ρ1i[-1, i + 1, j + 1] ρ1i[0, i]
    ρ1i[0, j] ρ1i[0, i + 1] ρ1i[0, j + 1]) ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}}
rhs = ∫(𝔼[π_i p_i + π_j p_j] ρ1i[0, i] ρ1i[0, j] ρ1i[0, i + 1] ρ1i[0, j + 1])
   ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}};
lhs == rhs
```

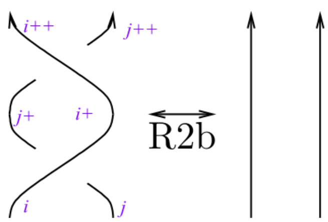
Out[∘]=

$$\frac{\mathbb{E}[p_{2+i}\,\pi_i + p_{2+j}\,\pi_j]}{16\,\pi^4}$$

Out[∘]=

True

Invariance Under Reidemeister 2b (no source terms)



Territory: $(x \mid p)_{(i|j)^+}$, $x_{i|j}$.

In[∘]:=
```
lhs = ∫(ρ1i[1, i, j] ρ1i[-1, i + 1, j + 1] ρ1i[0, i] ρ1i[0, j] ρ1i[0, i + 1] ρ1i[0, j + 1])
  ⅆ{x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}}
```

Out[∘]=

$$\frac{1}{4\,\pi^2}\,\mathbb{E}\Big[-p_i\,x_i + \epsilon\,p_i\,x_i + p_{2+i}\,x_i - \epsilon\,p_j\,x_i - T\,\epsilon\,p_{2+j}\,x_i + \frac{1}{2}\,(-1+T)\,\epsilon\,p_i\,p_j\,x_i^2 + \frac{1}{2}\,(1-T)\,\epsilon\,p_j^2\,x_i^2 + \frac{1}{2}\,(1-T)\,\epsilon\,p_{2+i}\,p_{2+j}\,x_i^2 + \frac{1}{2}\,(-1+T)\,\epsilon\,p_{2+j}^2\,x_i^2 - p_j\,x_j + p_{2+j}\,x_j + \epsilon\,p_{2+j}\,x_j - \epsilon\,p_i\,p_j\,x_i\,x_j + \epsilon\,p_j^2\,x_i\,x_j + \epsilon\,p_{2+i}\,p_{2+j}\,x_i\,x_j - \epsilon\,p_{2+j}^2\,x_i\,x_j\Big]$$

In[∘]:=
```
CF[lhs /. {p_i → (1 + ϵ) p_{i+2} - ϵ (1 + T) p_{j+2}, p_j → (1 + ϵ) p_{j+2}}] /. ϵ⁻ → 0
```

Out[∘]=

$$\frac{\mathbb{E}[0]}{4\,\pi^2}$$

In[∘]:=
```
rhs = ∫(ρ1i[0, i] ρ1i[0, j] ρ1i[0, i + 1] ρ1i[0, j + 1]) ⅆ{x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}}
```

Out[∘]=

$$\frac{\mathbb{E}[-p_i\,x_i + p_{2+i}\,x_i - p_j\,x_j + p_{2+j}\,x_j]}{4\,\pi^2}$$

In[∘]:=
```
CF[rhs /. {p_i → p_{i+2}, p_j → p_{j+2}}]
```

Out[∘]=

$$\frac{\mathbb{E}[0]}{4\,\pi^2}$$

In[∘]:=
```
Coefficient[(4 π^2 lhs)[[1]], ϵ, 0] == Coefficient[(4 π^2 rhs)[[1]], ϵ, 0]
```

Out[∘]=

True

In[∘]:=
```
diff = CF[Coefficient[(4 π^2 lhs)[[1]], ϵ, 0] +
    ϵ (Coefficient[(4 π^2 lhs)[[1]], ϵ, 1] - Coefficient[(4 π^2 rhs)[[1]], ϵ, 1])]
```

Out[∘]=

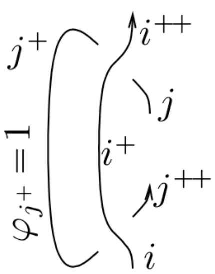
$$-p_i x_i + \epsilon\, p_i x_i + p_{2+i} x_i - \epsilon\, p_j x_i - T \epsilon\, p_{2+j} x_i + \frac{1}{2} (-1+T)\, \epsilon\, p_i p_j x_i^2 + \frac{1}{2} (1-T)\, \epsilon\, p_j^2 x_i^2 + \frac{1}{2} (1-T)\, \epsilon\, p_{2+i} p_{2+j} x_i^2 + \frac{1}{2} (-1+T)\, \epsilon\, p_{2+j}^2 x_i^2 - p_j x_j + p_{2+j} x_j + \epsilon\, p_{2+j} x_j - \epsilon\, p_i p_j x_i x_j + \epsilon\, p_j^2 x_i x_j + \epsilon\, p_{2+i} p_{2+j} x_i x_j - \epsilon\, p_{2+j}^2 x_i x_j$$

In[∘]:=
```
∫𝔼[diff + π_i p_i + π_j p_j] ⅆ{x_i, x_j, p_i, p_j}
```

Out[∘]=

$$\frac{\mathbb{E}[p_{2+i} \pi_i + p_{2+j} \pi_j]}{4 \pi^2}$$

Invariance Under R2c



In[∘]:=
```
lhs = ∫(𝔼[π_i p_i + π_j p_j] ρ1i[-1, i, j + 1] ρ1i[1, i + 1, j] ρ1i[0, i]
      ρ1i[0, j] ρ1i[0, i + 1] ρ1i[1, j + 1]) ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}}
```

Out[∘]=

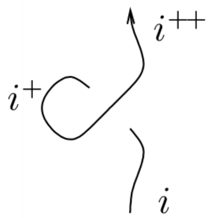
$$\frac{\sqrt{T}\, \mathbb{E}\left[-\frac{\epsilon}{2} + p_{2+i} \pi_i + p_{2+j} \pi_j - \epsilon\, p_{2+j} \pi_j\right]}{16 \pi^4}$$

In[∘]:=
```
rhs = ∫(𝔼[π_i p_i + π_j p_j] ρ1i[0, i] ρ1i[0, j] ρ1i[0, i + 1] ρ1i[1, j + 1])
     ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}};
lhs == rhs
```

Out[∘]=

True

Invariance Under R1l



In[ ]:=
```
lhs = ∫(𝔼[π_i p_i] ρ1i[1, i + 1, i] ρ1i[0, i] ρ1i[1, i + 1]) ⅆ{x_i, p_i, x_{i+1}, p_{i+1}}
```

Out[ ]=

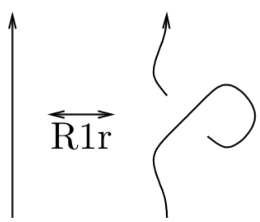
$$\frac{\mathbb{E}[p_{2+i}\,\pi_i]}{4\,\pi^2}$$

In[ ]:=
```
rhs = ∫(𝔼[π_i p_i] ρ1i[0, i] ρ1i[0, i + 1]) ⅆ{x_i, p_i, x_{i+1}, p_{i+1}};
lhs == rhs
```

Out[ ]=

True

Invariance Under R1r



In[ ]:=
```
lhs = ∫(𝔼[π_i p_i] ρ1i[1, i, i + 1] ρ1i[0, i] ρ1i[-1, i + 1]) ⅆ{x_i, p_i, x_{i+1}, p_{i+1}}
```

Out[ ]=

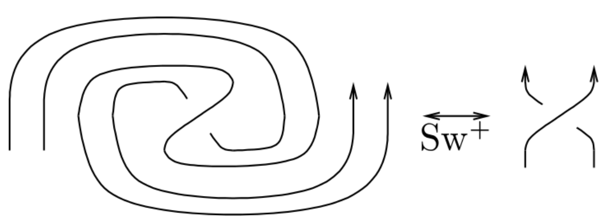
$$\frac{\mathbb{E}[p_{2+i}\,\pi_i]}{4\,\pi^2}$$

In[ ]:=
```
rhs = ∫(𝔼[π_i p_i] ρ1i[0, i] ρ1i[0, i + 1]) ⅆ{x_i, p_i, x_{i+1}, p_{i+1}};
lhs == rhs
```

Out[ ]=

True

Invariance Under Sw



In[∘]:= **CF /@ {ρ1i[1, j], ρ1i[1, i, j]}**

Out[∘]=

$$\left\{\sqrt{T}\ \mathbb{E}\left[\frac{\epsilon}{2} - p_j x_j - \epsilon\, p_j x_j + p_{1+j} x_j\right],\right.$$
$$\sqrt{T}\ \mathbb{E}\left[-\frac{\epsilon}{2} + \epsilon\, p_i x_i + (-1+T)\, p_{1+i} x_i - \epsilon\, p_j x_i + (1-T)\, p_{1+j} x_i + \frac{1}{2}(-1+T)\,\epsilon\, p_i p_j x_i^2 + \right.$$
$$\left.\left.\frac{1}{2}(1-T)\,\epsilon\, p_j^2 x_i^2 - \epsilon\, p_i p_j x_i x_j + \epsilon\, p_j^2 x_i x_j\right]\right\}$$

In[∘]:= **lhs = ∫(𝔼[π_i p_i + π_j p_j + ϵ π_{i+1} p_{i+1} + ϵ π_{j+1} p_{j+1} + ξ_{i+1} x_{i+1} + ξ_{j+1} x_{j+1}] ρ1i[1, i, j] ρ1i[-1, i] ρ1i[1, i + 1] ρ1i[-1, j] ρ1i[1, j + 1]) ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}}**

Out[∘]=

$$\frac{1}{16\,\pi^4}\sqrt{T}\ \mathbb{E}\left[T\, p_{2+i}\,\pi_i + \frac{1}{2} T\,\epsilon\, p_{2+i}\, p_{2+j}\,\pi_i\,(-\pi_i + T\,\pi_i - 2\,\pi_j) - \right.$$
$$\frac{1}{2} T\,\epsilon\, p_{2+j}^2\,\pi_i\,(-\pi_i + T\,\pi_i - 2\,\pi_j) + p_{2+j}\,(\pi_i - T\,\pi_i + \pi_j) + T\,\pi_i\,\xi_{1+i} + \pi_i\,\xi_{1+j} -$$
$$T\,\pi_i\,\xi_{1+j} + \pi_j\,\xi_{1+j} + \frac{1}{2}\,\epsilon\, p_{2+i}\left(2\,\pi_{1+i} - T\,\pi_i^2\,\xi_{1+j} + T^2\,\pi_i^2\,\xi_{1+j} - 2\,T\,\pi_i\,\pi_j\,\xi_{1+j}\right) +$$
$$\frac{1}{2}\,\epsilon\, p_{2+j}\left(2\,T\,\pi_i - 2\,\pi_j + 2\,\pi_{1+j} - T\,\pi_i^2\,\xi_{1+i} + T^2\,\pi_i^2\,\xi_{1+i} - 2\,T\,\pi_i\,\pi_j\,\xi_{1+i} + 2\,T\,\pi_i^2\,\xi_{1+j} - \right.$$
$$\left. 2\,T^2\,\pi_i^2\,\xi_{1+j} + 4\,T\,\pi_i\,\pi_j\,\xi_{1+j}\right) + \frac{1}{2}\,\epsilon\left(-1 + 2\,\pi_{1+i}\,\xi_{1+i} + 2\,T\,\pi_i\,\xi_{1+j} - 2\,\pi_j\,\xi_{1+j} + 2\,\pi_{1+j}\,\xi_{1+j} - \right.$$
$$\left.\left. T\,\pi_i^2\,\xi_{1+i}\,\xi_{1+j} + T^2\,\pi_i^2\,\xi_{1+i}\,\xi_{1+j} - 2\,T\,\pi_i\,\pi_j\,\xi_{1+i}\,\xi_{1+j} + T\,\pi_i^2\,\xi_{1+j}^2 - T^2\,\pi_i^2\,\xi_{1+j}^2 + 2\,T\,\pi_i\,\pi_j\,\xi_{1+j}^2\right)\right]$$

In[∘]:= **rhs = ∫(𝔼[π_i p_i + π_j p_j + ϵ π_{i+1} p_{i+1} + ϵ π_{j+1} p_{j+1} + ξ_{i+1} x_{i+1} + ξ_{j+1} x_{j+1}] ρ1i[1, i, j] ρ1i[0, i] ρ1i[0, i + 1] ρ1i[0, j] ρ1i[0, j + 1]) ⅆ{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}};**
lhs == rhs

Out[∘]=

True