

The Variables:

$$\begin{pmatrix} b_1 \setminus a_1 & x_{12} & x_{13} \\ y_{12} & b_2 \setminus a_2 & x_{23} \\ y_{13} & y_{23} & b_3 \setminus a_3 \end{pmatrix}, \quad \xi_-, \alpha_-, \beta_-, \eta_-.$$

We short “ x_1 ” for either of x_{12}, x_{23} and “ x_2 ” for x_{13} . Weights are intuitive on yb and 3-complementary on ax : wt: $b \rightarrow 0, y_1 \rightarrow 1, y_2 \rightarrow 2, a \rightarrow 3, x_1 \rightarrow 2, x_2 \rightarrow 1$. Weights are 3-complementary on the dual (greek) variables.

In $m[ij \rightarrow k]$:

At $\epsilon = 0$: $\dots, \xi_1 \eta_1 b, \xi_2 \eta_2 b, \xi_1' \xi_1'' x_2, \eta_2 \xi_1 y_1, \alpha \xi_i x_i \dots$

At ϵ/ϵ^2 : $\dots, \xi_1 \eta_1 a, \xi_2 \eta_2 a, \dots$

In $\Delta[i \rightarrow jk]$:

At $\epsilon = 0$: $\dots, \eta_i b y_i, \dots$

The Synthesis Equation.

$$Z_0 \sim V, \quad \partial_\lambda Z_\lambda \sim \sum Q_{ab}^{-1} (\partial_{ab} Z_\lambda + (\partial_a Z_\lambda)(\partial_b Z_\lambda)) =: S(Z)$$

In the best case S is nilpotent on V so Z_λ is a polynomial in λ .

The General Heisenberg Integrand.

$L = Q + V = Q + V_0 + V_{\geq 1} = Q + \sum_d \epsilon^d V_d$, where

$$Q \sim - \sum_{c:(s,i,j), \alpha} x_i^\alpha \left(p_i^\alpha - T^{s\alpha} p_{i+}^\alpha + (T^{s\alpha} - 1) p_{j+}^\alpha \right) + x_j^\alpha \left(p_j^\alpha - p_{j+}^\alpha \right),$$

$V_0 \sim p_i^{\alpha_3} x_{(i/j)+}^{\alpha_1} x_{(i/j)+}^{\alpha_2}$, with $\alpha_1 = (1 \ 0), \alpha_2 = (0 \ 1), \alpha_3 = (1 \ 1)$.

L is subject to some stable conditions:

1. L is weight-balanced, with wt $p^\alpha = \alpha$ and wt $x^\alpha = -\alpha$.
 - Should follow from both Lie reasons and Burau reasons.
2. $\deg_x V_0 - \deg_p V_0 \geq 1$.
 - For V_0 , this is “ V_0 is only trees and wheels”.
 - Comparing to \mathcal{A}^w , $p \sim$ “outgoing leg”, $x \sim$ “incoming leg”.
3. $\deg_p V \leq 1 + \deg_\epsilon V$.
 - Only holds for the connected case (taking a log).
 - Reflects “a connected arrow diagram with k out arrows must have at least $k - 1$ cobracket vertices”.

Some unstable conditions can be assumed too, if working mod ϵ^{d+1} :

1. $\deg_x V_1 \leq \deg_p V_1$.
 - Or else, given V_0 , the ϵ^1 integral vanishes. Yet terms with $\deg_x V_1 > \deg_p V_1$ may influence the $\epsilon^{>1}$ integrals.