$U(g) \otimes \mathbb{Q}[x] \longrightarrow \mathbb{Q}[x]$ $\mathbb{Q}[x]$ is a Verma module of $U(g)$:

$\mathbb{Q}[x] = U(g)/U(g)\langle y, a, b-t\rangle$

$$\begin{tikzcd} & \mathbb{H} \arrow[d] \\ U(g) \arrow[ru, "H"] \arrow[r] & \mathbb{Q}[x]^* \otimes \mathbb{Q}[x] \end{tikzcd}$$

The action

$\mathbb{H}_1 \otimes \mathbb{Q}[x]_2 \longrightarrow \mathbb{Q}[x]$

is

$\sim e^{(\xi_1+\xi_2)x - \pi_1 \xi_2}$

$$\begin{tikzcd} U(g)\otimes \mathbb{Q}[x] \arrow[d, "H"'] \arrow[d, shift left=8, "I"] \arrow[rd] & \\ \mathbb{H}\otimes \mathbb{Q}[x] \arrow[r] & \mathbb{Q}[x] \end{tikzcd}$$

$End(\mathbb{Q}[x]) \longrightarrow \mathbb{H}$

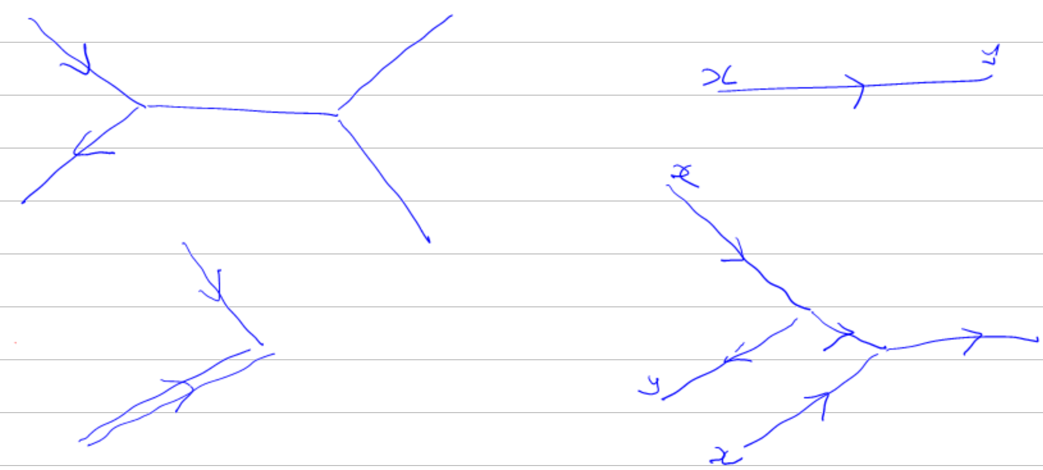
$\xi^i x^j \longrightarrow$

$e^{\beta p} e^{\gamma x p} \sim e^{\gamma x p} e^{\beta p} e^{\alpha \beta p}$

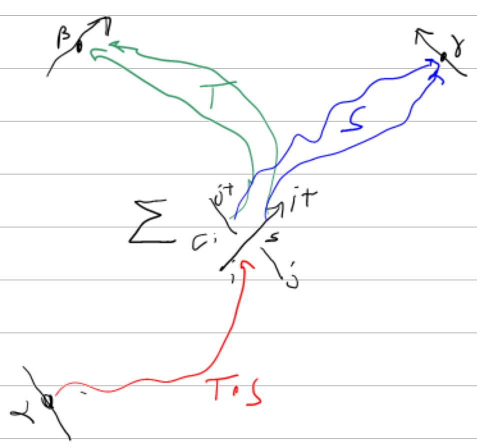
$G(H) = e^{\eta(tp - \epsilon x p^2)} e^{\beta(t + \epsilon x p)} e^{\alpha x p} e^{\xi x} // \sigma'$

at $\epsilon = 0$ this is $\sigma'(e^{\eta t p} e^{\beta t} e^{\alpha x p} e^{\xi x}) =$

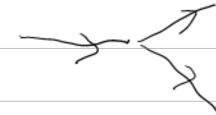
$\sigma'(e^{\beta t} e^{\eta t p} e^{\alpha p x - \alpha} e^{\xi x}) = e^{\beta t - \alpha} e^{\eta t p} e^{(\alpha p + \xi) x}$



$y_{\alpha\beta\gamma}$:



$$\mathcal{L} = \sum \mathcal{L}_c = \sum Q_c + V_c$$



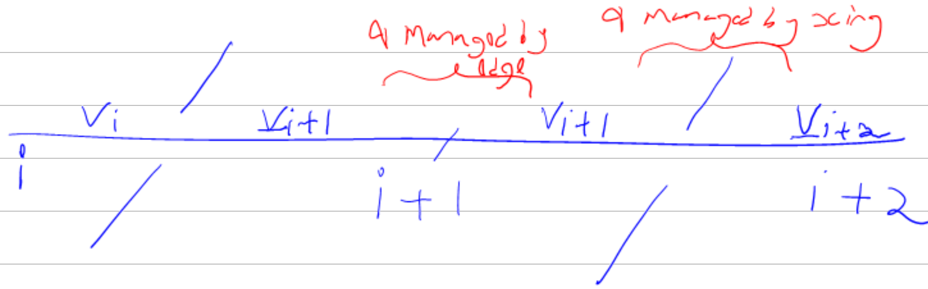
$\deg_{in} - \deg_{out} = -1$

1. $\mathcal{L}$ is weight-balanced.

2. In V, $\deg_{in} - \deg_{out} \geq 1 - 2\deg_{\epsilon}$

   * Preserved by integration/partial integration.

   *



$$q[s\_,\, i\_,\, j\_] := \mathrm{Sum}\Big[\, x_{v,i}\,(p_{v,i^+} - p_{v,i}) + x_{v,j}\,(p_{v,j^+} - p_{v,j}) + \big(T_v^s - 1\big)\, x_{v,i}\,(p_{v,i^+} - p_{v,j^+}),\ \{v, 3\}\Big];$$

$$\mathcal{L}[X_{i\_,j\_}[s\_]] := T_3^s\,\mathbb{E}\Big[q[s, i, j] + B^{-1}\, r_0[s, i, j] + \epsilon\, B\, r_1[s, i, j] + \epsilon\, r_{42}[s, i, j] + O[\epsilon]^2\Big];$$

(*γ1[φ_,k_]:=φ(3/2−$x_{1,k}p_{1,k}-x_{2,k}p_{2,k}-x_{3,k}p_{3,k}$);*)

$$\mathcal{L}[C_{k\_}[0]] := \mathbb{E}\Big[\mathrm{Sum}[x_{v,k}\,(p_{v,k^+} - p_{v,k}),\ \{v, 3\}] + O[\epsilon]^2\Big];$$

$$\mathcal{L}[C_{k\_}[\varphi\_]] := T_3^{\varphi}\,\mathbb{E}\Big[\mathrm{Sum}[x_{v,k}\,(p_{v,k^+} - p_{v,k}),\ \{v, 3\}] + B^{-1}\,\gamma_0[\varphi, k] + \epsilon\, B\,\gamma_1[\varphi, k] + \epsilon\,\gamma_{42}[\varphi, k] + O[\epsilon]^2\Big];$$

$q \to q$, $r_0 \to r_{0,12}$ $r_1 \to r_{1,21}$ $r_{42} \to r_{1,22}$

γ stays.

$$R_1[1, i\_, j\_] := \mathrm{Evaluate}\Big[\mathrm{px2g}[r_1[1, i, j]] + \Big(\mathrm{Coefficient}\big[r_1[1, i, j]\ /.\ t : (x \mid p)\_\_ \mapsto \lambda t,\ \lambda^3\big]\ /.\ x_{3,\alpha\_}\, p_{1,\beta\_}\, p_{2,\gamma\_} \mapsto y_{\alpha,\beta,\gamma}\Big)\Big]$$

$$R_1[-1, i\_, j\_] := \mathrm{Evaluate}\Big[\mathrm{px2g}[r_1[-1, i, j]] \ \ldots\ \mathrm{ient}\big[r_1[-1, i, j]\ /.\ t : (x \mid p)\_\_ \mapsto \lambda t,\ \lambda^3\big]\ /.\ x_{3,\alpha\_}\, p_{1,\beta\_}\, p_{2,\gamma\_} \mapsto y_{\alpha,\beta,\gamma}\Big)\Big]$$

No y's! No $\Theta[s, i, j, \alpha, \beta, \gamma]$. Instead,

Yes R_1, $\Theta[s1, i1, j1, s2, i2, j2]$

$$\theta[1, i\_, j\_, \alpha\_, \beta\_, \gamma\_] := \mathrm{Evaluate}[r_0[1, i, j]\ /.\ \{p_{3,j\_} \mapsto g_{3,j,\alpha},\ x_{1,i\_} \mapsto g_{1,\beta,i},\ x_{2,i\_} \mapsto g_{2,\gamma,i}\}];$$

(* The θ graph with light (pxx) vertex at (1,i,j) and unspecified heavy (xpp) vertex *)

$$\theta[-1, i\_, j\_, \alpha\_, \beta\_, \gamma\_] := \mathrm{Evaluate}[r_0[-1, i, j]\ /.\ \{p_{3,j\_} \mapsto g_{3,j,\alpha},\ x_{1,i\_} \mapsto g_{1,\beta,i},\ x_{2,i\_} \mapsto g_{2,\gamma,i}\}];$$

(* The θ graph with light (pxx) vertex at (-1,i,j) and unspecified heavy (xpp) vertex *)

$$\mathrm{gEval}[\mathcal{E}\_] := \mathrm{CF}[\mathcal{E}\ /.\ g_{v\_,\alpha\_,\beta\_} \mapsto (G[\![\alpha, \beta]\!]\ /.\ T \to T_v)];$$

$$\cancel{Y[\alpha\_, \beta\_, \gamma\_] := Y[\alpha, \beta, \gamma] = \mathrm{Sum}\big[\{s, i, j\} = c;\ \theta[s, i, j, \alpha, \beta, \gamma],\ \{c, Cs\}\big];}$$

(* The expectation value of $x_{3,\alpha}p_{1,\beta}p_{2,\gamma}$ *)

Include ~ double summation over

$$\mathrm{yEval}[\mathcal{E}\_] := \mathcal{E}\ /.\ y_{\alpha\_,\beta\_,\gamma\_} \mapsto Y[\alpha, \beta, \gamma];$$

$$\lambda 1 = \sum_{k=1}^{n} R_1\ @@\ Cs[\![k]\!] - \sum_{k=1}^{2n} \varphi[\![k]\!]\,(g_{1,k,k} + g_{2,k,k} + g_{3,k,k} - 3/2);$$

To do:

1. Finish the full-poly 14-xings stats.
2. Do 22/7 & 34/21 stats.
3. Write a hybrid theta program that can do numerical values quickly.
4. Work on the Beijing handout.