(Alt) In[ ]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\DunfieldKnots"];
Once[<< KnotTheory`];
<< ../Rot.m
T3 = T1 T2;
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

(Alt) In[ ]:=

```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | π | g)__, ∞] ⋃ {x, p, ϵ}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]];
```

Data

(from Talks/Beijing-2407/theta.nb)

(Alt) In[ ]:=

```
R1[1, i_, j_] = CF[
   1 / 2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
    T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
       T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];
```

(Alt) In[ ]:=

```
R1[-1, i_, j_] = CF[
   -1 / 2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
    T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
    (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
       g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)];
```

(Alt) In[ ]:=

```
θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
   T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

(Alt) In[ ]:=

```
θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
  (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
   (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

(Alt) In[ ]:=

```
θ[{-1, i0_, j0_}, {1, i1_, j1_}] = CF[
    T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 -
        T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 - g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1)];
```

(Alt) In[ ]:=

```
θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = CF[
    (1 - T3^-1) (-T1^-1 g1,j1,i0 g2,i1,i0 g3,j0,i1 +
        g1,j1,j0 g2,i1,i0 g3,j0,i1 + T1^-1 g1,j1,i0 g2,j1,i0 g3,j0,i1 - g1,j1,j0 g2,j1,i0 g3,j0,i1)];
```

(Alt) In[ ]:=

```
Γ1[φ_, k_] = -φ / 2 + φ g3,k,k;
```

The Programs

(Alt) In[ ]:=

```
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
    {Cs, φ} = Rot[K]; n = Length[Cs];
    A = IdentityMatrix[2 n + 1];
    Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
    Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
    G = Inverse[A]; gEval[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν)];
    z = gEval[Sum[Sum[θ[Cs[[k1]], Cs[[k2]]], {k2, 1, n}], {k1, 1, n}]];
    z += gEval[Sum[R1 @@ Cs[[k]], {k, 1, n}]];
    z += gEval[Sum[Γ1[φ[[k]], k], {k, 1, 2 n}]];
    {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // Factor];
```

(Alt) In[ ]:=

```
PolyPlot[0] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
  crs = CoefficientRules[T1^(m1=-Exponent[p,T1,Min]) T2^(m2=-Exponent[p,T2,Min]) p, {T1, T2}];
  maxc = N@Log@Max@Abs[Last /@ crs];
  minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
  If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
  hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
  Graphics[crs /. ({x1_, x2_} → c_) :> {
      If[c == 0, White, Lighter[If[c > 0, Red, Blue], 0.88 s[Abs@c]]],
      Polygon[({{1, -1/2}, {0, Sqrt[3]/2}}.{x1 + m1, x2 + m2} + #) & /@ hex]}]]
```

Testing

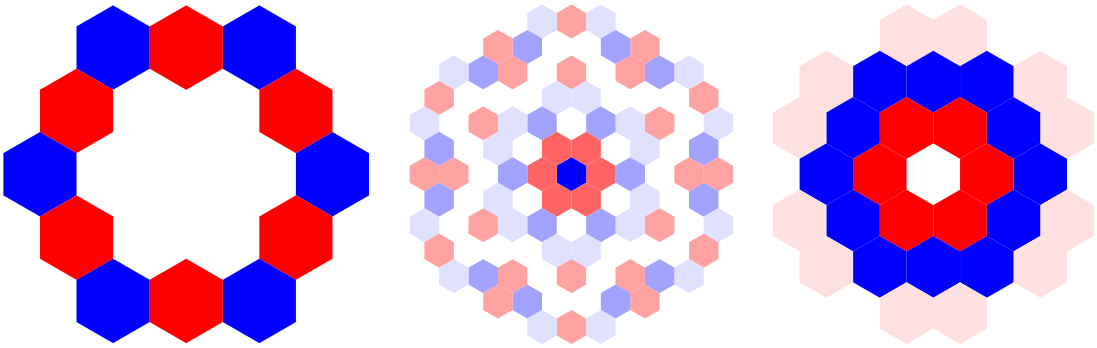
In[•]:=
```
GraphicsRow[PolyPlot[θ[Knot[#]][[2]]] &
  /@ {"3_1", "K11n34", "K11n42"}]
```

KnotTheory: Loading precomputed data in PD4Knots`.

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

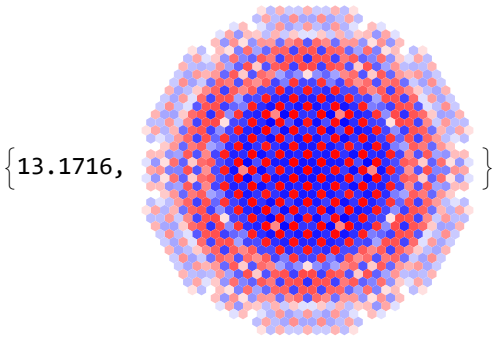
KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[•]=



In[•]:=
```
AbsoluteTiming@
 PolyPlot[θ[EPD[X14,1, X̄2,29, X3,40, X43,4, X̄26,5, X6,95, X96,7, X13,8, X̄9,28, X10,41, X42,11, X̄27,12,
     X30,15, X̄16,61, X̄17,72, X̄18,83, X19,34, X̄89,20, X̄21,92, X̄79,22, X̄68,23, X̄57,24, X̄25,56, X62,31,
     X73,32, X84,33, X̄50,35, X36,81, X37,70, X38,59, X̄39,54, X44,55, X58,45, X69,46, X80,47, X48,91,
     X90,49, X51,82, X52,71, X53,60, X̄63,74, X̄64,85, X̄76,65, X̄87,66, X̄67,94, X̄75,86, X̄88,77, X̄78,93]][[2]]]
```

Out[•]=



Run

(Alt) In[•]:=
```
DunfieldKnots =
  ReadList["../../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;
DK[n_] := DunfieldKnots[[n - 2]];
```

(Alt) In[ ]:=
```
DKString[n_] := StringDrop[ToString[1000 + n], 1]
```

In[ ]:=
```
DKString[76]
```

Out[ ]=
```
076
```

In[ ]:=
```
Do[
 If[FileExistsQ[from = "D" <> ToString[n] <> ".m"],
  RenameFile[from, "D" <> DKString[n] <> ".m"]];
 If[FileExistsQ[from = "PP" <> ToString[n] <> ".png"],
  RenameFile[from, "PP" <> DKString[n] <> ".png"]],
 {n, 3, 99}]
```

(Alt) In[ ]:=
```
Clear[at, pp];
Monitor[
 Do[
  If[(n = Prime[k]) > 1000, Abort[]];
  If[Not@FileExistsQ["D" <> DKString[n] <> ".m"],
   Put[
    ({at, th} = AbsoluteTiming[θ[DK[n]]]) /. {T₁ → T1, T₂ → T2},
    "D" <> DKString[n] <> ".m"];
   Export["PP" <> DKString[n] <> ".png", pp = PolyPlot[th[[2]]]]
  ];
  mon = {n, at, pp},
  {k, 3, 1000, 1}
 ],
 mon ~ Join ~ {n}
]
```