

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\Data"];
Once[<< KnotTheory`];
<< ../Rot.m
T3 = T1 T2;
```

C:\drorbn\AcademicPensieve\Projects\KnotTheory\KnotTheory

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

 SetDelayed: Tag Diff in Diff[K\_PD, rut\_, ag\_, n\_, m\_] is Protected.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[*]:= CCF[ε_] := ExpandDenominator@ExpandNumerator@Together[ε];
CCF[ε_] := Factor[ε];
CF[ε_List] := CF /@ ε;
CF[ε_] := Module[{vs = Cases[ε, {x | p | π | g}_, ∞] ∪ {x, p, ε}, ps, c},
  Total[CoefficientRules[Expand[ε], vs] /. (ps_ -> c_) => CCF[c] (Times @@ vs^ps)]];
```

```
In[*]:= R1[1, i_, j_] = CF[
  1/2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
  T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
  T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];
```

```
In[*]:= R1[-1, i_, j_] = CF[
  -1/2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
  T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
  (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
  g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)];
```

```
In[*]:= θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
  T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

```
In[*]:= θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
  (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
  (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

```
In[*]:= θ[{-1, i0_, j0_}, {1, i1_, j1_}] = CF[
  T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 -
  T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 - g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1)];
```

```
In[*]:=  $\Theta[-1, i\theta_-, j\theta_-], [-1, i1_-, j1_-] = \text{CF} \left[ \begin{aligned} & (1 - T_3^{-1}) \left( -T_1^{-1} g_{1,j1,i\theta} g_{2,i1,i\theta} g_{3,j\theta,i1} + \right. \\ & \left. g_{1,j1,j\theta} g_{2,i1,i\theta} g_{3,j\theta,i1} + T_1^{-1} g_{1,j1,i\theta} g_{2,j1,i\theta} g_{3,j\theta,i1} - g_{1,j1,j\theta} g_{2,j1,i\theta} g_{3,j\theta,i1} \right) \end{aligned} \right];$ 
```

```
In[*]:=  $\Gamma_1[\varphi_-, k_-] = -\varphi / 2 + \varphi g_{3,k,k};$ 
```

```
In[*]:=  $\Theta[K_-] := \text{Module} \left[ \{Cs, \varphi, n, A, s, i, j, k, \Delta, G, v, \alpha, \beta, gEval, c, z\}, \right.$ 
 $\{Cs, \varphi\} = \text{Rot}[K]; n = \text{Length}[Cs];$ 
 $A = \text{IdentityMatrix}[2n + 1];$ 
 $\text{Cases}[Cs, \{s_-, i_-, j_-\} \Rightarrow \left( A[[\{i, j\}, \{i + 1, j + 1\}] + = \begin{pmatrix} -T^s & T^s - 1 \\ \theta & -1 \end{pmatrix} \right)];$ 
 $\Delta = T^{(-\text{Total}[\varphi] - \text{Total}[Cs[[All, 1]]) / 2} \text{Det}[A];$ 
 $G = \text{Inverse}[A]; gEval[\mathcal{E}_-] := \text{Factor}[\mathcal{E} / . g_{v_-, \alpha_-, \beta_-} \Rightarrow (G[[\alpha, \beta]] / . T \rightarrow T_v)];$ 
 $z = gEval \left[ \sum_{k1=1}^n \sum_{k2=1}^n \Theta[Cs[[k1], Cs[[k2]]] \right];$ 
 $z += gEval \left[ \sum_{k=1}^n R_1 @ @ Cs[[k]] \right];$ 
 $z += gEval \left[ \sum_{k=1}^{2^n} \Gamma_1[\varphi[[k], k] \right];$ 
 $\{\Delta, (\Delta / . T \rightarrow T_1) (\Delta / . T \rightarrow T_2) (\Delta / . T \rightarrow T_3) z\} // \text{Factor} \left. \right];$ 
```

```
In[*]:= PolyPlot[ $\theta$ ] = Graphics[{}];
```

```
PolyPlot[ $p_-$ ] := Module[ {crs, m1, m2, maxc, minc, s, hex},
```

```
crs = CoefficientRules[ $T_1^{m1=-\text{Exponent}[p, T_1, \text{Min}]}$   $T_2^{m2=-\text{Exponent}[p, T_2, \text{Min}]}$   $p, \{T_1, T_2\}$ ];
```

```
maxc = N@Log@Max@Abs[Last /@ crs];
```

```
minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
```

```
If[minc == maxc, s[_] = 0, s[_] := s[c] = (maxc - Log@c) / (maxc - minc)];
```

```
hex = Table[{Cos[ $\alpha$ ], Sin[ $\alpha$ ]} / Cos[ $2\pi / 12$ ] / 2, { $\alpha, 2\pi / 12, 2\pi, 2\pi / 6$ }}];
```

```
Graphics[crs /. ({x1_-, x2_-}  $\rightarrow$  c_)  $\Rightarrow$  {
```

```
  If[c == 0, White, Lighter[Which[
```

```
    c > 0 ^ OddQ[c], Orange,
```

```
    c > 0 ^ EvenQ[c], Red,
```

```
    c < 0 ^ OddQ[c], Green,
```

```
    c < 0 ^ EvenQ[c], Blue
```

```
  ], 0.88 s[Abs@c]]],
```

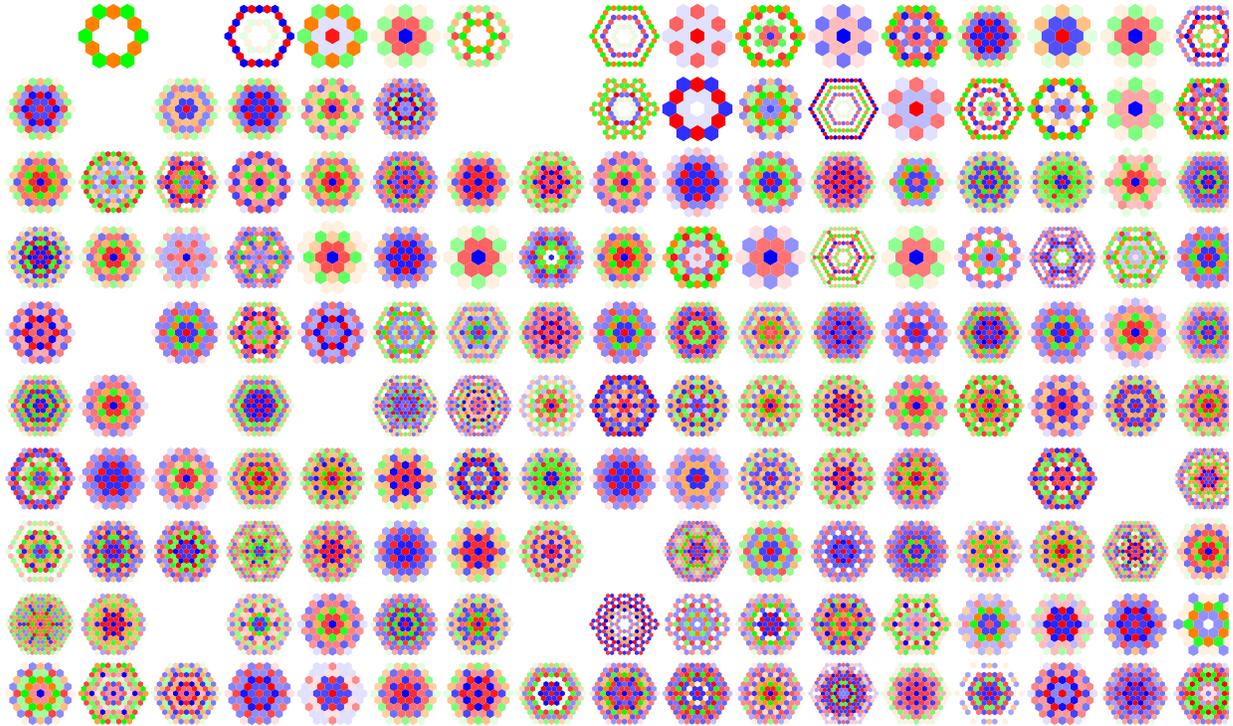
```
  Polygon[ $\left[ \left( \begin{pmatrix} 1 & -1/2 \\ \theta & \sqrt{3}/2 \end{pmatrix} \cdot \{x1 + m1, x2 + m2\} + \# \right) \& / @ hex \right] \right] ]];$ 
```

```
PolyPlot[{ $A_-$ ,  $\theta_-$ }] := PolyPlot[ $\theta$ ]
```

```
In[ ]:= tab250 = {0} ~ Join ~ Table[0[K] [[2]], {K, AllKnots[{3, 10}]}];
g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 25], Spacings -> 0]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[ ]:=



```
In[ ]:= Table[n -> NumberOfKnots[n], {n, 3, 15}]
```

Out[ ]:=

```
{3 -> 1, 4 -> 1, 5 -> 2, 6 -> 3, 7 -> 7, 8 -> 21, 9 -> 49,
10 -> 165, 11 -> 552, 12 -> 2176, 13 -> 9988, 14 -> 46972, 15 -> 253293}
```

In[\*]:=  $\Theta$ [Knot[15, Alternating, 20902]]

 KnotTheory: Loading precomputed data in KnotTheory/15A.dts.

 KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[\*]=

$$\left\{ -\frac{6 - 35 T + 91 T^2 - 125 T^3 + 91 T^4 - 35 T^5 + 6 T^6}{T^3}, \right. \\ \left. -\frac{1}{T_1^6 T_2^6} \left( 45 - 319 T_1 + 878 T_1^2 - 1219 T_1^3 + 878 T_1^4 - 319 T_1^5 + 45 T_1^6 - 319 T_2 + 1870 T_1 T_2 - 3824 T_1^2 T_2 + \right. \right. \\ \left. \left. 2356 T_1^3 T_2 + 2356 T_1^4 T_2 - 3824 T_1^5 T_2 + 1870 T_1^6 T_2 - 319 T_1^7 T_2 + 878 T_1^8 T_2 - 3824 T_1 T_2^2 + 2825 T_1^2 T_2^2 + \right. \right. \\ \left. \left. 10702 T_1^3 T_2^2 - 22185 T_1^4 T_2^2 + 10702 T_1^5 T_2^2 + 2825 T_1^6 T_2^2 - 3824 T_1^7 T_2^2 + 878 T_1^8 T_2^2 - 1219 T_2^3 + \right. \right. \\ \left. \left. 2356 T_1 T_2^3 + 10702 T_1^2 T_2^3 - 37286 T_1^3 T_2^3 + 27271 T_1^4 T_2^3 + 27271 T_1^5 T_2^3 - 37286 T_1^6 T_2^3 + 10702 T_1^7 T_2^3 + \right. \right. \\ \left. \left. 2356 T_1^8 T_2^3 - 1219 T_1^9 T_2^3 + 878 T_2^4 + 2356 T_1 T_2^4 - 22185 T_1^2 T_2^4 + 27271 T_1^3 T_2^4 + 45442 T_1^4 T_2^4 - \right. \right. \\ \left. \left. 116098 T_1^5 T_2^4 + 45442 T_1^6 T_2^4 + 27271 T_1^7 T_2^4 - 22185 T_1^8 T_2^4 + 2356 T_1^9 T_2^4 + 878 T_1^{10} T_2^4 - 319 T_2^5 - \right. \right. \\ \left. \left. 3824 T_1 T_2^5 + 10702 T_1^2 T_2^5 + 27271 T_1^3 T_2^5 - 116098 T_1^4 T_2^5 + 89308 T_1^5 T_2^5 + 89308 T_1^6 T_2^5 - \right. \right. \\ \left. \left. 116098 T_1^7 T_2^5 + 27271 T_1^8 T_2^5 + 10702 T_1^9 T_2^5 - 3824 T_1^{10} T_2^5 - 319 T_1^{11} T_2^5 + 45 T_2^6 + 1870 T_1 T_2^6 + \right. \right. \\ \left. \left. 2825 T_1^2 T_2^6 - 37286 T_1^3 T_2^6 + 45442 T_1^4 T_2^6 + 89308 T_1^5 T_2^6 - 220980 T_1^6 T_2^6 + 89308 T_1^7 T_2^6 + 45442 T_1^8 T_2^6 - \right. \right. \\ \left. \left. 37286 T_1^9 T_2^6 + 2825 T_1^{10} T_2^6 + 1870 T_1^{11} T_2^6 + 45 T_2^7 - 319 T_1 T_2^7 - 3824 T_1^2 T_2^7 + 10702 T_1^3 T_2^7 + \right. \right. \\ \left. \left. 27271 T_1^4 T_2^7 - 116098 T_1^5 T_2^7 + 89308 T_1^6 T_2^7 + 89308 T_1^7 T_2^7 - 116098 T_1^8 T_2^7 + 27271 T_1^9 T_2^7 + \right. \right. \\ \left. \left. 10702 T_1^{10} T_2^7 - 3824 T_1^{11} T_2^7 - 319 T_1^{12} T_2^7 + 878 T_2^8 + 2356 T_1 T_2^8 - 22185 T_1^2 T_2^8 + 27271 T_1^3 T_2^8 + \right. \right. \\ \left. \left. 45442 T_1^4 T_2^8 - 116098 T_1^5 T_2^8 + 45442 T_1^6 T_2^8 + 27271 T_1^7 T_2^8 - 22185 T_1^8 T_2^8 + 2356 T_1^9 T_2^8 + \right. \right. \\ \left. \left. 878 T_1^{10} T_2^8 - 1219 T_1^3 T_2^9 + 2356 T_1^4 T_2^9 + 10702 T_1^5 T_2^9 - 37286 T_1^6 T_2^9 + 27271 T_1^7 T_2^9 + 27271 T_1^8 T_2^9 - \right. \right. \\ \left. \left. 37286 T_1^9 T_2^9 + 10702 T_1^{10} T_2^9 + 2356 T_1^{11} T_2^9 - 1219 T_1^{12} T_2^9 + 878 T_1^4 T_2^{10} - 3824 T_1^5 T_2^{10} + 2825 T_1^6 T_2^{10} + \right. \right. \\ \left. \left. 10702 T_1^7 T_2^{10} - 22185 T_1^8 T_2^{10} + 10702 T_1^9 T_2^{10} + 2825 T_1^{10} T_2^{10} - 3824 T_1^{11} T_2^{10} + 878 T_1^{12} T_2^{10} - 319 T_1^5 T_2^{11} + \right. \right. \\ \left. \left. 1870 T_1^6 T_2^{11} - 3824 T_1^7 T_2^{11} + 2356 T_1^8 T_2^{11} + 2356 T_1^9 T_2^{11} - 3824 T_1^{10} T_2^{11} + 1870 T_1^{11} T_2^{11} - 319 T_1^{12} T_2^{11} + \right. \right. \\ \left. \left. 45 T_1^6 T_2^{12} - 319 T_1^7 T_2^{12} + 878 T_1^8 T_2^{12} - 1219 T_1^9 T_2^{12} + 878 T_1^{10} T_2^{12} - 319 T_1^{11} T_2^{12} + 45 T_1^{12} T_2^{12} \right) \right\}$$

In[\*]:=  $\Theta$ [Knot[15, Alternating, 20903]]

Out[\*]=

$$\left\{ \frac{(2 - 3 T + 2 T^2) (1 - 6 T + 16 T^2 - 23 T^3 + 25 T^4 - 23 T^5 + 16 T^6 - 6 T^7 + T^8)}{T^5}, \right. \\ \left. \frac{1}{T_1^{10} T_2^{10}} \left( 25 - 186 T_1 + 639 T_1^2 - 1291 T_1^3 + 1827 T_1^4 - 2015 T_1^5 + 1827 T_1^6 - 1291 T_1^7 + 639 T_1^8 - 186 T_1^9 + 25 T_1^{10} - \right. \right. \\ \left. \left. 186 T_2 + 1200 T_1 T_2 - 3382 T_1^2 T_2 + 4879 T_1^3 T_2 - 4021 T_1^4 T_2 + 1414 T_1^5 T_2 + 1414 T_1^6 T_2 - 4021 T_1^7 T_2 + \right. \right. \\ \left. \left. 4879 T_1^8 T_2 - 3382 T_1^9 T_2 + 1200 T_1^{10} T_2 - 186 T_1^{11} T_2 + 639 T_2^2 - 3382 T_1 T_2^2 + 6708 T_1^2 T_2^2 - 2333 T_1^3 T_2^2 - \right. \right. \\ \left. \left. 8819 T_1^4 T_2^2 + 17455 T_1^5 T_2^2 - 19176 T_1^6 T_2^2 + 17455 T_1^7 T_2^2 - 8819 T_1^8 T_2^2 - 2333 T_1^9 T_2^2 + 6708 T_1^{10} T_2^2 - \right. \right. \\ \left. \left. 3382 T_1^{11} T_2^2 + 639 T_1^{12} T_2^2 - 1291 T_2^3 + 4879 T_1 T_2^3 - 2333 T_1^2 T_2^3 - 21460 T_1^3 T_2^3 + 41279 T_1^4 T_2^3 - 33366 T_1^5 T_2^3 + \right. \right. \\ \left. \left. 9255 T_1^6 T_2^3 + 9255 T_1^7 T_2^3 - 33366 T_1^8 T_2^3 + 41279 T_1^9 T_2^3 - 21460 T_1^{10} T_2^3 - 2333 T_1^{11} T_2^3 + 4879 T_1^{12} T_2^3 - \right. \right. \\ \left. \left. 1291 T_1^{13} T_2^3 + 1827 T_2^4 - 4021 T_1 T_2^4 - 8819 T_1^2 T_2^4 + 41279 T_1^3 T_2^4 - 18452 T_1^4 T_2^4 - 56189 T_1^5 T_2^4 + \right. \right. \\ \left. \left. 109308 T_1^6 T_2^4 - 110634 T_1^7 T_2^4 + 109308 T_1^8 T_2^4 - 56189 T_1^9 T_2^4 - 18452 T_1^{10} T_2^4 + 41279 T_1^{11} T_2^4 - \right. \right. \\ \left. \left. 8819 T_1^{12} T_2^4 - 4021 T_1^{13} T_2^4 + 1827 T_2^5 + 1414 T_1 T_2^5 + 17455 T_1^2 T_2^5 - 33366 T_1^3 T_2^5 - \right. \right. \\ \left. \left. 56189 T_1^4 T_2^5 + 140578 T_1^5 T_2^5 - 110869 T_1^6 T_2^5 + 19830 T_1^7 T_2^5 + 19830 T_1^8 T_2^5 - 110869 T_1^9 T_2^5 + \right. \right. \\ \left. \left. 140578 T_1^{10} T_2^5 - 56189 T_1^{11} T_2^5 - 33366 T_1^{12} T_2^5 + 17455 T_1^{13} T_2^5 + 1414 T_1^{14} T_2^5 - 2015 T_1^{15} T_2^5 + 1827 T_2^6 + \right. \right. \\ \left. \left. 1414 T_1 T_2^6 - 19176 T_1^2 T_2^6 + 9255 T_1^3 T_2^6 + 109308 T_1^4 T_2^6 - 110869 T_1^5 T_2^6 - 18442 T_1^6 T_2^6 + 103490 T_1^7 T_2^6 - \right. \right. \\ \left. \left. 110869 T_1^8 T_2^6 + 140578 T_1^9 T_2^6 - 56189 T_1^{10} T_2^6 + 140578 T_1^{11} T_2^6 - 110869 T_1^{12} T_2^6 + 1414 T_1^{13} T_2^6 - 2015 T_1^{14} T_2^6 + 1827 T_1^{15} T_2^6 \right) \right\}$$

$$\begin{aligned}
 &64\,338\,T_1^8 T_2^6 + 103\,490\,T_1^9 T_2^6 - 18\,442\,T_1^{10} T_2^6 - 110\,869\,T_1^{11} T_2^6 + 109\,308\,T_1^{12} T_2^6 + 9255\,T_1^{13} T_2^6 - \\
 &19\,176\,T_1^{14} T_2^6 + 1414\,T_1^{15} T_2^6 + 1827\,T_1^{16} T_2^6 - 1291\,T_2^7 - 4021\,T_1 T_2^7 + 17\,455\,T_1^2 T_2^7 + 9255\,T_1^3 T_2^7 - \\
 &110\,634\,T_1^4 T_2^7 + 19\,830\,T_1^5 T_2^7 + 103\,490\,T_1^6 T_2^7 - 81\,860\,T_1^7 T_2^7 - 23\,747\,T_1^8 T_2^7 - 23\,747\,T_1^9 T_2^7 - 81\,860\,T_1^{10} T_2^7 + \\
 &103\,490\,T_1^{11} T_2^7 + 19\,830\,T_1^{12} T_2^7 - 110\,634\,T_1^{13} T_2^7 + 9255\,T_1^{14} T_2^7 + 17\,455\,T_1^{15} T_2^7 - 4021\,T_1^{16} T_2^7 - \\
 &1291\,T_1^{17} T_2^7 + 639\,T_2^8 + 4879\,T_1 T_2^8 - 8819\,T_1^2 T_2^8 - 33\,366\,T_1^3 T_2^8 + 109\,308\,T_1^4 T_2^8 + 19\,830\,T_1^5 T_2^8 - \\
 &64\,338\,T_1^6 T_2^8 - 23\,747\,T_1^7 T_2^8 + 87\,820\,T_1^8 T_2^8 + 11\,778\,T_1^9 T_2^8 + 87\,820\,T_1^{10} T_2^8 - 23\,747\,T_1^{11} T_2^8 - 64\,338\,T_1^{12} T_2^8 + \\
 &19\,830\,T_1^{13} T_2^8 + 109\,308\,T_1^{14} T_2^8 - 33\,366\,T_1^{15} T_2^8 - 8819\,T_1^{16} T_2^8 + 4879\,T_1^{17} T_2^8 + 639\,T_1^{18} T_2^8 - 186\,T_2^9 - \\
 &3382\,T_1 T_2^9 - 2333\,T_1^2 T_2^9 + 41\,279\,T_1^3 T_2^9 - 56\,189\,T_1^4 T_2^9 - 110\,869\,T_1^5 T_2^9 + 103\,490\,T_1^6 T_2^9 - 23\,747\,T_1^7 T_2^9 + \\
 &11\,778\,T_1^8 T_2^9 - 77\,420\,T_1^9 T_2^9 - 77\,420\,T_1^{10} T_2^9 + 11\,778\,T_1^{11} T_2^9 - 23\,747\,T_1^{12} T_2^9 + 103\,490\,T_1^{13} T_2^9 - \\
 &110\,869\,T_1^{14} T_2^9 - 56\,189\,T_1^{15} T_2^9 + 41\,279\,T_1^{16} T_2^9 - 2333\,T_1^{17} T_2^9 - 3382\,T_1^{18} T_2^9 - 186\,T_1^{19} T_2^9 + 25\,T_2^{10} + \\
 &1200\,T_1 T_2^{10} + 6708\,T_1^2 T_2^{10} - 21\,460\,T_1^3 T_2^{10} - 18\,452\,T_1^4 T_2^{10} + 140\,578\,T_1^5 T_2^{10} - 18\,442\,T_1^6 T_2^{10} - 81\,860\,T_1^7 T_2^{10} + \\
 &87\,820\,T_1^8 T_2^{10} - 77\,420\,T_1^9 T_2^{10} + 212\,052\,T_1^{10} T_2^{10} - 77\,420\,T_1^{11} T_2^{10} + 87\,820\,T_1^{12} T_2^{10} - 81\,860\,T_1^{13} T_2^{10} - \\
 &18\,442\,T_1^{14} T_2^{10} + 140\,578\,T_1^{15} T_2^{10} - 18\,452\,T_1^{16} T_2^{10} - 21\,460\,T_1^{17} T_2^{10} + 6708\,T_1^{18} T_2^{10} + 1200\,T_1^{19} T_2^{10} + \\
 &25\,T_2^{20} - 186\,T_1 T_2^{21} - 3382\,T_1^2 T_2^{21} - 2333\,T_1^3 T_2^{21} + 41\,279\,T_1^4 T_2^{21} - 56\,189\,T_1^5 T_2^{21} - 110\,869\,T_1^6 T_2^{21} + \\
 &103\,490\,T_1^7 T_2^{21} - 23\,747\,T_1^8 T_2^{21} + 11\,778\,T_1^9 T_2^{21} - 77\,420\,T_1^{10} T_2^{21} - 77\,420\,T_1^{11} T_2^{21} + 11\,778\,T_1^{12} T_2^{21} - \\
 &23\,747\,T_1^{13} T_2^{21} + 103\,490\,T_1^{14} T_2^{21} - 110\,869\,T_1^{15} T_2^{21} - 56\,189\,T_1^{16} T_2^{21} + 41\,279\,T_1^{17} T_2^{21} - 2333\,T_1^{18} T_2^{21} - \\
 &3382\,T_1^{19} T_2^{21} - 186\,T_1^{20} T_2^{21} + 639\,T_1^2 T_2^{22} + 4879\,T_1^3 T_2^{22} - 8819\,T_1^4 T_2^{22} - 33\,366\,T_1^5 T_2^{22} + 109\,308\,T_1^6 T_2^{22} + \\
 &19\,830\,T_1^7 T_2^{22} - 64\,338\,T_1^8 T_2^{22} - 23\,747\,T_1^9 T_2^{22} + 87\,820\,T_1^{10} T_2^{22} + 11\,778\,T_1^{11} T_2^{22} + 87\,820\,T_1^{12} T_2^{22} - \\
 &23\,747\,T_1^{13} T_2^{22} - 64\,338\,T_1^{14} T_2^{22} + 19\,830\,T_1^{15} T_2^{22} + 109\,308\,T_1^{16} T_2^{22} - 33\,366\,T_1^{17} T_2^{22} - 8819\,T_1^{18} T_2^{22} + \\
 &4879\,T_1^{19} T_2^{22} + 639\,T_1^{20} T_2^{22} - 1291\,T_1^3 T_2^{23} - 4021\,T_1^4 T_2^{23} + 17\,455\,T_1^5 T_2^{23} + 9255\,T_1^6 T_2^{23} - 110\,634\,T_1^7 T_2^{23} + \\
 &19\,830\,T_1^8 T_2^{23} + 103\,490\,T_1^9 T_2^{23} - 81\,860\,T_1^{10} T_2^{23} - 23\,747\,T_1^{11} T_2^{23} - 23\,747\,T_1^{12} T_2^{23} - 81\,860\,T_1^{13} T_2^{23} + \\
 &103\,490\,T_1^{14} T_2^{23} + 19\,830\,T_1^{15} T_2^{23} - 110\,634\,T_1^{16} T_2^{23} + 9255\,T_1^{17} T_2^{23} + 17\,455\,T_1^{18} T_2^{23} - 4021\,T_1^{19} T_2^{23} - \\
 &1291\,T_1^{20} T_2^{23} + 1827\,T_1^4 T_2^{24} + 1414\,T_1^5 T_2^{24} - 19\,176\,T_1^6 T_2^{24} + 9255\,T_1^7 T_2^{24} + 109\,308\,T_1^8 T_2^{24} - 110\,869\,T_1^9 T_2^{24} - \\
 &18\,442\,T_1^{10} T_2^{24} + 103\,490\,T_1^{11} T_2^{24} - 64\,338\,T_1^{12} T_2^{24} + 103\,490\,T_1^{13} T_2^{24} - 18\,442\,T_1^{14} T_2^{24} - 110\,869\,T_1^{15} T_2^{24} + \\
 &109\,308\,T_1^{16} T_2^{24} + 9255\,T_1^{17} T_2^{24} - 19\,176\,T_1^{18} T_2^{24} + 1414\,T_1^{19} T_2^{24} + 1827\,T_1^{20} T_2^{24} - 2015\,T_1^5 T_2^{25} + 1414\,T_1^6 T_2^{25} + \\
 &17\,455\,T_1^7 T_2^{25} - 33\,366\,T_1^8 T_2^{25} - 56\,189\,T_1^9 T_2^{25} + 140\,578\,T_1^{10} T_2^{25} - 110\,869\,T_1^{11} T_2^{25} + 19\,830\,T_1^{12} T_2^{25} + \\
 &19\,830\,T_1^{13} T_2^{25} - 110\,869\,T_1^{14} T_2^{25} + 140\,578\,T_1^{15} T_2^{25} - 56\,189\,T_1^{16} T_2^{25} - 33\,366\,T_1^{17} T_2^{25} + 17\,455\,T_1^{18} T_2^{25} + \\
 &1414\,T_1^{19} T_2^{25} - 2015\,T_1^{20} T_2^{25} + 1827\,T_1^6 T_2^{26} - 4021\,T_1^7 T_2^{26} - 8819\,T_1^8 T_2^{26} + 41\,279\,T_1^9 T_2^{26} - 18\,452\,T_1^{10} T_2^{26} - \\
 &56\,189\,T_1^{11} T_2^{26} + 109\,308\,T_1^{12} T_2^{26} - 110\,634\,T_1^{13} T_2^{26} + 109\,308\,T_1^{14} T_2^{26} - 56\,189\,T_1^{15} T_2^{26} - 18\,452\,T_1^{16} T_2^{26} + \\
 &41\,279\,T_1^{17} T_2^{26} - 8819\,T_1^{18} T_2^{26} - 4021\,T_1^{19} T_2^{26} + 1827\,T_1^{20} T_2^{26} - 1291\,T_1^7 T_2^{27} + 4879\,T_1^8 T_2^{27} - 2333\,T_1^9 T_2^{27} - \\
 &21\,460\,T_1^{10} T_2^{27} + 41\,279\,T_1^{11} T_2^{27} - 33\,366\,T_1^{12} T_2^{27} + 9255\,T_1^{13} T_2^{27} + 9255\,T_1^{14} T_2^{27} - 33\,366\,T_1^{15} T_2^{27} + \\
 &41\,279\,T_1^{16} T_2^{27} - 21\,460\,T_1^{17} T_2^{27} - 2333\,T_1^{18} T_2^{27} + 4879\,T_1^{19} T_2^{27} - 1291\,T_1^{20} T_2^{27} + 639\,T_1^8 T_2^{28} - 3382\,T_1^9 T_2^{28} + \\
 &6708\,T_1^{10} T_2^{28} - 2333\,T_1^{11} T_2^{28} - 8819\,T_1^{12} T_2^{28} + 17\,455\,T_1^{13} T_2^{28} - 19\,176\,T_1^{14} T_2^{28} + 17\,455\,T_1^{15} T_2^{28} - \\
 &8819\,T_1^{16} T_2^{28} - 2333\,T_1^{17} T_2^{28} + 6708\,T_1^{18} T_2^{28} - 3382\,T_1^{19} T_2^{28} + 639\,T_1^{20} T_2^{28} - 186\,T_1^9 T_2^{29} + 1200\,T_1^{10} T_2^{29} - \\
 &3382\,T_1^{11} T_2^{29} + 4879\,T_1^{12} T_2^{29} - 4021\,T_1^{13} T_2^{29} + 1414\,T_1^{14} T_2^{29} + 1414\,T_1^{15} T_2^{29} - 4021\,T_1^{16} T_2^{29} + 4879\,T_1^{17} T_2^{29} - \\
 &3382\,T_1^{18} T_2^{29} + 1200\,T_1^{19} T_2^{29} - 186\,T_1^{20} T_2^{29} + 25\,T_1^{10} T_2^{30} - 186\,T_1^{11} T_2^{30} + 639\,T_1^{12} T_2^{30} - 1291\,T_1^{13} T_2^{30} + \\
 &1827\,T_1^{14} T_2^{30} - 2015\,T_1^{15} T_2^{30} + 1827\,T_1^{16} T_2^{30} - 1291\,T_1^{17} T_2^{30} + 639\,T_1^{18} T_2^{30} - 186\,T_1^{19} T_2^{30} + 25\,T_1^{20} T_2^{30} \}
 \end{aligned}$$

In[\*]:=  $\theta[\text{Knot}[15, \text{Alternating}, 20904]]$

Out[ ]:=

$$\left\{ \frac{4 - 31 T + 109 T^2 - 217 T^3 + 271 T^4 - 217 T^5 + 109 T^6 - 31 T^7 + 4 T^8}{T^4}, \right.$$

$$\frac{1}{T_1^8 T_2^8} \left( 144 - 1104 T_1 + 3840 T_1^2 - 7580 T_1^3 + 9440 T_1^4 - 7580 T_1^5 + 3840 T_1^6 - 1104 T_1^7 + 144 T_1^8 - 1104 T_2 + \right.$$

$$7372 T_1 T_2 - 21032 T_1^2 T_2 + 28778 T_1^3 T_2 - 14318 T_1^4 T_2 - 14318 T_1^5 T_2 + 28778 T_1^6 T_2 - 21032 T_1^7 T_2 +$$

$$7372 T_1^8 T_2 - 1104 T_1^9 T_2 + 3840 T_2^2 - 21032 T_1 T_2^2 + 41413 T_1^2 T_2^2 - 5211 T_1^3 T_2^2 - 93916 T_1^4 T_2^2 +$$

$$154213 T_1^5 T_2^2 - 93916 T_1^6 T_2^2 - 5211 T_1^7 T_2^2 + 41413 T_1^8 T_2^2 - 21032 T_1^9 T_2^2 + 3840 T_1^{10} T_2^2 - 7580 T_2^3 +$$

$$28778 T_1 T_2^3 - 5211 T_1^2 T_2^3 - 172230 T_1^3 T_2^3 + 334651 T_1^4 T_2^3 - 188351 T_1^5 T_2^3 - 188351 T_1^6 T_2^3 +$$

$$334651 T_1^7 T_2^3 - 172230 T_1^8 T_2^3 - 5211 T_1^9 T_2^3 + 28778 T_1^{10} T_2^3 - 7580 T_1^{11} T_2^3 + 9440 T_2^4 - 14318 T_1 T_2^4 -$$

$$93916 T_1^2 T_2^4 + 334651 T_1^3 T_2^4 - 154881 T_1^4 T_2^4 - 611596 T_1^5 T_2^4 + 1123849 T_1^6 T_2^4 - 611596 T_1^7 T_2^4 -$$

$$154881 T_1^8 T_2^4 + 334651 T_1^9 T_2^4 - 93916 T_1^{10} T_2^4 - 14318 T_1^{11} T_2^4 + 9440 T_1^{12} T_2^4 - 7580 T_2^5 - 14318 T_1 T_2^5 +$$

$$154213 T_1^2 T_2^5 - 188351 T_1^3 T_2^5 - 611596 T_1^4 T_2^5 + 1498262 T_1^5 T_2^5 - 903551 T_1^6 T_2^5 - 903551 T_1^7 T_2^5 +$$

$$1498262 T_1^8 T_2^5 - 611596 T_1^9 T_2^5 - 188351 T_1^{10} T_2^5 + 154213 T_1^{11} T_2^5 - 14318 T_1^{12} T_2^5 - 7580 T_1^{13} T_2^5 +$$

$$3840 T_2^6 + 28778 T_1 T_2^6 - 93916 T_1^2 T_2^6 - 188351 T_1^3 T_2^6 + 1123849 T_1^4 T_2^6 - 903551 T_1^5 T_2^6 - 1249936 T_1^6 T_2^6 +$$

$$2819416 T_1^7 T_2^6 - 1249936 T_1^8 T_2^6 - 903551 T_1^9 T_2^6 + 1123849 T_1^{10} T_2^6 - 188351 T_1^{11} T_2^6 - 93916 T_1^{12} T_2^6 +$$

$$28778 T_1^{13} T_2^6 + 3840 T_1^{14} T_2^6 - 1104 T_2^7 - 21032 T_1 T_2^7 - 5211 T_1^2 T_2^7 + 334651 T_1^3 T_2^7 - 611596 T_1^4 T_2^7 -$$

$$903551 T_1^5 T_2^7 + 2819416 T_1^6 T_2^7 - 1794860 T_1^7 T_2^7 - 1794860 T_1^8 T_2^7 + 2819416 T_1^9 T_2^7 - 903551 T_1^{10} T_2^7 -$$

$$611596 T_1^{11} T_2^7 + 334651 T_1^{12} T_2^7 - 5211 T_1^{13} T_2^7 - 21032 T_1^{14} T_2^7 - 1104 T_1^{15} T_2^7 + 144 T_2^8 + 7372 T_1 T_2^8 +$$

$$41413 T_1^2 T_2^8 - 172230 T_1^3 T_2^8 - 154881 T_1^4 T_2^8 + 1498262 T_1^5 T_2^8 - 1249936 T_1^6 T_2^8 - 1794860 T_1^7 T_2^8 +$$

$$4059468 T_1^8 T_2^8 - 1794860 T_1^9 T_2^8 - 1249936 T_1^{10} T_2^8 + 1498262 T_1^{11} T_2^8 - 154881 T_1^{12} T_2^8 - 172230 T_1^{13} T_2^8 +$$

$$41413 T_1^{14} T_2^8 + 7372 T_1^{15} T_2^8 + 144 T_1^{16} T_2^8 - 1104 T_1 T_2^9 - 21032 T_1^2 T_2^9 - 5211 T_1^3 T_2^9 + 334651 T_1^4 T_2^9 -$$

$$611596 T_1^5 T_2^9 - 903551 T_1^6 T_2^9 + 2819416 T_1^7 T_2^9 - 1794860 T_1^8 T_2^9 - 1794860 T_1^9 T_2^9 + 2819416 T_1^{10} T_2^9 -$$

$$903551 T_1^{11} T_2^9 - 611596 T_1^{12} T_2^9 + 334651 T_1^{13} T_2^9 - 5211 T_1^{14} T_2^9 - 21032 T_1^{15} T_2^9 - 1104 T_1^{16} T_2^9 +$$

$$3840 T_1^2 T_2^{10} + 28778 T_1^3 T_2^{10} - 93916 T_1^4 T_2^{10} - 188351 T_1^5 T_2^{10} + 1123849 T_1^6 T_2^{10} - 903551 T_1^7 T_2^{10} -$$

$$1249936 T_1^8 T_2^{10} + 2819416 T_1^9 T_2^{10} - 1249936 T_1^{10} T_2^{10} - 903551 T_1^{11} T_2^{10} + 1123849 T_1^{12} T_2^{10} -$$

$$188351 T_1^{13} T_2^{10} - 93916 T_1^{14} T_2^{10} + 28778 T_1^{15} T_2^{10} + 3840 T_1^{16} T_2^{10} - 7580 T_1^3 T_2^{11} - 14318 T_1^4 T_2^{11} +$$

$$154213 T_1^5 T_2^{11} - 188351 T_1^6 T_2^{11} - 611596 T_1^7 T_2^{11} + 1498262 T_1^8 T_2^{11} - 903551 T_1^9 T_2^{11} - 903551 T_1^{10} T_2^{11} +$$

$$1498262 T_1^{11} T_2^{11} - 611596 T_1^{12} T_2^{11} - 188351 T_1^{13} T_2^{11} + 154213 T_1^{14} T_2^{11} - 14318 T_1^{15} T_2^{11} - 7580 T_1^{16} T_2^{11} +$$

$$9440 T_1^4 T_2^{12} - 14318 T_1^5 T_2^{12} - 93916 T_1^6 T_2^{12} + 334651 T_1^7 T_2^{12} - 154881 T_1^8 T_2^{12} - 611596 T_1^9 T_2^{12} +$$

$$1123849 T_1^{10} T_2^{12} - 611596 T_1^{11} T_2^{12} - 154881 T_1^{12} T_2^{12} + 334651 T_1^{13} T_2^{12} - 93916 T_1^{14} T_2^{12} - 14318 T_1^{15} T_2^{12} +$$

$$9440 T_1^{16} T_2^{12} - 7580 T_1^5 T_2^{13} + 28778 T_1^6 T_2^{13} - 5211 T_1^7 T_2^{13} - 172230 T_1^8 T_2^{13} + 334651 T_1^9 T_2^{13} -$$

$$188351 T_1^{10} T_2^{13} - 188351 T_1^{11} T_2^{13} + 334651 T_1^{12} T_2^{13} - 172230 T_1^{13} T_2^{13} - 5211 T_1^{14} T_2^{13} + 28778 T_1^{15} T_2^{13} -$$

$$7580 T_1^{16} T_2^{13} + 3840 T_1^6 T_2^{14} - 21032 T_1^7 T_2^{14} + 41413 T_1^8 T_2^{14} - 5211 T_1^9 T_2^{14} - 93916 T_1^{10} T_2^{14} +$$

$$154213 T_1^{11} T_2^{14} - 93916 T_1^{12} T_2^{14} - 5211 T_1^{13} T_2^{14} + 41413 T_1^{14} T_2^{14} - 21032 T_1^{15} T_2^{14} + 3840 T_1^{16} T_2^{14} -$$

$$1104 T_1^7 T_2^{15} + 7372 T_1^8 T_2^{15} - 21032 T_1^9 T_2^{15} + 28778 T_1^{10} T_2^{15} - 14318 T_1^{11} T_2^{15} - 14318 T_1^{12} T_2^{15} +$$

$$28778 T_1^{13} T_2^{15} - 21032 T_1^{14} T_2^{15} + 7372 T_1^{15} T_2^{15} - 1104 T_1^{16} T_2^{15} + 144 T_1^8 T_2^{16} - 1104 T_1^9 T_2^{16} +$$

$$3840 T_1^{10} T_2^{16} - 7580 T_1^{11} T_2^{16} + 9440 T_1^{12} T_2^{16} - 7580 T_1^{13} T_2^{16} + 3840 T_1^{14} T_2^{16} - 1104 T_1^{15} T_2^{16} + 144 T_1^{16} T_2^{16} \left. \right\}$$

In[ ]:=  $\theta$ [Knot[15, Alternating, 20905]]

Out[ ]:=

$$\left\{ \frac{1 - 10 T + 44 T^2 - 118 T^3 + 212 T^4 - 257 T^5 + 212 T^6 - 118 T^7 + 44 T^8 - 10 T^9 + T^{10}}{T^5}, \right.$$

$$\frac{1}{T_1^{10} T_2^{10}} \left( 1 - 10 T_1 + 44 T_1^2 - 118 T_1^3 + 212 T_1^4 - 257 T_1^5 + 212 T_1^6 - 118 T_1^7 + 44 T_1^8 - 10 T_1^9 + T_1^{10} - 10 T_2 + \right.$$

$$90 T_1 T_2 - 340 T_1^2 T_2 + 740 T_1^3 T_2 - 940 T_1^4 T_2 + 450 T_1^5 T_2 + 450 T_1^6 T_2 - 940 T_1^7 T_2 + 740 T_1^8 T_2 -$$

$$\begin{aligned}
 & 340 T_1^9 T_2 + 90 T_1^{10} T_2 - 10 T_1^{11} T_2 + 44 T_2^2 - 340 T_1 T_2^2 + 985 T_1^2 T_2^2 - 1269 T_1^3 T_2^2 - 401 T_1^4 T_2^2 + 4411 T_1^5 T_2^2 - \\
 & 6677 T_1^6 T_2^2 + 4411 T_1^7 T_2^2 - 401 T_1^8 T_2^2 - 1269 T_1^9 T_2^2 + 985 T_1^{10} T_2^2 - 340 T_1^{11} T_2^2 + 44 T_1^{12} T_2^2 - 118 T_2^3 + \\
 & 740 T_1 T_2^3 - 1269 T_1^2 T_2^3 - 914 T_1^3 T_2^3 + 7987 T_1^4 T_2^3 - 15 079 T_1^5 T_2^3 + 8131 T_1^6 T_2^3 + 8131 T_1^7 T_2^3 - 15 079 T_1^8 T_2^3 + \\
 & 7987 T_1^9 T_2^3 - 914 T_1^{10} T_2^3 - 1269 T_1^{11} T_2^3 + 740 T_1^{12} T_2^3 - 118 T_1^{13} T_2^3 + 212 T_2^4 - 940 T_1 T_2^4 - 401 T_1^2 T_2^4 + \\
 & 7987 T_1^3 T_2^4 - 18 980 T_1^4 T_2^4 + 15 896 T_1^5 T_2^4 + 19 541 T_1^6 T_2^4 - 42 394 T_1^7 T_2^4 + 19 541 T_1^8 T_2^4 + 15 896 T_1^9 T_2^4 - \\
 & 18 980 T_1^{10} T_2^4 + 7987 T_1^{11} T_2^4 - 401 T_1^{12} T_2^4 - 940 T_1^{13} T_2^4 + 212 T_1^{14} T_2^4 - 257 T_2^5 + 450 T_1 T_2^5 + 4411 T_1^2 T_2^5 - \\
 & 15 079 T_1^3 T_2^5 + 15 896 T_1^4 T_2^5 + 17 848 T_1^5 T_2^5 - 76 792 T_1^6 T_2^5 + 46 980 T_1^7 T_2^5 + 46 980 T_1^8 T_2^5 - 76 792 T_1^9 T_2^5 + \\
 & 17 848 T_1^{10} T_2^5 + 15 896 T_1^{11} T_2^5 - 15 079 T_1^{12} T_2^5 + 4411 T_1^{13} T_2^5 + 450 T_1^{14} T_2^5 - 257 T_1^{15} T_2^5 + 212 T_2^6 + \\
 & 450 T_1 T_2^6 - 6677 T_1^2 T_2^6 + 8131 T_1^3 T_2^6 + 19 541 T_1^4 T_2^6 - 76 792 T_1^5 T_2^6 + 92 385 T_1^6 T_2^6 + 67 072 T_1^7 T_2^6 - \\
 & 176 705 T_1^8 T_2^6 + 67 072 T_1^9 T_2^6 + 92 385 T_1^{10} T_2^6 - 76 792 T_1^{11} T_2^6 + 19 541 T_1^{12} T_2^6 + 8131 T_1^{13} T_2^6 - 6677 T_1^{14} T_2^6 + \\
 & 450 T_1^{15} T_2^6 + 212 T_1^{16} T_2^6 - 118 T_2^7 - 940 T_1 T_2^7 + 4411 T_1^2 T_2^7 + 8131 T_1^3 T_2^7 - 42 394 T_1^4 T_2^7 + 46 980 T_1^5 T_2^7 + \\
 & 67 072 T_1^6 T_2^7 - 296 378 T_1^7 T_2^7 + 181 777 T_1^8 T_2^7 + 181 777 T_1^9 T_2^7 - 296 378 T_1^{10} T_2^7 + 67 072 T_1^{11} T_2^7 + \\
 & 46 980 T_1^{12} T_2^7 - 42 394 T_1^{13} T_2^7 + 8131 T_1^{14} T_2^7 + 4411 T_1^{15} T_2^7 - 940 T_1^{16} T_2^7 - 118 T_1^{17} T_2^7 + 44 T_2^8 + \\
 & 740 T_1 T_2^8 - 401 T_1^2 T_2^8 - 15 079 T_1^3 T_2^8 + 19 541 T_1^4 T_2^8 + 46 980 T_1^5 T_2^8 - 176 705 T_1^6 T_2^8 + 181 777 T_1^7 T_2^8 + \\
 & 320 104 T_1^8 T_2^8 - 652 662 T_1^9 T_2^8 + 320 104 T_1^{10} T_2^8 + 181 777 T_1^{11} T_2^8 - 176 705 T_1^{12} T_2^8 + 46 980 T_1^{13} T_2^8 + \\
 & 19 541 T_1^{14} T_2^8 - 15 079 T_1^{15} T_2^8 - 401 T_1^{16} T_2^8 + 740 T_1^{17} T_2^8 + 44 T_1^{18} T_2^8 - 10 T_2^9 - 340 T_1 T_2^9 - 1269 T_1^2 T_2^9 + \\
 & 7987 T_1^3 T_2^9 + 15 896 T_1^4 T_2^9 - 76 792 T_1^5 T_2^9 + 67 072 T_1^6 T_2^9 + 181 777 T_1^7 T_2^9 - 652 662 T_1^8 T_2^9 + 391 094 T_1^9 T_2^9 + \\
 & 391 094 T_1^{10} T_2^9 - 652 662 T_1^{11} T_2^9 + 181 777 T_1^{12} T_2^9 + 67 072 T_1^{13} T_2^9 - 76 792 T_1^{14} T_2^9 + 15 896 T_1^{15} T_2^9 + \\
 & 7987 T_1^{16} T_2^9 - 1269 T_1^{17} T_2^9 - 340 T_1^{18} T_2^9 - 10 T_1^{19} T_2^9 + T_2^{10} + 90 T_1 T_2^{10} + 985 T_1^2 T_2^{10} - 914 T_1^3 T_2^{10} - \\
 & 18 980 T_1^4 T_2^{10} + 17 848 T_1^5 T_2^{10} + 92 385 T_1^6 T_2^{10} - 296 378 T_1^7 T_2^{10} + 320 104 T_1^8 T_2^{10} + 391 094 T_1^9 T_2^{10} - \\
 & 864 744 T_1^{10} T_2^{10} + 391 094 T_1^{11} T_2^{10} + 320 104 T_1^{12} T_2^{10} - 296 378 T_1^{13} T_2^{10} + 92 385 T_1^{14} T_2^{10} + 17 848 T_1^{15} T_2^{10} - \\
 & 18 980 T_1^{16} T_2^{10} - 914 T_1^{17} T_2^{10} + 985 T_1^{18} T_2^{10} + 90 T_1^{19} T_2^{10} + T_2^{20} - 10 T_1 T_2^{21} - 340 T_1^2 T_2^{21} - 1269 T_1^3 T_2^{21} + \\
 & 7987 T_1^4 T_2^{21} + 15 896 T_1^5 T_2^{21} - 76 792 T_1^6 T_2^{21} + 67 072 T_1^7 T_2^{21} + 181 777 T_1^8 T_2^{21} - 652 662 T_1^9 T_2^{21} + \\
 & 391 094 T_1^{10} T_2^{21} + 391 094 T_1^{11} T_2^{21} - 652 662 T_1^{12} T_2^{21} + 181 777 T_1^{13} T_2^{21} + 67 072 T_1^{14} T_2^{21} - 76 792 T_1^{15} T_2^{21} + \\
 & 15 896 T_1^{16} T_2^{21} + 7987 T_1^{17} T_2^{21} - 1269 T_1^{18} T_2^{21} - 340 T_1^{19} T_2^{21} - 10 T_1^{20} T_2^{21} + 44 T_1^{22} T_2^{21} + 740 T_1^3 T_2^{22} - \\
 & 401 T_1^4 T_2^{22} - 15 079 T_1^5 T_2^{22} + 19 541 T_1^6 T_2^{22} + 46 980 T_1^7 T_2^{22} - 176 705 T_1^8 T_2^{22} + 181 777 T_1^9 T_2^{22} + \\
 & 320 104 T_1^{10} T_2^{22} - 652 662 T_1^{11} T_2^{22} + 320 104 T_1^{12} T_2^{22} + 181 777 T_1^{13} T_2^{22} - 176 705 T_1^{14} T_2^{22} + 46 980 T_1^{15} T_2^{22} + \\
 & 19 541 T_1^{16} T_2^{22} - 15 079 T_1^{17} T_2^{22} - 401 T_1^{18} T_2^{22} + 740 T_1^{19} T_2^{22} + 44 T_1^{20} T_2^{22} - 118 T_1^3 T_2^{23} - 940 T_1^4 T_2^{23} + \\
 & 4411 T_1^5 T_2^{23} + 8131 T_1^6 T_2^{23} - 42 394 T_1^7 T_2^{23} + 46 980 T_1^8 T_2^{23} + 67 072 T_1^9 T_2^{23} - 296 378 T_1^{10} T_2^{23} + \\
 & 181 777 T_1^{11} T_2^{23} + 181 777 T_1^{12} T_2^{23} - 296 378 T_1^{13} T_2^{23} + 67 072 T_1^{14} T_2^{23} + 46 980 T_1^{15} T_2^{23} - 42 394 T_1^{16} T_2^{23} + \\
 & 8131 T_1^{17} T_2^{23} + 4411 T_1^{18} T_2^{23} - 940 T_1^{19} T_2^{23} - 118 T_1^{20} T_2^{23} + 212 T_1^4 T_2^{24} + 450 T_1^5 T_2^{24} - 6677 T_1^6 T_2^{24} + \\
 & 8131 T_1^7 T_2^{24} + 19 541 T_1^8 T_2^{24} - 76 792 T_1^9 T_2^{24} + 92 385 T_1^{10} T_2^{24} + 67 072 T_1^{11} T_2^{24} - 176 705 T_1^{12} T_2^{24} + \\
 & 67 072 T_1^{13} T_2^{24} + 92 385 T_1^{14} T_2^{24} - 76 792 T_1^{15} T_2^{24} + 19 541 T_1^{16} T_2^{24} + 8131 T_1^{17} T_2^{24} - 6677 T_1^{18} T_2^{24} + \\
 & 450 T_1^{19} T_2^{24} + 212 T_1^{20} T_2^{24} - 257 T_1^5 T_2^{25} + 450 T_1^6 T_2^{25} + 4411 T_1^7 T_2^{25} - 15 079 T_1^8 T_2^{25} + 15 896 T_1^9 T_2^{25} + \\
 & 17 848 T_1^{10} T_2^{25} - 76 792 T_1^{11} T_2^{25} + 46 980 T_1^{12} T_2^{25} + 46 980 T_1^{13} T_2^{25} - 76 792 T_1^{14} T_2^{25} + 17 848 T_1^{15} T_2^{25} + \\
 & 15 896 T_1^{16} T_2^{25} - 15 079 T_1^{17} T_2^{25} + 4411 T_1^{18} T_2^{25} + 450 T_1^{19} T_2^{25} - 257 T_1^{20} T_2^{25} + 212 T_1^6 T_2^{26} - 940 T_1^7 T_2^{26} - \\
 & 401 T_1^8 T_2^{26} + 7987 T_1^9 T_2^{26} - 18 980 T_1^{10} T_2^{26} + 15 896 T_1^{11} T_2^{26} + 19 541 T_1^{12} T_2^{26} - 42 394 T_1^{13} T_2^{26} + \\
 & 19 541 T_1^{14} T_2^{26} + 15 896 T_1^{15} T_2^{26} - 18 980 T_1^{16} T_2^{26} + 7987 T_1^{17} T_2^{26} - 401 T_1^{18} T_2^{26} - 940 T_1^{19} T_2^{26} + \\
 & 212 T_1^{20} T_2^{26} - 118 T_1^7 T_2^{27} + 740 T_1^8 T_2^{27} - 1269 T_1^9 T_2^{27} - 914 T_1^{10} T_2^{27} + 7987 T_1^{11} T_2^{27} - 15 079 T_1^{12} T_2^{27} + \\
 & 8131 T_1^{13} T_2^{27} + 8131 T_1^{14} T_2^{27} - 15 079 T_1^{15} T_2^{27} + 7987 T_1^{16} T_2^{27} - 914 T_1^{17} T_2^{27} - 1269 T_1^{18} T_2^{27} + 740 T_1^{19} T_2^{27} - \\
 & 118 T_1^{20} T_2^{27} + 44 T_1^8 T_2^{28} - 340 T_1^9 T_2^{28} + 985 T_1^{10} T_2^{28} - 1269 T_1^{11} T_2^{28} - 401 T_1^{12} T_2^{28} + 4411 T_1^{13} T_2^{28} - \\
 & 6677 T_1^{14} T_2^{28} + 4411 T_1^{15} T_2^{28} - 401 T_1^{16} T_2^{28} - 1269 T_1^{17} T_2^{28} + 985 T_1^{18} T_2^{28} - 340 T_1^{19} T_2^{28} + 44 T_1^{20} T_2^{28} - \\
 & 10 T_1^9 T_2^{29} + 90 T_1^{10} T_2^{29} - 340 T_1^{11} T_2^{29} + 740 T_1^{12} T_2^{29} - 940 T_1^{13} T_2^{29} + 450 T_1^{14} T_2^{29} + 450 T_1^{15} T_2^{29} - \\
 & 940 T_1^{16} T_2^{29} + 740 T_1^{17} T_2^{29} - 340 T_1^{18} T_2^{29} + 90 T_1^{19} T_2^{29} - 10 T_1^{20} T_2^{29} + T_1^{20} T_2^{20} - 10 T_1^{11} T_2^{20} + 44 T_1^{12} T_2^{20} - \\
 & 118 T_1^{13} T_2^{20} + 212 T_1^{14} T_2^{20} - 257 T_1^{15} T_2^{20} + 212 T_1^{16} T_2^{20} - 118 T_1^{17} T_2^{20} + 44 T_1^{18} T_2^{20} - 10 T_1^{19} T_2^{20} + T_1^{20} T_2^{20} \}
 \end{aligned}$$

```

In[*]:=  $\Theta$ [Knot[15, Alternating, 20905]] // LeafCount
Out[*]=
3871

In[*]:=  $\Theta$ [Knot[15, Alternating, 20905]] // ByteCount
Out[*]=
121264

In[*]:= ( $\Theta$ [Knot[15, Alternating, 20905]] /. {T1 → T1, T2 → T2}) // ByteCount
Out[*]=
75040

In[*]:= AbsoluteTiming@Monitor [
  Table[K → ( $\Theta$ [K] /. {T1 → T1, T2 → T2}), {K, AllKnots[{15, 15}}] >> "theta15-15.m",
  K
]
KnotTheory: Loading precomputed data in KnotTheory/15A.dts.
KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of
2005.

AbsoluteTiming@
Monitor[Table[K → HOMFLYPT[PD@K][a, z], {K, AllKnots[{3, 15}}] >> "HOMFLYPT3-15.m", K]

AbsoluteTiming@
Monitor[Table[K → Kh[PD@K][q, t], {K, AllKnots[{3, 15}}] >> "Data/Kh3-15.m", K]

```