```
SetDirectory["~/AcademicPensieve/Projects/HigherRank/Data"];
(*SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank\\Data"];*)
<< ../../KnotTheory/KnotTheory/init.m
AppendTo[$Path, KnotTheoryDirectory[] = "../../KnotTheory"];
Print[$Path = Cases[$Path, _String]];
<< ../Rot.m
T3 = T1 T2;
```

SetDirectory: Cannot set current directory to ~/AcademicPensieve/Projects/HigherRank/Data.

```
C:\drorbn\AcademicPensieve\Projects\KnotTheory\KnotTheory

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

SetDelayed: Tag Diff in Diff[K_PD, rut_, ag_, n_, m_] is Protected.

```
Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

In[•]:=
```
CCF[ℰ_] := ExpandDenominator@ExpandNumerator@Together[ℰ];
CCF[ℰ_] := Factor[ℰ];
CF[ℰ_List] := CF /@ ℰ;
CF[ℰ_] := Module[{vs = Cases[ℰ, (x | p | π | g)__, ∞] ⋃ {x, p, ϵ}, ps, c},
   Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> CCF[c] (Times @@ vs^ps)]];
```

## Data

(from Talks/Beijing-2407/theta.nb)

In[•]:=
```
R1[1, i_, j_] = CF[
   1/2 - T3 g1ji g2ji - g3ii + g2jj g3ii + T1 (T3 - 1) g1ji g3ji +
    T2 (T3 - 1) g2ji g3ji - T2 g2ji g3jj + (g1jj g2ii + (T3 - 1) g1jj g2ji -
       T1 g1ii g2jj - g1jj g3ii - T1 (T3 - 1) g1jj g3ji + T1 g1ii g3jj) / (T1 - 1)];
```

In[•]:=
```
R1[-1, i_, j_] = CF[
   -1/2 - T1^-1 g1ji g2ii - (1 - T1^-1 - T2^-1) g1ji g2ji - g1jj g2ji - g1ji g2jj + g3ii +
    T1^-1 g1ji g3ii - (1 - T2^-1) g2ji g3ii - g2jj g3ii + (1 - T3^-1) g1ji g3ji - (1 - T3^-1) g2ii g3ji +
    (2 - T2^-1) (1 - T3^-1) g2ji g3ji + (1 - T3^-1) g2jj g3ji + g1ji g3jj + g2ji g3jj + (T1 (1 - T2^-1) g1ii g2ji -
       g1jj g2ii + T1 g1ii g2jj + g1jj g3ii - T2^-1 (T3 - 1) g1ii g3ji - T1 g1ii g3jj) / (T1 - 1)];
```

In[•]:=
```
Θ[{1, i0_, j0_}, {1, i1_, j1_}] =
  -T1 (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 + (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 +
   T1 (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 - (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;
```

```
In[•]:= Θ[{1, i0_, j0_}, {-1, i1_, j1_}] =
          (T3 - 1) g1,j1,i0 g2,i1,i0 g3,j0,i1 - T1^-1 (T3 - 1) g1,j1,j0 g2,i1,i0 g3,j0,i1 -
           (T3 - 1) g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1^-1 (T3 - 1) g1,j1,j0 g2,j1,i0 g3,j0,i1;

In[•]:= Θ[{-1, i0_, j0_}, {1, i1_, j1_}] = CF[
          T1^-1 T2^-1 (T3 - 1) (g1,j1,i0 g2,i1,i0 g3,j0,i1 -
              T1 g1,j1,j0 g2,i1,i0 g3,j0,i1 - g1,j1,i0 g2,j1,i0 g3,j0,i1 + T1 g1,j1,j0 g2,j1,i0 g3,j0,i1)];

In[•]:= Θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = CF[
          (1 - T3^-1) (-T1^-1 g1,j1,i0 g2,i1,i0 g3,j0,i1 +
              g1,j1,j0 g2,i1,i0 g3,j0,i1 + T1^-1 g1,j1,i0 g2,j1,i0 g3,j0,i1 - g1,j1,j0 g2,j1,i0 g3,j0,i1)];

In[•]:= Γ1[φ_, k_] = -φ / 2 + φ g3,k,k;
```

## The Programs

```
In[•]:= Θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
           {Cs, φ} = Rot[K]; n = Length[Cs];
           A = IdentityMatrix[2 n + 1];
           Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
           Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
           G = Inverse[A]; gEval[ℰ_] := Factor[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν)];
           z = gEval[Sum[Θ[Cs[[k1]], Cs[[k2]]], {k1, n}, {k2, n}]];
           z += gEval@Sum[R1 @@ Cs[[k]], {k, n}];
           z += gEval@Sum[Γ1[φ[[k]], k], {k, 2 n}];
           {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // Factor];

In[•]:= Θ[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

```
Out[•]= {(1 - T + T^2)/T, -(1 - T1 + T1^2 - T2 - T1^3 T2 + T2^2 + T1^4 T2^2 - T1 T2^3 - T1^4 T2^3 + T1^2 T2^4 - T1^3 T2^4 + T1^4 T2^4)/(T1^2 T2^2)}
```

## The Run

```
In[•]:= Table[n → NumberOfKnots[n], {n, 3, 15}]

Out[•]= {3 → 1, 4 → 1, 5 → 2, 6 → 3, 7 → 7, 8 → 21, 9 → 49,
         10 → 165, 11 → 552, 12 → 2176, 13 → 9988, 14 → 46 972, 15 → 253 293}
```

```
AbsoluteTiming[
 Table[Echo[K] → (θ[K] /. {T1 → T1, T2 → T2}),
   {K, AllKnots[{15, 15}][[200001 ;; 253293]]}] >> "theta15(200001-253293).m"
]
```

Out[•]=

```
{0.134328, Null}
```

```
AbsoluteTiming[
 Table[Echo[K] → HOMFLYPT[PD@K][a, z], {K, AllKnots[{3, 15}]}] >> "HOMFLYPT3-15.m"
]
```

```
AbsoluteTiming[
 Table[Echo[K] → Kh[PD@K][q, t], {K, AllKnots[{3, 15}]}] >> "Data/Kh3-15.m"
]
```