In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`];
<< Rot.m
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

In[•]:=
```
{
  {r0,pxx[1, i_, j_], r0,pxx[-1, i_, j_]},
  {r1,ppx[1, i_, j_], r1,ppx[-1, i_, j_]},
  {r1,rest[1, i_, j_], r1,rest[-1, i_, j_]},
  ϒ1[φ_, k_]
 } = Get["px-data.m"]
```

Out[•]=

$$\left\{\left\{p_{3,j}\,x_{1,i}\,x_{2,i}-\frac{p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1},\ -\frac{p_{3,j}\,x_{1,i}\,x_{2,i}}{T_1^2\,T_2}+\frac{p_{3,j}\,x_{1,j}\,x_{2,i}}{T_1\,T_2}\right\},\right.$$

$$\left\{p_{1,j}\,p_{2,i}\,x_{3,i}-p_{1,j}\,p_{2,j}\,x_{3,i},\ -\frac{p_{1,j}\,p_{2,i}\,x_{3,i}}{T_1}+\frac{p_{1,j}\,p_{2,j}\,x_{3,i}}{T_1}\right\},$$

$$\left\{\frac{T_2\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{-1+T_1\,T_2}-\frac{p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}-\frac{p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1}+\frac{p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+\right.$$
$$\frac{p_{3,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}+\frac{p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{p_{1,j}\,p_{3,j}\,x_{1,j}\,x_{3,i}}{-1+T_1}-$$
$$\left.\frac{T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1}-\frac{p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{T_2\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{T_1\,(-1+T_1\,T_2)}\right\},$$

$$\frac{p_{1,j}\,p_{2,i}\,x_{1,i}\,x_{2,i}}{T_1^2\,(-1+T_1\,T_2)}-\frac{(-1+T_2)\,p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}+\frac{(-T_1-T_2+T_1\,T_2)\,p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,i}}{T_1^2\,T_2\,(-1+T_1\,T_2)}+$$
$$\frac{p_{1,j}\,p_{2,i}\,x_{1,j}\,x_{2,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{p_{1,j}\,p_{2,j}\,x_{1,j}\,x_{2,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{1,i}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+$$
$$\frac{p_{1,j}\,p_{2,j}\,x_{1,i}\,x_{2,j}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{3,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{1,j}\,p_{3,i}\,x_{1,i}\,x_{3,i}}{T_1^2\,(-1+T_1\,T_2)}+\frac{p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{(-1+T_1)\,T_1\,T_2}-$$
$$\frac{p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,i}}{T_1^2\,T_2}-\frac{p_{1,j}\,p_{3,i}\,x_{1,j}\,x_{3,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{(-1+T_2)\,p_{2,j}\,p_{3,i}\,x_{2,i}\,x_{3,i}}{T_1\,T_2\,(-1+T_1\,T_2)}+$$
$$\frac{p_{2,i}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1^2\,T_2}-\frac{(-1+2\,T_2)\,p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,i}}{T_1^2\,T_2^2}+\frac{p_{2,j}\,p_{3,i}\,x_{2,j}\,x_{3,i}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{2,j}\,p_{3,j}\,x_{2,j}\,x_{3,i}}{T_1^2\,T_2}+$$
$$\left.\frac{p_{1,i}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{p_{1,j}\,p_{3,j}\,x_{1,i}\,x_{3,j}}{T_1\,(-1+T_1\,T_2)}-\frac{p_{2,j}\,p_{3,j}\,x_{2,i}\,x_{3,j}}{T_1\,(-1+T_1\,T_2)}\right\},\ -\frac{\varphi\,p_{3,k}\,x_{3,k}}{T_1\,(-1+T_1\,T_2)}\Big\}$$

In[•]:=
```
r1,rest[1, 4, 5]
```

Out[•]=

$$\frac{T_2\, p_{1,5}\, p_{2,5}\, x_{1,4}\, x_{2,4}}{-1+T_1\,T_2} - \frac{p_{1,5}\, p_{2,4}\, x_{1,5}\, x_{2,4}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} - \frac{p_{1,5}\, p_{2,5}\, x_{1,5}\, x_{2,4}}{(-1+T_1)\,T_1} + \frac{p_{1,4}\, p_{2,5}\, x_{1,4}\, x_{2,5}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{p_{3,5}\, x_{3,4}}{T_1\,(-1+T_1\,T_2)} - p_{1,5}\, p_{3,5}\, x_{1,4}\, x_{3,4} + \frac{p_{1,5}\, p_{3,4}\, x_{1,5}\, x_{3,4}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} + \frac{p_{1,5}\, p_{3,5}\, x_{1,5}\, x_{3,4}}{-1+T_1} - \frac{T_2\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,4}}{T_1} - \frac{p_{2,5}\, p_{3,4}\, x_{2,5}\, x_{3,4}}{T_1\,(-1+T_1\,T_2)} - \frac{p_{1,4}\, p_{3,5}\, x_{1,4}\, x_{3,5}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{T_2\, p_{2,5}\, p_{3,5}\, x_{2,4}\, x_{3,5}}{T_1\,(-1+T_1\,T_2)}$$

In[•]:=
```
{p*, x*, π*, ξ*} = {π, ξ, p, x};  (u_i__)* := (u*)_i;
```

In[•]:=
```
Zip{}[ℰ_] := ℰ;
Zip{ξ_,ξs___}[ℰ_] := (Collect[ℰ // Zip{ξs}, ξ] /. f_. ξ^d_. :> (D[f, {ξ*, d}])) /. ξ* → 0
```

In[•]:=
```
px2g[ℰ_] := Module[{ps, xs, Q},
  ps = Union[Cases[ℰ, p__, ∞]]; xs = Union[Cases[ℰ, x__, ∞]];
  Q = Sum[p0* x0* gp0[[2]],x0[[2]],p0[[3]],x0[[3]], {p0, ps}, {x0, xs}];
  Expand[Zip_{ps∪xs}[ℰ e^Q] /. gα_,β_,i_,j_ :> If[α === β, gα,i,j, 0]]
 ]
```

In[•]:=
```
px2g[p2,j^2 x2,i x2,j]
```

Out[•]=

$$2\, g_{2,j,i}\, g_{2,j,j}$$

In[•]:=
```
R1[1, i_, j_] = px2g[r1,rest[1, i, j]]
```

Out[•]=

$$-\frac{g_{1,j,j}\, g_{2,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} + \frac{T_2\, g_{1,j,i}\, g_{2,j,i}}{-1+T_1\,T_2} - \frac{g_{1,j,j}\, g_{2,j,i}}{(-1+T_1)\,T_1} + \frac{g_{1,i,i}\, g_{2,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{g_{1,j,j}\, g_{3,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)} - \frac{g_{2,j,j}\, g_{3,i,i}}{T_1\,(-1+T_1\,T_2)} + \frac{g_{3,j,i}}{T_1\,(-1+T_1\,T_2)} - g_{1,j,i}\, g_{3,j,i} + \frac{g_{1,j,j}\, g_{3,j,i}}{-1+T_1} - \frac{T_2\, g_{2,j,i}\, g_{3,j,i}}{T_1} - \frac{g_{1,i,i}\, g_{3,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)} + \frac{T_2\, g_{2,j,i}\, g_{3,j,j}}{T_1\,(-1+T_1\,T_2)}$$

In[•]:=
```
R1[-1, i_, j_] = px2g[r1,rest[-1, i, j]]
```

Out[•]=

$$\frac{g_{1,j,i}\,g_{2,i,i}}{T_1^2\,(-1+T_1\,T_2)}+\frac{g_{1,j,j}\,g_{2,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}-\frac{g_{1,i,i}\,g_{2,j,i}}{(-1+T_1)\,(-1+T_1\,T_2)}+\frac{g_{1,i,i}\,g_{2,j,i}}{(-1+T_1)\,T_2\,(-1+T_1\,T_2)}-$$
$$\frac{g_{1,j,i}\,g_{2,j,i}}{T_1^2\,(-1+T_1\,T_2)}+\frac{g_{1,j,i}\,g_{2,j,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{1,j,i}\,g_{2,j,i}}{T_1\,T_2\,(-1+T_1\,T_2)}+\frac{g_{1,j,j}\,g_{2,j,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{1,i,i}\,g_{2,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)}+$$
$$\frac{g_{1,j,i}\,g_{2,j,j}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{1,j,i}\,g_{3,i,i}}{T_1^2\,(-1+T_1\,T_2)}-\frac{g_{1,j,j}\,g_{3,i,i}}{(-1+T_1)\,T_1\,(-1+T_1\,T_2)}+\frac{g_{2,j,i}\,g_{3,i,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{2,j,i}\,g_{3,i,i}}{T_1\,T_2\,(-1+T_1\,T_2)}+$$
$$\frac{g_{2,j,j}\,g_{3,i,i}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{3,j,i}}{T_1\,(-1+T_1\,T_2)}+\frac{g_{1,i,i}\,g_{3,j,i}}{(-1+T_1)\,T_1\,T_2}-\frac{g_{1,j,i}\,g_{3,j,i}}{T_1^2\,T_2}+\frac{g_{2,i,i}\,g_{3,j,i}}{T_1^2\,T_2}+\frac{g_{2,j,i}\,g_{3,j,i}}{T_1^2\,T_2^2}-$$
$$\frac{2\,g_{2,j,i}\,g_{3,j,i}}{T_1^2\,T_2}-\frac{g_{2,j,j}\,g_{3,j,i}}{T_1^2\,T_2}+\frac{g_{1,i,i}\,g_{3,j,j}}{(-1+T_1)\,(-1+T_1\,T_2)}-\frac{g_{1,j,i}\,g_{3,j,j}}{T_1\,(-1+T_1\,T_2)}-\frac{g_{2,j,i}\,g_{3,j,j}}{T_1\,(-1+T_1\,T_2)}$$

In[•]:=
```
px2g[r0,pxx[1, i0, j0] r1,ppx[1, i1, j1]]
```

Out[•]=

$$g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}-\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}-g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}$$

In[•]:=
```
Θ[{1, i0_, j0_}, {1, i1_, j1_}] = px2g[r0,pxx[1, i0, j0] r1,ppx[1, i1, j1]]
Θ[{1, i0_, j0_}, {-1, i1_, j1_}] = px2g[r0,pxx[1, i0, j0] r1,ppx[-1, i1, j1]]
Θ[{-1, i0_, j0_}, {1, i1_, j1_}] = px2g[r0,pxx[-1, i0, j0] r1,ppx[1, i1, j1]]
Θ[{-1, i0_, j0_}, {-1, i1_, j1_}] = px2g[r0,pxx[-1, i0, j0] r1,ppx[-1, i1, j1]]
```

Out[•]=

$$g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}-\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}-g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}+\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}$$

Out[•]=

$$-\frac{g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1}+\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2}+\frac{g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1}-\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2}$$

Out[•]=

$$-\frac{g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}+\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1\,T_2}+\frac{g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}-\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1\,T_2}$$

Out[•]=

$$\frac{g_{1,j1,i0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^3\,T_2}-\frac{g_{1,j1,j0}\,g_{2,i1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}-\frac{g_{1,j1,i0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^3\,T_2}+\frac{g_{1,j1,j0}\,g_{2,j1,i0}\,g_{3,j0,i1}}{T_1^2\,T_2}$$

In[•]:=
```
Γ1[φ_, k_] = px2g[γ1[φ, k]]
```

Out[•]=

$$-\frac{\varphi\,g_{3,k,k}}{T_1\,(-1+T_1\,T_2)}$$

In[•]:=
```
T3 = T1 T2;
CF[ℰ_] := Factor@Together[ℰ];
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, gEval, Y, yEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A[[{i, j}, {i + 1, j + 1}]] += ({{-T^s, T^s - 1}, {0, -1}}))];
  Δ = T^((-Total[φ] - Total[Cs[[All, 1]]])/2) Det[A];
  G = Inverse[A]; gEval[ℰ_] := CF[ℰ /. g_ν_,α_,β_ :> (G[[α, β]] /. T → T_ν)];
  z = gEval[∑_{k1=1}^n ∑_{k2=1}^n θ[Cs[[k1]], Cs[[k2]]]];
  z += gEval[∑_{k=1}^n R1 @@ Cs[[k]]];
  z += gEval[∑_{k=1}^{2 n} Γ1[φ[[k]], k]];
  {Δ, (Δ /. T → T1) (Δ /. T → T2) (Δ /. T → T3) z} // CF
 ];
```

In[•]:=
```
Timing[θ[Knot[3, 1]]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[•]=

$$\left\{0.,\ \left\{\frac{1-T+T^2}{T},\ \frac{1}{T_1^2\,T_2\,(-1+T_1\,T_2)}\right.\right.$$
$$\left.\left.(-1+T_1-T_1^2+T_2-T_1^2\,T_2+2\,T_1^3\,T_2-T_2^2-T_1\,T_2^2+T_1^2\,T_2^2-2\,T_1^3\,T_2^2+2\,T_1\,T_2^3-2\,T_1^2\,T_2^3+2\,T_1^3\,T_2^3)\right\}\right\}$$

In[•]:=
```
Timing[θ[Knot[4, 1]]]
```

Out[•]=

$$\left\{0.,\ \left\{-\frac{1-3\,T+T^2}{T},\ -\frac{\left(1-3\,T_1+T_1^2\right)\ (1+T_1\,T_2)\ \left(1-3\,T_2+T_2^2\right)}{T_1^3\,T_2^2}\right\}\right\}$$

In[•]:=
```
Timing[θ[Knot["K11n34"]]]
```

KnotTheory: Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory: The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

Out[•]=

$$\left\{0.03125,\ \left\{1,\ \frac{1}{T_1^7\,T_2^6\,(-1+T_1\,T_2)}\left(T_1^2-2\,T_1^3+T_1^4-2\,T_1\,T_2+2\,T_1^2\,T_2+2\,T_1^5\,T_2-2\,T_1^6\,T_2+T_2^2+2\,T_1\,T_2^2-2\,T_1^2\,T_2^2-\right.\right.\right.$$
$$2\,T_1^4\,T_2^2-2\,T_1^6\,T_2^2+2\,T_1^7\,T_2^2+T_1^8\,T_2^2-2\,T_2^3+T_1^4\,T_2^3+T_1^5\,T_2^3-2\,T_1^9\,T_2^3+T_2^4-2\,T_1^2\,T_2^4+T_1^3\,T_2^4+2\,T_1^4\,T_2^4+$$
$$2\,T_1^6\,T_2^4+T_1^7\,T_2^4-2\,T_1^8\,T_2^4+T_1^{10}\,T_2^4+2\,T_1\,T_2^5+T_1^3\,T_2^5-4\,T_1^5\,T_2^5-4\,T_1^6\,T_2^5+T_1^8\,T_2^5+2\,T_1^{10}\,T_2^5-2\,T_1\,T_2^6-$$
$$2\,T_1^2\,T_2^6+2\,T_1^4\,T_2^6-4\,T_1^5\,T_2^6+12\,T_1^6\,T_2^6-4\,T_1^7\,T_2^6+2\,T_1^8\,T_2^6-2\,T_1^{10}\,T_2^6-2\,T_1^{11}\,T_2^6+2\,T_1^2\,T_2^7+T_1^4\,T_2^7-$$
$$4\,T_1^6\,T_2^7-4\,T_1^7\,T_2^7+T_1^9\,T_2^7+2\,T_1^{11}\,T_2^7+T_1^2\,T_2^8-2\,T_1^4\,T_2^8+T_1^5\,T_2^8+2\,T_1^6\,T_2^8+2\,T_1^8\,T_2^8+T_1^9\,T_2^8-2\,T_1^{10}\,T_2^8+$$
$$\left.\left.\left.T_1^{12}\,T_2^8-2\,T_1^3\,T_2^9+T_1^7\,T_2^9+T_1^8\,T_2^9-2\,T_1^{12}\,T_2^9+T_1^4\,T_2^{10}+2\,T_1^5\,T_2^{10}-2\,T_1^6\,T_2^{10}-2\,T_1^8\,T_2^{10}-2\,T_1^{10}\,T_2^{10}+\right.\right.\right.$$
$$\left.\left.\left.2\,T_1^{11}\,T_2^{10}+T_1^{12}\,T_2^{10}-2\,T_1^6\,T_2^{11}+2\,T_1^7\,T_2^{11}+2\,T_1^{10}\,T_2^{11}-2\,T_1^{11}\,T_2^{11}+T_1^8\,T_2^{12}-2\,T_1^9\,T_2^{12}+T_1^{10}\,T_2^{12}\right)\right\}\right\}$$

In[•]:=
```
Timing[θ[Knot["K11n42"]]]
```

Out[•]=

$$\left\{0.03125,\ \left\{1,\ -\frac{1}{T_1^4\,T_2^3\,(-1+T_1\,T_2)}\left(T_1+T_1^2+T_2-2\,T_1\,T_2-2\,T_1^2\,T_2-2\,T_1^3\,T_2+T_1^4\,T_2+T_2^2-2\,T_1\,T_2^2+2\,T_1^2\,T_2^2+2\,T_1^3\,T_2^2-\right.\right.\right.$$
$$2\,T_1^4\,T_2^2+T_1^5\,T_2^2-2\,T_1\,T_2^3+2\,T_1^2\,T_2^3+2\,T_1^4\,T_2^3-2\,T_1^5\,T_2^3+T_1\,T_2^4-2\,T_1^2\,T_2^4+2\,T_1^3\,T_2^4+$$
$$\left.\left.\left.2\,T_1^4\,T_2^4-2\,T_1^5\,T_2^4+T_1^6\,T_2^4+T_1^2\,T_2^5-2\,T_1^3\,T_2^5-2\,T_1^4\,T_2^5-2\,T_1^5\,T_2^5+T_1^6\,T_2^5+T_1^4\,T_2^6+T_1^5\,T_2^6\right)\right\}\right\}$$

In[•]:=
```
PD[GST48] = PD[X[1, 15, 2, 14], X[29, 2, 30, 3], X[40, 4, 41, 3],
   X[4, 44, 5, 43], X[5, 26, 6, 27], X[95, 7, 96, 6], X[7, 1, 8, 96], X[8, 14, 9, 13],
   X[28, 9, 29, 10], X[41, 11, 42, 10], X[11, 43, 12, 42], X[12, 27, 13, 28],
   X[15, 31, 16, 30], X[61, 16, 62, 17], X[72, 17, 73, 18], X[83, 18, 84, 19],
   X[34, 20, 35, 19], X[20, 89, 21, 90], X[92, 21, 93, 22], X[22, 79, 23, 80],
   X[23, 68, 24, 69], X[24, 57, 25, 58], X[56, 25, 57, 26], X[31, 63, 32, 62],
   X[32, 74, 33, 73], X[33, 85, 34, 84], X[35, 50, 36, 51], X[81, 37, 82, 36],
   X[70, 38, 71, 37], X[59, 39, 60, 38], X[54, 39, 55, 40], X[55, 45, 56, 44],
   X[45, 59, 46, 58], X[46, 70, 47, 69], X[47, 81, 48, 80], X[91, 49, 92, 48],
   X[49, 91, 50, 90], X[82, 52, 83, 51], X[71, 53, 72, 52], X[60, 54, 61, 53],
   X[74, 63, 75, 64], X[85, 64, 86, 65], X[65, 76, 66, 77], X[66, 87, 67, 88],
   X[94, 67, 95, 68], X[86, 75, 87, 76], X[77, 88, 78, 89], X[93, 78, 94, 79]];
```

In[•]:=
```
AbsoluteTiming[gst48 = θ[GST48]]
```

Out[•]=

$$\left\{13.3561,\ \left\{-\frac{\left(-1+2\,T-T^2-T^3+2\,T^4-T^5+T^8\right)\left(-1+T^3-2\,T^4+T^5+T^6-2\,T^7+T^8\right)}{T^8},\ -\frac{1}{T_1^{21}\,T_2^{20}\,(-1+T_1\,T_2)}\right.\right.$$
$$\left(T_1^5-3\,T_1^6+4\,T_1^7-2\,T_1^8-2\,T_1^9+4\,T_1^{10}-2\,T_1^{11}-2\,T_1^{12}+4\,T_1^{13}-3\,T_1^{14}+T_1^{15}-3\,T_1^5\,T_2+6\,T_1^6\,T_2-3\,T_1^7\,T_2-\right.$$
$$\left.\left.6\,T_1^8\,T_2+12\,T_1^9\,T_2-6\,T_1^{10}\,T_2-6\,T_1^{11}\,T_2+12\,T_1^{12}\,T_2-6\,T_1^{13}\,T_2-3\,T_1^{14}\,T_2+6\,T_1^{15}\,T_2-3\,T_1^{16}\,T_2-T_1^3\,T_2^2+\right.\right.$$

$$3\,T_1^4\,T_2^2 - T_1^6\,T_2^2 - 4\,T_1^7\,T_2^2 + 9\,T_1^8\,T_2^2 - 7\,T_1^9\,T_2^2 - 3\,T_1^{10}\,T_2^2 + 8\,T_1^{11}\,T_2^2 - 3\,T_1^{12}\,T_2^2 - 7\,T_1^{13}\,T_2^2 + 9\,T_1^{14}\,T_2^2 -$$
$$4\,T_1^{15}\,T_2^2 - T_1^{16}\,T_2^2 + 3\,T_1^{18}\,T_2^2 - T_1^{19}\,T_2^2 - T_1^2\,T_2^3 + 6\,T_1^3\,T_2^3 - 10\,T_1^4\,T_2^3 + 3\,T_1^5\,T_2^3 + 2\,T_1^6\,T_2^3 - 3\,T_1^7\,T_2^3 +$$
$$4\,T_1^8\,T_2^3 - 2\,T_1^9\,T_2^3 + 2\,T_1^{10}\,T_2^3 - T_1^{11}\,T_2^3 - T_1^{12}\,T_2^3 + 2\,T_1^{13}\,T_2^3 - 2\,T_1^{14}\,T_2^3 + 4\,T_1^{15}\,T_2^3 - 3\,T_1^{16}\,T_2^3 + 2\,T_1^{17}\,T_2^3 +$$
$$3\,T_1^{18}\,T_2^3 - 10\,T_1^{19}\,T_2^3 + 6\,T_1^{20}\,T_2^3 - T_1^{21}\,T_2^3 + 3\,T_1^2\,T_2^4 - 10\,T_1^3\,T_2^4 + 3\,T_1^4\,T_2^4 + 17\,T_1^5\,T_2^4 - 19\,T_1^6\,T_2^4 +$$
$$10\,T_1^7\,T_2^4 - 7\,T_1^8\,T_2^4 + 6\,T_1^9\,T_2^4 - T_1^{10}\,T_2^4 - 18\,T_1^{11}\,T_2^4 + 35\,T_1^{12}\,T_2^4 - 18\,T_1^{13}\,T_2^4 - T_1^{14}\,T_2^4 + 6\,T_1^{15}\,T_2^4 - 7\,T_1^{16}\,T_2^4 +$$
$$10\,T_1^{17}\,T_2^4 - 19\,T_1^{18}\,T_2^4 + 17\,T_1^{19}\,T_2^4 + 3\,T_1^{20}\,T_2^4 - 10\,T_1^{21}\,T_2^4 + 3\,T_1^{22}\,T_2^4 + T_2^5 - 3\,T_1\,T_2^5 + 3\,T_1^3\,T_2^5 + 17\,T_1^4\,T_2^5 -$$
$$38\,T_1^5\,T_2^5 + 22\,T_1^6\,T_2^5 + 7\,T_1^7\,T_2^5 - 11\,T_1^8\,T_2^5 - 6\,T_1^9\,T_2^5 + 14\,T_1^{10}\,T_2^5 + 11\,T_1^{11}\,T_2^5 - 31\,T_1^{12}\,T_2^5 + 9\,T_1^{13}\,T_2^5 +$$
$$T_1^{14}\,T_2^5 + 8\,T_1^{16}\,T_2^5 - 15\,T_1^{17}\,T_2^5 + 9\,T_1^{18}\,T_2^5 + 16\,T_1^{19}\,T_2^5 - 32\,T_1^{20}\,T_2^5 + 15\,T_1^{21}\,T_2^5 + 3\,T_1^{22}\,T_2^5 - 3\,T_1^{24}\,T_2^5 +$$
$$T_1^{25}\,T_2^5 - 3\,T_2^6 + 6\,T_1\,T_2^6 - T_1^2\,T_2^6 + 2\,T_1^3\,T_2^6 - 19\,T_1^4\,T_2^6 + 22\,T_1^5\,T_2^6 + 24\,T_1^6\,T_2^6 - 68\,T_1^7\,T_2^6 + 43\,T_1^8\,T_2^6 +$$
$$9\,T_1^9\,T_2^6 - 29\,T_1^{10}\,T_2^6 + 2\,T_1^{11}\,T_2^6 - 12\,T_1^{12}\,T_2^6 + 28\,T_1^{13}\,T_2^6 - 42\,T_1^{14}\,T_2^6 + 26\,T_1^{15}\,T_2^6 - 29\,T_1^{16}\,T_2^6 - T_1^{17}\,T_2^6 +$$
$$45\,T_1^{18}\,T_2^6 - 64\,T_1^{19}\,T_2^6 + 24\,T_1^{20}\,T_2^6 + 18\,T_1^{21}\,T_2^6 - 17\,T_1^{22}\,T_2^6 + 2\,T_1^{23}\,T_2^6 - T_1^{24}\,T_2^6 + 6\,T_1^{25}\,T_2^6 - 3\,T_1^{26}\,T_2^6 +$$
$$4\,T_2^7 - 3\,T_1\,T_2^7 - 4\,T_1^2\,T_2^7 - 3\,T_1^3\,T_2^7 + 10\,T_1^4\,T_2^7 + 7\,T_1^5\,T_2^7 - 68\,T_1^6\,T_2^7 + 74\,T_1^7\,T_2^7 + 14\,T_1^8\,T_2^7 - 56\,T_1^9\,T_2^7 +$$
$$14\,T_1^{10}\,T_2^7 + 55\,T_1^{11}\,T_2^7 - 23\,T_1^{12}\,T_2^7 + 11\,T_1^{13}\,T_2^7 + 51\,T_1^{14}\,T_2^7 - 33\,T_1^{15}\,T_2^7 + 41\,T_1^{16}\,T_2^7 + 28\,T_1^{17}\,T_2^7 - 60\,T_1^{18}\,T_2^7 +$$
$$16\,T_1^{19}\,T_2^7 + 68\,T_1^{20}\,T_2^7 - 62\,T_1^{21}\,T_2^7 + 5\,T_1^{22}\,T_2^7 + 10\,T_1^{23}\,T_2^7 - 3\,T_1^{24}\,T_2^7 - 4\,T_1^{25}\,T_2^7 - 3\,T_1^{26}\,T_2^7 + 4\,T_1^{27}\,T_2^7 -$$
$$2\,T_2^8 - 6\,T_1\,T_2^8 + 9\,T_1^2\,T_2^8 + 4\,T_1^3\,T_2^8 - 7\,T_1^4\,T_2^8 - 11\,T_1^5\,T_2^8 + 43\,T_1^6\,T_2^8 + 14\,T_1^7\,T_2^8 - 123\,T_1^8\,T_2^8 + 133\,T_1^9\,T_2^8 -$$
$$36\,T_1^{10}\,T_2^8 - 89\,T_1^{11}\,T_2^8 + 136\,T_1^{12}\,T_2^8 - 127\,T_1^{13}\,T_2^8 + 31\,T_1^{14}\,T_2^8 - 31\,T_1^{15}\,T_2^8 + 16\,T_1^{16}\,T_2^8 - 33\,T_1^{17}\,T_2^8 -$$
$$28\,T_1^{18}\,T_2^8 + 109\,T_1^{19}\,T_2^8 - 115\,T_1^{20}\,T_2^8 + 14\,T_1^{21}\,T_2^8 + 51\,T_1^{22}\,T_2^8 - 27\,T_1^{23}\,T_2^8 + T_1^{24}\,T_2^8 + 4\,T_1^{25}\,T_2^8 + 9\,T_1^{26}\,T_2^8 -$$
$$6\,T_1^{27}\,T_2^8 - 2\,T_1^{28}\,T_2^8 - 2\,T_2^9 + 12\,T_1\,T_2^9 - 7\,T_1^2\,T_2^9 - 2\,T_1^3\,T_2^9 + 6\,T_1^4\,T_2^9 - 6\,T_1^5\,T_2^9 + 9\,T_1^6\,T_2^9 - 56\,T_1^7\,T_2^9 +$$
$$133\,T_1^8\,T_2^9 - 149\,T_1^9\,T_2^9 - 10\,T_1^{10}\,T_2^9 + 224\,T_1^{11}\,T_2^9 - 314\,T_1^{12}\,T_2^9 + 67\,T_1^{13}\,T_2^9 + 111\,T_1^{14}\,T_2^9 - 124\,T_1^{15}\,T_2^9 +$$
$$38\,T_1^{16}\,T_2^9 - 49\,T_1^{17}\,T_2^9 + 50\,T_1^{18}\,T_2^9 - 38\,T_1^{19}\,T_2^9 - 47\,T_1^{20}\,T_2^9 + 95\,T_1^{21}\,T_2^9 - 68\,T_1^{22}\,T_2^9 + 8\,T_1^{23}\,T_2^9 + 32\,T_1^{24}\,T_2^9 -$$
$$19\,T_1^{25}\,T_2^9 - 2\,T_1^{26}\,T_2^9 - 7\,T_1^{27}\,T_2^9 + 12\,T_1^{28}\,T_2^9 - 2\,T_1^{29}\,T_2^9 + 4\,T_2^{10} - 6\,T_1\,T_2^{10} - 3\,T_1^2\,T_2^{10} + 2\,T_1^3\,T_2^{10} - T_1^4\,T_2^{10} +$$
$$14\,T_1^5\,T_2^{10} - 29\,T_1^6\,T_2^{10} + 14\,T_1^7\,T_2^{10} - 36\,T_1^8\,T_2^{10} - 10\,T_1^9\,T_2^{10} + 240\,T_1^{10}\,T_2^{10} - 314\,T_1^{11}\,T_2^{10} + 74\,T_1^{12}\,T_2^{10} +$$
$$431\,T_1^{13}\,T_2^{10} - 386\,T_1^{14}\,T_2^{10} + 200\,T_1^{15}\,T_2^{10} + 34\,T_1^{16}\,T_2^{10} - 37\,T_1^{17}\,T_2^{10} + 186\,T_1^{18}\,T_2^{10} - 186\,T_1^{19}\,T_2^{10} +$$
$$136\,T_1^{20}\,T_2^{10} - 22\,T_1^{21}\,T_2^{10} - 12\,T_1^{22}\,T_2^{10} + 46\,T_1^{23}\,T_2^{10} - 93\,T_1^{24}\,T_2^{10} + 30\,T_1^{25}\,T_2^{10} + 11\,T_1^{26}\,T_2^{10} + 2\,T_1^{27}\,T_2^{10} -$$
$$3\,T_1^{28}\,T_2^{10} - 6\,T_1^{29}\,T_2^{10} + 4\,T_1^{30}\,T_2^{10} - 2\,T_2^{11} - 6\,T_1\,T_2^{11} + 8\,T_1^2\,T_2^{11} - T_1^3\,T_2^{11} - 18\,T_1^4\,T_2^{11} + 11\,T_1^5\,T_2^{11} +$$
$$2\,T_1^6\,T_2^{11} + 55\,T_1^7\,T_2^{11} - 89\,T_1^8\,T_2^{11} + 224\,T_1^9\,T_2^{11} - 314\,T_1^{10}\,T_2^{11} - 92\,T_1^{11}\,T_2^{11} + 764\,T_1^{12}\,T_2^{11} - 899\,T_1^{13}\,T_2^{11} +$$
$$273\,T_1^{14}\,T_2^{11} + 176\,T_1^{15}\,T_2^{11} - 382\,T_1^{16}\,T_2^{11} + 391\,T_1^{17}\,T_2^{11} - 420\,T_1^{18}\,T_2^{11} + 75\,T_1^{19}\,T_2^{11} + 212\,T_1^{20}\,T_2^{11} -$$
$$156\,T_1^{21}\,T_2^{11} - 46\,T_1^{22}\,T_2^{11} - 6\,T_1^{23}\,T_2^{11} + 65\,T_1^{24}\,T_2^{11} + 76\,T_1^{25}\,T_2^{11} - 107\,T_1^{26}\,T_2^{11} + 31\,T_1^{27}\,T_2^{11} - T_1^{28}\,T_2^{11} +$$
$$8\,T_1^{29}\,T_2^{11} - 6\,T_1^{30}\,T_2^{11} - 2\,T_1^{31}\,T_2^{11} - 2\,T_2^{12} + 12\,T_1\,T_2^{12} - 3\,T_1^2\,T_2^{12} - T_1^3\,T_2^{12} + 35\,T_1^4\,T_2^{12} - 31\,T_1^5\,T_2^{12} -$$
$$12\,T_1^6\,T_2^{12} - 23\,T_1^7\,T_2^{12} + 136\,T_1^8\,T_2^{12} - 314\,T_1^9\,T_2^{12} + 74\,T_1^{10}\,T_2^{12} + 764\,T_1^{11}\,T_2^{12} - 1304\,T_1^{12}\,T_2^{12} + 293\,T_1^{13}\,T_2^{12} +$$
$$744\,T_1^{14}\,T_2^{12} - 996\,T_1^{15}\,T_2^{12} + 616\,T_1^{16}\,T_2^{12} - 380\,T_1^{17}\,T_2^{12} - 68\,T_1^{18}\,T_2^{12} + 589\,T_1^{19}\,T_2^{12} - 596\,T_1^{20}\,T_2^{12} -$$
$$72\,T_1^{21}\,T_2^{12} + 294\,T_1^{22}\,T_2^{12} + 38\,T_1^{23}\,T_2^{12} - 64\,T_1^{24}\,T_2^{12} - 123\,T_1^{25}\,T_2^{12} + 60\,T_1^{26}\,T_2^{12} + 93\,T_1^{27}\,T_2^{12} - 69\,T_1^{28}\,T_2^{12} -$$
$$T_1^{29}\,T_2^{12} - 3\,T_1^{30}\,T_2^{12} + 12\,T_1^{31}\,T_2^{12} - 2\,T_1^{32}\,T_2^{12} + 4\,T_2^{13} - 6\,T_1\,T_2^{13} - 7\,T_1^2\,T_2^{13} + 2\,T_1^3\,T_2^{13} - 18\,T_1^4\,T_2^{13} +$$
$$9\,T_1^5\,T_2^{13} + 28\,T_1^6\,T_2^{13} + 11\,T_1^7\,T_2^{13} - 127\,T_1^8\,T_2^{13} + 67\,T_1^9\,T_2^{13} + 431\,T_1^{10}\,T_2^{13} - 899\,T_1^{11}\,T_2^{13} + 293\,T_1^{12}\,T_2^{13} +$$
$$1556\,T_1^{13}\,T_2^{13} - 1724\,T_1^{14}\,T_2^{13} + 887\,T_1^{15}\,T_2^{13} + 223\,T_1^{16}\,T_2^{13} - 480\,T_1^{17}\,T_2^{13} + 998\,T_1^{18}\,T_2^{13} - 905\,T_1^{19}\,T_2^{13} +$$
$$212\,T_1^{20}\,T_2^{13} + 686\,T_1^{21}\,T_2^{13} - 294\,T_1^{22}\,T_2^{13} - 313\,T_1^{23}\,T_2^{13} + 146\,T_1^{24}\,T_2^{13} + 24\,T_1^{25}\,T_2^{13} + 123\,T_1^{26}\,T_2^{13} -$$
$$238\,T_1^{27}\,T_2^{13} + 65\,T_1^{28}\,T_2^{13} + 45\,T_1^{29}\,T_2^{13} + 2\,T_1^{30}\,T_2^{13} - 7\,T_1^{31}\,T_2^{13} - 6\,T_1^{32}\,T_2^{13} + 4\,T_1^{33}\,T_2^{13} - 3\,T_2^{14} - 3\,T_1\,T_2^{14} +$$
$$9\,T_1^2\,T_2^{14} - 2\,T_1^3\,T_2^{14} - T_1^4\,T_2^{14} + T_1^5\,T_2^{14} - 42\,T_1^6\,T_2^{14} + 51\,T_1^7\,T_2^{14} + 31\,T_1^8\,T_2^{14} + 111\,T_1^9\,T_2^{14} - 386\,T_1^{10}\,T_2^{14} +$$
$$273\,T_1^{11}\,T_2^{14} + 744\,T_1^{12}\,T_2^{14} - 1724\,T_1^{13}\,T_2^{14} + 705\,T_1^{14}\,T_2^{14} + 482\,T_1^{15}\,T_2^{14} - 1315\,T_1^{16}\,T_2^{14} + 1061\,T_1^{17}\,T_2^{14} -$$
$$855\,T_1^{18}\,T_2^{14} - 140\,T_1^{19}\,T_2^{14} + 809\,T_1^{20}\,T_2^{14} - 758\,T_1^{21}\,T_2^{14} - 370\,T_1^{22}\,T_2^{14} + 595\,T_1^{23}\,T_2^{14} + 58\,T_1^{24}\,T_2^{14} -$$
$$229\,T_1^{25}\,T_2^{14} + T_1^{26}\,T_2^{14} + 95\,T_1^{27}\,T_2^{14} + 124\,T_1^{28}\,T_2^{14} - 151\,T_1^{29}\,T_2^{14} + 19\,T_1^{30}\,T_2^{14} - 2\,T_1^{31}\,T_2^{14} + 9\,T_1^{32}\,T_2^{14} -$$
$$3\,T_1^{33}\,T_2^{14} - 3\,T_1^{34}\,T_2^{14} + T_2^{15} + 6\,T_1\,T_2^{15} - 4\,T_1^2\,T_2^{15} + 4\,T_1^3\,T_2^{15} + 6\,T_1^4\,T_2^{15} + 26\,T_1^6\,T_2^{15} - 33\,T_1^7\,T_2^{15} - 31\,T_1^8\,T_2^{15} -$$
$$124\,T_1^9\,T_2^{15} + 200\,T_1^{10}\,T_2^{15} + 176\,T_1^{11}\,T_2^{15} - 996\,T_1^{12}\,T_2^{15} + 887\,T_1^{13}\,T_2^{15} + 482\,T_1^{14}\,T_2^{15} - 1534\,T_1^{15}\,T_2^{15} +$$
$$1712\,T_1^{16}\,T_2^{15} - 619\,T_1^{17}\,T_2^{15} - 569\,T_1^{18}\,T_2^{15} + 1420\,T_1^{19}\,T_2^{15} - 914\,T_1^{20}\,T_2^{15} - 229\,T_1^{21}\,T_2^{15} + 992\,T_1^{22}\,T_2^{15} -$$
$$257\,T_1^{23}\,T_2^{15} - 598\,T_1^{24}\,T_2^{15} + 440\,T_1^{25}\,T_2^{15} - 15\,T_1^{26}\,T_2^{15} - 50\,T_1^{27}\,T_2^{15} - 167\,T_1^{28}\,T_2^{15} + 92\,T_1^{29}\,T_2^{15} + 74\,T_1^{30}\,T_2^{15} -$$
$$49\,T_1^{31}\,T_2^{15} + 4\,T_1^{32}\,T_2^{15} - 4\,T_1^{33}\,T_2^{15} + 6\,T_1^{34}\,T_2^{15} + T_1^{35}\,T_2^{15} - 3\,T_1\,T_2^{16} - T_1^2\,T_2^{16} - 3\,T_1^3\,T_2^{16} - 7\,T_1^4\,T_2^{16} +$$

$$8\,T_1^5\,T_2^{16} - 29\,T_1^6\,T_2^{16} + 41\,T_1^7\,T_2^{16} + 16\,T_1^8\,T_2^{16} + 38\,T_1^9\,T_2^{16} + 34\,T_1^{10}\,T_2^{16} - 382\,T_1^{11}\,T_2^{16} + 616\,T_1^{12}\,T_2^{16} + 223\,T_1^{13}\,T_2^{16} - 1315\,T_1^{14}\,T_2^{16} + 1712\,T_1^{15}\,T_2^{16} - 720\,T_1^{16}\,T_2^{16} - 1180\,T_1^{17}\,T_2^{16} + 2146\,T_1^{18}\,T_2^{16} - 1310\,T_1^{19}\,T_2^{16} - 260\,T_1^{20}\,T_2^{16} + 1108\,T_1^{21}\,T_2^{16} - 545\,T_1^{22}\,T_2^{16} - 555\,T_1^{23}\,T_2^{16} + 792\,T_1^{24}\,T_2^{16} - 94\,T_1^{25}\,T_2^{16} - 350\,T_1^{26}\,T_2^{16} + 256\,T_1^{27}\,T_2^{16} - 24\,T_1^{28}\,T_2^{16} + 109\,T_1^{29}\,T_2^{16} - 189\,T_1^{30}\,T_2^{16} + 60\,T_1^{31}\,T_2^{16} + 17\,T_1^{32}\,T_2^{16} - 3\,T_1^{33}\,T_2^{16} - T_1^{34}\,T_2^{16} - 3\,T_1^{35}\,T_2^{16} + 2\,T_1^3\,T_2^{17} + 10\,T_1^4\,T_2^{17} - 15\,T_1^5\,T_2^{17} - T_1^6\,T_2^{17} + 28\,T_1^7\,T_2^{17} - 33\,T_1^8\,T_2^{17} - 49\,T_1^9\,T_2^{17} - 37\,T_1^{10}\,T_2^{17} + 391\,T_1^{11}\,T_2^{17} - 380\,T_1^{12}\,T_2^{17} - 480\,T_1^{13}\,T_2^{17} + 1061\,T_1^{14}\,T_2^{17} - 619\,T_1^{15}\,T_2^{17} - 1180\,T_1^{16}\,T_2^{17} + 2566\,T_1^{17}\,T_2^{17} - 1730\,T_1^{18}\,T_2^{17} - 591\,T_1^{19}\,T_2^{17} + 1520\,T_1^{20}\,T_2^{17} - 933\,T_1^{21}\,T_2^{17} - 265\,T_1^{22}\,T_2^{17} + 476\,T_1^{23}\,T_2^{17} + 123\,T_1^{24}\,T_2^{17} - 791\,T_1^{25}\,T_2^{17} + 681\,T_1^{26}\,T_2^{17} - 213\,T_1^{27}\,T_2^{17} - 82\,T_1^{28}\,T_2^{17} - 8\,T_1^{29}\,T_2^{17} + 74\,T_1^{30}\,T_2^{17} + 42\,T_1^{31}\,T_2^{17} - 59\,T_1^{32}\,T_2^{17} + 10\,T_1^{33}\,T_2^{17} + 2\,T_1^{34}\,T_2^{17} + 3\,T_1^2\,T_2^{18} + 3\,T_1^3\,T_2^{18} - 19\,T_1^4\,T_2^{18} + 9\,T_1^5\,T_2^{18} + 45\,T_1^6\,T_2^{18} - 60\,T_1^7\,T_2^{18} - 28\,T_1^8\,T_2^{18} + 50\,T_1^9\,T_2^{18} + 186\,T_1^{10}\,T_2^{18} - 420\,T_1^{11}\,T_2^{18} - 68\,T_1^{12}\,T_2^{18} + 998\,T_1^{13}\,T_2^{18} - 855\,T_1^{14}\,T_2^{18} - 569\,T_1^{15}\,T_2^{18} + 2146\,T_1^{16}\,T_2^{18} - 1730\,T_1^{17}\,T_2^{18} - 492\,T_1^{18}\,T_2^{18} + 2218\,T_1^{19}\,T_2^{18} - 1372\,T_1^{20}\,T_2^{18} - 146\,T_1^{21}\,T_2^{18} + 878\,T_1^{22}\,T_2^{18} - 163\,T_1^{23}\,T_2^{18} - 695\,T_1^{24}\,T_2^{18} + 872\,T_1^{25}\,T_2^{18} - 162\,T_1^{26}\,T_2^{18} - 458\,T_1^{27}\,T_2^{18} + 506\,T_1^{28}\,T_2^{18} - 208\,T_1^{29}\,T_2^{18} + 44\,T_1^{30}\,T_2^{18} - 100\,T_1^{31}\,T_2^{18} + 79\,T_1^{32}\,T_2^{18} - 19\,T_1^{33}\,T_2^{18} - 5\,T_1^{34}\,T_2^{18} + 3\,T_1^{35}\,T_2^{18} + 3\,T_1^{36}\,T_2^{18} - T_1^2\,T_2^{19} - 10\,T_1^3\,T_2^{19} + 17\,T_1^4\,T_2^{19} + 16\,T_1^5\,T_2^{19} - 64\,T_1^6\,T_2^{19} + 16\,T_1^7\,T_2^{19} + 109\,T_1^8\,T_2^{19} - 38\,T_1^9\,T_2^{19} - 186\,T_1^{10}\,T_2^{19} + 75\,T_1^{11}\,T_2^{19} + 589\,T_1^{12}\,T_2^{19} - 905\,T_1^{13}\,T_2^{19} - 140\,T_1^{14}\,T_2^{19} + 1420\,T_1^{15}\,T_2^{19} - 1310\,T_1^{16}\,T_2^{19} - 591\,T_1^{17}\,T_2^{19} + 2218\,T_1^{18}\,T_2^{19} - 2027\,T_1^{19}\,T_2^{19} + 155\,T_1^{20}\,T_2^{19} + 1033\,T_1^{21}\,T_2^{19} - 840\,T_1^{22}\,T_2^{19} - 49\,T_1^{23}\,T_2^{19} + 464\,T_1^{24}\,T_2^{19} + 37\,T_1^{25}\,T_2^{19} - 842\,T_1^{26}\,T_2^{19} + 972\,T_1^{27}\,T_2^{19} - 412\,T_1^{28}\,T_2^{19} - 44\,T_1^{29}\,T_2^{19} + 150\,T_1^{30}\,T_2^{19} - 21\,T_1^{31}\,T_2^{19} - 10\,T_1^{32}\,T_2^{19} - 42\,T_1^{33}\,T_2^{19} + 50\,T_1^{34}\,T_2^{19} - 13\,T_1^{35}\,T_2^{19} - 10\,T_1^{36}\,T_2^{19} - T_1^{37}\,T_2^{19} + 6\,T_1^3\,T_2^{20} + 3\,T_1^4\,T_2^{20} - 32\,T_1^5\,T_2^{20} + 24\,T_1^6\,T_2^{20} + 68\,T_1^7\,T_2^{20} - 115\,T_1^8\,T_2^{20} - 47\,T_1^9\,T_2^{20} + 136\,T_1^{10}\,T_2^{20} + 212\,T_1^{11}\,T_2^{20} - 596\,T_1^{12}\,T_2^{20} + 212\,T_1^{13}\,T_2^{20} + 809\,T_1^{14}\,T_2^{20} - 914\,T_1^{15}\,T_2^{20} - 260\,T_1^{16}\,T_2^{20} + 1520\,T_1^{17}\,T_2^{20} - 1372\,T_1^{18}\,T_2^{20} + 155\,T_1^{19}\,T_2^{20} + 1056\,T_1^{20}\,T_2^{20} - 1291\,T_1^{21}\,T_2^{20} + 674\,T_1^{22}\,T_2^{20} - 128\,T_1^{23}\,T_2^{20} - 56\,T_1^{24}\,T_2^{20} - 374\,T_1^{25}\,T_2^{20} + 603\,T_1^{26}\,T_2^{20} - 180\,T_1^{27}\,T_2^{20} - 504\,T_1^{28}\,T_2^{20} + 592\,T_1^{29}\,T_2^{20} - 340\,T_1^{30}\,T_2^{20} + 71\,T_1^{31}\,T_2^{20} - 39\,T_1^{32}\,T_2^{20} + 100\,T_1^{33}\,T_2^{20} - 60\,T_1^{34}\,T_2^{20} - 8\,T_1^{35}\,T_2^{20} + 19\,T_1^{36}\,T_2^{20} + 6\,T_1^{37}\,T_2^{20} - T_1^3\,T_2^{21} - 10\,T_1^4\,T_2^{21} + 15\,T_1^5\,T_2^{21} + 18\,T_1^6\,T_2^{21} - 62\,T_1^7\,T_2^{21} + 14\,T_1^8\,T_2^{21} + 95\,T_1^9\,T_2^{21} - 22\,T_1^{10}\,T_2^{21} - 156\,T_1^{11}\,T_2^{21} - 72\,T_1^{12}\,T_2^{21} + 686\,T_1^{13}\,T_2^{21} - 758\,T_1^{14}\,T_2^{21} - 229\,T_1^{15}\,T_2^{21} + 1108\,T_1^{16}\,T_2^{21} - 933\,T_1^{17}\,T_2^{21} - 146\,T_1^{18}\,T_2^{21} + 1033\,T_1^{19}\,T_2^{21} - 1291\,T_1^{20}\,T_2^{21} + 891\,T_1^{21}\,T_2^{21} - 152\,T_1^{22}\,T_2^{21} - 395\,T_1^{23}\,T_2^{21} + 328\,T_1^{24}\,T_2^{21} + 152\,T_1^{25}\,T_2^{21} - 52\,T_1^{26}\,T_2^{21} - 695\,T_1^{27}\,T_2^{21} + 1069\,T_1^{28}\,T_2^{21} - 559\,T_1^{29}\,T_2^{21} - 14\,T_1^{30}\,T_2^{21} + 166\,T_1^{31}\,T_2^{21} - 35\,T_1^{32}\,T_2^{21} - 12\,T_1^{33}\,T_2^{21} - 40\,T_1^{34}\,T_2^{21} + 52\,T_1^{35}\,T_2^{21} - 15\,T_1^{36}\,T_2^{21} - 10\,T_1^{37}\,T_2^{21} - T_1^{38}\,T_2^{21} + 3\,T_1^4\,T_2^{22} + 3\,T_1^5\,T_2^{22} - 17\,T_1^6\,T_2^{22} + 5\,T_1^7\,T_2^{22} + 51\,T_1^8\,T_2^{22} - 68\,T_1^9\,T_2^{22} - 12\,T_1^{10}\,T_2^{22} - 46\,T_1^{11}\,T_2^{22} + 294\,T_1^{12}\,T_2^{22} - 294\,T_1^{13}\,T_2^{22} - 370\,T_1^{14}\,T_2^{22} + 992\,T_1^{15}\,T_2^{22} - 545\,T_1^{16}\,T_2^{22} - 265\,T_1^{17}\,T_2^{22} + 878\,T_1^{18}\,T_2^{22} - 840\,T_1^{19}\,T_2^{22} + 674\,T_1^{20}\,T_2^{22} - 152\,T_1^{21}\,T_2^{22} - 206\,T_1^{22}\,T_2^{22} + 744\,T_1^{23}\,T_2^{22} - 390\,T_1^{24}\,T_2^{22} + 141\,T_1^{25}\,T_2^{22} - 385\,T_1^{26}\,T_2^{22} + 866\,T_1^{27}\,T_2^{22} - 464\,T_1^{28}\,T_2^{22} - 332\,T_1^{29}\,T_2^{22} + 614\,T_1^{30}\,T_2^{22} - 304\,T_1^{31}\,T_2^{22} + 60\,T_1^{32}\,T_2^{22} - 108\,T_1^{33}\,T_2^{22} + 85\,T_1^{34}\,T_2^{22} - 23\,T_1^{35}\,T_2^{22} - 3\,T_1^{36}\,T_2^{22} + 3\,T_1^{37}\,T_2^{22} + 3\,T_1^{38}\,T_2^{22} + 2\,T_1^6\,T_2^{23} + 10\,T_1^7\,T_2^{23} - 27\,T_1^8\,T_2^{23} + 8\,T_1^9\,T_2^{23} + 46\,T_1^{10}\,T_2^{23} - 6\,T_1^{11}\,T_2^{23} + 38\,T_1^{12}\,T_2^{23} - 313\,T_1^{13}\,T_2^{23} + 595\,T_1^{14}\,T_2^{23} - 257\,T_1^{15}\,T_2^{23} - 555\,T_1^{16}\,T_2^{23} + 476\,T_1^{17}\,T_2^{23} - 163\,T_1^{18}\,T_2^{23} - 49\,T_1^{19}\,T_2^{23} - 128\,T_1^{20}\,T_2^{23} - 395\,T_1^{21}\,T_2^{23} + 744\,T_1^{22}\,T_2^{23} - 1174\,T_1^{23}\,T_2^{23} + 198\,T_1^{24}\,T_2^{23} + 191\,T_1^{25}\,T_2^{23} - 109\,T_1^{26}\,T_2^{23} + 48\,T_1^{27}\,T_2^{23} - 668\,T_1^{28}\,T_2^{23} + 885\,T_1^{29}\,T_2^{23} - 489\,T_1^{30}\,T_2^{23} + 5\,T_1^{31}\,T_2^{23} + 19\,T_1^{32}\,T_2^{23} + 92\,T_1^{33}\,T_2^{23} + 51\,T_1^{34}\,T_2^{23} - 71\,T_1^{35}\,T_2^{23} + 10\,T_1^{36}\,T_2^{23} + 2\,T_1^{37}\,T_2^{23} - 3\,T_1^5\,T_2^{24} - T_1^6\,T_2^{24} - 3\,T_1^7\,T_2^{24} + T_1^8\,T_2^{24} + 32\,T_1^9\,T_2^{24} - 93\,T_1^{10}\,T_2^{24} + 65\,T_1^{11}\,T_2^{24} - 64\,T_1^{12}\,T_2^{24} + 146\,T_1^{13}\,T_2^{24} + 58\,T_1^{14}\,T_2^{24} - 598\,T_1^{15}\,T_2^{24} + 792\,T_1^{16}\,T_2^{24} + 123\,T_1^{17}\,T_2^{24} - 695\,T_1^{18}\,T_2^{24} + 464\,T_1^{19}\,T_2^{24} - 56\,T_1^{20}\,T_2^{24} + 328\,T_1^{21}\,T_2^{24} - 390\,T_1^{22}\,T_2^{24} + 198\,T_1^{23}\,T_2^{24} + 404\,T_1^{24}\,T_2^{24} - 140\,T_1^{25}\,T_2^{24} + 75\,T_1^{26}\,T_2^{24} - 655\,T_1^{27}\,T_2^{24} + 968\,T_1^{28}\,T_2^{24} - 310\,T_1^{29}\,T_2^{24} - 326\,T_1^{30}\,T_2^{24} + 364\,T_1^{31}\,T_2^{24} - 104\,T_1^{32}\,T_2^{24} + 133\,T_1^{33}\,T_2^{24} - 253\,T_1^{34}\,T_2^{24} + 84\,T_1^{35}\,T_2^{24} + 25\,T_1^{36}\,T_2^{24} - 3\,T_1^{37}\,T_2^{24} - T_1^{38}\,T_2^{24} - 3\,T_1^{39}\,T_2^{24} + T_1^5\,T_2^{25} + 6\,T_1^6\,T_2^{25} - 4\,T_1^7\,T_2^{25} + 4\,T_1^8\,T_2^{25} - 19\,T_1^9\,T_2^{25} + 30\,T_1^{10}\,T_2^{25} + 76\,T_1^{11}\,T_2^{25} - 123\,T_1^{12}\,T_2^{25} + 24\,T_1^{13}\,T_2^{25} - 229\,T_1^{14}\,T_2^{25} + 440\,T_1^{15}\,T_2^{25} - 94\,T_1^{16}\,T_2^{25} - 791\,T_1^{17}\,T_2^{25} + 872\,T_1^{18}\,T_2^{25} + 37\,T_1^{19}\,T_2^{25} - 374\,T_1^{20}\,T_2^{25} + 152\,T_1^{21}\,T_2^{25} + 141\,T_1^{22}\,T_2^{25} + 191\,T_1^{23}\,T_2^{25} - 140\,T_1^{24}\,T_2^{25} + 246\,T_1^{25}\,T_2^{25} - 674\,T_1^{26}\,T_2^{25} + 977\,T_1^{27}\,T_2^{25} - 52\,T_1^{28}\,T_2^{25} - 868\,T_1^{29}\,T_2^{25} + 680\,T_1^{30}\,T_2^{25} - 120\,T_1^{31}\,T_2^{25} + 5\,T_1^{32}\,T_2^{25} - 257\,T_1^{33}\,T_2^{25} + 142\,T_1^{34}\,T_2^{25} + 104\,T_1^{35}\,T_2^{25} - 74\,T_1^{36}\,T_2^{25} + 4\,T_1^{37}\,T_2^{25} - 4\,T_1^{38}\,T_2^{25} + 6\,T_1^{39}\,T_2^{25} + T_1^{40}\,T_2^{25} - 3\,T_1^6\,T_2^{26} -$$

$$3\,T_1^7\,T_2^{26}+9\,T_1^8\,T_2^{26}-2\,T_1^9\,T_2^{26}+11\,T_1^{10}\,T_2^{26}-107\,T_1^{11}\,T_2^{26}+60\,T_1^{12}\,T_2^{26}+123\,T_1^{13}\,T_2^{26}+T_1^{14}\,T_2^{26}-15\,T_1^{15}\,T_2^{26}-350\,T_1^{16}\,T_2^{26}+681\,T_1^{17}\,T_2^{26}-162\,T_1^{18}\,T_2^{26}-842\,T_1^{19}\,T_2^{26}+603\,T_1^{20}\,T_2^{26}-52\,T_1^{21}\,T_2^{26}-385\,T_1^{22}\,T_2^{26}-109\,T_1^{23}\,T_2^{26}+75\,T_1^{24}\,T_2^{26}-674\,T_1^{25}\,T_2^{26}+707\,T_1^{26}\,T_2^{26}+124\,T_1^{27}\,T_2^{26}-1276\,T_1^{28}\,T_2^{26}+1003\,T_1^{29}\,T_2^{26}+94\,T_1^{30}\,T_2^{26}-355\,T_1^{31}\,T_2^{26}-29\,T_1^{32}\,T_2^{26}+167\,T_1^{33}\,T_2^{26}+226\,T_1^{34}\,T_2^{26}-259\,T_1^{35}\,T_2^{26}+31\,T_1^{36}\,T_2^{26}-2\,T_1^{37}\,T_2^{26}+9\,T_1^{38}\,T_2^{26}-3\,T_1^{39}\,T_2^{26}-3\,T_1^{40}\,T_2^{26}+4\,T_1^7\,T_2^{27}-6\,T_1^8\,T_2^{27}-7\,T_1^9\,T_2^{27}+2\,T_1^{10}\,T_2^{27}+31\,T_1^{11}\,T_2^{27}+93\,T_1^{12}\,T_2^{27}-238\,T_1^{13}\,T_2^{27}+95\,T_1^{14}\,T_2^{27}-50\,T_1^{15}\,T_2^{27}+256\,T_1^{16}\,T_2^{27}-213\,T_1^{17}\,T_2^{27}-458\,T_1^{18}\,T_2^{27}+972\,T_1^{19}\,T_2^{27}-180\,T_1^{20}\,T_2^{27}-695\,T_1^{21}\,T_2^{27}+866\,T_1^{22}\,T_2^{27}+48\,T_1^{23}\,T_2^{27}-655\,T_1^{24}\,T_2^{27}+977\,T_1^{25}\,T_2^{27}+124\,T_1^{26}\,T_2^{27}-1524\,T_1^{27}\,T_2^{27}+1365\,T_1^{28}\,T_2^{27}+147\,T_1^{29}\,T_2^{27}-957\,T_1^{30}\,T_2^{27}+335\,T_1^{31}\,T_2^{27}+101\,T_1^{32}\,T_2^{27}+207\,T_1^{33}\,T_2^{27}-504\,T_1^{34}\,T_2^{27}+149\,T_1^{35}\,T_2^{27}+94\,T_1^{36}\,T_2^{27}+2\,T_1^{37}\,T_2^{27}-7\,T_1^{38}\,T_2^{27}-6\,T_1^{39}\,T_2^{27}+4\,T_1^{40}\,T_2^{27}-2\,T_1^8\,T_2^{28}+12\,T_1^9\,T_2^{28}-3\,T_1^{10}\,T_2^{28}-T_1^{11}\,T_2^{28}-69\,T_1^{12}\,T_2^{28}+65\,T_1^{13}\,T_2^{28}+124\,T_1^{14}\,T_2^{28}-167\,T_1^{15}\,T_2^{28}-24\,T_1^{16}\,T_2^{28}-82\,T_1^{17}\,T_2^{28}+506\,T_1^{18}\,T_2^{28}-412\,T_1^{19}\,T_2^{28}-504\,T_1^{20}\,T_2^{28}+1069\,T_1^{21}\,T_2^{28}-464\,T_1^{22}\,T_2^{28}-668\,T_1^{23}\,T_2^{28}+968\,T_1^{24}\,T_2^{28}-52\,T_1^{25}\,T_2^{28}-1276\,T_1^{26}\,T_2^{28}+1365\,T_1^{27}\,T_2^{28}+204\,T_1^{28}\,T_2^{28}-1248\,T_1^{29}\,T_2^{28}+726\,T_1^{30}\,T_2^{28}+270\,T_1^{31}\,T_2^{28}-224\,T_1^{32}\,T_2^{28}-267\,T_1^{33}\,T_2^{28}+196\,T_1^{34}\,T_2^{28}+189\,T_1^{35}\,T_2^{28}-173\,T_1^{36}\,T_2^{28}-T_1^{37}\,T_2^{28}-3\,T_1^{38}\,T_2^{28}+12\,T_1^{39}\,T_2^{28}-2\,T_1^{40}\,T_2^{28}-2\,T_1^9\,T_2^{29}-6\,T_1^{10}\,T_2^{29}+8\,T_1^{11}\,T_2^{29}-T_1^{12}\,T_2^{29}+45\,T_1^{13}\,T_2^{29}-151\,T_1^{14}\,T_2^{29}+92\,T_1^{15}\,T_2^{29}+109\,T_1^{16}\,T_2^{29}-8\,T_1^{17}\,T_2^{29}-208\,T_1^{18}\,T_2^{29}-44\,T_1^{19}\,T_2^{29}+592\,T_1^{20}\,T_2^{29}-559\,T_1^{21}\,T_2^{29}-332\,T_1^{22}\,T_2^{29}+885\,T_1^{23}\,T_2^{29}-310\,T_1^{24}\,T_2^{29}-868\,T_1^{25}\,T_2^{29}+1003\,T_1^{26}\,T_2^{29}+147\,T_1^{27}\,T_2^{29}-1248\,T_1^{28}\,T_2^{29}+896\,T_1^{29}\,T_2^{29}+114\,T_1^{30}\,T_2^{29}-478\,T_1^{31}\,T_2^{29}+75\,T_1^{32}\,T_2^{29}+119\,T_1^{33}\,T_2^{29}+166\,T_1^{34}\,T_2^{29}-269\,T_1^{35}\,T_2^{29}+94\,T_1^{36}\,T_2^{29}-T_1^{37}\,T_2^{29}+8\,T_1^{38}\,T_2^{29}-6\,T_1^{39}\,T_2^{29}-2\,T_1^{40}\,T_2^{29}+4\,T_1^{10}\,T_2^{30}-6\,T_1^{11}\,T_2^{30}-3\,T_1^{12}\,T_2^{30}+2\,T_1^{13}\,T_2^{30}+19\,T_1^{14}\,T_2^{30}+74\,T_1^{15}\,T_2^{30}-189\,T_1^{16}\,T_2^{30}+74\,T_1^{17}\,T_2^{30}+44\,T_1^{18}\,T_2^{30}+150\,T_1^{19}\,T_2^{30}-340\,T_1^{20}\,T_2^{30}-14\,T_1^{21}\,T_2^{30}+614\,T_1^{22}\,T_2^{30}-489\,T_1^{23}\,T_2^{30}-326\,T_1^{24}\,T_2^{30}+680\,T_1^{25}\,T_2^{30}+94\,T_1^{26}\,T_2^{30}-957\,T_1^{27}\,T_2^{30}+726\,T_1^{28}\,T_2^{30}+114\,T_1^{29}\,T_2^{30}-444\,T_1^{30}\,T_2^{30}+138\,T_1^{31}\,T_2^{30}+68\,T_1^{32}\,T_2^{30}+106\,T_1^{33}\,T_2^{30}-253\,T_1^{34}\,T_2^{30}+90\,T_1^{35}\,T_2^{30}+31\,T_1^{36}\,T_2^{30}+2\,T_1^{37}\,T_2^{30}-3\,T_1^{38}\,T_2^{30}-6\,T_1^{39}\,T_2^{30}+4\,T_1^{40}\,T_2^{30}-2\,T_1^{11}\,T_2^{31}+12\,T_1^{12}\,T_2^{31}-7\,T_1^{13}\,T_2^{31}-2\,T_1^{14}\,T_2^{31}-49\,T_1^{15}\,T_2^{31}+60\,T_1^{16}\,T_2^{31}+42\,T_1^{17}\,T_2^{31}-100\,T_1^{18}\,T_2^{31}-21\,T_1^{19}\,T_2^{31}+71\,T_1^{20}\,T_2^{31}+166\,T_1^{21}\,T_2^{31}-304\,T_1^{22}\,T_2^{31}+5\,T_1^{23}\,T_2^{31}+364\,T_1^{24}\,T_2^{31}-120\,T_1^{25}\,T_2^{31}-355\,T_1^{26}\,T_2^{31}+335\,T_1^{27}\,T_2^{31}+270\,T_1^{28}\,T_2^{31}-478\,T_1^{29}\,T_2^{31}+138\,T_1^{30}\,T_2^{31}+173\,T_1^{31}\,T_2^{31}-59\,T_1^{32}\,T_2^{31}-112\,T_1^{33}\,T_2^{31}+41\,T_1^{34}\,T_2^{31}+98\,T_1^{35}\,T_2^{31}-74\,T_1^{36}\,T_2^{31}-2\,T_1^{37}\,T_2^{31}-7\,T_1^{38}\,T_2^{31}+12\,T_1^{39}\,T_2^{31}-2\,T_1^{40}\,T_2^{31}-2\,T_1^{12}\,T_2^{32}-6\,T_1^{13}\,T_2^{32}+9\,T_1^{14}\,T_2^{32}+4\,T_1^{15}\,T_2^{32}+17\,T_1^{16}\,T_2^{32}-59\,T_1^{17}\,T_2^{32}+79\,T_1^{18}\,T_2^{32}-10\,T_1^{19}\,T_2^{32}-39\,T_1^{20}\,T_2^{32}-35\,T_1^{21}\,T_2^{32}+60\,T_1^{22}\,T_2^{32}+19\,T_1^{23}\,T_2^{32}-104\,T_1^{24}\,T_2^{32}+5\,T_1^{25}\,T_2^{32}-29\,T_1^{26}\,T_2^{32}+101\,T_1^{27}\,T_2^{32}-224\,T_1^{28}\,T_2^{32}+75\,T_1^{29}\,T_2^{32}+68\,T_1^{30}\,T_2^{32}-59\,T_1^{31}\,T_2^{32}-31\,T_1^{32}\,T_2^{32}-10\,T_1^{33}\,T_2^{32}+87\,T_1^{34}\,T_2^{32}-75\,T_1^{35}\,T_2^{32}+25\,T_1^{36}\,T_2^{32}+4\,T_1^{37}\,T_2^{32}+9\,T_1^{38}\,T_2^{32}-6\,T_1^{39}\,T_2^{32}-2\,T_1^{40}\,T_2^{32}+4\,T_1^{13}\,T_2^{33}-3\,T_1^{14}\,T_2^{33}-4\,T_1^{15}\,T_2^{33}-3\,T_1^{16}\,T_2^{33}+10\,T_1^{17}\,T_2^{33}-19\,T_1^{18}\,T_2^{33}-42\,T_1^{19}\,T_2^{33}+100\,T_1^{20}\,T_2^{33}-12\,T_1^{21}\,T_2^{33}-108\,T_1^{22}\,T_2^{33}+92\,T_1^{23}\,T_2^{33}+133\,T_1^{24}\,T_2^{33}-257\,T_1^{25}\,T_2^{33}+167\,T_1^{26}\,T_2^{33}+207\,T_1^{27}\,T_2^{33}-267\,T_1^{28}\,T_2^{33}+119\,T_1^{29}\,T_2^{33}+106\,T_1^{30}\,T_2^{33}-112\,T_1^{31}\,T_2^{33}-10\,T_1^{32}\,T_2^{33}+94\,T_1^{33}\,T_2^{33}-36\,T_1^{34}\,T_2^{33}-21\,T_1^{35}\,T_2^{33}+10\,T_1^{36}\,T_2^{33}-3\,T_1^{37}\,T_2^{33}-4\,T_1^{38}\,T_2^{33}-3\,T_1^{39}\,T_2^{33}+4\,T_1^{40}\,T_2^{33}-3\,T_1^{14}\,T_2^{34}+6\,T_1^{15}\,T_2^{34}-T_1^{16}\,T_2^{34}+2\,T_1^{17}\,T_2^{34}-5\,T_1^{18}\,T_2^{34}+50\,T_1^{19}\,T_2^{34}-60\,T_1^{20}\,T_2^{34}-40\,T_1^{21}\,T_2^{34}+85\,T_1^{22}\,T_2^{34}+51\,T_1^{23}\,T_2^{34}-253\,T_1^{24}\,T_2^{34}+142\,T_1^{25}\,T_2^{34}+226\,T_1^{26}\,T_2^{34}-504\,T_1^{27}\,T_2^{34}+196\,T_1^{28}\,T_2^{34}+166\,T_1^{29}\,T_2^{34}-253\,T_1^{30}\,T_2^{34}+41\,T_1^{31}\,T_2^{34}+87\,T_1^{32}\,T_2^{34}-36\,T_1^{33}\,T_2^{34}-60\,T_1^{34}\,T_2^{34}+46\,T_1^{35}\,T_2^{34}-3\,T_1^{36}\,T_2^{34}+2\,T_1^{37}\,T_2^{34}-T_1^{38}\,T_2^{34}+6\,T_1^{39}\,T_2^{34}-3\,T_1^{40}\,T_2^{34}+T_1^{15}\,T_2^{35}-3\,T_1^{16}\,T_2^{35}+3\,T_1^{18}\,T_2^{35}-13\,T_1^{19}\,T_2^{35}-8\,T_1^{20}\,T_2^{35}+52\,T_1^{21}\,T_2^{35}-23\,T_1^{22}\,T_2^{35}-71\,T_1^{23}\,T_2^{35}+84\,T_1^{24}\,T_2^{35}+104\,T_1^{25}\,T_2^{35}-259\,T_1^{26}\,T_2^{35}+149\,T_1^{27}\,T_2^{35}+189\,T_1^{28}\,T_2^{35}-269\,T_1^{29}\,T_2^{35}+90\,T_1^{30}\,T_2^{35}+98\,T_1^{31}\,T_2^{35}-75\,T_1^{32}\,T_2^{35}-21\,T_1^{33}\,T_2^{35}+46\,T_1^{34}\,T_2^{35}-2\,T_1^{35}\,T_2^{35}-15\,T_1^{36}\,T_2^{35}+3\,T_1^{37}\,T_2^{35}-3\,T_1^{39}\,T_2^{35}+T_1^{40}\,T_2^{35}+3\,T_1^{18}\,T_2^{36}-10\,T_1^{19}\,T_2^{36}+19\,T_1^{20}\,T_2^{36}-15\,T_1^{21}\,T_2^{36}-3\,T_1^{22}\,T_2^{36}+10\,T_1^{23}\,T_2^{36}+25\,T_1^{24}\,T_2^{36}-74\,T_1^{25}\,T_2^{36}+31\,T_1^{26}\,T_2^{36}+94\,T_1^{27}\,T_2^{36}-173\,T_1^{28}\,T_2^{36}+94\,T_1^{29}\,T_2^{36}+31\,T_1^{30}\,T_2^{36}-74\,T_1^{31}\,T_2^{36}+25\,T_1^{32}\,T_2^{36}+10\,T_1^{33}\,T_2^{36}-3\,T_1^{34}\,T_2^{36}-15\,T_1^{35}\,T_2^{36}+19\,T_1^{36}\,T_2^{36}-10\,T_1^{37}\,T_2^{36}+3\,T_1^{38}\,T_2^{36}-T_1^{19}\,T_2^{37}+6\,T_1^{20}\,T_2^{37}-10\,T_1^{21}\,T_2^{37}+3\,T_1^{22}\,T_2^{37}+2\,T_1^{23}\,T_2^{37}-3\,T_1^{24}\,T_2^{37}+4\,T_1^{25}\,T_2^{37}-2\,T_1^{26}\,T_2^{37}+2\,T_1^{27}\,T_2^{37}-T_1^{28}\,T_2^{37}-T_1^{29}\,T_2^{37}+2\,T_1^{30}\,T_2^{37}-2\,T_1^{31}\,T_2^{37}+4\,T_1^{32}\,T_2^{37}-3\,T_1^{33}\,T_2^{37}+2\,T_1^{34}\,T_2^{37}+3\,T_1^{35}\,T_2^{37}-10\,T_1^{36}\,T_2^{37}+6\,T_1^{37}\,T_2^{37}-T_1^{38}\,T_2^{37}-T_1^{21}\,T_2^{38}+3\,T_1^{22}\,T_2^{38}-T_1^{24}\,T_2^{38}-4\,T_1^{25}\,T_2^{38}+9\,T_1^{26}\,T_2^{38}-$$

$$7\,T_1^{27}\,T_2^{38}-3\,T_1^{28}\,T_2^{38}+8\,T_1^{29}\,T_2^{38}-3\,T_1^{30}\,T_2^{38}-7\,T_1^{31}\,T_2^{38}+9\,T_1^{32}\,T_2^{38}-4\,T_1^{33}\,T_2^{38}-T_1^{34}\,T_2^{38}+3\,T_1^{36}\,T_2^{38}-T_1^{37}\,T_2^{38}-3\,T_1^{24}\,T_2^{39}+6\,T_1^{25}\,T_2^{39}-3\,T_1^{26}\,T_2^{39}-6\,T_1^{27}\,T_2^{39}+12\,T_1^{28}\,T_2^{39}-6\,T_1^{29}\,T_2^{39}-6\,T_1^{30}\,T_2^{39}+12\,T_1^{31}\,T_2^{39}-6\,T_1^{32}\,T_2^{39}-3\,T_1^{33}\,T_2^{39}+6\,T_1^{34}\,T_2^{39}-3\,T_1^{35}\,T_2^{39}+T_1^{25}\,T_2^{40}-3\,T_1^{26}\,T_2^{40}+4\,T_1^{27}\,T_2^{40}-2\,T_1^{28}\,T_2^{40}-2\,T_1^{29}\,T_2^{40}+4\,T_1^{30}\,T_2^{40}-2\,T_1^{31}\,T_2^{40}-2\,T_1^{32}\,T_2^{40}+4\,T_1^{33}\,T_2^{40}-3\,T_1^{34}\,T_2^{40}+T_1^{35}\,T_2^{40}\Big)\Big\}\Big\}$$

In[•]:=
```
DuplicateFreeQ[θ /@ AllKnots[{3, 10}]]
```

Out[•]=

True

In[•]:=
```
DuplicateFreeQ[θ /@ AllKnots[{3, 12}]]
```

KnotTheory: Loading precomputed data in KnotTheory/12A.dts.

KnotTheory: Loading precomputed data in KnotTheory/12N.dts.

Out[•]=

False

In[•]:=
```
tab11 = Table[K → θ@K, {K, AllKnots[{3, 11}] }]
```

Out[•]=

$$\Big\{\text{Knot}[3,1]\to\Big\{\frac{1-T+T^2}{T},\ \frac{-1+T_1-T_1^2+T_2-T_1^2T_2+2T_1^3T_2-T_2^2-T_1T_2^2+T_1^2T_2^2-2T_1^3T_2^2+2T_1T_2^3-2T_1^2T_2^3+2T_1^3T_2^3}{T_1^2T_2\,(-1+T_1T_2)}\Big\},$$

$$\text{Knot}[4,1]\to\Big\{-\frac{1-3T+T^2}{T},\ -\frac{(1-3T_1+T_1^2)\,(1+T_1T_2)\,(1-3T_2+T_2^2)}{T_1^3T_2^2}\Big\},\ \text{Knot}[5,1]\to\Big\{\frac{1-T+T^2-T^3+T^4}{T^2},\ \frac{-1+\langle\!\langle 67\rangle\!\rangle+4T_1^7T_2^7}{T_1^4T_2^3\,(-1+T_1T_2)}\Big\},$$

$$\langle\!\langle 795\rangle\!\rangle,\ \text{Knot}[11,\text{NonAlternating},183]\to\Big\{\frac{\langle\!\langle 1\rangle\!\rangle}{T^3},\ -\langle\!\langle 1\rangle\!\rangle\Big\},$$

$$\text{Knot}[11,\text{NonAlternating},184]\to\Big\{\frac{(1-T+T^2)\,(2-7T+11T^2-7T^3+2T^4)}{T^3},\ \frac{-33+149T_1-332T_1^2+\langle\!\langle 177\rangle\!\rangle+148T_1^{10}T_2^{12}-67T_1^{11}T_2^{12}+15T_1^{12}T_2^{12}}{T_1^7T_2^6\,(-1+T_1T_2)}\Big\},$$

$$\text{Knot}[11,\text{NonAlternating},185]\to\Big\{-\frac{(1-3T+T^2)\,(1-T+T^2)\,(2-3T+2T^2)}{T^3},$$
$$-\frac{1}{T_1^7T_2^6\,(-1+T_1T_2)}\big(-41+225T_1-490T_1^2+633T_1^3-490T_1^4+225T_1^5-41T_1^6+225T_2-1054T_1T_2+1693T_1^2T_2-1311T_1^3T_2-101T_1^4T_2+923T_1^5T_2-768T_1^6T_2+\langle\!\langle 139\rangle\!\rangle+5T_1^5T_2^{11}-64T_1^6T_2^{11}+263T_1^7T_2^{11}-541T_1^8T_2^{11}+669T_1^9T_2^{11}-507T_1^{10}T_2^{11}+222T_1^{11}T_2^{11}-39T_1^{12}T_2^{11}+7T_1^6T_2^{12}-39T_1^7T_2^{12}+86T_1^8T_2^{12}-111T_1^9T_2^{12}+86T_1^{10}T_2^{12}-39T_1^{11}T_2^{12}+7T_1^{12}T_2^{12}\big)\Big\}\Big\}$$

Full expression not available (original memory size: 36.2 MB)

In[•]:=
```
Gather[tab11, Last[#1] === Last[#2] &]
```

Out[•]=

$$\Big\{\Big\{\text{Knot}[3,1]\to\Big\{\frac{1-T+T^2}{T},\ \frac{-1+T_1-T_1^2+T_2-T_1^2T_2+2T_1^3T_2-T_2^2-T_1T_2^2+T_1^2T_2^2-2T_1^3T_2^2+2T_1T_2^3-2T_1^2T_2^3+2T_1^3T_2^3}{T_1^2T_2\,(-1+T_1T_2)}\Big\}\Big\},$$

$$\Big\{\text{Knot}[4,1]\to\Big\{-\frac{1-3T+T^2}{T},\ -\frac{(1-3T_1+T_1^2)\,(1+T_1T_2)\,(1-3T_2+T_2^2)}{T_1^3T_2^2}\Big\}\Big\},\ \Big\{\text{Knot}[5,1]\to\Big\{\frac{1-T+T^2-T^3+T^4}{T^2},\ \frac{-1+\langle\!\langle 67\rangle\!\rangle+4T_1^7T_2^7}{T_1^4T_2^3\,(-1+T_1T_2)}\Big\}\Big\},$$

$$\langle\!\langle 792\rangle\!\rangle,\ \Big\{\text{Knot}[11,\text{NonAlternating},183]\to\Big\{\frac{\langle\!\langle 1\rangle\!\rangle}{T^3},\ -\langle\!\langle 1\rangle\!\rangle\Big\}\Big\},$$

$$\Big\{\text{Knot}[11,\text{NonAlternating},184]\to\Big\{\frac{(1-T+T^2)\,(2-7T+11T^2-7T^3+2T^4)}{T^3},\ \frac{-33+149T_1-332T_1^2+415T_1^3-332T_1^4+\langle\!\langle 174\rangle\!\rangle+148T_1^{10}T_2^{12}-67T_1^{11}T_2^{12}+15T_1^{12}T_2^{12}}{T_1^7T_2^6\,(-1+T_1T_2)}\Big\}\Big\},$$

$$\Big\{\text{Knot}[11,\text{NonAlternating},185]\to$$
$$\Big\{-\frac{(1-3T+T^2)\,(1-T+T^2)\,(2-3T+2T^2)}{T^3},\ -\frac{1}{T_1^7T_2^6\,(-1+T_1T_2)}\big(-41+225T_1-490T_1^2+633T_1^3-490T_1^4+225T_1^5-41T_1^6+225T_2-1054T_1T_2+1693T_1^2T_2-1311T_1^3T_2-101T_1^4T_2+923T_1^5T_2-768T_1^6T_2+\langle\!\langle 142\rangle\!\rangle+263T_1^7T_2^{11}-541T_1^8T_1^{11}+669T_1^9T_2^{11}-507T_1^{10}T_2^{11}+222T_1^{11}T_2^{11}-39T_1^{12}T_2^{11}+7T_1^6T_2^{12}-39T_1^7T_2^{12}+86T_1^8T_2^{12}-111T_1^9T_2^{12}+86T_1^{10}T_2^{12}-39T_1^{11}T_2^{12}+7T_1^{12}T_2^{12}\big)\Big\}\Big\}\Big\}$$

Stored in notebook: 36.2 MB | + Show more | Show all | Iconize

In[•]:=
```
Select[Gather[tab11, Last[#1] === Last[#2] &], Length[#] > 1 &]
```

Out[•]=

$$\left\{\left\{\text{Knot}[11,\text{Alternating},44]\to\left\{\frac{\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^4},\frac{1}{T_1^9T_2^8\left(-1+T_1T_2\right)}\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\right.\right.$$
$$\left(1-T_1T_2+T_1^2T_2^2\right)\left(-4+16T_1-36T_1^2+44T_1^3-36T_1^4+16T_1^5-4T_1^6+16T_2-53T_1T_2+100T_1^2T_2-\right.$$
$$77T_1^3T_2+23T_1^4T_2+35T_1^5T_2-28T_1^6T_2+11T_1^7T_2-36T_2^2+100T_1T_2^2-161T_1^2T_2^2+54T_1^3T_2^2+$$
$$39T_1^4T_2^2-107T_1^5T_2^2+23T_1^6T_2^2+8T_1^7T_2^2-13T_1^8T_2^2+44T_2^3-77T_1T_2^3+54T_1^2T_2^3+189T_1^3T_2^3-$$
$$215T_1^4T_2^3+153T_1^5T_2^3+106T_1^6T_2^3-107T_1^7T_2^3+52T_1^8T_2^3-36T_2^4+23T_1T_2^4+39T_1^2T_2^4-$$
$$215T_1^3T_2^4-23T_1^4T_2^4+238T_1^5T_2^4-478T_1^6T_2^4+214T_1^7T_2^4-4T_1^8T_2^4-52T_1^9T_2^4+13T_1^{10}T_2^4+$$
$$16T_2^5+35T_1T_2^5-107T_1^2T_2^5+153T_1^3T_2^5+238T_1^4T_2^5-485T_1^5T_2^5+547T_1^6T_2^5-52T_1^7T_2^5-$$
$$194T_1^8T_2^5+111T_1^9T_2^5-8T_1^{10}T_2^5-11T_1^{11}T_2^5-4T_2^6-28T_1T_2^6+23T_1^2T_2^6+106T_1^3T_2^6-478T_1^4T_2^6+$$
$$547T_1^5T_2^6-192T_1^6T_2^6-419T_1^7T_2^6+498T_1^8T_2^6-126T_1^9T_2^6-23T_1^{10}T_2^6+28T_1^{11}T_2^6+4T_1^{12}T_2^6+$$
$$11T_1T_2^7+8T_1^2T_2^7-107T_1^3T_2^7+214T_1^4T_2^7-52T_1^5T_2^7-419T_1^6T_2^7+613T_1^7T_2^7-342T_1^8T_2^7-$$
$$133T_1^9T_2^7+111T_1^{10}T_2^7-35T_1^{11}T_2^7-16T_1^{12}T_2^7-13T_1^2T_2^8+52T_1^3T_2^8-4T_1^4T_2^8-194T_1^5T_2^8+$$
$$498T_1^6T_2^8-342T_1^7T_2^8+43T_1^8T_2^8+235T_1^9T_2^8-47T_1^{10}T_2^8-23T_1^{11}T_2^8+36T_1^{12}T_2^8-52T_1^4T_2^9+$$
$$111T_1^5T_2^9-126T_1^6T_2^9-133T_1^7T_2^9+235T_1^8T_2^9-209T_1^9T_2^9-50T_1^{10}T_2^9+77T_1^{11}T_2^9-44T_1^{12}T_2^9+$$
$$13T_1^4T_2^{10}-8T_1^5T_2^{10}-23T_1^6T_2^{10}+111T_1^7T_2^{10}-47T_1^8T_2^{10}-50T_1^9T_2^{10}+161T_1^{10}T_2^{10}-100T_1^{11}T_2^{10}+$$
$$36T_1^{12}T_2^{10}-11T_1^5T_2^{11}+28T_1^6T_2^{11}-35T_1^7T_2^{11}-23T_1^8T_2^{11}+77T_1^9T_2^{11}-100T_1^{10}T_2^{11}+53T_1^{11}T_2^{11}-$$
$$\left.\left.16T_1^{12}T_2^{11}+4T_1^6T_2^{12}-16T_1^7T_2^{12}+36T_1^8T_2^{12}-44T_1^9T_2^{12}+36T_1^{10}T_2^{12}-16T_1^{11}T_2^{12}+4T_1^{12}T_2^{12}\right)\right\},$$

$$\text{Knot}[11,\text{Alternating},47]\to\left\{\frac{\left(1-T+T^2\right)^2\left(1-3T+5T^2-3T^3+T^4\right)}{T^4},\right.$$
$$\frac{1}{T_1^9T_2^8\left(-1+T_1T_2\right)}\left(1-T_1+T_1^2\right)\left(1-T_2+T_2^2\right)\left(1-T_1T_2+T_1^2T_2^2\right)$$
$$\left(-4+16T_1-36T_1^2+44T_1^3-36T_1^4+16T_1^5-4T_1^6+16T_2-53T_1T_2+100T_1^2T_2-77T_1^3T_2+\right.$$
$$23T_1^4T_2+35T_1^5T_2-28T_1^6T_2+11T_1^7T_2-36T_2^2+100T_1T_2^2-161T_1^2T_2^2+54T_1^3T_2^2+$$
$$39T_1^4T_2^2-107T_1^5T_2^2+23T_1^6T_2^2+8T_1^7T_2^2-13T_1^8T_2^2+44T_2^3-77T_1T_2^3+54T_1^2T_2^3+189T_1^3T_2^3-$$
$$215T_1^4T_2^3+153T_1^5T_2^3+106T_1^6T_2^3-107T_1^7T_2^3+52T_1^8T_2^3-36T_2^4+23T_1T_2^4+39T_1^2T_2^4-$$
$$215T_1^3T_2^4-23T_1^4T_2^4+238T_1^5T_2^4-478T_1^6T_2^4+214T_1^7T_2^4-4T_1^8T_2^4-52T_1^9T_2^4+13T_1^{10}T_2^4+$$
$$16T_2^5+35T_1T_2^5-107T_1^2T_2^5+153T_1^3T_2^5+238T_1^4T_2^5-485T_1^5T_2^5+547T_1^6T_2^5-52T_1^7T_2^5-$$
$$194T_1^8T_2^5+111T_1^9T_2^5-8T_1^{10}T_2^5-11T_1^{11}T_2^5-4T_2^6-28T_1T_2^6+23T_1^2T_2^6+106T_1^3T_2^6-478T_1^4T_2^6+$$
$$547T_1^5T_2^6-192T_1^6T_2^6-419T_1^7T_2^6+498T_1^8T_2^6-126T_1^9T_2^6-23T_1^{10}T_2^6+28T_1^{11}T_2^6+4T_1^{12}T_2^6+$$
$$11T_1T_2^7+8T_1^2T_2^7-107T_1^3T_2^7+214T_1^4T_2^7-52T_1^5T_2^7-419T_1^6T_2^7+613T_1^7T_2^7-342T_1^8T_2^7-$$
$$133T_1^9T_2^7+111T_1^{10}T_2^7-35T_1^{11}T_2^7-16T_1^{12}T_2^7-13T_1^2T_2^8+52T_1^3T_2^8-4T_1^4T_2^8-194T_1^5T_2^8+$$
$$498T_1^6T_2^8-342T_1^7T_2^8+43T_1^8T_2^8+235T_1^9T_2^8-47T_1^{10}T_2^8-23T_1^{11}T_2^8+36T_1^{12}T_2^8-52T_1^4T_2^9+$$
$$111T_1^5T_2^9-126T_1^6T_2^9-133T_1^7T_2^9+235T_1^8T_2^9-209T_1^9T_2^9-50T_1^{10}T_2^9+77T_1^{11}T_2^9-44T_1^{12}T_2^9+$$
$$13T_1^4T_2^{10}-8T_1^5T_2^{10}-23T_1^6T_2^{10}+111T_1^7T_2^{10}-47T_1^8T_2^{10}-50T_1^9T_2^{10}+161T_1^{10}T_2^{10}-100T_1^{11}T_2^{10}+$$
$$36T_1^{12}T_2^{10}-11T_1^5T_2^{11}+28T_1^6T_2^{11}-35T_1^7T_2^{11}-23T_1^8T_2^{11}+77T_1^9T_2^{11}-100T_1^{10}T_2^{11}+53T_1^{11}T_2^{11}-$$
$$\left.\left.\left.16T_1^{12}T_2^{11}+4T_1^6T_2^{12}-16T_1^7T_2^{12}+36T_1^8T_2^{12}-44T_1^9T_2^{12}+36T_1^{10}T_2^{12}-16T_1^{11}T_2^{12}+4T_1^{12}T_2^{12}\right)\right\}\right\},$$

$$\left\{\text{Knot}[11,\text{Alternating},57]\to\left\{-\frac{\left(1-T+T^2\right)^2\left(1-3T+3T^2-3T^3+T^4\right)}{T^4},\right.\right.$$

$$-\frac{1}{T_1^9\,T_2^8\,(-1+T_1\,T_2)}\,\left(1-T_1+T_1^2\right)\,\left(1-T_2+T_2^2\right)\,\left(1-T_1\,T_2+T_1^2\,T_2^2\right)$$
$$\left(-3+12\,T_1-21\,T_1^2+27\,T_1^3-21\,T_1^4+12\,T_1^5-3\,T_1^6+12\,T_2-41\,T_1\,T_2+56\,T_1^2\,T_2-59\,T_1^3\,T_2+21\,T_1^4\,T_2+\right.$$
$$T_1^5\,T_2-16\,T_1^6\,T_2+7\,T_1^7\,T_2-21\,T_2^2+56\,T_1\,T_2^2-37\,T_1^2\,T_2^2+5\,T_1^3\,T_2^2+85\,T_1^4\,T_2^2-90\,T_1^5\,T_2^2+77\,T_1^6\,T_2^2-$$
$$20\,T_1^7\,T_2^2-2\,T_1^8\,T_2^2+27\,T_2^3-59\,T_1\,T_2^3+5\,T_1^2\,T_2^3+39\,T_1^3\,T_2^3-152\,T_1^4\,T_2^3+108\,T_1^5\,T_2^3-52\,T_1^6\,T_2^3-$$
$$42\,T_1^7\,T_2^3+44\,T_1^8\,T_2^3-9\,T_1^9\,T_2^3-21\,T_2^4+21\,T_1\,T_2^4+85\,T_1^2\,T_2^4-152\,T_1^3\,T_2^4+257\,T_1^4\,T_2^4-118\,T_1^5\,T_2^4-$$
$$16\,T_1^6\,T_2^4+109\,T_1^7\,T_2^4-52\,T_1^8\,T_2^4-28\,T_1^9\,T_2^4+16\,T_1^{10}\,T_2^4+12\,T_2^5+T_1\,T_2^5-90\,T_1^2\,T_2^5+108\,T_1^3\,T_2^5-$$
$$118\,T_1^4\,T_2^5-55\,T_1^5\,T_2^5+133\,T_1^6\,T_2^5-68\,T_1^7\,T_2^5-35\,T_1^8\,T_2^5+92\,T_1^9\,T_2^5-4\,T_1^{10}\,T_2^5-15\,T_1^{11}\,T_2^5-3\,T_2^6-$$
$$16\,T_1\,T_2^6+77\,T_1^2\,T_2^6-52\,T_1^3\,T_2^6-16\,T_1^4\,T_2^6+133\,T_1^5\,T_2^6-156\,T_1^6\,T_2^6-T_1^7\,T_2^6+4\,T_1^8\,T_2^6-12\,T_1^9\,T_2^6-$$
$$93\,T_1^{10}\,T_2^6+40\,T_1^{11}\,T_2^6+5\,T_1^{12}\,T_2^6+7\,T_1\,T_2^7-20\,T_1^2\,T_2^7-42\,T_1^3\,T_2^7+109\,T_1^4\,T_2^7-68\,T_1^5\,T_2^7-T_1^6\,T_2^7+$$
$$187\,T_1^7\,T_2^7-18\,T_1^8\,T_2^7-34\,T_1^9\,T_2^7+140\,T_1^{10}\,T_2^7-25\,T_1^{11}\,T_2^7-20\,T_1^{12}\,T_2^7-2\,T_1^2\,T_2^8+44\,T_1^3\,T_2^8-52\,T_1^4\,T_2^8-$$
$$35\,T_1^5\,T_2^8+4\,T_1^6\,T_2^8-18\,T_1^7\,T_2^8-269\,T_1^8\,T_2^8+226\,T_1^9\,T_2^8-189\,T_1^{10}\,T_2^8-5\,T_1^{11}\,T_2^8+35\,T_1^{12}\,T_2^8-9\,T_1^3\,T_2^9-$$
$$28\,T_1^4\,T_2^9+92\,T_1^5\,T_2^9-12\,T_1^6\,T_2^9-34\,T_1^7\,T_2^9+226\,T_1^8\,T_2^9-103\,T_1^9\,T_2^9+45\,T_1^{10}\,T_2^9+75\,T_1^{11}\,T_2^9-45\,T_1^{12}\,T_2^9+$$
$$16\,T_1^4\,T_2^{10}-4\,T_1^5\,T_2^{10}-93\,T_1^6\,T_2^{10}+140\,T_1^7\,T_2^{10}-189\,T_1^8\,T_2^{10}+45\,T_1^9\,T_2^{10}+21\,T_1^{10}\,T_2^{10}-80\,T_1^{11}\,T_2^{10}+$$
$$35\,T_1^{12}\,T_2^{10}-15\,T_1^5\,T_2^{11}+40\,T_1^6\,T_2^{11}-25\,T_1^7\,T_2^{11}-5\,T_1^8\,T_2^{11}+75\,T_1^9\,T_2^{11}-80\,T_1^{10}\,T_2^{11}+65\,T_1^{11}\,T_2^{11}-$$
$$\left.20\,T_1^{12}\,T_2^{11}+5\,T_1^6\,T_2^{12}-20\,T_1^7\,T_2^{12}+35\,T_1^8\,T_2^{12}-45\,T_1^9\,T_2^{12}+35\,T_1^{10}\,T_2^{12}-20\,T_1^{11}\,T_2^{12}+5\,T_1^{12}\,T_2^{12}\right)\Big\},$$

$$\text{Knot}[11,\text{Alternating},231]\to\left\{-\frac{\left(1-T+T^2\right)^2\,\left(1-3\,T+3\,T^2-3\,T^3+T^4\right)}{T^4},\right.$$
$$-\frac{1}{T_1^9\,T_2^8\,(-1+T_1\,T_2)}\,\left(1-T_1+T_1^2\right)\,\left(1-T_2+T_2^2\right)\,\left(1-T_1\,T_2+T_1^2\,T_2^2\right)$$
$$\left(-3+12\,T_1-21\,T_1^2+27\,T_1^3-21\,T_1^4+12\,T_1^5-3\,T_1^6+12\,T_2-41\,T_1\,T_2+56\,T_1^2\,T_2-59\,T_1^3\,T_2+21\,T_1^4\,T_2+\right.$$
$$T_1^5\,T_2-16\,T_1^6\,T_2+7\,T_1^7\,T_2-21\,T_2^2+56\,T_1\,T_2^2-37\,T_1^2\,T_2^2+5\,T_1^3\,T_2^2+85\,T_1^4\,T_2^2-90\,T_1^5\,T_2^2+77\,T_1^6\,T_2^2-$$
$$20\,T_1^7\,T_2^2-2\,T_1^8\,T_2^2+27\,T_2^3-59\,T_1\,T_2^3+5\,T_1^2\,T_2^3+39\,T_1^3\,T_2^3-152\,T_1^4\,T_2^3+108\,T_1^5\,T_2^3-52\,T_1^6\,T_2^3-$$
$$42\,T_1^7\,T_2^3+44\,T_1^8\,T_2^3-9\,T_1^9\,T_2^3-21\,T_2^4+21\,T_1\,T_2^4+85\,T_1^2\,T_2^4-152\,T_1^3\,T_2^4+257\,T_1^4\,T_2^4-118\,T_1^5\,T_2^4-$$
$$16\,T_1^6\,T_2^4+109\,T_1^7\,T_2^4-52\,T_1^8\,T_2^4-28\,T_1^9\,T_2^4+16\,T_1^{10}\,T_2^4+12\,T_2^5+T_1\,T_2^5-90\,T_1^2\,T_2^5+108\,T_1^3\,T_2^5-$$
$$118\,T_1^4\,T_2^5-55\,T_1^5\,T_2^5+133\,T_1^6\,T_2^5-68\,T_1^7\,T_2^5-35\,T_1^8\,T_2^5+92\,T_1^9\,T_2^5-4\,T_1^{10}\,T_2^5-15\,T_1^{11}\,T_2^5-3\,T_2^6-$$
$$16\,T_1\,T_2^6+77\,T_1^2\,T_2^6-52\,T_1^3\,T_2^6-16\,T_1^4\,T_2^6+133\,T_1^5\,T_2^6-156\,T_1^6\,T_2^6-T_1^7\,T_2^6+4\,T_1^8\,T_2^6-12\,T_1^9\,T_2^6-$$
$$93\,T_1^{10}\,T_2^6+40\,T_1^{11}\,T_2^6+5\,T_1^{12}\,T_2^6+7\,T_1\,T_2^7-20\,T_1^2\,T_2^7-42\,T_1^3\,T_2^7+109\,T_1^4\,T_2^7-68\,T_1^5\,T_2^7-T_1^6\,T_2^7+$$
$$187\,T_1^7\,T_2^7-18\,T_1^8\,T_2^7-34\,T_1^9\,T_2^7+140\,T_1^{10}\,T_2^7-25\,T_1^{11}\,T_2^7-20\,T_1^{12}\,T_2^7-2\,T_1^2\,T_2^8+44\,T_1^3\,T_2^8-52\,T_1^4\,T_2^8-$$
$$35\,T_1^5\,T_2^8+4\,T_1^6\,T_2^8-18\,T_1^7\,T_2^8-269\,T_1^8\,T_2^8+226\,T_1^9\,T_2^8-189\,T_1^{10}\,T_2^8-5\,T_1^{11}\,T_2^8+35\,T_1^{12}\,T_2^8-9\,T_1^3\,T_2^9-$$
$$28\,T_1^4\,T_2^9+92\,T_1^5\,T_2^9-12\,T_1^6\,T_2^9-34\,T_1^7\,T_2^9+226\,T_1^8\,T_2^9-103\,T_1^9\,T_2^9+45\,T_1^{10}\,T_2^9+75\,T_1^{11}\,T_2^9-45\,T_1^{12}\,T_2^9+$$
$$16\,T_1^4\,T_2^{10}-4\,T_1^5\,T_2^{10}-93\,T_1^6\,T_2^{10}+140\,T_1^7\,T_2^{10}-189\,T_1^8\,T_2^{10}+45\,T_1^9\,T_2^{10}+21\,T_1^{10}\,T_2^{10}-80\,T_1^{11}\,T_2^{10}+$$
$$35\,T_1^{12}\,T_2^{10}-15\,T_1^5\,T_2^{11}+40\,T_1^6\,T_2^{11}-25\,T_1^7\,T_2^{11}-5\,T_1^8\,T_2^{11}+75\,T_1^9\,T_2^{11}-80\,T_1^{10}\,T_2^{11}+65\,T_1^{11}\,T_2^{11}-$$
$$\left.\left.20\,T_1^{12}\,T_2^{11}+5\,T_1^6\,T_2^{12}-20\,T_1^7\,T_2^{12}+35\,T_1^8\,T_2^{12}-45\,T_1^9\,T_2^{12}+35\,T_1^{10}\,T_2^{12}-20\,T_1^{11}\,T_2^{12}+5\,T_1^{12}\,T_2^{12}\right)\right\}\right\},$$

$$\left\{\text{Knot}[11,\text{NonAlternating},73]\to\left\{\frac{\left(1-T+T^2\right)^2}{T^2},\ \frac{1}{T_1^5\,T_2^4\,(-1+T_1\,T_2)}\right.\right.$$
$$2\,\left(1-T_1+T_1^2\right)\,\left(1-T_2+T_2^2\right)\,\left(1-T_1\,T_2+T_1^2\,T_2^2\right)$$
$$\left.\left(-1+T_1-T_1^2+T_2+2\,T_1^2\,T_2-T_2^2+2\,T_1\,T_2^2-6\,T_1^2\,T_2^2+T_1^4\,T_2^2+2\,T_1^3\,T_2^3-T_1^4\,T_2^3+T_1^2\,T_2^4-T_1^3\,T_2^4+T_1^4\,T_2^4\right)\right\},$$
$$\text{Knot}[11,\text{NonAlternating},74]\to$$
$$\left\{\frac{\left(1-T+T^2\right)^2}{T^2},\right.$$

$$\frac{1}{T_1^5\,T_2^4\,(-1+T_1\,T_2)\,2\,\left(1-T_1+T_1^2\right)\,\left(1-T_2+T_2^2\right)\,\left(1-T_1\,T_2+T_1^2\,T_2^2\right)\left(-1+T_1-T_1^2+T_2+2\,T_1^2\,T_2-T_2^2+2\,T_1\,T_2^2-6\,T_1^2\,T_2^2+T_1^4\,T_2^2+2\,T_1^3\,T_2^3-T_1^4\,T_2^3+T_1^2\,T_2^4-T_1^3\,T_2^4+T_1^4\,T_2^4\right)\Big\}\Big\}\Big\}$$

In[•]:=
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tab12 = Table[K → θ@K, {K, AllKnots[{3, 12}] }]
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Out[•]=

$\Big\{$Knot[3, 1] → $\Big\{\frac{1-T+T^2}{T},\ \frac{-1+T_1-T_1^2+T_2-T_1^2\,T_2+2\,T_1^3\,T_2-T_2^2-T_1\,T_2^2+T_1^2\,T_2^2-2\,T_1^3\,T_2^2+2\,T_1\,T_2^3-2\,T_1^2\,T_2^3+2\,T_1^3\,T_2^3}{T_1^2\,T_2\,(-1+T_1\,T_2)}\Big\}$,

Knot[4, 1] → $\Big\{-\frac{1-3\,T+T^2}{T},\ -\frac{(1-3\,T_1+T_1^2)\,(1+T_1\,T_2)\,(1-3\,T_2+T_2^2)}{T_1^3\,T_2^2}\Big\}$, Knot[5, 1] → $\Big\{\frac{1-T+T^2-T^3+T^4}{T^2},\ \frac{-1+\langle\!\langle 67 \rangle\!\rangle+4\,T_1^7\,T_2^7}{T_1^4\,T_2^3\,\left(-1+\langle\!\langle 1 \rangle\!\rangle\right)}\Big\}$,

$\langle\!\langle 2971 \rangle\!\rangle$, Knot[12, NonAlternating, 886] → $\Big\{\langle\!\langle 1 \rangle\!\rangle,\ \langle\!\langle 1 \rangle\!\rangle\Big\}$,

Knot[12, NonAlternating, 887] → $\Big\{\frac{1-6\,T+16\,T^2-25\,T^3+29\,T^4-25\,T^5+16\,T^6-6\,T^7+T^8}{T^4},\ \frac{-6+36\,T_1-96\,T_1^2+\langle\!\langle 325 \rangle\!\rangle+32\,T_1^{14}\,T_2^{16}-12\,T_1^{15}\,T_2^{16}+2\,T_1^{16}\,T_2^{16}}{T_1^9\,T_2^8\,(-1+T_1\,T_2)}\Big\}$,

Knot[12, NonAlternating, 888] → $\Big\{\frac{(1-T+T^2)^2\,(1+T-2\,T^2+T^3-2\,T^4+T^5+T^6)}{T^5},\ \frac{1}{T_1^{11}\,T_2^{10}\,(-1+T_1\,T_2)}$

$\left(1-T_1+T_1^2\right)\,\left(1-T_2+T_2^2\right)\,\left(\langle\!\langle 1 \rangle\!\rangle\right)\,\left(-10+20\,T_1^2-40\,T_1^3+50\,T_1^4-40\,T_1^5+20\,T_1^6-10\,T_1^8-T_1\,T_2+2\,T_1^3\,T_2-4\,T_1^4\,T_2+5\,T_1^5\,T_2-4\,T_1^6\,T_2+2\,T_1^7\,T_2-T_1^9\,T_2+20\,T_2^2-24\,T_1^2\,T_2^2+\langle\!\langle 213 \rangle\!\rangle+3\,T_1^6\,T_2^{14}-7\,T_1^8\,T_2^{14}+11\,T_1^9\,T_2^{14}-9\,T_1^{10}\,T_2^{14}+3\,T_1^{11}\,T_2^{14}-T_1^{13}\,T_2^{14}+2\,T_1^{14}\,T_2^{14}+T_1^7\,T_2^{15}-2\,T_1^9\,T_2^{15}+4\,T_1^{10}\,T_2^{15}-5\,T_1^{11}\,T_2^{15}+4\,T_1^{12}\,T_2^{15}-2\,T_1^{13}\,T_2^{15}+T_1^{15}\,T_2^{15}\right)\Big\}\Big\}$

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In[•]:=
```
dup12 = Map[First, Select[Gather[tab12, Last[#1] === Last[#2] &], Length[#] > 1 &], {2}]
```

Out[•]=

```
{{Knot[10, 106], Knot[12, NonAlternating, 369]},
 {Knot[11, Alternating, 44], Knot[11, Alternating, 47]},
 {Knot[11, Alternating, 57], Knot[11, Alternating, 231]},
 {Knot[11, NonAlternating, 73], Knot[11, NonAlternating, 74]},
 {Knot[12, Alternating, 30], Knot[12, Alternating, 33]},
 {Knot[12, Alternating, 122], Knot[12, Alternating, 182]},
 {Knot[12, Alternating, 164], Knot[12, Alternating, 166]},
 {Knot[12, Alternating, 167], Knot[12, Alternating, 692]},
 {Knot[12, Alternating, 273], Knot[12, Alternating, 890]},
 {Knot[12, Alternating, 341], Knot[12, Alternating, 627]},
 {Knot[12, Alternating, 427], Knot[12, Alternating, 435], Knot[12, Alternating, 990]},
 {Knot[12, Alternating, 458], Knot[12, Alternating, 887]},
 {Knot[12, Alternating, 510], Knot[12, Alternating, 821]},
 {Knot[12, NonAlternating, 56], Knot[12, NonAlternating, 57]},
 {Knot[12, NonAlternating, 60], Knot[12, NonAlternating, 61]},
 {Knot[12, NonAlternating, 62], Knot[12, NonAlternating, 66]},
 {Knot[12, NonAlternating, 144], Knot[12, NonAlternating, 507]},
 {Knot[12, NonAlternating, 313], Knot[12, NonAlternating, 430]}}
```

In[•]:=
```
Length /@ dup12
```

Out[•]=

```
{2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}
```

```
In[•]:= Total[(Length /@ dup12) - 1]
```

```
Out[•]=
19
```

```
In[•]:= Length /@ Select[
  Gather[tab12 /. {T1 → 22 / 7, T2 → 13 / 21}, Last[#1] === Last[#2] &], Length[#] > 1 &]
```

```
Out[•]=
{2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 2, 2, 2, 2, 2, 2, 2}
```

```
In[•]:= Put[tab12 /. {T1 → T1, T2 → T2}, "Data12.m"]
```

```
In[•]:= Table[K → θ[K],
 {K, {Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27], Knot[9, 41],
   Knot[9, 46], Knot[10, 3], Knot[10, 22], Knot[10, 35], Knot[10, 42], Knot[10, 48],
   Knot[10, 75], Knot[10, 87], Knot[10, 99], Knot[10, 123], Knot[10, 129],
   Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]}}]
```

```
Out[•]=
{Knot[6, 1] → {-((-2 + T) (-1 + 2 T))/T,
  -1/(T1^3 T2^2 (-1 + T1 T2)) (-7 + 17 T1 - 7 T1^2 + 17 T2 - 44 T1 T2 + 26 T1^2 T2 - 3 T1^3 T2 - 7 T2^2 + 26 T1 T2^2 - 24 T1^2 T2^2 -
    14 T1^3 T2^2 + 9 T1^4 T2^2 - 3 T1 T2^3 - 14 T1^2 T2^3 + 56 T1^3 T2^3 - 23 T1^4 T2^3 + 9 T1^2 T2^4 - 23 T1^3 T2^4 + 9 T1^4 T2^4)},
 Knot[8, 8] → {(2 - 2 T + T^2) (1 - 2 T + 2 T^2)/T^2, 1/(T1^5 T2^4 (-1 + T1 T2))
   (-17 + 51 T1 - 77 T1^2 + 51 T1^3 - 17 T1^4 + 51 T2 - 126 T1 T2 + 150 T1^2 T2 - 30 T1^3 T2 - 30 T1^4 T2 +
    27 T1^5 T2 - 77 T2^2 + 150 T1 T2^2 - 117 T1^2 T2^2 - 113 T1^3 T2^2 + 135 T1^4 T2^2 - 66 T1^5 T2^2 - 5 T1^6 T2^2 + 51 T2^3 -
    30 T1 T2^3 - 113 T1^2 T2^3 + 336 T1^3 T2^3 - 186 T1^4 T2^3 - 5 T1^5 T2^3 + 78 T1^6 T2^3 - 21 T1^7 T2^3 - 17 T2^4 - 30 T1 T2^4 +
    135 T1^2 T2^4 - 186 T1^3 T2^4 - 60 T1^4 T2^4 + 222 T1^5 T2^4 - 153 T1^6 T2^4 + 18 T1^7 T2^4 + 15 T1^8 T2^4 + 27 T1 T2^5 -
    66 T1^2 T2^5 - 5 T1^3 T2^5 + 222 T1^4 T2^5 - 300 T1^5 T2^5 + 103 T1^6 T2^5 + 42 T1^7 T2^5 - 45 T1^8 T2^5 - 5 T1^2 T2^6 + 78 T1^3 T2^6 -
    153 T1^4 T2^6 + 103 T1^5 T2^6 + 99 T1^6 T2^6 - 138 T1^7 T2^6 + 67 T1^8 T2^6 - 21 T1^3 T2^7 + 18 T1^4 T2^7 + 42 T1^5 T2^7 -
    138 T1^6 T2^7 + 114 T1^7 T2^7 - 45 T1^8 T2^7 + 15 T1^4 T2^8 - 45 T1^5 T2^8 + 67 T1^6 T2^8 - 45 T1^7 T2^8 + 15 T1^8 T2^8)},
 Knot[8, 9] → {-((-1 + T - 2 T^2 + T^3) (-1 + 2 T - T^2 + T^3))/T^3,
  -1/(T1^7 T2^6) (-1 + T1 - 2 T1^2 + T1^3) (-1 + 2 T1 - T1^2 + T1^3) (1 + T1 T2) (-1 + T2 - 2 T2^2 + T2^3)
    (-1 + 2 T2 - T2^2 + T2^3) (3 - 6 T1 T2 + 8 T1^2 T2^2 - 6 T1^3 T2^3 + 3 T1^4 T2^4)},
 Knot[8, 20] → {(1 - T + T^2)^2/T^2, 1/(T1^5 T2^4 (-1 + T1 T2)) 2 (-1 + 2 T1 - 3 T1^2 + 2 T1^3 - T1^4 + 2 T2 - 3 T1 T2 + 4 T1^2 T2 -
    T1^3 T2 + T1^5 T2 - 3 T2^2 + 4 T1 T2^2 - 2 T1^2 T2^2 - 4 T1^3 T2^2 + 4 T1^4 T2^2 - 2 T1^5 T2^2 + 2 T2^3 - T1 T2^3 - 4 T1^2 T2^3 + 5 T1^3 T2^3 -
    T1^4 T2^3 - 4 T1^5 T2^3 + 2 T1^6 T2^3 - T1^7 T2^3 - T2^4 + 4 T1^2 T2^4 - T1^3 T2^4 + 3 T1^5 T2^4 + 2 T1^6 T2^4 + T1^8 T2^4 + T1 T2^5 - 2 T1^2 T2^5 -
    4 T1^3 T2^5 + 3 T1^4 T2^5 - 3 T1^5 T2^5 - 4 T1^6 T2^5 + T1^7 T2^5 - 2 T1^8 T2^5 + 2 T1^3 T2^6 + 2 T1^4 T2^6 - 4 T1^5 T2^6 + 8 T1^6 T2^6 - 4 T1^7 T2^6 +
    3 T1^8 T2^6 - T1^3 T2^7 + T1^5 T2^7 - 4 T1^6 T2^7 + 3 T1^7 T2^7 - 2 T1^8 T2^7 + T1^4 T2^8 - 2 T1^5 T2^8 + 3 T1^6 T2^8 - 2 T1^7 T2^8 + T1^8 T2^8)},
```

$$\text{Knot}[9,\,27] \to \left\{ -\frac{\left(-1+2\,T-3\,T^2+T^3\right)\,\left(-1+3\,T-2\,T^2+T^3\right)}{T^3},\; -\frac{1}{T_1^7\,T_2^6\,\left(-1+T_1\,T_2\right)} \right.$$
$$\left(-3+15\,T_1-33\,T_1^2+45\,T_1^3-33\,T_1^4+15\,T_1^5-3\,T_1^6+15\,T_2-65\,T_1\,T_2+115\,T_1^2\,T_2-115\,T_1^3\,T_2+15\,T_1^4\,T_2+\right.$$
$$35\,T_1^5\,T_2-35\,T_1^6\,T_2+10\,T_1^7\,T_2-33\,T_2^2+115\,T_1\,T_2^2-125\,T_1^2\,T_2^2+T_1^3\,T_2^2+265\,T_1^4\,T_2^2-219\,T_1^5\,T_2^2+$$
$$95\,T_1^6\,T_2^2+5\,T_1^7\,T_2^2-11\,T_1^8\,T_2^2+45\,T_2^3-115\,T_1\,T_2^3+T_1^2\,T_2^3+268\,T_1^3\,T_2^3-554\,T_1^4\,T_2^3+156\,T_1^5\,T_2^3+$$
$$108\,T_1^6\,T_2^3-164\,T_1^7\,T_2^3+55\,T_1^8\,T_2^3-33\,T_2^4+15\,T_1\,T_2^4+265\,T_1^2\,T_2^4-554\,T_1^3\,T_2^4+518\,T_1^4\,T_2^4+342\,T_1^5\,T_2^4-$$
$$542\,T_1^6\,T_2^4+286\,T_1^7\,T_2^4-T_1^8\,T_2^4-55\,T_1^9\,T_2^4+11\,T_1^{10}\,T_2^4+15\,T_2^5+35\,T_1\,T_2^5-219\,T_1^2\,T_2^5+156\,T_1^3\,T_2^5+$$
$$342\,T_1^4\,T_2^5-1095\,T_1^5\,T_2^5+705\,T_1^6\,T_2^5+67\,T_1^7\,T_2^5-294\,T_1^8\,T_2^5+166\,T_1^9\,T_2^5-5\,T_1^{10}\,T_2^5-10\,T_1^{11}\,T_2^5-3\,T_2^6-$$
$$35\,T_1\,T_2^6+95\,T_1^2\,T_2^6+108\,T_1^3\,T_2^6-542\,T_1^4\,T_2^6+705\,T_1^5\,T_2^6+12\,T_1^6\,T_2^6-745\,T_1^7\,T_2^6+444\,T_1^8\,T_2^6-92\,T_1^9\,T_2^6-$$
$$97\,T_1^{10}\,T_2^6+35\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+10\,T_1\,T_2^7+5\,T_1^2\,T_2^7-164\,T_1^3\,T_2^7+286\,T_1^4\,T_2^7+67\,T_1^5\,T_2^7-745\,T_1^6\,T_2^7+$$
$$1055\,T_1^7\,T_2^7-208\,T_1^8\,T_2^7-164\,T_1^9\,T_2^7+221\,T_1^{10}\,T_2^7-35\,T_1^{11}\,T_2^7-15\,T_1^{12}\,T_2^7-11\,T_1^2\,T_2^8+55\,T_1^3\,T_2^8-T_1^4\,T_2^8-$$
$$294\,T_1^5\,T_2^8+444\,T_1^6\,T_2^8-208\,T_1^7\,T_2^8-616\,T_1^8\,T_2^8+546\,T_1^9\,T_2^8-267\,T_1^{10}\,T_2^8-15\,T_1^{11}\,T_2^8+33\,T_1^{12}\,T_2^8-$$
$$55\,T_1^4\,T_2^9+166\,T_1^5\,T_2^9-92\,T_1^6\,T_2^9-164\,T_1^7\,T_2^9+546\,T_1^8\,T_2^9-252\,T_1^9\,T_2^9+T_1^{10}\,T_2^9+115\,T_1^{11}\,T_2^9-45\,T_1^{12}\,T_2^9+$$
$$11\,T_1^4\,T_2^{10}-5\,T_1^5\,T_2^{10}-97\,T_1^6\,T_2^{10}+221\,T_1^7\,T_2^{10}-267\,T_1^8\,T_2^{10}+T_1^9\,T_2^{10}+123\,T_1^{10}\,T_2^{10}-115\,T_1^{11}\,T_2^{10}+$$
$$33\,T_1^{12}\,T_2^{10}-10\,T_1^5\,T_2^{11}+35\,T_1^6\,T_2^{11}-35\,T_1^7\,T_2^{11}-15\,T_1^8\,T_2^{11}+115\,T_1^9\,T_2^{11}-115\,T_1^{10}\,T_2^{11}+65\,T_1^{11}\,T_2^{11}-$$
$$\left.\left.15\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-15\,T_1^7\,T_2^{12}+33\,T_1^8\,T_2^{12}-45\,T_1^9\,T_2^{12}+33\,T_1^{10}\,T_2^{12}-15\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\right)\right\},$$

$$\text{Knot}[9,\,41] \to \left\{ \frac{\left(3-3\,T+T^2\right)\,\left(1-3\,T+3\,T^2\right)}{T^2},\; \frac{1}{T_1^5\,T_2^4\,\left(-1+T_1\,T_2\right)} \right.$$
$$\left(-51+201\,T_1-315\,T_1^2+201\,T_1^3-51\,T_1^4+201\,T_2-698\,T_1\,T_2+880\,T_1^2\,T_2-236\,T_1^3\,T_2-\right.$$
$$158\,T_1^4\,T_2+93\,T_1^5\,T_2-315\,T_2^2+880\,T_1\,T_2^2-519\,T_1^2\,T_2^2-1035\,T_1^3\,T_2^2+1305\,T_1^4\,T_2^2-488\,T_1^5\,T_2^2+$$
$$27\,T_1^6\,T_2^2+201\,T_2^3-236\,T_1\,T_2^3-1035\,T_1^2\,T_2^3+2964\,T_1^3\,T_2^3-2484\,T_1^4\,T_2^3+333\,T_1^5\,T_2^3+376\,T_1^6\,T_2^3-$$
$$123\,T_1^7\,T_2^3-51\,T_2^4-158\,T_1\,T_2^4+1305\,T_1^2\,T_2^4-2484\,T_1^3\,T_2^4+1188\,T_1^4\,T_2^4+1692\,T_1^5\,T_2^4-$$
$$1467\,T_1^6\,T_2^4+274\,T_1^7\,T_2^4+57\,T_1^8\,T_2^4+93\,T_1\,T_2^5-488\,T_1^2\,T_2^5+333\,T_1^3\,T_2^5+1692\,T_1^4\,T_2^5-$$
$$3756\,T_1^5\,T_2^5+1701\,T_1^6\,T_2^5+124\,T_1^7\,T_2^5-231\,T_1^8\,T_2^5+27\,T_1^2\,T_2^6+376\,T_1^3\,T_2^6-1467\,T_1^4\,T_2^6+$$
$$1701\,T_1^5\,T_2^6+357\,T_1^6\,T_2^6-992\,T_1^7\,T_2^6+369\,T_1^8\,T_2^6-123\,T_1^3\,T_2^7+274\,T_1^4\,T_2^7+124\,T_1^5\,T_2^7-$$
$$\left.\left.992\,T_1^6\,T_2^7+814\,T_1^7\,T_2^7-231\,T_1^8\,T_2^7+57\,T_1^4\,T_2^8-231\,T_1^5\,T_2^8+369\,T_1^6\,T_2^8-231\,T_1^7\,T_2^8+57\,T_1^8\,T_2^8\right)\right\},$$

$$\text{Knot}[9,\,46] \to \left\{ -\frac{(-2+T)\,(-1+2\,T)}{T},\; -\frac{1}{T_1^3\,T_2^2\,\left(-1+T_1\,T_2\right)} \right.$$
$$\left.\left(-5+11\,T_1-5\,T_1^2+11\,T_2-32\,T_1\,T_2+38\,T_1^2\,T_2-9\,T_1^3\,T_2-5\,T_2^2+38\,T_1\,T_2^2-72\,T_1^2\,T_2^2-2\,T_1^3\,T_2^2+\right.\right.$$
$$\left.\left.11\,T_1^4\,T_2^2-9\,T_1\,T_2^3-2\,T_1^2\,T_2^3+68\,T_1^3\,T_2^3-29\,T_1^4\,T_2^3+11\,T_1^2\,T_2^4-29\,T_1^3\,T_2^4+11\,T_1^4\,T_2^4\right)\right\},$$

$$\text{Knot}[10,\,3] \to \left\{ -\frac{(-3+2\,T)\,(-2+3\,T)}{T},\; -\frac{1}{T_1^3\,T_2^2\,\left(-1+T_1\,T_2\right)}\,\left(-171+367\,T_1-171\,T_1^2+\right.\right.$$
$$\left.\left.367\,T_2-888\,T_1\,T_2+594\,T_1^2\,T_2-101\,T_1^3\,T_2-171\,T_2^2+594\,T_1\,T_2^2-420\,T_1^2\,T_2^2-342\,T_1^3\,T_2^2+\right.\right.$$
$$\left.\left.261\,T_1^4\,T_2^2-101\,T_1\,T_2^3-342\,T_1^2\,T_2^3+1140\,T_1^3\,T_2^3-569\,T_1^4\,T_2^3+261\,T_1^2\,T_2^4-569\,T_1^3\,T_2^4+261\,T_1^4\,T_2^4\right)\right\},$$

$$\text{Knot}[10,\,22] \to \left\{ -\frac{\left(-2+2\,T-2\,T^2+T^3\right)\,\left(-1+2\,T-2\,T^2+2\,T^3\right)}{T^3},\right.$$
$$\left. -\frac{1}{T_1^7\,T_2^6\,\left(-1+T_1\,T_2\right)}\,\left(-25+75\,T_1-125\,T_1^2+163\,T_1^3-125\,T_1^4+75\,T_1^5-25\,T_1^6+75\,T_2-174\,T_1\,T_2+\right.\right.$$
$$222\,T_1^2\,T_2-234\,T_1^3\,T_2+42\,T_1^4\,T_2+30\,T_1^5\,T_2-78\,T_1^6\,T_2+51\,T_1^7\,T_2-125\,T_2^2+222\,T_1\,T_2^2-211\,T_1^2\,T_2^2+$$
$$189\,T_1^3\,T_2^2+141\,T_1^4\,T_2^2-91\,T_1^5\,T_2^2+109\,T_1^6\,T_2^2-18\,T_1^7\,T_2^2-45\,T_1^8\,T_2^2+163\,T_2^3-234\,T_1\,T_2^3+189\,T_1^2\,T_2^3-$$
$$204\,T_1^3\,T_2^3-184\,T_1^4\,T_2^3-52\,T_1^5\,T_2^3+6\,T_1^6\,T_2^3-111\,T_1^7\,T_2^3+114\,T_1^8\,T_2^3+7\,T_1^9\,T_2^3-125\,T_2^4+42\,T_1\,T_2^4+$$
$$\left.\left.141\,T_1^2\,T_2^4-184\,T_1^3\,T_2^4+557\,T_1^4\,T_2^4-83\,T_1^5\,T_2^4+105\,T_1^6\,T_2^4+24\,T_1^7\,T_2^4+5\,T_1^8\,T_2^4-126\,T_1^9\,T_2^4+\right.\right.$$

$$35\,T_1^{10}\,T_2^4+75\,T_2^5+30\,T_1\,T_2^5-91\,T_1^2\,T_2^5-52\,T_1^3\,T_2^5-83\,T_1^4\,T_2^5-416\,T_1^5\,T_2^5+174\,T_1^6\,T_2^5-63\,T_1^7\,T_2^5-32\,T_1^8\,T_2^5+129\,T_1^9\,T_2^5+30\,T_1^{10}\,T_2^5-45\,T_1^{11}\,T_2^5-25\,T_2^6-78\,T_1\,T_2^6+109\,T_1^2\,T_2^6+6\,T_1^3\,T_2^6+105\,T_1^4\,T_2^6+174\,T_1^5\,T_2^6+108\,T_1^6\,T_2^6-218\,T_1^7\,T_2^6+73\,T_1^8\,T_2^6-66\,T_1^9\,T_2^6-115\,T_1^{10}\,T_2^6+66\,T_1^{11}\,T_2^6+23\,T_1^{12}\,T_2^6+51\,T_1\,T_2^7-18\,T_1^2\,T_2^7-111\,T_1^3\,T_2^7+24\,T_1^4\,T_2^7-63\,T_1^5\,T_2^7-218\,T_1^6\,T_2^7+372\,T_1^7\,T_2^7-43\,T_1^8\,T_2^7+44\,T_1^9\,T_2^7+109\,T_1^{10}\,T_2^7-18\,T_1^{11}\,T_2^7-69\,T_1^{12}\,T_2^7-45\,T_1^2\,T_2^8+114\,T_1^3\,T_2^8+5\,T_1^4\,T_2^8-32\,T_1^5\,T_2^8+73\,T_1^6\,T_2^8-43\,T_1^7\,T_2^8-379\,T_1^8\,T_2^8+176\,T_1^9\,T_2^8-131\,T_1^{10}\,T_2^8-54\,T_1^{11}\,T_2^8+115\,T_1^{12}\,T_2^8+7\,T_1^3\,T_2^9-126\,T_1^4\,T_2^9+129\,T_1^5\,T_2^9-66\,T_1^6\,T_2^9+44\,T_1^7\,T_2^9+176\,T_1^8\,T_2^9+144\,T_1^9\,T_2^9-171\,T_1^{10}\,T_2^9+222\,T_1^{11}\,T_2^9-149\,T_1^{12}\,T_2^9+35\,T_1^4\,T_2^{10}+30\,T_1^5\,T_2^{10}-115\,T_1^6\,T_2^{10}+109\,T_1^7\,T_2^{10}-131\,T_1^8\,T_2^{10}-171\,T_1^9\,T_2^{10}+205\,T_1^{10}\,T_2^{10}-210\,T_1^{11}\,T_2^{10}+115\,T_1^{12}\,T_2^{10}-45\,T_1^5\,T_2^{11}+66\,T_1^6\,T_2^{11}-18\,T_1^7\,T_2^{11}-54\,T_1^8\,T_2^{11}+222\,T_1^9\,T_2^{11}-210\,T_1^{10}\,T_2^{11}+162\,T_1^{11}\,T_2^{11}-69\,T_1^{12}\,T_2^{11}+23\,T_1^6\,T_2^{12}-69\,T_1^7\,T_2^{12}+115\,T_1^8\,T_2^{12}-149\,T_1^9\,T_2^{12}+115\,T_1^{10}\,T_2^{12}-69\,T_1^{11}\,T_2^{12}+23\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,35]\to\left\{\frac{\left(2-4\,T+T^2\right)\,\left(1-4\,T+2\,T^2\right)}{T^2},\ \frac{1}{T_1^5\,T_2^4\,\left(-1+T_1\,T_2\right)}\right.$$

$$\Big(-17+103\,T_1-181\,T_1^2+103\,T_1^3-17\,T_1^4+103\,T_2-570\,T_1\,T_2+762\,T_1^2\,T_2-30\,T_1^3\,T_2-234\,T_1^4\,T_2+55\,T_1^5\,T_2-181\,T_2^2+762\,T_1\,T_2^2+112\,T_1^2\,T_2^2-2442\,T_1^3\,T_2^2+1708\,T_1^4\,T_2^2-246\,T_1^5\,T_2^2-13\,T_1^6\,T_2^2+103\,T_2^3-30\,T_1\,T_2^3-2442\,T_1^2\,T_2^3+5136\,T_1^3\,T_2^3-2172\,T_1^4\,T_2^3-426\,T_1^5\,T_2^3+330\,T_1^6\,T_2^3-41\,T_1^7\,T_2^3-17\,T_2^4-234\,T_1\,T_2^4+1708\,T_1^2\,T_2^4-2172\,T_1^3\,T_2^4-1392\,T_1^4\,T_2^4+3108\,T_1^5\,T_2^4-1412\,T_1^6\,T_2^4+150\,T_1^7\,T_2^4+15\,T_1^8\,T_2^4+55\,T_1\,T_2^5-246\,T_1^2\,T_2^5-426\,T_1^3\,T_2^5+3108\,T_1^4\,T_2^5-4200\,T_1^5\,T_2^5+1590\,T_1^6\,T_2^5+114\,T_1^7\,T_2^5-89\,T_1^8\,T_2^5-13\,T_1^2\,T_2^6+330\,T_1^3\,T_2^6-1412\,T_1^4\,T_2^6+1590\,T_1^5\,T_2^6+184\,T_1^6\,T_2^6-678\,T_1^7\,T_2^6+155\,T_1^8\,T_2^6-41\,T_1^3\,T_2^7+150\,T_1^4\,T_2^7+114\,T_1^5\,T_2^7-678\,T_1^6\,T_2^7+486\,T_1^7\,T_2^7-89\,T_1^8\,T_2^7+15\,T_1^4\,T_2^8-89\,T_1^5\,T_2^8+155\,T_1^6\,T_2^8-89\,T_1^7\,T_2^8+15\,T_1^8\,T_2^8\Big)\Big\},$$

$$\text{Knot}[10,42]\to\left\{-\frac{\left(-1+3\,T-4\,T^2+T^3\right)\,\left(-1+4\,T-3\,T^2+T^3\right)}{T^3},\right.$$

$$-\frac{1}{T_1^7\,T_2^6\,\left(-1+T_1\,T_2\right)}\,\Big(-3+21\,T_1-57\,T_1^2+81\,T_1^3-57\,T_1^4+21\,T_1^5-3\,T_1^6+21\,T_2-133\,T_1\,T_2+301\,T_1^2\,T_2-301\,T_1^3\,T_2+21\,T_1^4\,T_2+119\,T_1^5\,T_2-77\,T_1^6\,T_2+14\,T_1^7\,T_2-57\,T_2^2+301\,T_1\,T_2^2-422\,T_1^2\,T_2^2-166\,T_1^3\,T_2^2+1164\,T_1^4\,T_2^2-926\,T_1^5\,T_2^2+262\,T_1^6\,T_2^2+35\,T_1^7\,T_2^2-19\,T_1^8\,T_2^2+81\,T_2^3-301\,T_1\,T_2^3-166\,T_1^2\,T_2^3+1864\,T_1^3\,T_2^3-3062\,T_1^4\,T_2^3+950\,T_1^5\,T_2^3+674\,T_1^6\,T_2^3-641\,T_1^7\,T_2^3+133\,T_1^8\,T_2^3-57\,T_2^4+21\,T_1\,T_2^4+1164\,T_1^2\,T_2^4-3062\,T_1^3\,T_2^4+2392\,T_1^4\,T_2^4+2552\,T_1^5\,T_2^4-3800\,T_1^6\,T_2^4+1690\,T_1^7\,T_2^4-38\,T_1^8\,T_2^4-133\,T_1^9\,T_2^4+19\,T_1^{10}\,T_2^4+21\,T_2^5+119\,T_1\,T_2^5-926\,T_1^2\,T_2^5+950\,T_1^3\,T_2^5+2552\,T_1^4\,T_2^5-7805\,T_1^5\,T_2^5+5699\,T_1^6\,T_2^5-279\,T_1^7\,T_2^5-1582\,T_1^8\,T_2^5+689\,T_1^9\,T_2^5-35\,T_1^{10}\,T_2^5-14\,T_1^{11}\,T_2^5-3\,T_2^6-77\,T_1\,T_2^6+262\,T_1^2\,T_2^6+674\,T_1^3\,T_2^6-3800\,T_1^4\,T_2^6+5699\,T_1^5\,T_2^6-552\,T_1^6\,T_2^6-5255\,T_1^7\,T_2^6+4018\,T_1^8\,T_2^6-818\,T_1^9\,T_2^6-274\,T_1^{10}\,T_2^6+77\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+14\,T_1\,T_2^7+35\,T_1^2\,T_2^7-641\,T_1^3\,T_2^7+1690\,T_1^4\,T_2^7-279\,T_1^5\,T_2^7-5255\,T_1^6\,T_2^7+8249\,T_1^7\,T_2^7-3110\,T_1^8\,T_2^7-842\,T_1^9\,T_2^7+974\,T_1^{10}\,T_2^7-119\,T_1^{11}\,T_2^7-21\,T_1^{12}\,T_2^7-19\,T_1^2\,T_2^8+133\,T_1^3\,T_2^8-38\,T_1^4\,T_2^8-1582\,T_1^5\,T_2^8+4018\,T_1^6\,T_2^8-3110\,T_1^7\,T_2^8-2174\,T_1^8\,T_2^8+3170\,T_1^9\,T_2^8-1240\,T_1^{10}\,T_2^8-21\,T_1^{11}\,T_2^8+57\,T_1^{12}\,T_2^8-133\,T_1^4\,T_2^9+689\,T_1^5\,T_2^9-818\,T_1^6\,T_2^9-842\,T_1^7\,T_2^9+3170\,T_1^8\,T_2^9-2008\,T_1^9\,T_2^9+214\,T_1^{10}\,T_2^9+301\,T_1^{11}\,T_2^9-81\,T_1^{12}\,T_2^9+19\,T_1^4\,T_2^{10}-35\,T_1^5\,T_2^{10}-274\,T_1^6\,T_2^{10}+974\,T_1^7\,T_2^{10}-1240\,T_1^8\,T_2^{10}+214\,T_1^9\,T_2^{10}+410\,T_1^{10}\,T_2^{10}-301\,T_1^{11}\,T_2^{10}+57\,T_1^{12}\,T_2^{10}-14\,T_1^5\,T_2^{11}+77\,T_1^6\,T_2^{11}-119\,T_1^7\,T_2^{11}-21\,T_1^8\,T_2^{11}+301\,T_1^9\,T_2^{11}-301\,T_1^{10}\,T_2^{11}+133\,T_1^{11}\,T_2^{11}-21\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-21\,T_1^7\,T_2^{12}+57\,T_1^8\,T_2^{12}-81\,T_1^9\,T_2^{12}+57\,T_1^{10}\,T_2^{12}-21\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,48]\to\left\{\frac{\left(1-T+2\,T^2-2\,T^3+T^4\right)\,\left(1-2\,T+2\,T^2-T^3+T^4\right)}{T^4},\right.$$

$$\frac{1}{T_1^9\,T_2^8\,\left(-1+T_1\,T_2\right)}$$

$$\Big(-4+12\,T_1-24\,T_1^2+36\,T_1^3-44\,T_1^4+36\,T_1^5-24\,T_1^6+12\,T_1^7-4\,T_1^8+12\,T_2-27\,T_1\,T_2+45\,T_1^2\,T_2-54\,T_1^3\,T_2+51\,T_1^4\,T_2-9\,T_1^5\,T_2-9\,T_1^6\,T_2+18\,T_1^7\,T_2-15\,T_1^8\,T_2+9\,T_1^9\,T_2-24\,T_2^2+45\,T_1\,T_2^2-74\,T_1^2\,T_2^2+$$

$$88\,T_1^3\,T_2^2-91\,T_1^4\,T_2^2+26\,T_1^5\,T_2^2-31\,T_1^6\,T_2^2+16\,T_1^7\,T_2^2-14\,T_1^8\,T_2^2+9\,T_1^9\,T_2^2-12\,T_1^{10}\,T_2^2+36\,T_2^3-54\,T_1\,T_2^3+88\,T_1^2\,T_2^3-97\,T_1^3\,T_2^3+100\,T_1^4\,T_2^3-3\,T_1^5\,T_2^3+42\,T_1^6\,T_2^3-8\,T_1^7\,T_2^3+11\,T_1^8\,T_2^3-2\,T_1^9\,T_2^3+9\,T_1^{10}\,T_2^3+9\,T_1^{11}\,T_2^3-44\,T_2^4+51\,T_1\,T_2^4-91\,T_1^2\,T_2^4+100\,T_1^3\,T_2^4-115\,T_1^4\,T_2^4-83\,T_1^6\,T_2^4+6\,T_1^7\,T_2^4+T_1^8\,T_2^4-26\,T_1^9\,T_2^4+11\,T_1^{10}\,T_2^4-27\,T_1^{11}\,T_2^4+36\,T_2^5-9\,T_1\,T_2^5+26\,T_1^2\,T_2^5-3\,T_1^3\,T_2^5+110\,T_1^5\,T_2^5+44\,T_1^6\,T_2^5+44\,T_1^7\,T_2^5-16\,T_1^8\,T_2^5+9\,T_1^9\,T_2^5+24\,T_1^{10}\,T_2^5-T_1^{11}\,T_2^5+27\,T_1^{12}\,T_2^5-9\,T_1^{13}\,T_2^5-24\,T_2^6-9\,T_1\,T_2^6-31\,T_1^2\,T_2^6+42\,T_1^3\,T_2^6-83\,T_1^4\,T_2^6+44\,T_1^5\,T_2^6-212\,T_1^6\,T_2^6+38\,T_1^7\,T_2^6-20\,T_1^8\,T_2^6+8\,T_1^9\,T_2^6+T_1^{10}\,T_2^6-30\,T_1^{11}\,T_2^6-7\,T_1^{12}\,T_2^6-9\,T_1^{13}\,T_2^6+12\,T_1^{14}\,T_2^6+12\,T_2^7+18\,T_1\,T_2^7+16\,T_1^2\,T_2^7-8\,T_1^3\,T_2^7+6\,T_1^4\,T_2^7+44\,T_1^5\,T_2^7+38\,T_1^6\,T_2^7+158\,T_1^7\,T_2^7-118\,T_1^8\,T_2^7+29\,T_1^9\,T_2^7-28\,T_1^{10}\,T_2^7+9\,T_1^{11}\,T_2^7+28\,T_1^{12}\,T_2^7-2\,T_1^{13}\,T_2^7-9\,T_1^{14}\,T_2^7-9\,T_1^{15}\,T_2^7-4\,T_2^8-15\,T_1\,T_2^8-14\,T_1^2\,T_2^8+11\,T_1^3\,T_2^8+T_1^4\,T_2^8-16\,T_1^5\,T_2^8-20\,T_1^6\,T_2^8-118\,T_1^7\,T_2^8+48\,T_1^8\,T_2^8+74\,T_1^9\,T_2^8+10\,T_1^{10}\,T_2^8+20\,T_1^{11}\,T_2^8-19\,T_1^{12}\,T_2^8-7\,T_1^{13}\,T_2^8+16\,T_1^{14}\,T_2^8+15\,T_1^{15}\,T_2^8+4\,T_1^{16}\,T_2^8+9\,T_1\,T_2^9+9\,T_1^2\,T_2^9-2\,T_1^3\,T_2^9-26\,T_1^4\,T_2^9+9\,T_1^5\,T_2^9+8\,T_1^6\,T_2^9+29\,T_1^7\,T_2^9+74\,T_1^8\,T_2^9-202\,T_1^9\,T_2^9+20\,T_1^{10}\,T_2^9-64\,T_1^{11}\,T_2^9+12\,T_1^{12}\,T_2^9+10\,T_1^{13}\,T_2^9-20\,T_1^{14}\,T_2^9-18\,T_1^{15}\,T_2^9-12\,T_1^{16}\,T_2^9-12\,T_1^2\,T_2^{10}+9\,T_1^3\,T_2^{10}+11\,T_1^4\,T_2^{10}+24\,T_1^5\,T_2^{10}+T_1^6\,T_2^{10}-28\,T_1^7\,T_2^{10}+10\,T_1^8\,T_2^{10}+20\,T_1^9\,T_2^{10}+202\,T_1^{10}\,T_2^{10}-64\,T_1^{11}\,T_2^{10}+85\,T_1^{12}\,T_2^{10}-48\,T_1^{13}\,T_2^{10}+35\,T_1^{14}\,T_2^{10}+9\,T_1^{15}\,T_2^{10}+24\,T_1^{16}\,T_2^{10}+9\,T_1^3\,T_2^{11}-27\,T_1^4\,T_2^{11}-T_1^5\,T_2^{11}-30\,T_1^6\,T_2^{11}+9\,T_1^7\,T_2^{11}+20\,T_1^8\,T_2^{11}-64\,T_1^9\,T_2^{11}-64\,T_1^{10}\,T_2^{11}-106\,T_1^{11}\,T_2^{11}+18\,T_1^{12}\,T_2^{11}-3\,T_1^{13}\,T_2^{11}-28\,T_1^{14}\,T_2^{11}+9\,T_1^{15}\,T_2^{11}-36\,T_1^{16}\,T_2^{11}+27\,T_1^5\,T_2^{12}-7\,T_1^6\,T_2^{12}+28\,T_1^7\,T_2^{12}-19\,T_1^8\,T_2^{12}+12\,T_1^9\,T_2^{12}+85\,T_1^{10}\,T_2^{12}+18\,T_1^{11}\,T_2^{12}+97\,T_1^{12}\,T_2^{12}-98\,T_1^{13}\,T_2^{12}+95\,T_1^{14}\,T_2^{12}-51\,T_1^{15}\,T_2^{12}+44\,T_1^{16}\,T_2^{12}-9\,T_1^5\,T_2^{13}-9\,T_1^6\,T_2^{13}-2\,T_1^7\,T_2^{13}-7\,T_1^8\,T_2^{13}+10\,T_1^9\,T_2^{13}-48\,T_1^{10}\,T_2^{13}-3\,T_1^{11}\,T_2^{13}-98\,T_1^{12}\,T_2^{13}+101\,T_1^{13}\,T_2^{13}-92\,T_1^{14}\,T_2^{13}+54\,T_1^{15}\,T_2^{13}-36\,T_1^{16}\,T_2^{13}+12\,T_1^6\,T_2^{14}-9\,T_1^7\,T_2^{14}+16\,T_1^8\,T_2^{14}-20\,T_1^9\,T_2^{14}+35\,T_1^{10}\,T_2^{14}-28\,T_1^{11}\,T_2^{14}+95\,T_1^{12}\,T_2^{14}-92\,T_1^{13}\,T_2^{14}+76\,T_1^{14}\,T_2^{14}-45\,T_1^{15}\,T_2^{14}+24\,T_1^{16}\,T_2^{14}-9\,T_1^7\,T_2^{15}+15\,T_1^8\,T_2^{15}-18\,T_1^9\,T_2^{15}+9\,T_1^{10}\,T_2^{15}+9\,T_1^{11}\,T_2^{15}-51\,T_1^{12}\,T_2^{15}+54\,T_1^{13}\,T_2^{15}-45\,T_1^{14}\,T_2^{15}+27\,T_1^{15}\,T_2^{15}-12\,T_1^{16}\,T_2^{15}+4\,T_1^8\,T_2^{16}-12\,T_1^9\,T_2^{16}+24\,T_1^{10}\,T_2^{16}-36\,T_1^{11}\,T_2^{16}+44\,T_1^{12}\,T_2^{16}-36\,T_1^{13}\,T_2^{16}+24\,T_1^{14}\,T_2^{16}-12\,T_1^{15}\,T_2^{16}+4\,T_1^{16}\,T_2^{16}\Big)\Big\},$$

$$\text{Knot}[10,75]\to\Big\{-\frac{\left(-1+3\,T-4\,T^2+T^3\right)\,\left(-1+4\,T-3\,T^2+T^3\right)}{T^3},$$

$$-\frac{1}{T_1^7\,T_2^6\,\left(-1+T_1\,T_2\right)}\,\Big(-3+21\,T_1-57\,T_1^2+81\,T_1^3-57\,T_1^4+21\,T_1^5-3\,T_1^6+21\,T_2-133\,T_1\,T_2+301\,T_1^2\,T_2-301\,T_1^3\,T_2+21\,T_1^4\,T_2+119\,T_1^5\,T_2-77\,T_1^6\,T_2+14\,T_1^7\,T_2-57\,T_2^2+301\,T_1\,T_2^2-414\,T_1^2\,T_2^2-198\,T_1^3\,T_2^2+1218\,T_1^4\,T_2^2-958\,T_1^5\,T_2^2+270\,T_1^6\,T_2^2+35\,T_1^7\,T_2^2-19\,T_1^8\,T_2^2+81\,T_2^3-301\,T_1\,T_2^3-198\,T_1^2\,T_2^3+1954\,T_1^3\,T_2^3-3148\,T_1^4\,T_2^3+864\,T_1^5\,T_2^3+764\,T_1^6\,T_2^3-673\,T_1^7\,T_2^3+133\,T_1^8\,T_2^3-57\,T_2^4+21\,T_1\,T_2^4+1218\,T_1^2\,T_2^4-3148\,T_1^3\,T_2^4+2358\,T_1^4\,T_2^4+2906\,T_1^5\,T_2^4-3834\,T_1^6\,T_2^4+1604\,T_1^7\,T_2^4+16\,T_1^8\,T_2^4-133\,T_1^9\,T_2^4+19\,T_1^{10}\,T_2^4+21\,T_2^5+119\,T_1\,T_2^5-958\,T_1^2\,T_2^5+864\,T_1^3\,T_2^5+2906\,T_1^4\,T_2^5-8283\,T_1^5\,T_2^5+5221\,T_1^6\,T_2^5+75\,T_1^7\,T_2^5-1668\,T_1^8\,T_2^5+657\,T_1^9\,T_2^5-35\,T_1^{10}\,T_2^5-14\,T_1^{11}\,T_2^5-3\,T_2^6-77\,T_1\,T_2^6+270\,T_1^2\,T_2^6+764\,T_1^3\,T_2^6-3834\,T_1^4\,T_2^6+5221\,T_1^5\,T_2^6+900\,T_1^6\,T_2^6-5733\,T_1^7\,T_2^6+3984\,T_1^8\,T_2^6-728\,T_1^9\,T_2^6-266\,T_1^{10}\,T_2^6+77\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+14\,T_1\,T_2^7+35\,T_1^2\,T_2^7-673\,T_1^3\,T_2^7+1604\,T_1^4\,T_2^7+75\,T_1^5\,T_2^7-5733\,T_1^6\,T_2^7+7771\,T_1^7\,T_2^7-2756\,T_1^8\,T_2^7-928\,T_1^9\,T_2^7+942\,T_1^{10}\,T_2^7-119\,T_1^{11}\,T_2^7-21\,T_1^{12}\,T_2^7-19\,T_1^2\,T_2^8+133\,T_1^3\,T_2^8+16\,T_1^4\,T_2^8-1668\,T_1^5\,T_2^8+3984\,T_1^6\,T_2^8-2756\,T_1^7\,T_2^8-2208\,T_1^8\,T_2^8+3084\,T_1^9\,T_2^8-1186\,T_1^{10}\,T_2^8-21\,T_1^{11}\,T_2^8+57\,T_1^{12}\,T_2^8-133\,T_1^4\,T_2^9+657\,T_1^5\,T_2^9-728\,T_1^6\,T_2^9-928\,T_1^7\,T_2^9+3084\,T_1^8\,T_2^9-1918\,T_1^9\,T_2^9+182\,T_1^{10}\,T_2^9+301\,T_1^{11}\,T_2^9-81\,T_1^{12}\,T_2^9+19\,T_1^4\,T_2^{10}-35\,T_1^5\,T_2^{10}-266\,T_1^6\,T_2^{10}+942\,T_1^7\,T_2^{10}-1186\,T_1^8\,T_2^{10}+182\,T_1^9\,T_2^{10}+418\,T_1^{10}\,T_2^{10}-301\,T_1^{11}\,T_2^{10}+57\,T_1^{12}\,T_2^{10}-14\,T_1^5\,T_2^{11}+77\,T_1^6\,T_2^{11}-119\,T_1^7\,T_2^{11}-21\,T_1^8\,T_2^{11}+301\,T_1^9\,T_2^{11}-301\,T_1^{10}\,T_2^{11}+133\,T_1^{11}\,T_2^{11}-21\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-21\,T_1^7\,T_2^{12}+57\,T_1^8\,T_2^{12}-81\,T_1^9\,T_2^{12}+57\,T_1^{10}\,T_2^{12}-21\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,87]\to\Big\{-\frac{(-2+T)\,(-1+2\,T)\,\left(1-T+T^2\right)^2}{T^3},$$

$$-\frac{1}{T_1^7\,T_2^6\,\left(-1+T_1\,T_2\right)}\,\Big(-25+112\,T_1-224\,T_1^2+287\,T_1^3-224\,T_1^4+112\,T_1^5-25\,T_1^6+112\,T_2-426\,T_1\,T_2+$$

$$664\,T_1^2\,T_2 - 606\,T_1^3\,T_2 + 132\,T_1^4\,T_2 + 178\,T_1^5\,T_2 - 228\,T_1^6\,T_2 + 76\,T_1^7\,T_2 - 224\,T_2^2 + 664\,T_1\,T_2^2 - 570\,T_1^2\,T_2^2 - 95\,T_1^3\,T_2^2 + 1152\,T_1^4\,T_2^2 - 1103\,T_1^5\,T_2^2 + 582\,T_1^6\,T_2^2 + 16\,T_1^7\,T_2^2 - 80\,T_1^8\,T_2^2 + 287\,T_2^3 - 606\,T_1\,T_2^3 - 95\,T_1^2\,T_2^3 + 1138\,T_1^3\,T_2^3 - 2061\,T_1^4\,T_2^3 + 855\,T_1^5\,T_2^3 + 214\,T_1^6\,T_2^3 - 797\,T_1^7\,T_2^3 + 312\,T_1^8\,T_2^3 + 11\,T_1^9\,T_2^3 - 224\,T_2^4 + 132\,T_1\,T_2^4 + 1152\,T_1^2\,T_2^4 - 2061\,T_1^3\,T_2^4 + 2114\,T_1^4\,T_2^4 + 358\,T_1^5\,T_2^4 - 1378\,T_1^6\,T_2^4 + 1251\,T_1^7\,T_2^4 + 18\,T_1^8\,T_2^4 - 336\,T_1^9\,T_2^4 + 64\,T_1^{10}\,T_2^4 + 112\,T_2^5 + 178\,T_1\,T_2^5 - 1103\,T_1^2\,T_2^5 + 855\,T_1^3\,T_2^5 + 358\,T_1^4\,T_2^5 - 2758\,T_1^5\,T_2^5 + 2084\,T_1^6\,T_2^5 - 290\,T_1^7\,T_2^5 - 1269\,T_1^8\,T_2^5 + 823\,T_1^9\,T_2^5 + 16\,T_1^{10}\,T_2^5 - 68\,T_1^{11}\,T_2^5 - 25\,T_2^6 - 228\,T_1\,T_2^6 + 582\,T_1^2\,T_2^6 + 214\,T_1^3\,T_2^6 - 1378\,T_1^4\,T_2^6 + 2084\,T_1^5\,T_2^6 + 72\,T_1^6\,T_2^6 - 2020\,T_1^7\,T_2^6 + 1970\,T_1^8\,T_2^6 - 338\,T_1^9\,T_2^6 - 606\,T_1^{10}\,T_2^6 + 204\,T_1^{11}\,T_2^6 + 23\,T_1^{12}\,T_2^6 + 76\,T_1\,T_2^7 + 16\,T_1^2\,T_2^7 - 797\,T_1^3\,T_2^7 + 1251\,T_1^4\,T_2^7 - 290\,T_1^5\,T_2^7 - 2020\,T_1^6\,T_2^7 + 2822\,T_1^7\,T_2^7 - 938\,T_1^8\,T_2^7 - 873\,T_1^9\,T_2^7 + 1129\,T_1^{10}\,T_2^7 - 146\,T_1^{11}\,T_2^7 - 104\,T_1^{12}\,T_2^7 - 80\,T_1^2\,T_2^8 + 312\,T_1^3\,T_2^8 + 18\,T_1^4\,T_2^8 - 1269\,T_1^5\,T_2^8 + 1970\,T_1^6\,T_2^8 - 938\,T_1^7\,T_2^8 - 1522\,T_1^8\,T_2^8 + 2043\,T_1^9\,T_2^8 - 1116\,T_1^{10}\,T_2^8 - 156\,T_1^{11}\,T_2^8 + 208\,T_1^{12}\,T_2^8 + 11\,T_1^3\,T_2^9 - 336\,T_1^4\,T_2^9 + 823\,T_1^5\,T_2^9 - 338\,T_1^6\,T_2^9 - 873\,T_1^7\,T_2^9 + 2043\,T_1^8\,T_2^9 - 1262\,T_1^9\,T_2^9 + 121\,T_1^{10}\,T_2^9 + 582\,T_1^{11}\,T_2^9 - 265\,T_1^{12}\,T_2^9 + 64\,T_1^4\,T_2^{10} + 16\,T_1^5\,T_2^{10} - 606\,T_1^6\,T_2^{10} + 1129\,T_1^7\,T_2^{10} - 1116\,T_1^8\,T_2^{10} + 121\,T_1^9\,T_2^{10} + 546\,T_1^{10}\,T_2^{10} - 632\,T_1^{11}\,T_2^{10} + 208\,T_1^{12}\,T_2^{10} - 68\,T_1^5\,T_2^{11} + 204\,T_1^6\,T_2^{11} - 146\,T_1^7\,T_2^{11} - 156\,T_1^8\,T_2^{11} + 582\,T_1^9\,T_2^{11} - 632\,T_1^{10}\,T_2^{11} + 402\,T_1^{11}\,T_2^{11} - 104\,T_1^{12}\,T_2^{11} + 23\,T_1^6\,T_2^{12} - 104\,T_1^7\,T_2^{12} + 208\,T_1^8\,T_2^{12} - 265\,T_1^9\,T_2^{12} + 208\,T_1^{10}\,T_2^{12} - 104\,T_1^{11}\,T_2^{12} + 23\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10, 99] \to \left\{\frac{\left(1 - T + T^2\right)^4}{T^4},\ \frac{4\left(1 - T_1 + T_1^2\right)^4 (1 + T_1\,T_2)\left(1 - T_2 + T_2^2\right)^4 \left(1 - T_1\,T_2 + T_1^2\,T_2^2\right)^3}{T_1^9\,T_2^8}\right\},$$

$$\text{Knot}[10, 123] \to \left\{\frac{\left(1 - 3\,T + 3\,T^2 - 3\,T^3 + T^4\right)^2}{T^4},\ \frac{1}{T_1^9\,T_2^8}\,2\left(1 - 3\,T_1 + 3\,T_1^2 - 3\,T_1^3 + T_1^4\right)^2 (1 + T_1\,T_2)\left(2 - 3\,T_1\,T_2 + 2\,T_1^2\,T_2^2\right)\left(1 - 3\,T_2 + 3\,T_2^2 - 3\,T_2^3 + T_2^4\right)^2 \left(1 - 3\,T_1\,T_2 + 3\,T_1^2\,T_2^2 - 3\,T_1^3\,T_2^3 + T_1^4\,T_2^4\right)\right\},$$

$$\text{Knot}[10, 129] \to \left\{\frac{\left(2 - 2\,T + T^2\right)\left(1 - 2\,T + 2\,T^2\right)}{T^2},\ \frac{1}{T_1^5\,T_2^4\,(-1 + T_1\,T_2)}\Big(-17 + 50\,T_1 - 75\,T_1^2 + 50\,T_1^3 - 17\,T_1^4 + 50\,T_2 - 124\,T_1\,T_2 + 146\,T_1^2\,T_2 - 34\,T_1^3\,T_2 - 28\,T_1^4\,T_2 + 26\,T_1^5\,T_2 - 75\,T_2^2 + 146\,T_1\,T_2^2 - 77\,T_1^2\,T_2^2 - 151\,T_1^3\,T_2^2 + 175\,T_1^4\,T_2^2 - 70\,T_1^5\,T_2^2 - 3\,T_1^6\,T_2^2 + 50\,T_2^3 - 34\,T_1\,T_2^3 - 151\,T_1^2\,T_2^3 + 332\,T_1^3\,T_2^3 - 190\,T_1^4\,T_2^3 - 43\,T_1^5\,T_2^3 + 74\,T_1^6\,T_2^3 - 22\,T_1^7\,T_2^3 - 17\,T_2^4 - 28\,T_1\,T_2^4 + 175\,T_1^2\,T_2^4 - 190\,T_1^3\,T_2^4 - 12\,T_1^4\,T_2^4 + 218\,T_1^5\,T_2^4 - 113\,T_1^6\,T_2^4 + 20\,T_1^7\,T_2^4 + 15\,T_1^8\,T_2^4 + 26\,T_1\,T_2^5 - 70\,T_1^2\,T_2^5 - 43\,T_1^3\,T_2^5 + 218\,T_1^4\,T_2^5 - 304\,T_1^5\,T_2^5 + 65\,T_1^6\,T_2^5 + 38\,T_1^7\,T_2^5 - 46\,T_1^8\,T_2^5 - 3\,T_1^2\,T_2^6 + 74\,T_1^3\,T_2^6 - 113\,T_1^4\,T_2^6 + 65\,T_1^5\,T_2^6 + 139\,T_1^6\,T_2^6 - 142\,T_1^7\,T_2^6 + 69\,T_1^8\,T_2^6 - 22\,T_1^3\,T_2^7 + 20\,T_1^4\,T_2^7 + 38\,T_1^5\,T_2^7 - 142\,T_1^6\,T_2^7 + 116\,T_1^7\,T_2^7 - 46\,T_1^8\,T_2^7 + 15\,T_1^4\,T_2^8 - 46\,T_1^5\,T_2^8 + 69\,T_1^6\,T_2^8 - 46\,T_1^7\,T_2^8 + 15\,T_1^8\,T_2^8\Big)\right\},$$

$$\text{Knot}[10, 137] \to \left\{\frac{\left(1 - 3\,T + T^2\right)^2}{T^2},\ \frac{1}{T_1^5\,T_2^4\,(-1 + T_1\,T_2)}\,2\left(1 - 3\,T_1 + T_1^2\right)\left(1 - 3\,T_2 + T_2^2\right)\left(1 - 3\,T_1\,T_2 + T_1^2\,T_2^2\right)\left(-1 + 3\,T_1 - T_1^2 + 3\,T_2 - 8\,T_1\,T_2 + 4\,T_1^2\,T_2 - T_2^2 + 4\,T_1\,T_2^2 - 6\,T_1^2\,T_2^2 - 2\,T_1^3\,T_2^2 + T_1^4\,T_2^2 - 2\,T_1^2\,T_2^3 + 10\,T_1^3\,T_2^3 - 3\,T_1^4\,T_2^3 + T_1^2\,T_2^4 - 3\,T_1^3\,T_2^4 + T_1^4\,T_2^4\right)\right\},$$

$$\text{Knot}[10, 140] \to \left\{\frac{\left(1 - T + T^2\right)^2}{T^2},\ \frac{1}{T_1^5\,T_2^4\,(-1 + T_1\,T_2)}\,2\left(-1 + 2\,T_1 - 3\,T_1^2 + 2\,T_1^3 - T_1^4 + 2\,T_2 - 3\,T_1\,T_2 + 4\,T_1^2\,T_2 - T_1^3\,T_2 + T_1^5\,T_2 - 3\,T_2^2 + 4\,T_1\,T_2^2 + T_1^2\,T_2^2 - 8\,T_1^3\,T_2^2 + 7\,T_1^4\,T_2^2 - 2\,T_1^5\,T_2^2 + 2\,T_2^3 - T_1\,T_2^3 -$$

$$8\,T_1^2\,T_2^3+6\,T_1^3\,T_2^3-8\,T_1^5\,T_2^3+2\,T_1^6\,T_2^3-T_1^7\,T_2^3-T_2^4+7\,T_1^2\,T_2^4+4\,T_1^5\,T_2^4+5\,T_1^6\,T_2^4+T_1^8\,T_2^4+T_1\,T_2^5-2\,T_1^2\,T_2^5-8\,T_1^3\,T_2^5+4\,T_1^4\,T_2^5-2\,T_1^5\,T_2^5-8\,T_1^6\,T_2^5+T_1^7\,T_2^5-2\,T_1^8\,T_2^5+2\,T_1^3\,T_2^6+5\,T_1^4\,T_2^6-8\,T_1^5\,T_2^6+11\,T_1^6\,T_2^6-4\,T_1^7\,T_2^6+3\,T_1^8\,T_2^6-T_1^3\,T_2^7+T_1^5\,T_2^7-4\,T_1^6\,T_2^7+3\,T_1^7\,T_2^7-2\,T_1^8\,T_2^7+T_1^4\,T_2^8-2\,T_1^5\,T_2^8+3\,T_1^6\,T_2^8-2\,T_1^7\,T_2^8+T_1^8\,T_2^8\Big)\Big\},$$

$$\text{Knot}[10,153]\to\Big\{\frac{\left(1-T+T^3\right)\left(1-T^2+T^3\right)}{T^3},\ \frac{1}{T_1^7\,T_2^6\,(-1+T_1\,T_2)}\Big(-2+2\,T_1+2\,T_1^2-6\,T_1^3+2\,T_1^4+2\,T_1^5-2\,T_1^6+2\,T_2-3\,T_1\,T_2+3\,T_1^3\,T_2-T_1^4\,T_2-T_1^6\,T_2+T_1^7\,T_2+2\,T_2^2-2\,T_1^2\,T_2^2+6\,T_1^3\,T_2^2-2\,T_1^5\,T_2^2+2\,T_1^6\,T_2^2+2\,T_1^7\,T_2^2-6\,T_2^3+3\,T_1\,T_2^3+6\,T_1^2\,T_2^3-14\,T_1^3\,T_2^3-2\,T_1^4\,T_2^3+2\,T_1^5\,T_2^3+4\,T_1^6\,T_2^3-5\,T_1^7\,T_2^3-5\,T_1^8\,T_2^3+3\,T_1^9\,T_2^3+2\,T_2^4-T_1\,T_2^4-2\,T_1^3\,T_2^4+5\,T_1^4\,T_2^4+7\,T_1^5\,T_2^4-11\,T_1^6\,T_2^4-2\,T_1^7\,T_2^4+10\,T_1^8\,T_2^4-3\,T_1^9\,T_2^4-2\,T_1^{10}\,T_2^4+2\,T_2^5-2\,T_1^2\,T_2^5+2\,T_1^3\,T_2^5+7\,T_1^4\,T_2^5-5\,T_1^5\,T_2^5-7\,T_1^6\,T_2^5+18\,T_1^7\,T_2^5-10\,T_1^8\,T_2^5-3\,T_1^9\,T_2^5+8\,T_1^{10}\,T_2^5-3\,T_1^{11}\,T_2^5-2\,T_2^6-T_1\,T_2^6+2\,T_1^2\,T_2^6+4\,T_1^3\,T_2^6-11\,T_1^4\,T_2^6-7\,T_1^5\,T_2^6+24\,T_1^6\,T_2^6-13\,T_1^7\,T_2^6-9\,T_1^8\,T_2^6+16\,T_1^9\,T_2^6-6\,T_1^{10}\,T_2^6-3\,T_1^{11}\,T_2^6+4\,T_1^{12}\,T_2^6+T_1\,T_2^7+2\,T_1^2\,T_2^7-5\,T_1^3\,T_2^7-2\,T_1^4\,T_2^7+18\,T_1^5\,T_2^7-13\,T_1^6\,T_2^7-15\,T_1^7\,T_2^7+29\,T_1^8\,T_2^7-14\,T_1^9\,T_2^7-6\,T_1^{10}\,T_2^7+10\,T_1^{11}\,T_2^7-4\,T_1^{12}\,T_2^7-5\,T_1^3\,T_2^8+10\,T_1^4\,T_2^8-10\,T_1^5\,T_2^8-9\,T_1^6\,T_2^8+29\,T_1^7\,T_2^8-25\,T_1^8\,T_2^8-10\,T_1^9\,T_2^8+20\,T_1^{10}\,T_2^8-7\,T_1^{11}\,T_2^8-4\,T_1^{12}\,T_2^8+3\,T_1^3\,T_2^9-3\,T_1^4\,T_2^9-3\,T_1^5\,T_2^9+16\,T_1^6\,T_2^9-14\,T_1^7\,T_2^9-10\,T_1^8\,T_2^9+34\,T_1^9\,T_2^9-14\,T_1^{10}\,T_2^9-11\,T_1^{11}\,T_2^9+12\,T_1^{12}\,T_2^9-2\,T_1^4\,T_2^{10}+8\,T_1^5\,T_2^{10}-6\,T_1^6\,T_2^{10}-6\,T_1^7\,T_2^{10}+20\,T_1^8\,T_2^{10}-14\,T_1^9\,T_2^{10}-2\,T_1^{10}\,T_2^{10}+10\,T_1^{11}\,T_2^{10}-4\,T_1^{12}\,T_2^{10}-3\,T_1^5\,T_2^{11}-3\,T_1^6\,T_2^{11}+10\,T_1^7\,T_2^{11}-7\,T_1^8\,T_2^{11}-11\,T_1^9\,T_2^{11}+10\,T_1^{10}\,T_2^{11}-T_1^{11}\,T_2^{11}-4\,T_1^{12}\,T_2^{11}+4\,T_1^6\,T_2^{12}-4\,T_1^7\,T_2^{12}-4\,T_1^8\,T_2^{12}+12\,T_1^9\,T_2^{12}-4\,T_1^{10}\,T_2^{12}-4\,T_1^{11}\,T_2^{12}+4\,T_1^{12}\,T_2^{12}\Big)\Big\},$$

$$\text{Knot}[10,155]\to\Big\{-\frac{\left(-1+T-2\,T^2+T^3\right)\left(-1+2\,T-T^2+T^3\right)}{T^3},\ -\frac{1}{T_1^7\,T_2^6\,(-1+T_1\,T_2)}\Big(-3+9\,T_1-15\,T_1^2+21\,T_1^3-15\,T_1^4+9\,T_1^5-3\,T_1^6+9\,T_2-21\,T_1\,T_2+27\,T_1^2\,T_2-33\,T_1^3\,T_2+3\,T_1^4\,T_2+3\,T_1^5\,T_2-9\,T_1^6\,T_2+6\,T_1^7\,T_2-15\,T_2^2+27\,T_1\,T_2^2-28\,T_1^2\,T_2^2+38\,T_1^3\,T_2^2+16\,T_1^4\,T_2^2-2\,T_1^5\,T_2^2+12\,T_1^6\,T_2^2-3\,T_1^7\,T_2^2-5\,T_1^8\,T_2^2+21\,T_2^3-33\,T_1\,T_2^3+38\,T_1^2\,T_2^3-64\,T_1^3\,T_2^3-24\,T_1^4\,T_2^3-12\,T_1^5\,T_2^3-16\,T_1^6\,T_2^3-7\,T_1^7\,T_2^3+15\,T_1^8\,T_2^3-15\,T_2^4+3\,T_1\,T_2^4+16\,T_1^2\,T_2^4-24\,T_1^3\,T_2^4+151\,T_1^4\,T_2^4-53\,T_1^5\,T_2^4+63\,T_1^6\,T_2^4+4\,T_1^7\,T_2^4-10\,T_1^8\,T_2^4-15\,T_1^9\,T_2^4+5\,T_1^{10}\,T_2^4+9\,T_2^5+3\,T_1\,T_2^5-2\,T_1^2\,T_2^5-12\,T_1^3\,T_2^5-53\,T_1^4\,T_2^5-91\,T_1^5\,T_2^5+29\,T_1^6\,T_2^5-48\,T_1^7\,T_2^5+8\,T_1^8\,T_2^5+23\,T_1^9\,T_2^5+3\,T_1^{10}\,T_2^5-6\,T_1^{11}\,T_2^5-3\,T_2^6-9\,T_1\,T_2^6+12\,T_1^2\,T_2^6-16\,T_1^3\,T_2^6+63\,T_1^4\,T_2^6+29\,T_1^5\,T_2^6+60\,T_1^6\,T_2^6-45\,T_1^7\,T_2^6+41\,T_1^8\,T_2^6-28\,T_1^9\,T_2^6-16\,T_1^{10}\,T_2^6+9\,T_1^{11}\,T_2^6+3\,T_1^{12}\,T_2^6+6\,T_1\,T_2^7-3\,T_1^2\,T_2^7-7\,T_1^3\,T_2^7+4\,T_1^4\,T_2^7-48\,T_1^5\,T_2^7-45\,T_1^6\,T_2^7+75\,T_1^7\,T_2^7-43\,T_1^8\,T_2^7+24\,T_1^9\,T_2^7+18\,T_1^{10}\,T_2^7-3\,T_1^{11}\,T_2^7-9\,T_1^{12}\,T_2^7-5\,T_1^2\,T_2^8+15\,T_1^3\,T_2^8-10\,T_1^4\,T_2^8+8\,T_1^5\,T_2^8+41\,T_1^6\,T_2^8-43\,T_1^7\,T_2^8-47\,T_1^8\,T_2^8+36\,T_1^9\,T_2^8-36\,T_1^{10}\,T_2^8-3\,T_1^{11}\,T_2^8+15\,T_1^{12}\,T_2^8-15\,T_1^4\,T_2^9+23\,T_1^5\,T_2^9-28\,T_1^6\,T_2^9+24\,T_1^7\,T_2^9+36\,T_1^8\,T_2^9+20\,T_1^9\,T_2^9-22\,T_1^{10}\,T_2^9+33\,T_1^{11}\,T_2^9-21\,T_1^{12}\,T_2^9+5\,T_1^4\,T_2^{10}+3\,T_1^5\,T_2^{10}-16\,T_1^6\,T_2^{10}+18\,T_1^7\,T_2^{10}-36\,T_1^8\,T_2^{10}-22\,T_1^9\,T_2^{10}+24\,T_1^{10}\,T_2^{10}-27\,T_1^{11}\,T_2^{10}+15\,T_1^{12}\,T_2^{10}-6\,T_1^5\,T_2^{11}+9\,T_1^6\,T_2^{11}-3\,T_1^7\,T_2^{11}-3\,T_1^8\,T_2^{11}+33\,T_1^9\,T_2^{11}-27\,T_1^{10}\,T_2^{11}+21\,T_1^{11}\,T_2^{11}-9\,T_1^{12}\,T_2^{11}+3\,T_1^6\,T_2^{12}-9\,T_1^7\,T_2^{12}+15\,T_1^8\,T_2^{12}-21\,T_1^9\,T_2^{12}+15\,T_1^{10}\,T_2^{12}-9\,T_1^{11}\,T_2^{12}+3\,T_1^{12}\,T_2^{12}\Big)\Big\}\Big\}$$

```
In[•]:= DunfieldKnots = ReadList["../../People/Dunfield/nmd_random_knots"] /. k_Integer :> k + 1;

In[•]:= (Crossings /@ DunfieldKnots) == Range[3, 1000]

Out[•]=
True

In[•]:= DK[n_] := DunfieldKnots[[n - 2]]

In[•]:= Crossings[DK[576]]

Out[•]=
576
```

```
In[•]:= AbsoluteTiming[θ[DK[3]]]
```

Out[•]=

$$\left\{0.0625038,\ \left\{\frac{1-T+T^2}{T},\ \frac{-2+2\,T_1-2\,T_1^2+2\,T_2-T_1\,T_2+T_1^2\,T_2+T_1^3\,T_2-2\,T_2^2+T_1\,T_2^2-T_1^3\,T_2^2+T_1\,T_2^3-T_1^2\,T_2^3+T_1^3\,T_2^3}{T_1^3\,T_2^2\,(-1+T_1\,T_2)}\right\}\right\}$$

```
In[•]:= AbsoluteTiming[θ[DK[30]];]
```

Out[•]=

```
{4.88079, Null}
```

```
In[•]:= AbsoluteTiming[θ[DK[60]];]
```

Out[•]=

```
{44.9012, Null}
```

```
In[•]:= AbsoluteTiming[θ[DK[90]];]
```

Out[•]=

```
{314.553, Null}
```

```
In[•]:= AbsoluteTiming[θ120 = θ[DK[120]];]
```

Out[•]=

```
{761.428, Null}
```

```
In[•]:= Put[θ120, "Theta4DK120.m"]
```

```
In[•]:= AbsoluteTiming[θ[DK[150]];]
```

Out[•]=

```
{2357.39, Null}
```

(during the previous computation I biked home, so the AbsoluteTiming is too much)

```
In[•]:= AbsoluteTiming[θ[DK[180]];]
```

Out[•]=

```
{5391.24, Null}
```

```
In[•]:= AbsoluteTiming[θ[DK[210]];]
```

Out[•]=

```
{9613.68, Null}
```

```
In[•]:= AbsoluteTiming[θ[DK[240]];]
```

Out[•]=

```
{22462.4, Null}
```

```
In[•]:= AbsoluteTiming[θ[DK[270]];]
```

Mathematica crashed while trying the above computation.

```
In[•]:= AbsoluteTiming[θ[DK[300]];]
```