

Pensieve header: Finding the A_2 $\mathcal{S}d=1$ invariant using undetermined coefficients.

Searching for $Q + p_{xx} + \epsilon(ppx + 1 + px + ppxx)$ solutions.

Initialization

```
In[*]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory` ; << Rot.m];
<< FormalGaussianIntegration.m;
i_+ := i + 1;
```

Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.

Read more at <http://katlas.org/wiki/KnotTheory>.

Loading Rot.m from <http://drorbn.net/AP/Projects/HigherRank> to compute rotation numbers.

```
In[*]:= Features[Knot[8, 17]]
```

 KnotTheory: Loading precomputed data in PD4Knots`.

Out[*]=

```
Features[18,
C6[-1] C14[-1] X1,7[1] X3,9[-1] X5,13[-1] X8,16[1] X10,4[-1] X12,18[1] X15,2[-1] X17,11[1]]
```

```
In[*]:= T3 = T1 T2;
S = {x_, p_};
q[s_, i_, j_] := Sum[
  xv,i (pv,i+ - pv,i) + xv,j (pv,j+ - pv,j) + (Tv^S - 1) xv,i (pv,i+ - pv,j+),
  {v, 3}];
L[Xi_,j_[s_]] := T3^S E[q[s, i, j] + r0[s, i, j] + e r1[s, i, j] + O[epsilon]^1];
(* gamma1[phi_, k_] := phi (3/2 - X1,k p1,k - X2,k p2,k - X3,k p3,k); *)
L[Cr_[phi_]] := T3^phi E[Sum[xv,k (pv,k+ - pv,k), {v, 3}] + e gamma1[phi, k] + O[epsilon]^1];
ps_i := Sequence[p1,i, p2,i, p3,i];
xs_i := Sequence[x1,i, x2,i, x3,i];
vs_i := Sequence[ps_i, xs_i];
F[is___] := E[Sum[pi_v,i pv,i, {i, {is}}, {v, 3}]];
L[K_] := (2 pi)^{-Features[K][[1]]} CF[L/@Features[K][[2]]];
vs[K_] := Union@@Table[{vs_i}, {i, Features[K][[1]]}]
```

```
In[*]:= vs_i
```

Out[*]=

```
Sequence[p1,i, p2,i, p3,i, x1,i, x2,i, x3,i]
```

The pxx Terms (r_0)

```

In[*]:=  $x = \mathbf{0}$ ;
r0[1, i_, j_] := Evaluate[Sum[
  a_{++x} p_{3,k3} x_{1,k1} x_{2,k2},
  {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
]];
r0[1, i, j]

Out[*]=
a1 p_{3,i} x_{1,i} x_{2,i} + a2 p_{3,j} x_{1,i} x_{2,i} + a5 p_{3,i} x_{1,j} x_{2,i} + a6 p_{3,j} x_{1,j} x_{2,i} +
a3 p_{3,i} x_{1,i} x_{2,j} + a4 p_{3,j} x_{1,i} x_{2,j} + a7 p_{3,i} x_{1,j} x_{2,j} + a8 p_{3,j} x_{1,j} x_{2,j}

In[*]:=  $\mathcal{L}[X_{i,j}[s]] := T_3^s \mathbb{E}[q[s, i, j] + r_0[s, i, j]]$ ;
 $\mathcal{L}[X_{i,j}[1]]$ 

Out[*]=
T1 T2
 $\mathbb{E} [ (-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + (-p_{1,j} + p_{1,1+j}) x_{1,j} + (-p_{2,i} + p_{2,1+i}) x_{2,i} +$ 
 $(-1 + T_2) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + a_1 p_{3,i} x_{1,i} x_{2,i} + a_2 p_{3,j} x_{1,i} x_{2,i} + a_5 p_{3,i} x_{1,j} x_{2,i} +$ 
 $a_6 p_{3,j} x_{1,j} x_{2,i} + (-p_{2,j} + p_{2,1+j}) x_{2,j} + a_3 p_{3,i} x_{1,i} x_{2,j} + a_4 p_{3,j} x_{1,i} x_{2,j} + a_7 p_{3,i} x_{1,j} x_{2,j} +$ 
 $a_8 p_{3,j} x_{1,j} x_{2,j} + (-p_{3,i} + p_{3,1+i}) x_{3,i} + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j} ]$ 

```

Reidemeister 3 for pxx (r_0)

```

In[*]:= {lhs} = Cases[ $\int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^*,k}[1] X_{j^*,k^*}[1]) \mathbb{d}\{vs_i, vs_j, vs_k, vs_{i^*}, vs_{j^*}, vs_{k^*}\},$ 
   $\mathbb{E}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty$ ];

In[*]:= {rhs} = Cases[ $\int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{j,k}[1] X_{i,k^*}[1] X_{i^*,j^*}[1]) \mathbb{d}\{vs_i, vs_j, vs_k, vs_{i^*}, vs_{j^*}, vs_{k^*}\},$ 
   $\mathbb{E}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty$ ];

In[*]:= eqn = CF[lhs - rhs];

In[*]:= cvs = Union@Cases[eqn, p__ |  $\pi_{__}, \infty$ ]

Out[*]=
{p_{3,2+i}, p_{3,2+j}, p_{3,2+k},  $\pi_{1,i}, \pi_{1,j}, \pi_{1,k}, \pi_{2,i}, \pi_{2,j}, \pi_{2,k}$ }

In[*]:= eqns = CoefficientRules[eqn, cvs] /. ( $\_ \rightarrow c\_ \Rightarrow (c == \mathbf{0})$ );

In[*]:= vars = Union@Cases[r0[1, i, j], a_,  $\infty$ ]

Out[*]=
{a1, a2, a3, a4, a5, a6, a7, a8}

```

In[*]:= `{sol} = Solve[eqns, vars]`

 **Solve:** Equations may not give solutions for all "solve" variables. 

Out[*]=

$$\left\{ \left\{ a_1 \rightarrow 0, a_3 \rightarrow 0, a_5 \rightarrow 0, a_6 \rightarrow -\frac{a_2}{T_1} - \frac{a_4 T_2}{T_1}, a_7 \rightarrow 0, a_8 \rightarrow 0 \right\} \right\}$$

In[*]:= `sol /. (v_ -> val_) -> (v = CF[val]);`

In[*]:= `r0[1, i, j]`

Out[*]=

$$a_2 p_{3,j} x_{1,i} x_{2,i} - \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} + a_4 p_{3,j} x_{1,i} x_{2,j}$$

The ppx Terms (r_1)

In[*]:= `x = 0;`

`r1[1, i_, j_] := Evaluate[Sum[
 b++x x3,k3 p1,k1 p2,k2,
 {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}
]];
r1[1, i, j]`

Out[*]=

$$b_1 p_{1,i} p_{2,i} x_{3,i} + b_5 p_{1,j} p_{2,i} x_{3,i} + b_3 p_{1,i} p_{2,j} x_{3,i} + b_7 p_{1,j} p_{2,j} x_{3,i} + b_2 p_{1,i} p_{2,i} x_{3,j} + b_6 p_{1,j} p_{2,i} x_{3,j} + b_4 p_{1,i} p_{2,j} x_{3,j} + b_8 p_{1,j} p_{2,j} x_{3,j}$$

In[*]:= `L[Xi_,j_][s_] := T3^5 E[q[s, i, j] + e r1[s, i, j] + O[e]^2];
 L[Xi,j][1]`

Out[*]=

$$T_1 T_2 E[\epsilon \text{Series}[(-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + (-p_{1,j} + p_{1,1+j}) x_{1,j} + (-p_{2,i} + p_{2,1+i}) x_{2,i} + (-1 + T_2) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + (-p_{2,j} + p_{2,1+j}) x_{2,j} + (-p_{3,i} + p_{3,1+i}) x_{3,i} + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j}, b_1 p_{1,i} p_{2,i} x_{3,i} + b_5 p_{1,j} p_{2,i} x_{3,i} + b_3 p_{1,i} p_{2,j} x_{3,i} + b_7 p_{1,j} p_{2,j} x_{3,i} + b_2 p_{1,i} p_{2,i} x_{3,j} + b_6 p_{1,j} p_{2,i} x_{3,j} + b_4 p_{1,i} p_{2,j} x_{3,j} + b_8 p_{1,j} p_{2,j} x_{3,j}]]$$

Reidemeister 3 for ppx (r_1)

In[*]:= `{lhs} = Cases[
 ∫ $\mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^*,k}[1] X_{j^*,k^*}[1]) \text{d}\{vs_i, vs_j, vs_k, vs_{i^*}, vs_{j^*}, vs_{k^*}\},$
 eSeries[_ , e_] -> e, ∞];`

In[*]:= `{rhs} = Cases[
 ∫ $\mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{j,k}[1] X_{i,k^*}[1] X_{i^*,j^*}[1]) \text{d}\{vs_i, vs_j, vs_k, vs_{i^*}, vs_{j^*}, vs_{k^*}\},$
 eSeries[_ , e_] -> e, ∞];`

In[*]:= `eqn = CF[lhs - rhs];`

```

In[*]:= cvs = Union@Cases[eqn, p__ |  $\pi$ __,  $\infty$ ]
Out[*]= {p1,2+i, p1,2+j, p1,2+k, p2,2+i, p2,2+j, p2,2+k,  $\pi$ 3,i,  $\pi$ 3,j,  $\pi$ 3,k}

In[*]:= eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0);
In[*]:= vars = Union@Cases[r1[1, i, j], b_,  $\infty$ ]
Out[*]= {b1, b2, b3, b4, b5, b6, b7, b8}

In[*]:= {sol} = Solve[eqns, vars]
(* Solve: Equations may not give solutions for all "solve" variables. *)
Out[*]= {{b1 -> 0, b2 -> 0, b4 -> 0, b6 -> 0, b7 -> -b3 - b5, b8 -> 0}}

In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);
In[*]:= r1[1, i, j]
Out[*]= b5 p1,j p2,i x3,i + b3 p1,i p2,j x3,i + (-b3 - b5) p1,j p2,j x3,i

```

Reidemeister 3 with pxx and ppx

```

In[*]:=  $\mathcal{L}[X_{i,j}[s_]] := T_3^s \mathbb{E}[q[s, i, j] + r_0[s, i, j] + \epsilon r_1[s, i, j] + 0[\epsilon]^2];$ 
 $\mathcal{L}[X_{i,j}[1]]$ 
Out[*]= T1 T2  $\mathbb{E}\left[ eSeries\left[ \begin{aligned} &(-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + (-p_{1,j} + p_{1,1+j}) x_{1,j} + (-p_{2,i} + p_{2,1+i}) x_{2,i} + \\ &(-1 + T_2) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + a_2 p_{3,j} x_{1,i} x_{2,i} - \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} + (-p_{2,j} + p_{2,1+j}) x_{2,j} + \\ &a_4 p_{3,j} x_{1,i} x_{2,j} + (-p_{3,i} + p_{3,1+i}) x_{3,i} + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j}, \\ &b_5 p_{1,j} p_{2,i} x_{3,i} + b_3 p_{1,i} p_{2,j} x_{3,i} + (-b_3 - b_5) p_{1,j} p_{2,j} x_{3,i} \end{aligned} \right] \right]$ 

In[*]:= {lhs} = Cases[ $\int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^+,k}[1] X_{j^+,k^+}[1]) \mathbb{d}\{vs_i, vs_j, vs_k, vs_{i^+}, vs_{j^+}, vs_{k^+}\},$ 
 $\mathbb{E}[\mathcal{E}_-] :-> \mathcal{E}, \infty$ ];

In[*]:= {rhs} = Cases[ $\int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{j,k}[1] X_{i,k^+}[1] X_{i^+,j^+}[1]) \mathbb{d}\{vs_i, vs_j, vs_k, vs_{i^+}, vs_{j^+}, vs_{k^+}\},$ 
 $\mathbb{E}[\mathcal{E}_-] :-> \mathcal{E}, \infty$ ];

In[*]:= Echo /@ Short /@ (CF@CoefficientList[CF[lhs - rhs][[2]] /. {ai ->  $\lambda$  ai, bi ->  $\lambda$  bi},  $\lambda$ );

```

» 0

» 0

$$\begin{aligned}
 & \gg a_2 b_5 T_2 p_{1,2+k} p_{2,2+j} \pi_{1,i} \pi_{2,i} + \ll 45 \gg + \frac{b_5 \ll 5 \gg \pi_{\ll 1 \gg}}{T_1} - \\
 & \frac{(a_2 + a_4 T_2) (-b_3 + b_3 T_2 + b_3 T_1 T_2 + b_5 T_2^2) \ll 1 \gg \ll 1 \gg \pi_{\ll 1 \gg} \pi_{3,j}}{T_1} \\
 & \gg - \frac{a_2 (a_2 b_5 - a_4 b_3 T_1 + a_4 b_5 T_2) p_{3,2+k} \pi_{1,i} \pi_{2,i}}{T_1} - \\
 & a_2 a_4 b_3 (-1 + T_1) T_1 p_{1,2+j} p_{3,2+k} \pi_{1,i}^2 \pi_{2,i} + a_2 \ll 6 \gg \pi_{\ll 1 \gg} + \ll 72 \gg \\
 & \gg \frac{a_2 a_4 (-1 + T_1) (-1 + T_2) (a_2 + \ll 1 \gg) (b_3 + b_5 + b_3 T_1 + b_5 T_2) p_{3,2+k}^2 \pi_{1,i}^2 \pi_{2,i}^2}{T_1} + \\
 & \ll 26 \gg + a_4^3 b_3 p_{3,2+k}^2 \pi_{1,i} \pi_{1,j} \pi_{2,j} \pi_{2,k}
 \end{aligned}$$

In[*]:= **err** = **CF@Coefficient**[**CF**[**lhs** - **rhs**][[2]] /. {**a_i** -> **λ a_i**, **b_i** -> **λ b_i**}, **λ²**]

Out[*]=

$$\begin{aligned}
 & a_2 b_5 T_2 p_{1,2+k} p_{2,2+j} \pi_{1,i} \pi_{2,i} + a_2 b_3 T_1 p_{1,2+j} p_{2,2+k} \pi_{1,i} \pi_{2,i} - a_2 (b_3 T_1 + b_5 T_2) p_{1,2+k} p_{2,2+k} \pi_{1,i} \pi_{2,i} - \\
 & \frac{b_5 T_2 (a_2 + a_4 T_2) p_{1,2+k} p_{2,2+j} \pi_{1,j} \pi_{2,i}}{T_1} - b_3 (a_2 + a_4 T_2) p_{1,2+j} p_{2,2+k} \pi_{1,j} \pi_{2,i} + \\
 & \frac{(a_2 + a_4 T_2) (b_3 T_1 + b_5 T_2) p_{1,2+k} p_{2,2+k} \pi_{1,j} \pi_{2,i}}{T_1} + a_4 b_5 T_2 p_{1,2+k} p_{2,2+j} \pi_{1,i} \pi_{2,j} + \\
 & \frac{a_4 b_3 T_1 p_{1,2+j} p_{2,2+k} \pi_{1,i} \pi_{2,j} - a_4 (b_3 T_1 + b_5 T_2) p_{1,2+k} p_{2,2+k} \pi_{1,i} \pi_{2,j} -}{(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2) p_{3,2+k} \pi_{3,i}} + \\
 & \frac{b_5 T_1 (-a_4 - a_2 T_1 + a_4 T_1) T_2 p_{1,2+k} p_{3,2+j} \pi_{1,i} \pi_{3,i} +}{T_1 (a_2 b_5 - a_4 b_5 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1^2 T_2) p_{1,2+j} p_{3,2+k} \pi_{1,i} \pi_{3,i} -} \\
 & (-a_4 b_5 - a_2 b_5 T_1 + 2 a_4 b_5 T_1 + a_4 b_3 T_1^2) (-1 + T_1 T_2) p_{1,2+k} p_{3,2+k} \pi_{1,i} \pi_{3,i} + \\
 & b_5 T_1 T_2 (a_2 + a_4 T_2) p_{1,2+k} p_{3,2+j} \pi_{1,j} \pi_{3,i} + b_3 T_1^2 (a_2 + a_4 T_2) p_{1,2+i} p_{3,2+k} \pi_{1,j} \pi_{3,i} + \\
 & (-a_2 b_5 + a_4 b_3 T_1 - a_2 b_3 T_1^2 - a_4 b_5 T_2 + a_4 b_5 T_1 T_2 - a_4 b_3 T_1^2 T_2) p_{1,2+j} p_{3,2+k} \pi_{1,j} \pi_{3,i} + \\
 & (a_2 b_5 - a_4 b_5 - a_4 b_3 T_1 + a_4 b_5 T_2 - a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2^2) p_{1,2+k} p_{3,2+k} \pi_{1,j} \pi_{3,i} - \\
 & b_3 T_1 (a_2 + a_4 T_2) p_{1,2+i} p_{3,2+k} \pi_{1,k} \pi_{3,i} + b_3 T_1 (a_2 + a_4 T_2) p_{1,2+j} p_{3,2+k} \pi_{1,k} \pi_{3,i} - \\
 & b_3 T_2 (-a_2 + a_2 T_2 - a_4 T_2 + a_2 T_1 T_2 + a_4 T_2^2) p_{2,2+k} p_{3,2+j} \pi_{2,i} \pi_{3,i} + \\
 & \frac{T_2 (a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2 - a_2 b_5 T_1 T_2^2 - a_4 b_5 T_1 T_2^3) p_{2,2+j} p_{3,2+k} \pi_{2,i} \pi_{3,i}}{T_1} + \\
 & \frac{1}{T_1} (-1 + T_1 T_2) (-a_2 b_3 + 2 a_2 b_3 T_2 - a_4 b_3 T_2 + a_2 b_3 T_1 T_2 + 2 a_4 b_3 T_2^2 + a_2 b_5 T_2^2 + a_4 b_5 T_2^3) \\
 & p_{2,2+k} p_{3,2+k} \pi_{2,i} \pi_{3,i} - a_4 b_3 T_1 T_2^2 p_{2,2+k} p_{3,2+j} \pi_{2,j} \pi_{3,i} - a_4 b_5 T_2^3 p_{2,2+i} p_{3,2+k} \pi_{2,j} \pi_{3,i} - \\
 & \frac{T_2 (a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2^2) p_{2,2+j} p_{3,2+k} \pi_{2,j} \pi_{3,i}}{T_1} + \\
 & \frac{(a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2 + a_4 b_3 T_1^2 T_2^2) p_{2,2+k} p_{3,2+k} \pi_{2,j} \pi_{3,i}}{T_1} + \\
 & a_4 b_5 T_2^2 p_{2,2+i} p_{3,2+k} \pi_{2,k} \pi_{3,i} - a_4 b_5 T_2^2 p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,i} - a_4 b_5 T_1 T_2 p_{1,2+k} p_{3,2+j} \pi_{1,i} \pi_{3,j} - \\
 & a_4 b_3 T_1^2 p_{1,2+j} p_{3,2+k} \pi_{1,i} \pi_{3,j} + a_4 (-b_5 + b_5 T_1 + b_3 T_1^2 + b_5 T_1 T_2) p_{1,2+k} p_{3,2+k} \pi_{1,i} \pi_{3,j} + \\
 & b_3 T_2 (a_2 + a_4 T_2) p_{2,2+k} p_{3,2+j} \pi_{2,i} \pi_{3,j} + \frac{b_5 T_2^2 (a_2 + a_4 T_2) p_{2,2+j} p_{3,2+k} \pi_{2,i} \pi_{3,j}}{T_1} - \\
 & \frac{(a_2 + a_4 T_2) (-b_3 + b_3 T_2 + b_3 T_1 T_2 + b_5 T_2^2) p_{2,2+k} p_{3,2+k} \pi_{2,i} \pi_{3,j}}{T_1}
 \end{aligned}$$

Reidemeister 3 with ppx and px added

```
In[*]:= x = 0;
r42[1, i_, j_] = Evaluate[Plus[
  Sum[
    C++x Xv1,k1 Pv1,k2 Xv2,k3 Pv2,k4,
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1 + 1, 3}
  ],
  Sum[
    C++x Xv,k1 Pv,k2,
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
  ]
]]
```

```
Out[*]=
C49 p1,i X1,i + C52 p1,j X1,i + C55 p1,i X1,j + C58 p1,j X1,j + C50 p2,i X2,i + C53 p2,j X2,i +
C1 p1,i p2,i X1,i X2,i + C13 p1,j p2,i X1,i X2,i + C4 p1,i p2,j X1,i X2,i + C16 p1,j p2,j X1,i X2,i +
C25 p1,i p2,i X1,j X2,i + C37 p1,j p2,i X1,j X2,i + C28 p1,i p2,j X1,j X2,i + C40 p1,j p2,j X1,j X2,i +
C56 p2,i X2,j + C59 p2,j X2,j + C7 p1,i p2,i X1,i X2,j + C19 p1,j p2,i X1,i X2,j + C10 p1,i p2,j X1,i X2,j +
C22 p1,j p2,j X1,i X2,j + C31 p1,i p2,i X1,j X2,j + C43 p1,j p2,i X1,j X2,j + C34 p1,i p2,j X1,j X2,j +
C46 p1,j p2,j X1,j X2,j + C51 p3,i X3,i + C54 p3,j X3,i + C2 p1,i p3,i X1,i X3,i + C14 p1,j p3,i X1,i X3,i +
C5 p1,i p3,j X1,i X3,i + C17 p1,j p3,j X1,i X3,i + C26 p1,i p3,i X1,j X3,i + C38 p1,j p3,i X1,j X3,i +
C29 p1,i p3,j X1,j X3,i + C41 p1,j p3,j X1,j X3,i + C3 p2,i p3,i X2,i X3,i + C15 p2,j p3,i X2,i X3,i +
C6 p2,i p3,j X2,i X3,i + C18 p2,j p3,j X2,i X3,i + C27 p2,i p3,i X2,j X3,i + C39 p2,j p3,i X2,j X3,i +
C30 p2,i p3,j X2,j X3,i + C42 p2,j p3,j X2,j X3,i + C57 p3,i X3,j + C60 p3,j X3,j +
C8 p1,i p3,i X1,i X3,j + C20 p1,j p3,i X1,i X3,j + C11 p1,i p3,j X1,i X3,j + C23 p1,j p3,j X1,i X3,j +
C32 p1,i p3,i X1,j X3,j + C44 p1,j p3,i X1,j X3,j + C35 p1,i p3,j X1,j X3,j + C47 p1,j p3,j X1,j X3,j +
C9 p2,i p3,i X2,i X3,j + C21 p2,j p3,i X2,i X3,j + C12 p2,i p3,j X2,i X3,j + C24 p2,j p3,j X2,i X3,j +
C33 p2,i p3,i X2,j X3,j + C45 p2,j p3,i X2,j X3,j + C36 p2,i p3,j X2,j X3,j + C48 p2,j p3,j X2,j X3,j
```

In[*]:= $\mathcal{L}[X_{i,j}[s_-]] := T_3^s \mathbb{E}[q[s, i, j] + \epsilon r_{42}[s, i, j] + 0[\epsilon]^2];$
 $\mathcal{L}[X_{i,j}[1]]$


Out[*]=

$T_1 T_2 \mathbb{E}[\epsilon \text{Series} [(-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + (-p_{1,j} + p_{1,1+j}) x_{1,j} +$
 $(-p_{2,i} + p_{2,1+i}) x_{2,i} + (-1 + T_2) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + (-p_{2,j} + p_{2,1+j}) x_{2,j} +$
 $(-p_{3,i} + p_{3,1+i}) x_{3,i} + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j},$
 $C_{49} p_{1,i} x_{1,i} + C_{52} p_{1,j} x_{1,i} + C_{55} p_{1,i} x_{1,j} + C_{58} p_{1,j} x_{1,j} + C_{50} p_{2,i} x_{2,i} + C_{53} p_{2,j} x_{2,i} +$
 $C_1 p_{1,i} p_{2,i} x_{1,i} x_{2,i} + C_{13} p_{1,j} p_{2,i} x_{1,i} x_{2,i} + C_4 p_{1,i} p_{2,j} x_{1,i} x_{2,i} + C_{16} p_{1,j} p_{2,j} x_{1,i} x_{2,i} +$
 $C_{25} p_{1,i} p_{2,i} x_{1,j} x_{2,i} + C_{37} p_{1,j} p_{2,i} x_{1,j} x_{2,i} + C_{28} p_{1,i} p_{2,j} x_{1,j} x_{2,i} + C_{40} p_{1,j} p_{2,j} x_{1,j} x_{2,i} +$
 $C_{56} p_{2,i} x_{2,j} + C_{59} p_{2,j} x_{2,j} + C_7 p_{1,i} p_{2,i} x_{1,i} x_{2,j} + C_{19} p_{1,j} p_{2,i} x_{1,i} x_{2,j} + C_{10} p_{1,i} p_{2,j} x_{1,i} x_{2,j} +$
 $C_{22} p_{1,j} p_{2,j} x_{1,i} x_{2,j} + C_{31} p_{1,i} p_{2,i} x_{1,j} x_{2,j} + C_{43} p_{1,j} p_{2,i} x_{1,j} x_{2,j} + C_{34} p_{1,i} p_{2,j} x_{1,j} x_{2,j} +$
 $C_{46} p_{1,j} p_{2,j} x_{1,j} x_{2,j} + C_{51} p_{3,i} x_{3,i} + C_{54} p_{3,j} x_{3,i} + C_2 p_{1,i} p_{3,i} x_{1,i} x_{3,i} + C_{14} p_{1,j} p_{3,i} x_{1,i} x_{3,i} +$
 $C_5 p_{1,i} p_{3,j} x_{1,i} x_{3,i} + C_{17} p_{1,j} p_{3,j} x_{1,i} x_{3,i} + C_{26} p_{1,i} p_{3,i} x_{1,j} x_{3,i} + C_{38} p_{1,j} p_{3,i} x_{1,j} x_{3,i} +$
 $C_{29} p_{1,i} p_{3,j} x_{1,j} x_{3,i} + C_{41} p_{1,j} p_{3,j} x_{1,j} x_{3,i} + C_3 p_{2,i} p_{3,i} x_{2,i} x_{3,i} + C_{15} p_{2,j} p_{3,i} x_{2,i} x_{3,i} +$
 $C_6 p_{2,i} p_{3,j} x_{2,i} x_{3,i} + C_{18} p_{2,j} p_{3,j} x_{2,i} x_{3,i} + C_{27} p_{2,i} p_{3,i} x_{2,j} x_{3,i} + C_{39} p_{2,j} p_{3,i} x_{2,j} x_{3,i} +$
 $C_{30} p_{2,i} p_{3,j} x_{2,j} x_{3,i} + C_{42} p_{2,j} p_{3,j} x_{2,j} x_{3,i} + C_{57} p_{3,i} x_{3,j} + C_{60} p_{3,j} x_{3,j} +$
 $C_8 p_{1,i} p_{3,i} x_{1,i} x_{3,j} + C_{20} p_{1,j} p_{3,i} x_{1,i} x_{3,j} + C_{11} p_{1,i} p_{3,j} x_{1,i} x_{3,j} + C_{23} p_{1,j} p_{3,j} x_{1,i} x_{3,j} +$
 $C_{32} p_{1,i} p_{3,i} x_{1,j} x_{3,j} + C_{44} p_{1,j} p_{3,i} x_{1,j} x_{3,j} + C_{35} p_{1,i} p_{3,j} x_{1,j} x_{3,j} + C_{47} p_{1,j} p_{3,j} x_{1,j} x_{3,j} +$
 $C_9 p_{2,i} p_{3,i} x_{2,i} x_{3,j} + C_{21} p_{2,j} p_{3,i} x_{2,i} x_{3,j} + C_{12} p_{2,i} p_{3,j} x_{2,i} x_{3,j} + C_{24} p_{2,j} p_{3,j} x_{2,i} x_{3,j} +$
 $C_{33} p_{2,i} p_{3,i} x_{2,j} x_{3,j} + C_{45} p_{2,j} p_{3,i} x_{2,j} x_{3,j} + C_{36} p_{2,i} p_{3,j} x_{2,j} x_{3,j} + C_{48} p_{2,j} p_{3,j} x_{2,j} x_{3,j}]$

In[*]:= $\{\text{lhs}\} = \text{Cases} \left[\int \mathcal{F}[i, j, k] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^+,k}[1] X_{j^+,k^+}[1]) \text{d} \{ \text{vs}_i, \text{vs}_j, \text{vs}_k, \text{vs}_{i^+}, \text{vs}_{j^+}, \text{vs}_{k^+} \}, \right.$
 $\left. \mathbb{E}[\mathcal{E}_-] \rightarrow \mathcal{E}, \infty \right]$

Out[*]=

$\{ \epsilon \text{Series} [T_1^2 p_{1,2+i} \pi_{1,i} - (-1 + T_1) T_1 p_{1,2+j} \pi_{1,i} + (1 - T_1) p_{1,2+k} \pi_{1,i} + T_1 p_{1,2+j} \pi_{1,j} + (1 - T_1) p_{1,2+k} \pi_{1,j} + p_{1,2+k} \pi_{1,k} +$
 $T_2^2 p_{2,2+i} \pi_{2,i} - (-1 + T_2) T_2 p_{2,2+j} \pi_{2,i} + (1 - T_2) p_{2,2+k} \pi_{2,i} + T_2 p_{2,2+j} \pi_{2,j} + (1 - T_2) p_{2,2+k} \pi_{2,j} + p_{2,2+k} \pi_{2,k} +$
 $T_1^2 T_2^2 p_{3,2+i} \pi_{3,i} - T_1 T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{3,i} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,i} + T_1 T_2 p_{3,2+j} \pi_{3,j} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,j} + p_{3,2+k} \pi_{3,k},$
 $3 (C_1 + C_2 + C_3 + C_{10} + C_{11} + C_{12} + C_{37} + C_{38} + C_{39} + C_{46} + C_{47} + C_{48} + C_{49} + C_{50} + C_{51} + C_{58} + C_{59} + C_{60}) +$
 $2 (C_1 + C_2 + C_{10} + C_{11} + C_{49}) T_1^2 p_{1,2+i} \pi_{1,i} - T_1 (-2 C_1 - 2 C_2 - 2 C_{10} - 2 C_{11} - C_{13} - C_{14} - C_{22} - C_{23} - 2 C_{49} - C_{52} + 2 C_1 T_1 + 2 C_2 T_1 +$
 $2 C_{10} T_1 + 2 C_{11} T_1 - C_{25} T_1 - C_{26} T_1 - C_{34} T_1 - C_{35} T_1 + 2 C_{49} T_1 - C_{55} T_1 + C_{25} T_1^2 + C_{26} T_1^2 + C_{34} T_1^2 + C_{35} T_1^2 + C_{55} T_1^2) p_{1,2+j} \pi_{1,i} +$
 $\dots 338 \dots + 2 (C_{33} + C_{36} + C_{45} + C_{48} - C_{33} T_2 - C_{36} T_2 - C_{33} T_1 T_2 - C_{45} T_1 T_2 + C_{33} T_1 T_2^2) p_{2,2+k} p_{3,2+k} \pi_{2,k} \pi_{3,k}] \}$

Full expression not available (original memory size: 0.7 MB) 

In[*]:= {rhs} = Cases [$\int \mathcal{F}[\mathbf{i}, \mathbf{j}, \mathbf{k}] \times \mathcal{L} / @ (X_{j,k}[1] X_{i,k^*}[1] X_{i^*,j^*}[1]) \mathcal{d} \{ \mathbf{vS}_i, \mathbf{vS}_j, \mathbf{vS}_k, \mathbf{vS}_{i^*}, \mathbf{vS}_{j^*}, \mathbf{vS}_{k^*} \},$
 $\mathbb{E}[\mathcal{E}_-] \mapsto \mathcal{E}, \infty]$

Out[*]=

$$\left\{ \text{Series} \left[T_1^2 p_{1,2+i} \pi_{1,i} - (-1 + T_1) T_1 p_{1,2+j} \pi_{1,i} + (1 - T_1) p_{1,2+k} \pi_{1,i} + T_1 p_{1,2+j} \pi_{1,j} + (1 - T_1) p_{1,2+k} \pi_{1,j} + p_{1,2+k} \pi_{1,k} + \right. \right.$$

$$T_2^2 p_{2,2+i} \pi_{2,i} - (-1 + T_2) T_2 p_{2,2+j} \pi_{2,i} + (1 - T_2) p_{2,2+k} \pi_{2,i} + T_2 p_{2,2+j} \pi_{2,j} + (1 - T_2) p_{2,2+k} \pi_{2,j} + p_{2,2+k} \pi_{2,k} +$$

$$T_1^2 T_2^2 p_{3,2+i} \pi_{3,i} - T_1 T_2 (-1 + T_1 T_2) p_{3,2+j} \pi_{3,i} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,i} + T_1 T_2 p_{3,2+j} \pi_{3,j} + (1 - T_1 T_2) p_{3,2+k} \pi_{3,j} + p_{3,2+k} \pi_{3,k},$$

$$3 (C_1 + C_2 + C_3 + C_{10} + C_{11} + C_{12} + C_{37} + C_{38} + C_{39} + C_{46} + C_{47} + C_{48} + C_{49} + C_{50} + C_{51} + C_{58} + C_{59} + C_{60}) +$$

$$2 (C_1 + C_2 + C_{10} + C_{11} + C_{49}) T_1^2 p_{1,2+i} \pi_{1,i} -$$

$$T_1 (-2 C_1 - 2 C_2 - 2 C_{10} - 2 C_{11} - C_{13} - C_{14} - C_{22} - C_{23} - 2 C_{49} - C_{52} + 2 C_1 T_1 + 2 C_2 T_1 + 2 C_{10} T_1 + 2 C_{11} T_1 + 2 C_{49} T_1) p_{1,2+j} \pi_{1,i} +$$

$$\dots 391 \dots + (-2 + T_2) T_2 (-C_{33} - C_{36} + C_{33} T_1 T_2) p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,k} +$$

$$2 (C_{33} + C_{36} + C_{45} + C_{48} - C_{33} T_2 - C_{36} T_2 - C_{33} T_1 T_2 - C_{45} T_1 T_2 + C_{33} T_1 T_2^2) p_{2,2+k} p_{3,2+k} \pi_{2,k} \pi_{3,k} \left. \right\}$$

Full expression not available (original memory size: 0.6 MB)

In[*]:= eqn = CF [(lhs - rhs) [[2]] - err]

Out[*]=

$$- \left((C_{25} + C_{26} + C_{34} + C_{35} + C_{55}) (-1 + T_1) T_1^2 p_{1,2+j} \pi_{1,i} \right) -$$

$$(-1 + T_1) (C_1 + C_2 + C_{10} + C_{11} + C_{13} + C_{14} + C_{22} + C_{23} + C_{49} + C_{52} + C_{25} T_1 + C_{26} T_1 + C_{34} T_1 + C_{35} T_1 +$$

$$C_{37} T_1 + C_{38} T_1 + C_{46} T_1 + C_{47} T_1 + C_{55} T_1 + C_{58} T_1 - C_{25} T_1^2 - C_{26} T_1^2 - C_{34} T_1^2 - C_{35} T_1^2 - C_{55} T_1^2) p_{1,2+k} \pi_{1,i} +$$

$$(C_{25} + C_{26} + C_{34} + C_{35} + C_{55}) (-1 + T_1) T_1^2 p_{1,2+i} \pi_{1,j} - (C_{25} + C_{26} + C_{34} + C_{35} + C_{55}) (-1 + T_1)^2 T_1 p_{1,2+j} \pi_{1,j} -$$

$$(C_{25} + C_{26} + C_{34} + C_{35} + C_{55}) (-1 + T_1) T_1 p_{1,2+k} \pi_{1,j} - (C_{25} + C_{26} + C_{34} + C_{35} + C_{55}) (-1 + T_1) T_1 p_{1,2+i} \pi_{1,k} +$$

$$\dots 365 \dots + C_{33} T_1^2 (-1 + T_2) T_2^3 p_{2,2+j} p_{3,2+i} \pi_{2,k} \pi_{3,k} + T_1 T_2 (-C_{33} - C_{45} + C_{33} T_2) (-1 + T_1 T_2) p_{2,2+k} p_{3,2+i} \pi_{2,k} \pi_{3,k} +$$

$$C_{33} T_1 T_2^2 (-1 + T_1 T_2) p_{2,2+i} p_{3,2+j} \pi_{2,k} \pi_{3,k} - C_{33} T_1 (-1 + T_2) T_2^2 (-1 + T_1 T_2) p_{2,2+j} p_{3,2+j} \pi_{2,k} \pi_{3,k} -$$

$$T_1 T_2 (-C_{33} - C_{45} + C_{33} T_2) (-1 + T_1 T_2) p_{2,2+k} p_{3,2+j} \pi_{2,k} \pi_{3,k} +$$

$$(-1 + T_2) T_2 (-C_{33} - C_{36} + C_{33} T_1 T_2) p_{2,2+i} p_{3,2+k} \pi_{2,k} \pi_{3,k} - (-1 + T_2) T_2 (-C_{33} - C_{36} + C_{33} T_1 T_2) p_{2,2+j} p_{3,2+k} \pi_{2,k} \pi_{3,k}$$

Full expression not available (original memory size: 0.8 MB)

In[*]:= cvs = Union@Cases [eqn, p_ | \pi_ , \infty]

Out[*]=

{p_{1,2+i}, p_{1,2+j}, p_{1,2+k}, p_{2,2+i}, p_{2,2+j}, p_{2,2+k}, p_{3,2+i},
 p_{3,2+j}, p_{3,2+k}, $\pi_{1,i}$, $\pi_{1,j}$, $\pi_{1,k}$, $\pi_{2,i}$, $\pi_{2,j}$, $\pi_{2,k}$, $\pi_{3,i}$, $\pi_{3,j}$, $\pi_{3,k}$ }

In[*]:= eqns = CoefficientRules [eqn, cvs] /. (_ -> c_) \mapsto (c == 0)

Out[*]=

$$\left\{ -c_7 T_1^2 T_2^2 + c_7 T_1^2 T_2^3 = 0, c_7 T_1^2 T_2 - c_7 T_1^2 T_2^2 = 0, -c_{25} T_1^2 T_2^2 + c_{25} T_1^3 T_2^2 = 0, -c_{31} T_1^2 T_2^2 + c_{31} T_1^3 T_2^2 + c_{31} T_1^2 T_2^3 - c_{31} T_1^3 T_2^3 = 0, \right.$$

$$\dots 245 \dots, -c_8 T_1 T_2 - c_9 T_1 T_2 - c_{44} T_1 T_2 - c_{45} T_1 T_2 - c_{57} T_1 T_2 + c_8 T_1^2 T_2^2 + c_9 T_1^2 T_2^2 + c_{44} T_1^2 T_2^2 + c_{45} T_1^2 T_2^2 + c_{57} T_1^2 T_2^2 = 0,$$

$$a_2 b_3 - a_4 b_3 + c_2 + c_3 + c_5 + c_6 + c_{38} + c_{39} + c_{41} + c_{42} + c_{51} + c_{54} + \frac{a_2 b_5}{T_1} + a_4 b_3 T_2 - a_4 b_5 T_2 + \frac{a_4 b_5 T_2}{T_1} - c_2 T_1 T_2 - c_3 T_1 T_2 - c_5 T_1 T_2 -$$

$$c_6 T_1 T_2 + c_8 T_1 T_2 + c_9 T_1 T_2 + c_{11} T_1 T_2 + c_{12} T_1 T_2 - c_{38} T_1 T_2 - c_{39} T_1 T_2 - c_{41} T_1 T_2 - c_{42} T_1 T_2 + c_{44} T_1 T_2 + c_{45} T_1 T_2 +$$

$$c_{47} T_1 T_2 + c_{48} T_1 T_2 - c_{51} T_1 T_2 - c_{54} T_1 T_2 + c_{57} T_1 T_2 + c_{60} T_1 T_2 - 2 c_8 T_1^2 T_2^2 - 2 c_9 T_1^2 T_2^2 - c_{11} T_1^2 T_2^2 - c_{12} T_1^2 T_2^2 - 2 c_{44} T_1^2 T_2^2 -$$

$$2 c_{45} T_1^2 T_2^2 - c_{47} T_1^2 T_2^2 - c_{48} T_1^2 T_2^2 - 2 c_{57} T_1^2 T_2^2 - c_{60} T_1^2 T_2^2 + c_8 T_1^3 T_2^3 + c_9 T_1^3 T_2^3 + c_{44} T_1^3 T_2^3 + c_{45} T_1^3 T_2^3 + c_{57} T_1^3 T_2^3 = 0,$$

$$c_8 T_1 T_2 + c_9 T_1 T_2 + c_{44} T_1 T_2 + c_{45} T_1 T_2 + c_{57} T_1 T_2 - c_8 T_1^2 T_2^2 - c_9 T_1^2 T_2^2 - c_{44} T_1^2 T_2^2 - c_{45} T_1^2 T_2^2 - c_{57} T_1^2 T_2^2 = 0 \left. \right\}$$

Full expression not available (original memory size: 1 MB)

In[*]:= vars = Union@Cases[r₄₂[1, i, j], c_, ∞]

Out[*]=

{C₁, C₂, C₃, C₄, C₅, C₆, C₇, C₈, C₉, C₁₀, C₁₁, C₁₂, C₁₃, C₁₄, C₁₅, C₁₆, C₁₇, C₁₈, C₁₉, C₂₀, C₂₁, C₂₂,
C₂₃, C₂₄, C₂₅, C₂₆, C₂₇, C₂₈, C₂₉, C₃₀, C₃₁, C₃₂, C₃₃, C₃₄, C₃₅, C₃₆, C₃₇, C₃₈, C₃₉, C₄₀, C₄₁,
C₄₂, C₄₃, C₄₄, C₄₅, C₄₆, C₄₇, C₄₈, C₄₉, C₅₀, C₅₁, C₅₂, C₅₃, C₅₄, C₅₅, C₅₆, C₅₇, C₅₈, C₅₉, C₆₀}

In[*]:= {sol} = Solve[eqns, vars]

 Solve: Equations may not give solutions for all "solve" variables. 

Out[*]=

$$\left\{ \begin{aligned} &C_1 \rightarrow 0, C_2 \rightarrow 0, C_3 \rightarrow 0, C_7 \rightarrow 0, C_8 \rightarrow 0, C_9 \rightarrow 0, C_{10} \rightarrow -\frac{C_{13}}{1-T_1} - \frac{C_4}{-1+T_2} - \\ &\frac{a_2 b_3 - a_4 b_3 - a_2 b_5 + a_4 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2}{(-1+T_1)(-1+T_2)(-1+T_1 T_2)}, \\ &C_{11} \rightarrow -\frac{C_{14}}{1-T_1} - \frac{C_5}{-1+T_1 T_2} - \frac{-a_2 b_5 + a_4 b_5 - a_4 b_5 T_2}{(-1+T_1)(-1+T_1 T_2)}, \\ &C_{12} \rightarrow -\frac{C_{15}}{1-T_2} - \frac{C_6}{-1+T_1 T_2} - \frac{-a_2 b_3 - a_4 b_3 T_2 + a_4 b_3 T_1 T_2}{T_1(-1+T_2)(-1+T_1 T_2)}, \\ &C_{16} \rightarrow -C_4(1-T_1) - C_{13}(1-T_2) - \frac{a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2}{-1+T_1 T_2}, \\ &C_{17} \rightarrow -C_5(1-T_1) - C_{14}(1-T_1 T_2) - \\ &\frac{-a_2 b_3 + a_2 b_5 - a_4 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - a_4 b_5 T_2^2}{-1+T_2}, \\ &C_{18} \rightarrow -C_6(1-T_2) - C_{15}(1-T_1 T_2) - \frac{1}{(-1+T_1) T_1} (a_2 b_3 - a_2 b_3 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - \\ &2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2), \\ &C_{19} \rightarrow 0, C_{20} \rightarrow 0, C_{21} \rightarrow 0, C_{22} \rightarrow -\frac{a_4 b_3}{-1+T_2} - \frac{C_4(-1+T_1)}{-1+T_2}, \\ &C_{23} \rightarrow -\frac{C_5(-1+T_1)}{-1+T_1 T_2} - \frac{a_2 b_3 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_4 b_3 T_2 - 2 a_4 b_3 T_1 T_2}{(-1+T_2)(-1+T_1 T_2)}, \\ &C_{24} \rightarrow -\frac{C_6(-1+T_2)}{-1+T_1 T_2} - \frac{T_2(a_2 b_5 + a_4 b_5 T_2)}{T_1(-1+T_1 T_2)} - \frac{a_4 b_5(-T_2 + T_2^2)}{(-1+T_1)(-1+T_1 T_2)}, \\ &C_{25} \rightarrow 0, C_{26} \rightarrow 0, C_{27} \rightarrow 0, C_{28} \rightarrow 0, C_{29} \rightarrow -\frac{a_2 b_3 + a_4 b_3 T_2}{T_1(-1+T_2)}, \\ &C_{30} \rightarrow \frac{a_4 b_5}{-1+T_1}, C_{31} \rightarrow 0, C_{32} \rightarrow 0, C_{33} \rightarrow 0, C_{34} \rightarrow 0, C_{35} \rightarrow 0, C_{36} \rightarrow 0, \\ &C_{37} \rightarrow -\frac{C_{13}}{-1+T_1} - \frac{C_4}{1-T_2} - \left(-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - \right. \\ &\left. a_4 b_5 T_1 T_2 - a_4 b_3 T_1^2 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2 \right) / \left((-1+T_1) T_1 (-1+T_2) (-1+T_1 T_2) \right), \\ &C_{38} \rightarrow -\frac{C_{14}}{-1+T_1} - \frac{C_5}{1-T_1 T_2} - \left(a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_4 b_3 T_2 + \right. \\ &\left. a_2 b_5 T_2 - a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2 \right) / \end{aligned} \right.$$

$$\begin{aligned}
 & ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)), c_{39} \rightarrow -\frac{c_{15}}{-1 + T_2} - \frac{c_6}{1 - T_1 T_2} - \\
 & (-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - \\
 & a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) / \\
 & ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)), c_{40} \rightarrow -\frac{c_{13} (-1 + T_2)}{-1 + T_1} - \frac{-a_2 b_5 - a_4 b_5 T_2}{(-1 + T_1) T_1}, \\
 c_{41} & \rightarrow \frac{a_2 b_3 + a_4 b_3 T_2}{T_1 (-1 + T_2)} - \frac{a_2 b_5 + a_4 b_5 T_2}{-1 + T_1} - \frac{c_{14} (-1 + T_1 T_2)}{-1 + T_1}, \\
 c_{42} & \rightarrow -\frac{a_4 b_5}{-1 + T_1} + \frac{a_4 b_3 T_2}{-1 + T_2} - \frac{c_{15} (-1 + T_1 T_2)}{-1 + T_2}, c_{43} \rightarrow \theta, \\
 c_{44} & \rightarrow \theta, c_{45} \rightarrow \theta, c_{46} \rightarrow \theta, c_{47} \rightarrow \theta, c_{48} \rightarrow \theta, c_{55} \rightarrow \theta, \\
 c_{56} & \rightarrow \theta, c_{57} \rightarrow \theta, c_{58} \rightarrow -\frac{c_{49}}{T_1} - \frac{c_{52}}{T_1}, c_{59} \rightarrow -\frac{c_{50}}{T_2} - \frac{c_{53}}{T_2}, \\
 c_{60} & \rightarrow -\frac{c_{51}}{T_1 T_2} - \frac{c_{54}}{T_1 T_2} - \frac{a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2}{T_1^2 T_2 (-1 + T_1 T_2)} \}
 \end{aligned}$$

In[*]:= sol /. (v_ -> val_) -> (v = CF[val]);

In[*]:= CF[r42[1, i, j]]

Out[*]=

$$\begin{aligned}
 & c_{49} p_{1,i} x_{1,i} + c_{52} p_{1,j} x_{1,i} - \frac{(c_{49} + c_{52}) p_{1,j} x_{1,j}}{T_1} + c_{50} p_{2,i} x_{2,i} + \\
 & c_{53} p_{2,j} x_{2,i} + c_{13} p_{1,j} p_{2,i} x_{1,i} x_{2,i} + c_4 p_{1,i} p_{2,j} x_{1,i} x_{2,i} - \frac{1}{-1 + T_1 T_2} \\
 & (a_2 b_3 - c_4 - c_{13} + c_4 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 + c_{13} T_2 - a_4 b_3 T_1 T_2 + c_4 T_1 T_2 + c_{13} T_1 T_2 - \\
 & c_4 T_1^2 T_2 + a_4 b_5 T_2^2 - c_{13} T_1 T_2^2) p_{1,j} p_{2,j} x_{1,i} x_{2,i} - \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & (-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - c_4 T_1 + c_{13} T_1 + c_4 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - \\
 & a_4 b_5 T_1 T_2 - c_{13} T_1 T_2 - a_4 b_3 T_1^2 T_2 + c_4 T_1^2 T_2 - c_{13} T_1^2 T_2 - c_4 T_1^3 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2 + c_{13} T_1^2 T_2^2) \\
 & p_{1,j} p_{2,i} x_{1,j} x_{2,i} - \frac{(-a_2 b_5 - c_{13} T_1 - a_4 b_5 T_2 + c_{13} T_1 T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1} - \\
 & \frac{(c_{50} + c_{53}) p_{2,j} x_{2,j}}{T_2} + \frac{1}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} \\
 & (-a_2 b_3 + a_4 b_3 + a_2 b_5 - a_4 b_5 - c_4 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 + c_4 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 - \\
 & c_{13} T_2 + a_4 b_3 T_1 T_2 + c_4 T_1 T_2 - c_{13} T_1 T_2 - c_4 T_1^2 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2) p_{1,i} p_{2,j} x_{1,i} x_{2,j} - \\
 & \frac{(a_4 b_3 - c_4 + c_4 T_1) p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{-1 + T_2} + c_{51} p_{3,i} x_{3,i} + c_{54} p_{3,j} x_{3,i} + c_{14} p_{1,j} p_{3,i} x_{1,i} x_{3,i} + \\
 & c_5 p_{1,i} p_{3,j} x_{1,i} x_{3,i} - \frac{1}{-1 + T_2} \\
 & (-a_2 b_3 + a_2 b_5 - a_4 b_5 - c_5 - c_{14} + a_2 b_3 T_1 - a_4 b_3 T_1 + c_5 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 + \\
 & c_5 T_2 + c_{14} T_2 + 2 a_4 b_3 T_1 T_2 - c_5 T_1 T_2 + c_{14} T_1 T_2 - a_4 b_5 T_2^2 - c_{14} T_1 T_2^2) p_{1,j} p_{3,j} x_{1,i} x_{3,i} - \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} (a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + c_{14} T_1 + a_2 b_3 T_1^2 -
 \end{aligned}$$

$$\begin{aligned}
 & a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_5 T_1 T_2 - c_{14} T_1 T_2 + \\
 & 2 a_4 b_3 T_1^2 T_2 - c_5 T_1^2 T_2 - c_{14} T_1^2 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2 + c_{14} T_1^2 T_2^2) p_{1,j} p_{3,i} x_{1,j} x_{3,i} - \\
 & \frac{b_3 (a_2 + a_4 T_2) p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2)} + \frac{1}{(-1 + T_1) T_1 (-1 + T_2)} \\
 & (- a_2 b_3 + a_2 b_3 T_1 + a_2 b_5 T_1 - c_{14} T_1 - a_4 b_3 T_2 + a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + \\
 & a_4 b_5 T_1 T_2 + c_{14} T_1 T_2 + c_{14} T_1^2 T_2 - a_4 b_5 T_1 T_2^2 - c_{14} T_1^2 T_2^2) p_{1,j} p_{3,j} x_{1,j} x_{3,i} + \\
 & \frac{1}{(-1 + T_1) T_1} \\
 & c_{15} p_{2,j} p_{3,i} x_{2,i} x_{3,i} + c_6 p_{2,i} p_{3,j} x_{2,i} x_{3,i} + \frac{1}{(-1 + T_1) T_1} \\
 & (- a_2 b_3 + a_2 b_3 T_1 + c_6 T_1 + c_{15} T_1 - c_6 T_1^2 - c_{15} T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - \\
 & a_4 b_5 T_1 T_2 - c_6 T_1 T_2 - a_4 b_3 T_1^2 T_2 + c_6 T_1^2 T_2 - c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2) \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & p_{2,j} p_{3,j} x_{2,i} x_{3,i} - \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & (- a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 + c_{15} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_6 T_1^2 - c_{15} T_1^2 + \\
 & a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + a_4 b_3 T_1^2 T_2 - c_6 T_1^2 T_2 - \\
 & c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{2,j} p_{3,i} x_{2,j} x_{3,i} + \frac{a_4 b_5 p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_1} - \\
 & \frac{(- a_4 b_5 + c_{15} - c_{15} T_1 + a_4 b_3 T_2 + a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - c_{15} T_1 T_2 + c_{15} T_1^2 T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{(-1 + T_1) (-1 + T_2)} - \\
 & \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \\
 & (a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - c_{51} T_1 - c_{54} T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 + c_{51} T_1^2 T_2 + c_{54} T_1^2 T_2) \\
 & p_{3,j} x_{3,j} + \frac{(a_2 b_5 - a_4 b_5 + c_5 - c_{14} - c_5 T_1 + a_4 b_5 T_2 + c_{14} T_1 T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} + \\
 & \frac{(- a_2 b_3 - c_5 + a_2 b_3 T_1 - a_4 b_3 T_1 + c_5 T_1 - a_4 b_3 T_2 + c_5 T_2 + 2 a_4 b_3 T_1 T_2 - c_5 T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_2) (-1 + T_1 T_2)} - \\
 & \frac{(- a_2 b_3 - c_6 T_1 + c_{15} T_1 - a_4 b_3 T_2 + a_4 b_3 T_1 T_2 + c_6 T_1 T_2 - c_{15} T_1^2 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} - \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \\
 & (c_6 T_1 - c_6 T_1^2 - a_2 b_5 T_2 + a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2 - c_6 T_1 T_2 + c_6 T_1^2 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2) \\
 & p_{2,j} p_{3,j} x_{2,i} x_{3,j}
 \end{aligned}$$

Testing

In[*]:= $\mathcal{L}[X_{i,j}[s_-]] := T_3^5 \mathbb{E}[q[s, i, j] + r_\theta[s, i, j] + e r_1[s, i, j] - e r_{42}[s, i, j] + 0[e]^2];$
 $\mathcal{L}[X_{i,j}[1]] // \text{Short}$

Out[*]//Short=

$$T_1 T_2 \mathbb{E} \left[\text{Series} \left[(-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + \right. \right. \\
 \left. \left. \ll 9 \gg + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j}, \ll 52 \gg + \frac{\ll 1 \gg}{\ll 1 \gg} \right] \right]$$

$$\text{In[*]:= } \{\text{lhs}\} = \text{Cases} \left[\int \mathcal{F}[\mathbf{i}, \mathbf{j}, \mathbf{k}] \times \mathcal{L} / @ (\mathbf{X}_{\mathbf{i},\mathbf{j}}[1] \mathbf{X}_{\mathbf{i}^+,\mathbf{k}}[1] \mathbf{X}_{\mathbf{j}^+,\mathbf{k}^+}[1]) \text{ d} \{\mathbf{v}_{\mathbf{S}_i}, \mathbf{v}_{\mathbf{S}_j}, \mathbf{v}_{\mathbf{S}_k}, \mathbf{v}_{\mathbf{S}_{i^+}}, \mathbf{v}_{\mathbf{S}_{j^+}}, \mathbf{v}_{\mathbf{S}_{k^+}}\}, \right. \\ \left. \mathbb{E}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty \right];$$

$$\text{In[*]:= } \{\text{rhs}\} = \text{Cases} \left[\int \mathcal{F}[\mathbf{i}, \mathbf{j}, \mathbf{k}] \times \mathcal{L} / @ (\mathbf{X}_{\mathbf{j},\mathbf{k}}[1] \mathbf{X}_{\mathbf{i},\mathbf{k}^+}[1] \mathbf{X}_{\mathbf{i}^+,\mathbf{j}^+}[1]) \text{ d} \{\mathbf{v}_{\mathbf{S}_i}, \mathbf{v}_{\mathbf{S}_j}, \mathbf{v}_{\mathbf{S}_k}, \mathbf{v}_{\mathbf{S}_{i^+}}, \mathbf{v}_{\mathbf{S}_{j^+}}, \mathbf{v}_{\mathbf{S}_{k^+}}\}, \right. \\ \left. \mathbb{E}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty \right];$$

$$\text{In[*]:= } \text{CF}[\text{lhs} - \text{rhs}] \text{ [[1]]}$$

$$\text{Out[*]= } \\ 0$$

$$\text{In[*]:= } \text{Echo} / @ \text{Short} / @ (\text{CF} @ \text{CoefficientList}[\text{CF}[\text{lhs} - \text{rhs}] \text{ [[2]]} / . \{\mathbf{a}_{i_-} \Rightarrow \lambda \mathbf{a}_i, \mathbf{b}_{i_-} \Rightarrow \lambda \mathbf{b}_i\}, \lambda]);$$

» 0

$$\text{» } - \frac{a_2 T_1 T_2 (2 c_5 + \langle\langle 49 \rangle\rangle + c_{51} T_1^2 T_2^2) p_{3,2+j} \pi_{1,i} \pi_{2,i}}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} + \langle\langle 178 \rangle\rangle$$

$$\text{» } \langle\langle 65 \rangle\rangle + \langle\langle 1 \rangle\rangle - \frac{a_4^2 (c_4 + \langle\langle 23 \rangle\rangle) p_3^2 \langle\langle 1 \rangle\rangle^2 \langle\langle 1 \rangle\rangle^2 \langle\langle 1 \rangle\rangle^2 \pi_{1,i} \pi_{1,j} \pi_{2,k}^2}{(-1 + T_2) (-1 + T_1 T_2)}$$

$$\text{» } \langle\langle 155 \rangle\rangle + \frac{(\langle\langle 1 \rangle\rangle) p_{\langle\langle 1 \rangle\rangle}^2 \pi_{\langle\langle 1 \rangle\rangle} \pi_{\langle\langle 1 \rangle\rangle} \pi_{3,k}}{(-1 + T_1) T_1 (\langle\langle 1 \rangle\rangle) (-1 + T_1 T_2)}$$

$$\text{» } - \frac{T_2 (-2 a_2^3 b_3 + \langle\langle 111 \rangle\rangle + a_4^3 b_5 T_2^5 - a_4^3 b_5 T_1 T_2^5) p_{\langle\langle 1 \rangle\rangle} \langle\langle 1 \rangle\rangle \pi_{1,\langle\langle 1 \rangle\rangle}^2 \langle\langle 1 \rangle\rangle^2 \pi_{2,i}^2}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} +$$

$$\langle\langle 61 \rangle\rangle + \frac{a_4^2 b_3 \langle\langle 4 \rangle\rangle \langle\langle 1 \rangle\rangle \pi_{2,\langle\langle 1 \rangle\rangle}^2 \langle\langle 1 \rangle\rangle^2}{(-1 + T_2) (-1 + \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle)}$$

$$\text{In[*]:= } \text{Coefficient}[\text{CF} @ \text{CoefficientList}[\text{CF}[\text{lhs} - \text{rhs}] \text{ [[2]]} / . \{\mathbf{a}_{i_-} \Rightarrow \lambda \mathbf{a}_i, \mathbf{b}_{i_-} \Rightarrow \lambda \mathbf{b}_i\}, \lambda] / . \\ \{\mathbf{p}_{i_-} \Rightarrow \lambda \mathbf{p}_i, \pi_{i_-} \Rightarrow \lambda^{-1} \pi_i\}, \lambda, 0]$$

$$\text{Out[*]= } \\ 0$$

$$\text{In[*]:= } \text{Coefficient}[\text{CF} @ \text{CoefficientList}[\text{CF}[\text{lhs} - \text{rhs}] \text{ [[2]]} / . \{\mathbf{a}_{i_-} \Rightarrow \lambda \mathbf{a}_i, \mathbf{b}_{i_-} \Rightarrow \lambda \mathbf{b}_i\}, \lambda] / . \\ \{\mathbf{p}_{i_-} \Rightarrow \lambda \mathbf{p}_i, \pi_{i_-} \Rightarrow \lambda^{-1} \pi_i\}, \lambda, 1]$$

$$\text{Out[*]= } \\ 0$$

$$\text{In[*]:= } \text{Coefficient}[\text{CF} @ \text{CoefficientList}[\text{CF}[\text{lhs} - \text{rhs}] \text{ [[2]]} / . \{\mathbf{a}_{i_-} \Rightarrow \lambda \mathbf{a}_i, \mathbf{b}_{i_-} \Rightarrow \lambda \mathbf{b}_i\}, \lambda] / . \\ \{\mathbf{p}_{i_-} \Rightarrow \lambda \mathbf{p}_i, \pi_{i_-} \Rightarrow \lambda^{-1} \pi_i\}, \lambda, 2]$$

$$\text{Out[*]= } \\ 0$$

In[*]:= CF@Coefficient[CF[lhs - rhs][[2]] /. {a_i -> λ a_i, b_i -> λ b_i}, λ²]

Out[*]=

$$\frac{T_2 \dots 4 \dots \pi_{2,i}^2}{(-1+T_1) (-1+T_2)} - \frac{1}{(-1+T_1) T_1 (-1+T_2) (-1+T_1 T_2)}$$

$$\left(-a_2 a_4 c_4 - a_2^2 c_5 - a_2 a_4 c_{13} + 4 a_2 a_4 c_4 T_1 + 2 a_2^2 c_5 T_1 + a_2 a_4 c_6 T_1 - a_2^2 c_{13} T_1 + 3 a_2 a_4 c_{13} T_1 - \right.$$

$$\left. 5 a_2 a_4 c_4 T_1^2 - a_2^2 c_5 T_1^2 - 2 a_2 a_4 c_6 T_1^2 + \dots 206 \dots + 5 a_2^2 c_4 T_1^2 T_2^5 - 2 a_2 a_4 c_{13} T_1^2 T_2^5 + 4 a_2^2 c_{13} T_1^3 T_2^5 - \right.$$

$$\left. 4 a_2^2 c_4 T_1^4 T_2^5 + a_2 a_4 c_{13} T_1^4 T_2^5 - 4 a_2^2 c_{13} T_1^4 T_2^5 + a_2^2 c_4 T_1^5 T_2^5 + a_2^2 c_{13} T_1^2 T_2^6 - 2 a_2^2 c_{13} T_1^3 T_2^6 + a_2^2 c_{13} T_1^4 T_2^6 \right)$$

$$\dots 1 \dots \dots 1 \dots \dots 1 \dots \pi^2 \dots 1 \dots - \frac{T_2 \dots 5 \dots \pi_{2,i}^2}{(-1+T_1) T_1 (-1+T_2) (-1+T_1 T_2)} - \frac{\dots 1 \dots}{\dots 1 \dots} + \dots 58 \dots +$$

$$\frac{a_2^2 T_1 \left(c_4 + c_5 - c_{13} - c_{14} - c_4 T_1 - c_5 T_1 - c_5 T_2 + c_{13} T_2 + c_{14} T_2 - c_4 T_1 T_2 + c_5 T_1 T_2 + c_{13} T_1 T_2 + c_{14} T_1 T_2 + c_4 T_1^2 T_2 - c_{13} T_1 T_2^2 - c_{14} T_1 T_2^2 \right) \dots 1 \dots \pi_{1,i}^2 \pi_{2,k}^2}{(-1+T_2) (-1+T_1 T_2)}$$

$$\frac{a_2^2 \left(c_4 + c_5 - c_{13} - c_{14} - c_4 T_1 - c_5 T_1 - c_5 T_2 + c_{13} T_2 + c_{14} T_2 - c_4 T_1 T_2 + c_5 T_1 T_2 + c_{13} T_1 T_2 + c_{14} T_1 T_2 + c_4 T_1^2 T_2 - c_{13} T_1 T_2^2 - c_{14} T_1 T_2^2 \right) p_{2,2-k}^2 \pi_{1,i} \pi_{1,j} \pi_{2,k}^2}{(-1+T_2) (-1+T_1 T_2)}$$

Full expression not available (original memory size: 1.7 MB) ⚙

In[*]:= Coefficient[CF@Coefficient[CF[lhs - rhs][[2]] /. {a_i -> λ a_i, b_i -> λ b_i}, λ²] /. {p_i -> λ p_i, π_i -> λ⁻¹ π_i}, λ, 0]

Out[*]=

0

Playing

In[*]:= CF[r₄₂[1, i, j] /. {a₄ -> 0, b₃ -> 0, b₅ -> (T₁ - 1) (T₂ - 1) (T₃ - 1) a₂⁻¹, c_{4|5|6|13|14|15|49|50|51|52|53|54} -> 0}]

Out[*]=

$$- \left((-1 + T_1) (-1 + T_2) T_2 p_{1,j} p_{2,j} x_{1,i} x_{2,i} \right) + \frac{(-1 + T_2) p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{T_1} +$$

$$\frac{(-1 + T_2) (-1 + T_1 T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{T_1} + (1 - T_2) p_{1,i} p_{2,j} x_{1,i} x_{2,j} +$$

$$(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,i} - \frac{(-1 + T_2) p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{T_1} -$$

$$(-1 + T_2) (-1 + T_1 T_2) p_{1,j} p_{3,j} x_{1,j} x_{3,i} + \frac{(-1 + T_1) (-1 + T_2) T_2 (-1 + T_1 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{T_1} +$$

$$\frac{(-1 + T_1) (-1 + T_2) p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{T_1} - \frac{(-1 + T_1) (-1 + T_2) p_{3,j} x_{3,j}}{T_1^2 T_2} +$$

$$(-1 + T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j} - \frac{(-1 + T_1) (-1 + T_2) T_2 p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_1}$$

In[*]:= CF[(r₀[1, i, j] - (r₀[1, i, j] /. {T₁ -> T₂, T₂ -> T₁, p_{1,i} -> p_{2,i}, p_{2,i} -> p_{1,i}, x_{1,i} -> x_{2,i}, x_{2,i} -> x_{1,i}})] /. a₂ -> -(T₁ + T₂) a₄]

Out[*]=

0

$$\text{In[*]:= CF}[(r_1[1, i, j] - (r_1[1, i, j] / \{T_1 \rightarrow T_2, T_2 \rightarrow T_1, p_{1,i} \rightarrow p_{2,i}, p_{2,i} \rightarrow p_{1,i}, x_{1,i} \rightarrow x_{2,i}, x_{2,i} \rightarrow x_{1,i}\})) / \cdot b_5 \rightarrow b_3]$$

Out[*]=
0

$$\text{In[*]:= CF}[(r_{42}[1, i, j] - (r_{42}[1, i, j] / \{T_1 \rightarrow T_2, T_2 \rightarrow T_1, p_{1,i} \rightarrow p_{2,i}, p_{2,i} \rightarrow p_{1,i}, x_{1,i} \rightarrow x_{2,i}, x_{2,i} \rightarrow x_{1,i}\})) / \cdot \{a_2 \rightarrow - (T_1 + T_2) a_4, b_5 \rightarrow b_3\}]$$

$$\begin{aligned} \text{Out[*]=} & (C_{49} - C_{50}) p_{1,i} x_{1,i} + (C_{52} - C_{53}) p_{1,j} x_{1,i} - \frac{(C_{49} - C_{50} + C_{52} - C_{53}) p_{1,j} x_{1,j}}{T_1} + \\ & (-C_{49} + C_{50}) p_{2,i} x_{2,i} + (-C_{52} + C_{53}) p_{2,j} x_{2,i} + (-C_4 + C_{13}) p_{1,j} p_{2,i} x_{1,i} x_{2,i} + \\ & (C_4 - C_{13}) p_{1,i} p_{2,j} x_{1,i} x_{2,i} + (C_4 - C_{13}) (T_1 - T_2) p_{1,j} p_{2,j} x_{1,i} x_{2,i} + \\ & \frac{(C_4 - C_{13}) (-2 + T_1 + T_2) p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{(-1 + T_1) (-1 + T_2)} + \frac{(C_4 - C_{13}) (-1 + T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{-1 + T_1} + \\ & \frac{(C_{49} - C_{50} + C_{52} - C_{53}) p_{2,j} x_{2,j}}{T_2} - \frac{(C_4 - C_{13}) (-2 + T_1 + T_2) p_{1,i} p_{2,j} x_{1,i} x_{2,j}}{(-1 + T_1) (-1 + T_2)} - \\ & \frac{(C_4 - C_{13}) (-1 + T_1) p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{-1 + T_2} + (C_{14} - C_{15}) p_{1,j} p_{3,i} x_{1,i} x_{3,i} + (C_5 - C_6) p_{1,i} p_{3,j} x_{1,i} x_{3,i} + \\ & \frac{(-C_5 + C_6 - C_{14} + C_{15} + C_5 T_1 - C_6 T_1 + C_{14} T_1 T_2 - C_{15} T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,i} +}{(-1 + T_1) (-1 + T_1 T_2)} \\ & \frac{(-C_5 + C_6 + C_{14} - C_{15} + C_5 T_1 - C_6 T_1 - C_{14} T_1 T_2 + C_{15} T_1 T_2) p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{(-1 + T_1) (-1 + T_1 T_2)} - \\ & \frac{(C_{14} - C_{15}) (-1 + T_1 T_2) p_{1,j} p_{3,j} x_{1,j} x_{3,i}}{-1 + T_1} + (-C_{14} + C_{15}) p_{2,j} p_{3,i} x_{2,i} x_{3,i} + \\ & \frac{(-C_5 + C_6) p_{2,i} p_{3,j} x_{2,i} x_{3,i} + (C_5 - C_6 + C_{14} - C_{15} - C_5 T_2 + C_6 T_2 - C_{14} T_1 T_2 + C_{15} T_1 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,i} -}{(-1 + T_2) (-1 + T_1 T_2)} \\ & \frac{(-C_5 + C_6 + C_{14} - C_{15} + C_5 T_2 - C_6 T_2 - C_{14} T_1 T_2 + C_{15} T_1 T_2) p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{(-1 + T_2) (-1 + T_1 T_2)} + \\ & \frac{(C_{14} - C_{15}) (-1 + T_1 T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_2} - \\ & \frac{(-C_5 + C_6 + C_{14} - C_{15} + C_5 T_1 - C_6 T_1 - C_{14} T_1 T_2 + C_{15} T_1 T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \\ & \frac{(C_5 - C_6) (-1 + T_1) p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{-1 + T_1 T_2} + \\ & \frac{(-C_5 + C_6 + C_{14} - C_{15} + C_5 T_2 - C_6 T_2 - C_{14} T_1 T_2 + C_{15} T_1 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j}}{(-1 + T_2) (-1 + T_1 T_2)} + \\ & \frac{(C_5 - C_6) (-1 + T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{-1 + T_1 T_2} \end{aligned}$$

$$\text{In[*]:= CF}[(r_{42}[1, i, j] - (r_{42}[1, i, j] / \{T_1 \rightarrow T_2, T_2 \rightarrow T_1, p_{1,i} \rightarrow p_{2,i}, p_{2,i} \rightarrow p_{1,i}, x_{1,i} \rightarrow x_{2,i}, x_{2,i} \rightarrow x_{1,i}\})) / \cdot \{a_2 \rightarrow - (T_1 + T_2) a_4, b_5 \rightarrow b_3, c_{50} \rightarrow c_{49}, c_{53} \rightarrow c_{52}, c_{13} \rightarrow c_4, c_6 \rightarrow c_5, c_{15} \rightarrow c_{14}\}]$$

Out[*]=
0

$$\text{In[*]} := \text{CF}[\mathbf{r}_{42}[1, \mathbf{i}, \mathbf{j}] / \cdot \{ \mathbf{a}_2 \rightarrow -(\mathbf{T}_1 + \mathbf{T}_2) \mathbf{a}_4, \mathbf{b}_5 \rightarrow \mathbf{b}_3, \mathbf{c}_{50} \rightarrow \mathbf{c}_{49}, \mathbf{c}_{53} \rightarrow \mathbf{c}_{52}, \mathbf{c}_{13} \rightarrow \mathbf{c}_4, \mathbf{c}_6 \rightarrow \mathbf{c}_5, \mathbf{c}_{15} \rightarrow \mathbf{c}_{14} \} / \cdot \{ \mathbf{c}_{4|5|14|49|51|52|54} \rightarrow \mathbf{0} \}]$$

Out[*]=

$$\begin{aligned} & \frac{\mathbf{a}_4 \mathbf{b}_3 (\mathbf{T}_1 + \mathbf{T}_2 + 2 \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,i}}{-1 + \mathbf{T}_1 \mathbf{T}_2} + \frac{\mathbf{a}_4 \mathbf{b}_3 (\mathbf{T}_1 - \mathbf{T}_2) (1 + \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{1,j} \mathbf{x}_{2,i}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,j} \mathbf{x}_{2,i}}{-1 + \mathbf{T}_1} - \frac{\mathbf{a}_4 \mathbf{b}_3 (1 + \mathbf{T}_1) (\mathbf{T}_1 - \mathbf{T}_2) \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j}}{-1 + \mathbf{T}_2} - \frac{\mathbf{a}_4 \mathbf{b}_3 (-1 - \mathbf{T}_1 - \mathbf{T}_1^2 + \mathbf{T}_2 + 2 \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,i}}{-1 + \mathbf{T}_2} + \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 (-1 - \mathbf{T}_1 + \mathbf{T}_1^2 + \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,i} \mathbf{x}_{1,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \frac{\mathbf{a}_4 \mathbf{b}_3 \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,j} \mathbf{x}_{3,i}}{-1 + \mathbf{T}_2} + \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 (1 - 2 \mathbf{T}_1 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2)} - \frac{\mathbf{a}_4 \mathbf{b}_3 (-1 + \mathbf{T}_1 - \mathbf{T}_2 + 2 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,i}}{-1 + \mathbf{T}_1} + \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 (-1 + \mathbf{T}_1 + \mathbf{T}_1^2 - \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,i} \mathbf{x}_{2,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \frac{\mathbf{a}_4 \mathbf{b}_3 \mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,j} \mathbf{x}_{3,i}}{-1 + \mathbf{T}_1} + \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 (1 - 2 \mathbf{T}_2 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,j} \mathbf{x}_{3,i}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2)} + \frac{\mathbf{a}_4 \mathbf{b}_3 (2 + \mathbf{T}_1 + \mathbf{T}_2) \mathbf{p}_{3,j} \mathbf{x}_{3,j}}{\mathbf{T}_1 \mathbf{T}_2 (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 (1 + \mathbf{T}_1) \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,j}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \frac{\mathbf{a}_4 \mathbf{b}_3 \mathbf{T}_1 (\mathbf{T}_1 - \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,j}}{(-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} - \\ & \frac{\mathbf{a}_4 \mathbf{b}_3 (1 + \mathbf{T}_2) \mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,j}}{(-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2)} + \frac{\mathbf{a}_4 \mathbf{b}_3 (\mathbf{T}_1 - \mathbf{T}_2) \mathbf{T}_2 \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,j}}{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2)} \end{aligned}$$

$$\text{In[*]} := \text{CF}[\mathbf{r}_{42}[1, \mathbf{i}, \mathbf{j}] / \cdot \{ \mathbf{a}_2 \rightarrow -(\mathbf{T}_1 + \mathbf{T}_2) \mathbf{a}_4, \mathbf{b}_5 \rightarrow \mathbf{b}_3, \mathbf{c}_{50} \rightarrow \mathbf{c}_{49}, \mathbf{c}_{53} \rightarrow \mathbf{c}_{52}, \mathbf{c}_{13} \rightarrow \mathbf{c}_4, \mathbf{c}_6 \rightarrow \mathbf{c}_5, \mathbf{c}_{15} \rightarrow \mathbf{c}_{14} \} / \cdot \{ \mathbf{c}_{4|5|14|49|51|52|54} \rightarrow \mathbf{0} \} / \cdot \{ \mathbf{a}_4 \rightarrow \mathbf{b}_3^{-1} (-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2) \}]$$

Out[*]=

$$\begin{aligned} & (-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (\mathbf{T}_1 + \mathbf{T}_2 + 2 \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,i} + \\ & (\mathbf{T}_1 - \mathbf{T}_2) (1 + \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{1,j} \mathbf{x}_{2,i} - (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,j} \mathbf{x}_{2,i} - \\ & (1 + \mathbf{T}_1) (\mathbf{T}_1 - \mathbf{T}_2) \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j} - (-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{1,i} \mathbf{x}_{2,j} - \\ & (-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2) (-1 - \mathbf{T}_1 - \mathbf{T}_1^2 + \mathbf{T}_2 + 2 \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,i} + \\ & (-1 - \mathbf{T}_1 + \mathbf{T}_1^2 + \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_2^2) \mathbf{p}_{1,j} \mathbf{p}_{3,i} \mathbf{x}_{1,j} \mathbf{x}_{3,i} + \\ & (-1 + \mathbf{T}_1) (-1 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,j} \mathbf{x}_{3,i} + (-1 + \mathbf{T}_1 \mathbf{T}_2) (1 - 2 \mathbf{T}_1 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,j} \mathbf{x}_{3,i} - \\ & (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2) (-1 + \mathbf{T}_1 - \mathbf{T}_2 + 2 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,i} + \\ & (-1 + \mathbf{T}_1 + \mathbf{T}_1^2 - \mathbf{T}_2 - \mathbf{T}_1 \mathbf{T}_2 + \mathbf{T}_2^2) \mathbf{p}_{2,j} \mathbf{p}_{3,i} \mathbf{x}_{2,j} \mathbf{x}_{3,i} + (-1 + \mathbf{T}_2) (-1 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,j} \mathbf{x}_{3,i} + \\ & (-1 + \mathbf{T}_1 \mathbf{T}_2) (1 - 2 \mathbf{T}_2 + \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,j} \mathbf{x}_{3,i} + \frac{(-1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) (2 + \mathbf{T}_1 + \mathbf{T}_2) \mathbf{p}_{3,j} \mathbf{x}_{3,j}}{\mathbf{T}_1 \mathbf{T}_2} - \\ & (1 + \mathbf{T}_1) (-1 + \mathbf{T}_2) \mathbf{p}_{1,i} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,j} - (-1 + \mathbf{T}_1) \mathbf{T}_1 (\mathbf{T}_1 - \mathbf{T}_2) \mathbf{p}_{1,j} \mathbf{p}_{3,j} \mathbf{x}_{1,i} \mathbf{x}_{3,j} - \\ & (-1 + \mathbf{T}_1) (1 + \mathbf{T}_2) \mathbf{p}_{2,i} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,j} + (\mathbf{T}_1 - \mathbf{T}_2) (-1 + \mathbf{T}_2) \mathbf{T}_2 \mathbf{p}_{2,j} \mathbf{p}_{3,j} \mathbf{x}_{2,i} \mathbf{x}_{3,j} \end{aligned}$$

Reidemeister 2b for r_0

In[*]:= $\mathbf{x} = \mathbf{0}$;

$r_0[-1, i_, j_] := \text{Evaluate}[\text{Sum}[\mathbf{d}_{++\mathbf{x}} \mathbf{p}_{3,k3} \mathbf{x}_{1,k1} \mathbf{x}_{2,k2}, \{\mathbf{k1}, \{i, j\}\}, \{\mathbf{k2}, \{i, j\}\}, \{\mathbf{k3}, \{i, j\}\}]];$
 $r_0[-1, i, j]$

Out[*]=

$d_1 p_{3,i} x_{1,i} x_{2,i} + d_2 p_{3,j} x_{1,i} x_{2,i} + d_5 p_{3,i} x_{1,j} x_{2,i} + d_6 p_{3,j} x_{1,j} x_{2,i} +$
 $d_3 p_{3,i} x_{1,i} x_{2,j} + d_4 p_{3,j} x_{1,i} x_{2,j} + d_7 p_{3,i} x_{1,j} x_{2,j} + d_8 p_{3,j} x_{1,j} x_{2,j}$

In[*]:= $\mathcal{L}[\mathbf{X}_{i_,j_}[\mathbf{s}_]] := \mathbf{T}_3^s \mathbb{E}[\mathbf{q}[\mathbf{s}, i, j] + r_0[\mathbf{s}, i, j] + \mathbf{0}[\mathbf{e}]];$
 $\mathcal{L}[\mathbf{X}_{i,j}[-1]]$

Out[*]=

$\frac{1}{T_1 T_2} \mathbb{E}[\epsilon \text{Series}[\mathbf{(-p}_{1,i} + \mathbf{p}_{1,1+i}) x_{1,i} + \left(-1 + \frac{1}{T_1}\right) (\mathbf{p}_{1,1+i} - \mathbf{p}_{1,1+j}) x_{1,i} + (\mathbf{-p}_{1,j} + \mathbf{p}_{1,1+j}) x_{1,j} + (\mathbf{-p}_{2,i} + \mathbf{p}_{2,1+i}) x_{2,i} +$
 $\left(-1 + \frac{1}{T_2}\right) (\mathbf{p}_{2,1+i} - \mathbf{p}_{2,1+j}) x_{2,i} + d_1 p_{3,i} x_{1,i} x_{2,i} + d_2 p_{3,j} x_{1,i} x_{2,i} + d_5 p_{3,i} x_{1,j} x_{2,i} +$
 $d_6 p_{3,j} x_{1,j} x_{2,i} + (\mathbf{-p}_{2,j} + \mathbf{p}_{2,1+j}) x_{2,j} + d_3 p_{3,i} x_{1,i} x_{2,j} + d_4 p_{3,j} x_{1,i} x_{2,j} + d_7 p_{3,i} x_{1,j} x_{2,j} +$
 $d_8 p_{3,j} x_{1,j} x_{2,j} + (\mathbf{-p}_{3,i} + \mathbf{p}_{3,1+i}) x_{3,i} + \left(-1 + \frac{1}{T_1 T_2}\right) (\mathbf{p}_{3,1+i} - \mathbf{p}_{3,1+j}) x_{3,i} + (\mathbf{-p}_{3,j} + \mathbf{p}_{3,1+j}) x_{3,j}]]]$

$$In[] := \{lhs\} = \text{Cases} \left[\int \mathcal{F}[i, j] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^+,j^+}[-1]) \, d\{vs_i, vs_j, vs_{i^+}, vs_{j^+}\}, \text{eSeries}[\mathcal{E}_-] \Rightarrow \mathcal{E}, \infty \right]$$

Out[] =

$$\left\{ \begin{aligned} & p_{1,2+i} \pi_{1,i} + p_{1,2+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + \\ & \frac{1}{T_1 T_2} (d_7 + d_3 T_1 - d_7 T_1 + d_5 T_2 - d_7 T_2 + d_1 T_1 T_2 - d_3 T_1 T_2 - d_5 T_1 T_2 + d_7 T_1 T_2) p_{3,2+i} \pi_{1,i} \pi_{2,i} + \\ & \frac{1}{T_1 T_2} \left(-d_7 - d_3 T_1 + d_7 T_1 - d_5 T_2 + d_7 T_2 + a_2 T_1 T_2 - d_1 T_1 T_2 + d_3 T_1 T_2 + d_5 T_1 T_2 + d_8 T_1 T_2 + \right. \\ & \quad d_3 T_1^2 T_2 + d_4 T_1^2 T_2 - d_7 T_1^2 T_2 - d_8 T_1^2 T_2 + d_5 T_1 T_2^2 + d_6 T_1 T_2^2 - d_7 T_1 T_2^2 - d_8 T_1 T_2^2 + \\ & \quad \left. d_1 T_1^2 T_2^2 + d_2 T_1^2 T_2^2 - d_3 T_1^2 T_2^2 - d_4 T_1^2 T_2^2 - d_5 T_1^2 T_2^2 - d_6 T_1^2 T_2^2 + d_7 T_1^2 T_2^2 + d_8 T_1^2 T_2^2 \right) \\ & p_{3,2+j} \pi_{1,i} \pi_{2,i} + \frac{(d_7 + d_5 T_2 - d_7 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i}}{T_1 T_2} - \\ & \frac{1}{T_1 T_2} (d_7 + a_2 T_2 + d_5 T_2 - d_7 T_2 - d_7 T_1 T_2 - d_8 T_1 T_2 + a_4 T_2^2 - d_5 T_1 T_2^2 - d_6 T_1 T_2^2 + d_7 T_1 T_2^2 + d_8 T_1 T_2^2) \\ & p_{3,2+j} \pi_{1,j} \pi_{2,i} + p_{2,2+j} \pi_{2,j} + \frac{(d_7 + d_3 T_1 - d_7 T_1) p_{3,2+i} \pi_{1,i} \pi_{2,j}}{T_1 T_2} + \\ & \frac{1}{T_1 T_2} \left(-d_7 - d_3 T_1 + d_7 T_1 + a_4 T_1 T_2 + d_7 T_1 T_2 + d_8 T_1 T_2 + d_3 T_1^2 T_2 + d_4 T_1^2 T_2 - d_7 T_1^2 T_2 - d_8 T_1^2 T_2 \right) \\ & p_{3,2+j} \pi_{1,i} \pi_{2,j} + \frac{d_7 p_{3,2+i} \pi_{1,j} \pi_{2,j}}{T_1 T_2} + \\ & \left. \frac{(-d_7 + d_7 T_1 T_2 + d_8 T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,j}}{T_1 T_2} + p_{3,2+i} \pi_{3,i} + p_{3,2+j} \pi_{3,j} \right\} \end{aligned} \right.$$

$$In[] := \text{eqn} = \text{CF}[\text{lhs} - (p_{1,2+i} \pi_{1,i} + p_{1,2+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + p_{2,2+j} \pi_{2,j} + p_{3,2+i} \pi_{3,i} + p_{3,2+j} \pi_{3,j})]$$

Out[] =

$$\left\{ \begin{aligned} & \frac{1}{T_1 T_2} (d_7 + d_3 T_1 - d_7 T_1 + d_5 T_2 - d_7 T_2 + d_1 T_1 T_2 - d_3 T_1 T_2 - d_5 T_1 T_2 + d_7 T_1 T_2) p_{3,2+i} \pi_{1,i} \pi_{2,i} + \\ & \frac{1}{T_1 T_2} \left(-d_7 - d_3 T_1 + d_7 T_1 - d_5 T_2 + d_7 T_2 + a_2 T_1 T_2 - d_1 T_1 T_2 + d_3 T_1 T_2 + d_5 T_1 T_2 + \right. \\ & \quad d_8 T_1 T_2 + d_3 T_1^2 T_2 + d_4 T_1^2 T_2 - d_7 T_1^2 T_2 - d_8 T_1^2 T_2 + d_5 T_1 T_2^2 + d_6 T_1 T_2^2 - d_7 T_1 T_2^2 - d_8 T_1 T_2^2 + \\ & \quad \left. d_1 T_1^2 T_2^2 + d_2 T_1^2 T_2^2 - d_3 T_1^2 T_2^2 - d_4 T_1^2 T_2^2 - d_5 T_1^2 T_2^2 - d_6 T_1^2 T_2^2 + d_7 T_1^2 T_2^2 + d_8 T_1^2 T_2^2 \right) \\ & p_{3,2+j} \pi_{1,i} \pi_{2,i} + \frac{(d_7 + d_5 T_2 - d_7 T_2) p_{3,2+i} \pi_{1,j} \pi_{2,i}}{T_1 T_2} - \\ & \frac{1}{T_1 T_2} (d_7 + a_2 T_2 + d_5 T_2 - d_7 T_2 - d_7 T_1 T_2 - d_8 T_1 T_2 + a_4 T_2^2 - d_5 T_1 T_2^2 - d_6 T_1 T_2^2 + d_7 T_1 T_2^2 + d_8 T_1 T_2^2) \\ & p_{3,2+j} \pi_{1,j} \pi_{2,i} + \frac{(d_7 + d_3 T_1 - d_7 T_1) p_{3,2+i} \pi_{1,i} \pi_{2,j}}{T_1 T_2} + \\ & \frac{1}{T_1 T_2} \left(-d_7 - d_3 T_1 + d_7 T_1 + a_4 T_1 T_2 + d_7 T_1 T_2 + d_8 T_1 T_2 + d_3 T_1^2 T_2 + d_4 T_1^2 T_2 - d_7 T_1^2 T_2 - d_8 T_1^2 T_2 \right) \\ & p_{3,2+j} \pi_{1,i} \pi_{2,j} + \frac{d_7 p_{3,2+i} \pi_{1,j} \pi_{2,j}}{T_1 T_2} + \frac{(-d_7 + d_7 T_1 T_2 + d_8 T_1 T_2) p_{3,2+j} \pi_{1,j} \pi_{2,j}}{T_1 T_2} \end{aligned} \right.$$

In[*]:= **cvs = Union@Cases [eqn, p__ | π __, ∞]**

Out[*]=
 $\{p_{3,2+i}, p_{3,2+j}, \pi_{1,i}, \pi_{1,j}, \pi_{2,i}, \pi_{2,j}\}$

In[*]:= **eqns = CoefficientRules [eqn, cvs] /. (_ -> c_) :-> (c == 0)**

Out[*]=

$$\left\{ \begin{aligned} & d_1 - d_3 - d_5 + d_7 + \frac{d_5}{T_1} - \frac{d_7}{T_1} + \frac{d_3}{T_2} - \frac{d_7}{T_2} + \frac{d_7}{T_1 T_2} == 0, \frac{d_3}{T_2} - \frac{d_7}{T_2} + \frac{d_7}{T_1 T_2} == 0, \frac{d_5}{T_1} - \frac{d_7}{T_1} + \frac{d_7}{T_1 T_2} == 0, \frac{d_7}{T_1 T_2} == 0, \\ & a_2 - d_1 + d_3 + d_5 + d_8 - \frac{d_5}{T_1} + \frac{d_7}{T_1} + d_3 T_1 + d_4 T_1 - d_7 T_1 - d_8 T_1 - \frac{d_3}{T_2} + \frac{d_7}{T_2} - \frac{d_7}{T_1 T_2} + d_5 T_2 + d_6 T_2 - d_7 T_2 - \\ & d_8 T_2 + d_1 T_1 T_2 + d_2 T_1 T_2 - d_3 T_1 T_2 - d_4 T_1 T_2 - d_5 T_1 T_2 - d_6 T_1 T_2 + d_7 T_1 T_2 + d_8 T_1 T_2 == 0, \\ & a_4 + d_7 + d_8 + d_3 T_1 + d_4 T_1 - d_7 T_1 - d_8 T_1 - \frac{d_3}{T_2} + \frac{d_7}{T_2} - \frac{d_7}{T_1 T_2} == 0, \\ & d_7 + d_8 - \frac{a_2}{T_1} - \frac{d_5}{T_1} + \frac{d_7}{T_1} - \frac{d_7}{T_1 T_2} + d_5 T_2 + d_6 T_2 - d_7 T_2 - d_8 T_2 - \frac{a_4 T_2}{T_1} == 0, d_7 + d_8 - \frac{d_7}{T_1 T_2} == 0 \end{aligned} \right\}$$

In[*]:= **vars = Union@Cases [r_0[-1, i, j], d_, ∞]**

Out[*]=
 $\{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8\}$

In[*]:= **{sol} = Solve [eqns, vars]**

Out[*]=

$$\left\{ \left\{ d_1 \rightarrow 0, d_2 \rightarrow -\frac{a_2 - a_4 T_1 + a_4 T_2}{T_1^2 T_2}, d_3 \rightarrow 0, d_4 \rightarrow -\frac{a_4}{T_1}, d_5 \rightarrow 0, d_6 \rightarrow -\frac{-a_2 - a_4 T_2}{T_1 T_2}, d_7 \rightarrow 0, d_8 \rightarrow 0 \right\} \right\}$$

In[*]:= **sol /. (v_ -> val_) :-> (v = CF [val]);**

In[*]:= **r_0[-1, i, j]**

Out[*]=

$$\frac{(-a_2 + a_4 T_1 - a_4 T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} - \frac{a_4 p_{3,j} x_{1,i} x_{2,j}}{T_1}$$

Reidemeister 2b for r_1

In[*]:= **x = 0;**

r_1[-1, i_, j_] := Evaluate [Sum [

$$e_{++x} x_{3,k3} p_{1,k1} p_{2,k2},$$

$$\{k1, \{i, j\}\}, \{k2, \{i, j\}\}, \{k3, \{i, j\}\}$$

$$]];$$

r_1[-1, i, j]

Out[*]=

$$e_1 p_{1,i} p_{2,i} x_{3,i} + e_5 p_{1,j} p_{2,i} x_{3,i} + e_3 p_{1,i} p_{2,j} x_{3,i} + e_7 p_{1,j} p_{2,j} x_{3,i} +$$

$$e_2 p_{1,i} p_{2,i} x_{3,j} + e_6 p_{1,j} p_{2,i} x_{3,j} + e_4 p_{1,i} p_{2,j} x_{3,j} + e_8 p_{1,j} p_{2,j} x_{3,j}$$

$$\text{In[*]} := \mathcal{L}[\mathbf{X}_{i,j}[\mathbf{s}_-]] := \mathbf{T}_3^s \mathbb{E}[\mathbf{q}[\mathbf{s}, \mathbf{i}, \mathbf{j}] + \mathbf{e} \mathbf{r}_1[\mathbf{s}, \mathbf{i}, \mathbf{j}] + \mathbf{0}[\mathbf{e}]^2];$$

$$\mathcal{L}[\mathbf{X}_{i,j}[\mathbf{1}]]$$

$$\mathcal{L}[\mathbf{X}_{i,j}[-\mathbf{1}]]$$

Out[*]=

$$\mathbf{T}_1 \mathbf{T}_2 \mathbb{E}[\text{Series}[(-\mathbf{p}_{1,i} + \mathbf{p}_{1,1+i}) \mathbf{x}_{1,i} + (-\mathbf{1} + \mathbf{T}_1) (\mathbf{p}_{1,1+i} - \mathbf{p}_{1,1+j}) \mathbf{x}_{1,i} + (-\mathbf{p}_{1,j} + \mathbf{p}_{1,1+j}) \mathbf{x}_{1,j} +$$

$$(-\mathbf{p}_{2,i} + \mathbf{p}_{2,1+i}) \mathbf{x}_{2,i} + (-\mathbf{1} + \mathbf{T}_2) (\mathbf{p}_{2,1+i} - \mathbf{p}_{2,1+j}) \mathbf{x}_{2,i} + (-\mathbf{p}_{2,j} + \mathbf{p}_{2,1+j}) \mathbf{x}_{2,j} +$$

$$(-\mathbf{p}_{3,i} + \mathbf{p}_{3,1+i}) \mathbf{x}_{3,i} + (-\mathbf{1} + \mathbf{T}_1 \mathbf{T}_2) (\mathbf{p}_{3,1+i} - \mathbf{p}_{3,1+j}) \mathbf{x}_{3,i} + (-\mathbf{p}_{3,j} + \mathbf{p}_{3,1+j}) \mathbf{x}_{3,j},$$

$$\mathbf{b}_5 \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{3,i} + \mathbf{b}_3 \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{3,i} + (-\mathbf{b}_3 - \mathbf{b}_5) \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{3,i}]]]$$

Out[*]=

$$\frac{1}{\mathbf{T}_1 \mathbf{T}_2} \mathbb{E}[\text{Series}[(-\mathbf{p}_{1,i} + \mathbf{p}_{1,1+i}) \mathbf{x}_{1,i} + \left(-\mathbf{1} + \frac{1}{\mathbf{T}_1}\right) (\mathbf{p}_{1,1+i} - \mathbf{p}_{1,1+j}) \mathbf{x}_{1,i} + (-\mathbf{p}_{1,j} + \mathbf{p}_{1,1+j}) \mathbf{x}_{1,j} +$$

$$(-\mathbf{p}_{2,i} + \mathbf{p}_{2,1+i}) \mathbf{x}_{2,i} + \left(-\mathbf{1} + \frac{1}{\mathbf{T}_2}\right) (\mathbf{p}_{2,1+i} - \mathbf{p}_{2,1+j}) \mathbf{x}_{2,i} + (-\mathbf{p}_{2,j} + \mathbf{p}_{2,1+j}) \mathbf{x}_{2,j} +$$

$$(-\mathbf{p}_{3,i} + \mathbf{p}_{3,1+i}) \mathbf{x}_{3,i} + \left(-\mathbf{1} + \frac{1}{\mathbf{T}_1 \mathbf{T}_2}\right) (\mathbf{p}_{3,1+i} - \mathbf{p}_{3,1+j}) \mathbf{x}_{3,i} + (-\mathbf{p}_{3,j} + \mathbf{p}_{3,1+j}) \mathbf{x}_{3,j},$$

$$\mathbf{e}_1 \mathbf{p}_{1,i} \mathbf{p}_{2,i} \mathbf{x}_{3,i} + \mathbf{e}_5 \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{3,i} + \mathbf{e}_3 \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{3,i} + \mathbf{e}_7 \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{3,i} +$$

$$\mathbf{e}_2 \mathbf{p}_{1,i} \mathbf{p}_{2,i} \mathbf{x}_{3,j} + \mathbf{e}_6 \mathbf{p}_{1,j} \mathbf{p}_{2,i} \mathbf{x}_{3,j} + \mathbf{e}_4 \mathbf{p}_{1,i} \mathbf{p}_{2,j} \mathbf{x}_{3,j} + \mathbf{e}_8 \mathbf{p}_{1,j} \mathbf{p}_{2,j} \mathbf{x}_{3,j}]]]$$

$$\text{In[*]} := \{\text{eqn}\} = \{\text{lhs}\} =$$

$$\text{Cases}\left[\int \mathcal{F}[\mathbf{i}, \mathbf{j}] \times \mathcal{L} / @ (\mathbf{X}_{i,j}[\mathbf{1}] \mathbf{X}_{i^+,j^+}[-\mathbf{1}]) \text{d}\{\mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_{i^+}, \mathbf{vs}_{j^+}\}, \text{Series}[_, \mathcal{E}_-] \mapsto \mathcal{E}, \infty\right]$$

Out[*]=

$$\left\{ \frac{(\mathbf{e}_2 + \mathbf{e}_1 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{e}_2 \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,2+i} \mathbf{p}_{2,2+i} \pi_{3,i}}{\mathbf{T}_1 \mathbf{T}_2} + \right.$$

$$\frac{1}{\mathbf{T}_1 \mathbf{T}_2} (-\mathbf{e}_2 + \mathbf{e}_2 \mathbf{T}_1 + \mathbf{e}_6 \mathbf{T}_1 + \mathbf{b}_5 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{e}_1 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_1 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{e}_2 \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{e}_5 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{e}_6 \mathbf{T}_1^2 \mathbf{T}_2)$$

$$\mathbf{p}_{1,2+j} \mathbf{p}_{2,2+i} \pi_{3,i} +$$

$$\frac{1}{\mathbf{T}_1 \mathbf{T}_2} (-\mathbf{e}_2 + \mathbf{e}_2 \mathbf{T}_2 + \mathbf{e}_4 \mathbf{T}_2 + \mathbf{b}_3 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{e}_1 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_1 \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{e}_2 \mathbf{T}_1 \mathbf{T}_2^2 + \mathbf{e}_3 \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{e}_4 \mathbf{T}_1 \mathbf{T}_2^2) \mathbf{p}_{1,2+i}$$

$$\mathbf{p}_{2,2+j} \pi_{3,i} - \frac{1}{\mathbf{T}_1 \mathbf{T}_2} (-\mathbf{e}_2 + \mathbf{e}_2 \mathbf{T}_1 + \mathbf{e}_6 \mathbf{T}_1 + \mathbf{e}_2 \mathbf{T}_2 + \mathbf{e}_4 \mathbf{T}_2 + \mathbf{b}_3 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{b}_5 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{e}_1 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{e}_4 \mathbf{T}_1 \mathbf{T}_2 -$$

$$\mathbf{e}_6 \mathbf{T}_1 \mathbf{T}_2 - \mathbf{e}_8 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_1 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{e}_2 \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{e}_5 \mathbf{T}_1^2 \mathbf{T}_2 - \mathbf{e}_6 \mathbf{T}_1^2 \mathbf{T}_2 + \mathbf{e}_1 \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{e}_2 \mathbf{T}_1 \mathbf{T}_2^2 + \mathbf{e}_3 \mathbf{T}_1 \mathbf{T}_2^2 - \mathbf{e}_4 \mathbf{T}_1 \mathbf{T}_2^2 -$$

$$\mathbf{e}_1 \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{e}_2 \mathbf{T}_1^2 \mathbf{T}_2^2 - \mathbf{e}_3 \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{e}_4 \mathbf{T}_1^2 \mathbf{T}_2^2 - \mathbf{e}_5 \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{e}_6 \mathbf{T}_1^2 \mathbf{T}_2^2 - \mathbf{e}_7 \mathbf{T}_1^2 \mathbf{T}_2^2 + \mathbf{e}_8 \mathbf{T}_1^2 \mathbf{T}_2^2) \mathbf{p}_{1,2+j} \mathbf{p}_{2,2+j} \pi_{3,i} +$$

$$\frac{\mathbf{e}_2 \mathbf{p}_{1,2+i} \mathbf{p}_{2,2+i} \pi_{3,j}}{\mathbf{T}_1 \mathbf{T}_2} + \frac{(-\mathbf{e}_2 + \mathbf{e}_2 \mathbf{T}_1 + \mathbf{e}_6 \mathbf{T}_1) \mathbf{p}_{1,2+j} \mathbf{p}_{2,2+i} \pi_{3,j}}{\mathbf{T}_1 \mathbf{T}_2} +$$

$$\frac{(-\mathbf{e}_2 + \mathbf{e}_2 \mathbf{T}_2 + \mathbf{e}_4 \mathbf{T}_2) \mathbf{p}_{1,2+i} \mathbf{p}_{2,2+j} \pi_{3,j}}{\mathbf{T}_1 \mathbf{T}_2} +$$

$$\left. \frac{1}{\mathbf{T}_1 \mathbf{T}_2} (\mathbf{e}_2 - \mathbf{e}_2 \mathbf{T}_1 - \mathbf{e}_6 \mathbf{T}_1 - \mathbf{e}_2 \mathbf{T}_2 - \mathbf{e}_4 \mathbf{T}_2 + \mathbf{e}_2 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_4 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_6 \mathbf{T}_1 \mathbf{T}_2 + \mathbf{e}_8 \mathbf{T}_1 \mathbf{T}_2) \mathbf{p}_{1,2+j} \mathbf{p}_{2,2+j} \pi_{3,j} \right\}$$

$$\text{In[*]} := \text{cvs} = \text{Union}@\text{Cases}[\text{eqn}, \mathbf{p}_{__} | \pi_{__}, \infty]$$

Out[*]=

$$\{\mathbf{p}_{1,2+i}, \mathbf{p}_{1,2+j}, \mathbf{p}_{2,2+i}, \mathbf{p}_{2,2+j}, \pi_{3,i}, \pi_{3,j}\}$$

In[*]:= eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0)

Out[*]=

$$\left\{ \begin{aligned} e_1 - e_2 + \frac{e_2}{T_1 T_2} &= 0, \quad \frac{e_2}{T_1 T_2} = 0, \\ b_3 - e_1 + e_2 + \frac{e_2}{T_1} + \frac{e_4}{T_1} - \frac{e_2}{T_1 T_2} + e_1 T_2 - e_2 T_2 + e_3 T_2 - e_4 T_2 &= 0, \quad \frac{e_2}{T_1} + \frac{e_4}{T_1} - \frac{e_2}{T_1 T_2} = 0, \\ b_5 - e_1 + e_2 + e_1 T_1 - e_2 T_1 + e_5 T_1 - e_6 T_1 + \frac{e_2}{T_2} + \frac{e_6}{T_2} - \frac{e_2}{T_1 T_2} &= 0, \quad \frac{e_2}{T_2} + \frac{e_6}{T_2} - \frac{e_2}{T_1 T_2} = 0, \\ -b_3 - b_5 + e_1 + e_4 + e_6 + e_8 - \frac{e_2}{T_1} - \frac{e_4}{T_1} - e_1 T_1 + e_2 T_1 - e_5 T_1 + e_6 T_1 - \frac{e_2}{T_2} - \frac{e_6}{T_2} + \frac{e_2}{T_1 T_2} - e_1 T_2 + e_2 T_2 - \\ e_3 T_2 + e_4 T_2 + e_1 T_1 T_2 - e_2 T_1 T_2 + e_3 T_1 T_2 - e_4 T_1 T_2 + e_5 T_1 T_2 - e_6 T_1 T_2 + e_7 T_1 T_2 - e_8 T_1 T_2 &= 0, \\ e_2 + e_4 + e_6 + e_8 - \frac{e_2}{T_1} - \frac{e_4}{T_1} - \frac{e_2}{T_2} - \frac{e_6}{T_2} + \frac{e_2}{T_1 T_2} &= 0 \end{aligned} \right\}$$

In[*]:= vars = Union@Cases[r1[-1, i, j], e_, ∞]

Out[*]=

$$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$$

In[*]:= {sol} = Solve[eqns, vars]

Out[*]=

$$\left\{ \left\{ e_1 \rightarrow 0, e_2 \rightarrow 0, e_3 \rightarrow -\frac{b_3}{T_2}, e_4 \rightarrow 0, e_5 \rightarrow -\frac{b_5}{T_1}, e_6 \rightarrow 0, e_7 \rightarrow -\frac{-b_3 T_1 - b_5 T_2}{T_1 T_2}, e_8 \rightarrow 0 \right\} \right\}$$

In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);

In[*]:= r1[-1, i, j]

Out[*]=

$$-\frac{b_5 p_{1,j} p_{2,i} x_{3,i}}{T_1} - \frac{b_3 p_{1,i} p_{2,j} x_{3,i}}{T_2} + \frac{(b_3 T_1 + b_5 T_2) p_{1,j} p_{2,j} x_{3,i}}{T_1 T_2}$$

Reidemeister 2b in full

$$\text{In[*]} := \mathcal{L}[\mathbf{X}_{i,j}[s_-]] := T_3^5 \mathbb{E}[\mathbf{q}[s, i, j] + r_0[s, i, j] + \epsilon r_1[s, i, j] + \mathbf{0}[\epsilon]^2];$$

$$\mathcal{L}[\mathbf{X}_{i,j}[\mathbf{1}]]$$

$$\mathcal{L}[\mathbf{X}_{i,j}[-\mathbf{1}]]$$

Out[*]=

$$T_1 T_2 \mathbb{E} \left[\epsilon \text{Series} \left[\begin{aligned} &(-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + (-p_{1,j} + p_{1,1+j}) x_{1,j} + (-p_{2,i} + p_{2,1+i}) x_{2,i} + \\ &(-1 + T_2) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + a_2 p_{3,j} x_{1,i} x_{2,i} - \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1} + (-p_{2,j} + p_{2,1+j}) x_{2,j} + \\ &a_4 p_{3,j} x_{1,i} x_{2,j} + (-p_{3,i} + p_{3,1+i}) x_{3,i} + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j}, \\ &b_5 p_{1,j} p_{2,i} x_{3,i} + b_3 p_{1,i} p_{2,j} x_{3,i} + (-b_3 - b_5) p_{1,j} p_{2,j} x_{3,i} \end{aligned} \right] \right]$$

Out[*]=

$$\frac{1}{T_1 T_2} \mathbb{E} \left[\epsilon \text{Series} \left[\begin{aligned} &(-p_{1,i} + p_{1,1+i}) x_{1,i} + \left(-1 + \frac{1}{T_1}\right) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + \\ &(-p_{1,j} + p_{1,1+j}) x_{1,j} + (-p_{2,i} + p_{2,1+i}) x_{2,i} + \left(-1 + \frac{1}{T_2}\right) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + \\ &\frac{(-a_2 + a_4 T_1 - a_4 T_2) p_{3,j} x_{1,i} x_{2,i}}{T_1^2 T_2} + \frac{(a_2 + a_4 T_2) p_{3,j} x_{1,j} x_{2,i}}{T_1 T_2} + (-p_{2,j} + p_{2,1+j}) x_{2,j} - \\ &\frac{a_4 p_{3,j} x_{1,i} x_{2,j}}{T_1} + (-p_{3,i} + p_{3,1+i}) x_{3,i} + \left(-1 + \frac{1}{T_1 T_2}\right) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j}, \\ &\frac{(-b_5 T_2 p_{1,j} p_{2,i} - b_3 T_1 p_{1,i} p_{2,j} + b_3 T_1 p_{1,j} p_{2,j} + b_5 T_2 p_{1,j} p_{2,j}) x_{3,i}}{T_1 T_2} \end{aligned} \right] \right]$$

$$\text{In[*]} := \int \mathcal{F}[\mathbf{i}, \mathbf{j}] \times \mathcal{L} / @ (\mathbf{X}_{i,j}[\mathbf{1}] \mathbf{X}_{i^+,j^+}[-\mathbf{1}]) \, d\{\mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_{i^+}, \mathbf{vs}_{j^+}\}$$

Out[*]=

$$\mathbb{E} \left[\epsilon \text{Series} \left[\begin{aligned} &p_{1,2+i} \pi_{1,i} + p_{1,2+j} \pi_{1,j} + p_{2,2+i} \pi_{2,i} + p_{2,2+j} \pi_{2,j} + p_{3,2+i} \pi_{3,i} + p_{3,2+j} \pi_{3,j}, \\ &\frac{(-a_2 b_5 + a_4 b_3 T_1 - a_4 b_5 T_2) p_{3,2+j} \pi_{3,i}}{T_1} + a_4 b_3 p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{3,i} + (-a_4 b_3 + a_2 b_5 - a_4 b_5) p_{1,2+j} \\ &p_{3,2+j} \pi_{1,i} \pi_{3,i} - \frac{b_5 (a_2 + a_4 T_2) p_{1,2+j} p_{3,2+j} \pi_{1,j} \pi_{3,i}}{T_1} - \frac{b_5 (a_2 + a_4 T_2) p_{2,2+i} p_{3,2+j} \pi_{2,i} \pi_{3,i}}{T_1} + \\ &\frac{(a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 + a_4 b_3 T_2 + a_4 b_5 T_2) p_{2,2+j} p_{3,2+j} \pi_{2,i} \pi_{3,i}}{T_1} + \\ &\frac{1}{T_1} (-a_2 a_4 b_3 + a_2^2 b_5 - a_2 a_4 b_5 - a_2 a_4 b_3 T_1 - a_4^2 b_3 T_2 + a_2 a_4 b_5 T_2 - a_4^2 b_5 T_2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,i} \pi_{3,i} - \\ &\frac{b_5 (a_2 + a_4 T_2)^2 p_{3,2+j}^2 \pi_{1,j} \pi_{2,i} \pi_{3,i}}{T_1^2} + a_4 b_3 p_{2,2+j} p_{3,2+j} \pi_{2,j} \pi_{3,i} - a_4^2 b_3 p_{3,2+j}^2 \pi_{1,i} \pi_{2,j} \pi_{3,i} \end{aligned} \right] \right]$$

In[*]:= Echo /@ (CF@CoefficientList [Cases [∫ ℱ[i, j] × ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) dℓ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}}, eSeries[_ , ε₋] ⇒ ε, ∞] [[1]] /. {a_{i₋} ⇒ λ a_i, b_{i₋} ⇒ λ b_i}, λ]);

» 0

» 0

$$\begin{aligned} & \gg \frac{(-a_2 b_5 + a_4 b_3 T_1 - a_4 b_5 T_2) p_{3,2+j} \pi_{3,i}}{T_1} + a_4 b_3 p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{3,i} + (-a_4 b_3 + a_2 b_5 - a_4 b_5) p_{1,2+j} p_{3,2+j} \pi_{1,i} \pi_{3,i} - \\ & \frac{b_5 (a_2 + a_4 T_2) p_{1,2+j} p_{3,2+j} \pi_{1,j} \pi_{3,i}}{T_1} - \frac{b_5 (a_2 + a_4 T_2) p_{2,2+i} p_{3,2+j} \pi_{2,i} \pi_{3,i}}{T_1} + \\ & \frac{(a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 + a_4 b_3 T_2 + a_4 b_5 T_2) p_{2,2+j} p_{3,2+j} \pi_{2,i} \pi_{3,i}}{T_1} + a_4 b_3 p_{2,2+j} p_{3,2+j} \pi_{2,j} \pi_{3,i} \\ & \gg \frac{1}{T_1} (-a_2 a_4 b_3 + a_2^2 b_5 - a_2 a_4 b_5 - a_2 a_4 b_3 T_1 - a_4^2 b_3 T_2 + a_2 a_4 b_5 T_2 - a_4^2 b_5 T_2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,i} \pi_{3,i} - \\ & \frac{b_5 (a_2 + a_4 T_2)^2 p_{3,2+j}^2 \pi_{1,j} \pi_{2,i} \pi_{3,i}}{T_1^2} - a_4^2 b_3 p_{3,2+j}^2 \pi_{1,i} \pi_{2,j} \pi_{3,i} \end{aligned}$$

In[*]:= err = CF@Coefficient [Cases [∫ ℱ[i, j] × ℒ /@ (X_{i,j}[1] X_{i⁺,j⁺}[-1]) dℓ{vs_i, vs_j, vs_{i⁺}, vs_{j⁺}}, eSeries[_ , ε₋] ⇒ ε, ∞] [[1]] /. {a_{i₋} ⇒ λ a_i, b_{i₋} ⇒ λ b_i}, λ²]

Out[*]=

$$\begin{aligned} & \frac{(-a_2 b_5 + a_4 b_3 T_1 - a_4 b_5 T_2) p_{3,2+j} \pi_{3,i}}{T_1} + \\ & a_4 b_3 p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{3,i} + (-a_4 b_3 + a_2 b_5 - a_4 b_5) p_{1,2+j} p_{3,2+j} \pi_{1,i} \pi_{3,i} - \\ & \frac{b_5 (a_2 + a_4 T_2) p_{1,2+j} p_{3,2+j} \pi_{1,j} \pi_{3,i}}{T_1} - \frac{b_5 (a_2 + a_4 T_2) p_{2,2+i} p_{3,2+j} \pi_{2,i} \pi_{3,i}}{T_1} + \\ & \frac{(a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 + a_4 b_3 T_2 + a_4 b_5 T_2) p_{2,2+j} p_{3,2+j} \pi_{2,i} \pi_{3,i}}{T_1} + a_4 b_3 p_{2,2+j} p_{3,2+j} \pi_{2,j} \pi_{3,i} \end{aligned}$$

```
In[*]:= x = 0;
r42[-1, i_, j_] = Evaluate[Plus[
  Sum[
    f_{++x} X_{v1,k1} P_{v1,k2} X_{v2,k3} P_{v2,k4},
    {k1, {i, j}}, {k2, {i, j}}, {k3, {i, j}}, {k4, {i, j}}, {v1, 2}, {v2, v1 + 1, 3}
  ],
  Sum[
    f_{++x} X_{v,k1} P_{v,k2},
    {k1, {i, j}}, {k2, {i, j}}, {v, 3}
  ]
]]
```

Out[*]=

$$\begin{aligned}
 & f_{49} p_{1,i} x_{1,i} + f_{52} p_{1,j} x_{1,i} + f_{55} p_{1,i} x_{1,j} + f_{58} p_{1,j} x_{1,j} + f_{50} p_{2,i} x_{2,i} + f_{53} p_{2,j} x_{2,i} + \\
 & f_1 p_{1,i} p_{2,i} x_{1,i} x_{2,i} + f_{13} p_{1,j} p_{2,i} x_{1,i} x_{2,i} + f_4 p_{1,i} p_{2,j} x_{1,i} x_{2,i} + f_{16} p_{1,j} p_{2,j} x_{1,i} x_{2,i} + \\
 & f_{25} p_{1,i} p_{2,i} x_{1,j} x_{2,i} + f_{37} p_{1,j} p_{2,i} x_{1,j} x_{2,i} + f_{28} p_{1,i} p_{2,j} x_{1,j} x_{2,i} + f_{40} p_{1,j} p_{2,j} x_{1,j} x_{2,i} + \\
 & f_{56} p_{2,i} x_{2,j} + f_{59} p_{2,j} x_{2,j} + f_7 p_{1,i} p_{2,i} x_{1,i} x_{2,j} + f_{19} p_{1,j} p_{2,i} x_{1,i} x_{2,j} + f_{10} p_{1,i} p_{2,j} x_{1,i} x_{2,j} + \\
 & f_{22} p_{1,j} p_{2,j} x_{1,i} x_{2,j} + f_{31} p_{1,i} p_{2,i} x_{1,j} x_{2,j} + f_{43} p_{1,j} p_{2,i} x_{1,j} x_{2,j} + f_{34} p_{1,i} p_{2,j} x_{1,j} x_{2,j} + \\
 & f_{46} p_{1,j} p_{2,j} x_{1,j} x_{2,j} + f_{51} p_{3,i} x_{3,i} + f_{54} p_{3,j} x_{3,i} + f_2 p_{1,i} p_{3,i} x_{1,i} x_{3,i} + f_{14} p_{1,j} p_{3,i} x_{1,i} x_{3,i} + \\
 & f_5 p_{1,i} p_{3,j} x_{1,i} x_{3,i} + f_{17} p_{1,j} p_{3,j} x_{1,i} x_{3,i} + f_{26} p_{1,i} p_{3,i} x_{1,j} x_{3,i} + f_{38} p_{1,j} p_{3,i} x_{1,j} x_{3,i} + \\
 & f_{29} p_{1,i} p_{3,j} x_{1,j} x_{3,i} + f_{41} p_{1,j} p_{3,j} x_{1,j} x_{3,i} + f_3 p_{2,i} p_{3,i} x_{2,i} x_{3,i} + f_{15} p_{2,j} p_{3,i} x_{2,i} x_{3,i} + \\
 & f_6 p_{2,i} p_{3,j} x_{2,i} x_{3,i} + f_{18} p_{2,j} p_{3,j} x_{2,i} x_{3,i} + f_{27} p_{2,i} p_{3,i} x_{2,j} x_{3,i} + f_{39} p_{2,j} p_{3,i} x_{2,j} x_{3,i} + \\
 & f_{30} p_{2,i} p_{3,j} x_{2,j} x_{3,i} + f_{42} p_{2,j} p_{3,j} x_{2,j} x_{3,i} + f_{57} p_{3,i} x_{3,j} + f_{60} p_{3,j} x_{3,j} + \\
 & f_8 p_{1,i} p_{3,i} x_{1,i} x_{3,j} + f_{20} p_{1,j} p_{3,i} x_{1,i} x_{3,j} + f_{11} p_{1,i} p_{3,j} x_{1,i} x_{3,j} + f_{23} p_{1,j} p_{3,j} x_{1,i} x_{3,j} + \\
 & f_{32} p_{1,i} p_{3,i} x_{1,j} x_{3,j} + f_{44} p_{1,j} p_{3,i} x_{1,j} x_{3,j} + f_{35} p_{1,i} p_{3,j} x_{1,j} x_{3,j} + f_{47} p_{1,j} p_{3,j} x_{1,j} x_{3,j} + \\
 & f_9 p_{2,i} p_{3,i} x_{2,i} x_{3,j} + f_{21} p_{2,j} p_{3,i} x_{2,i} x_{3,j} + f_{12} p_{2,i} p_{3,j} x_{2,i} x_{3,j} + f_{24} p_{2,j} p_{3,j} x_{2,i} x_{3,j} + \\
 & f_{33} p_{2,i} p_{3,i} x_{2,j} x_{3,j} + f_{45} p_{2,j} p_{3,i} x_{2,j} x_{3,j} + f_{36} p_{2,i} p_{3,j} x_{2,j} x_{3,j} + f_{48} p_{2,j} p_{3,j} x_{2,j} x_{3,j}
 \end{aligned}$$

```
In[*]:= L[X_{i_,j_}[s_]] := T_3^5 E[q[s, i, j] + e r42[s, i, j] + O[e]^2];
L[X_{i,j}[1]]
L[X_{i,j}[-1]]
```

Out[*]=

$$\begin{aligned}
 & T_1 T_2 E \left[e \text{Series} \left[(-p_{1,i} + p_{1,1+i}) x_{1,i} + (-1 + T_1) (p_{1,1+i} - p_{1,1+j}) x_{1,i} + (-p_{1,j} + p_{1,1+j}) x_{1,j} + \right. \right. \\
 & \quad (-p_{2,i} + p_{2,1+i}) x_{2,i} + (-1 + T_2) (p_{2,1+i} - p_{2,1+j}) x_{2,i} + (-p_{2,j} + p_{2,1+j}) x_{2,j} + \\
 & \quad \left. (-p_{3,i} + p_{3,1+i}) x_{3,i} + (-1 + T_1 T_2) (p_{3,1+i} - p_{3,1+j}) x_{3,i} + (-p_{3,j} + p_{3,1+j}) x_{3,j}, \right. \\
 & \quad \left. C_{49} p_{1,i} x_{1,i} + C_{52} p_{1,j} x_{1,i} - \frac{(C_{49} + C_{52}) p_{1,j} x_{1,j}}{T_1} + C_{50} p_{2,i} x_{2,i} + C_{53} p_{2,j} x_{2,i} + C_{13} p_{1,j} p_{2,i} x_{1,i} x_{2,i} + \right. \\
 & \quad \left. C_4 p_{1,i} p_{2,j} x_{1,i} x_{2,i} + \frac{1}{-1 + T_1 T_2} (-a_2 b_3 + c_4 + c_{13} - c_4 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 - c_{13} T_2 + \right. \\
 & \quad \left. a_4 b_3 T_1 T_2 - c_4 T_1 T_2 - c_{13} T_1 T_2 + c_4 T_1^2 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2) p_{1,j} p_{2,j} x_{1,i} x_{2,i} - \right. \\
 & \quad \left((-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - c_4 T_1 + c_{13} T_1 + c_4 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - a_4 b_5 \right. \\
 & \quad \left. T_1 T_2 - c_{13} T_1 T_2 - a_4 b_3 T_1^2 T_2 + c_4 T_1^2 T_2 - c_{13} T_1^3 T_2 - c_4 T_1^3 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2 + c_{13} T_1^2 T_2^2) \right. \\
 & \quad \left. p_{1,j} p_{2,i} x_{1,j} x_{2,i} \right) / ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) - \\
 & \quad \left. \frac{(-a_2 b_5 - c_{13} T_1 - a_4 b_5 T_2 + c_{13} T_1 T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{(-1 + T_1) T_1} - \frac{(C_{50} + C_{53}) p_{2,j} x_{2,j}}{T_2} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \left((-a_2 b_3 + a_4 b_3 + a_2 b_5 - a_4 b_5 - c_4 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 + c_4 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 - \right. \\
 & \quad \left. c_{13} T_2 + a_4 b_3 T_1 T_2 + c_4 T_1 T_2 - c_{13} T_1 T_2 - c_4 T_1^2 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2) p_{1,i} p_{2,j} x_{1,i} x_{2,j} \right) / \\
 & \quad \left((-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) \right) - \frac{(a_4 b_3 - c_4 + c_4 T_1) p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{-1 + T_2} + \\
 & c_{51} p_{3,i} x_{3,i} + c_{54} p_{3,j} x_{3,i} + c_{14} p_{1,j} p_{3,i} x_{1,i} x_{3,i} + c_5 p_{1,i} p_{3,j} x_{1,i} x_{3,i} + \\
 & \frac{1}{-1 + T_2} \left(a_2 b_3 - a_2 b_5 + a_4 b_5 + c_5 + c_{14} - a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_5 T_2 - \right. \\
 & \quad \left. c_5 T_2 - c_{14} T_2 - 2 a_4 b_3 T_1 T_2 + c_5 T_1 T_2 - c_{14} T_1 T_2 + a_4 b_5 T_2^2 + c_{14} T_1 T_2^2 \right) p_{1,j} p_{3,j} x_{1,i} x_{3,i} - \\
 & \left((a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + c_{14} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 + \right. \\
 & \quad \left. a_2 b_5 T_2 - a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_5 T_1 T_2 - c_{14} T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 - \right. \\
 & \quad \left. c_5 T_1^2 T_2 - c_{14} T_1^2 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2 + c_{14} T_1^2 T_2^2) p_{1,j} p_{3,i} x_{1,j} x_{3,i} \right) / \\
 & \quad \left((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2) \right) - \frac{b_3 (a_2 + a_4 T_2) p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2)} - \\
 & \left((a_2 b_3 - a_2 b_3 T_1 - a_2 b_5 T_1 + c_{14} T_1 + a_4 b_3 T_2 - a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2 - \right. \\
 & \quad \left. c_{14} T_1 T_2 - c_{14} T_1^2 T_2 + a_4 b_5 T_1 T_2^2 + c_{14} T_1^2 T_2^2) p_{1,j} p_{3,j} x_{1,j} x_{3,i} \right) / \\
 & \quad \left((-1 + T_1) T_1 (-1 + T_2) \right) + c_{15} p_{2,j} p_{3,i} x_{2,i} x_{3,i} + c_6 p_{2,i} p_{3,j} x_{2,i} x_{3,i} + \\
 & \frac{1}{(-1 + T_1) T_1} \left(-a_2 b_3 + a_2 b_3 T_1 + c_6 T_1 + c_{15} T_1 - c_6 T_1^2 - c_{15} T_1^2 - a_4 b_3 T_2 - \right. \\
 & \quad \left. a_2 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2 - c_6 T_1 T_2 - a_4 b_3 T_1^2 T_2 + \right. \\
 & \quad \left. c_6 T_1^2 T_2 - c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2 \right) p_{2,j} p_{3,j} x_{2,i} x_{3,i} - \\
 & \left((-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 + c_{15} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + \right. \\
 & \quad \left. c_6 T_1^2 - c_{15} T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + \right. \\
 & \quad \left. a_4 b_3 T_1^2 T_2 - c_6 T_1^2 T_2 - c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{2,j} p_{3,i} x_{2,j} x_{3,i} \right) / \\
 & \quad \left((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2) \right) + \frac{a_4 b_5 p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{-1 + T_1} - \frac{1}{(-1 + T_1) (-1 + T_2)} \\
 & \left(-a_4 b_5 + c_{15} - c_{15} T_1 + a_4 b_3 T_2 + a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - c_{15} T_1 T_2 + c_{15} T_1^2 T_2 \right) p_{2,j} p_{3,j} x_{2,j} x_{3,i} - \\
 & \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \left(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - c_{51} T_1 - c_{54} T_1 + a_4 b_5 T_2 + \right. \\
 & \quad \left. a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 + c_{51} T_1^2 T_2 + c_{54} T_1^2 T_2 \right) p_{3,j} x_{3,j} + \\
 & \frac{(a_2 b_5 - a_4 b_5 + c_5 - c_{14} - c_5 T_1 + a_4 b_5 T_2 + c_{14} T_1 T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \\
 & \left((a_2 b_3 + c_5 - a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + a_4 b_3 T_2 - c_5 T_2 - 2 a_4 b_3 T_1 T_2 + c_5 T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j} \right) / \\
 & \quad \left((-1 + T_2) (-1 + T_1 T_2) \right) + \\
 & \left((a_2 b_3 + c_6 T_1 - c_{15} T_1 + a_4 b_3 T_2 - a_4 b_3 T_1 T_2 - c_6 T_1 T_2 + c_{15} T_1^2 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,j} \right) / \\
 & \quad \left(T_1 (-1 + T_2) (-1 + T_1 T_2) \right) - \\
 & \left((c_6 T_1 - c_6 T_1^2 - a_2 b_5 T_2 + a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2 - c_6 T_1 T_2 + c_6 T_1^2 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2) \right. \\
 & \quad \left. p_{2,j} p_{3,j} x_{2,i} x_{3,j} \right) / \left((-1 + T_1) T_1 (-1 + T_1 T_2) \right) \Big]
 \end{aligned}$$

Out[*]=

$$\frac{1}{T_1 T_2} \mathbb{E} \left[\text{Series} \left[\left(-p_{1,i} + p_{1,1+i} \right) x_{1,i} + \left(-1 + \frac{1}{T_1} \right) \left(p_{1,1+i} - p_{1,1+j} \right) x_{1,i} + \left(-p_{1,j} + p_{1,1+j} \right) x_{1,j} + \right. \right. \\ \left. \left. \left(-p_{2,i} + p_{2,1+i} \right) x_{2,i} + \left(-1 + \frac{1}{T_2} \right) \left(p_{2,1+i} - p_{2,1+j} \right) x_{2,i} + \left(-p_{2,j} + p_{2,1+j} \right) x_{2,j} + \right. \right. \\ \left. \left. \left(-p_{3,i} + p_{3,1+i} \right) x_{3,i} + \left(-1 + \frac{1}{T_1 T_2} \right) \left(p_{3,1+i} - p_{3,1+j} \right) x_{3,i} + \left(-p_{3,j} + p_{3,1+j} \right) x_{3,j}, \right. \right. \\ f_{49} p_{1,i} x_{1,i} + f_{52} p_{1,j} x_{1,i} + f_{55} p_{1,i} x_{1,j} + f_{58} p_{1,j} x_{1,j} + f_{50} p_{2,i} x_{2,i} + f_{53} p_{2,j} x_{2,i} + \\ f_1 p_{1,i} p_{2,i} x_{1,i} x_{2,i} + f_{13} p_{1,j} p_{2,i} x_{1,i} x_{2,i} + f_4 p_{1,i} p_{2,j} x_{1,i} x_{2,i} + f_{16} p_{1,j} p_{2,j} x_{1,i} x_{2,i} + \\ f_{25} p_{1,i} p_{2,i} x_{1,j} x_{2,i} + f_{37} p_{1,j} p_{2,i} x_{1,j} x_{2,i} + f_{28} p_{1,i} p_{2,j} x_{1,j} x_{2,i} + f_{40} p_{1,j} p_{2,j} x_{1,j} x_{2,i} + \\ f_{56} p_{2,i} x_{2,j} + f_{59} p_{2,j} x_{2,j} + f_7 p_{1,i} p_{2,i} x_{1,i} x_{2,j} + f_{19} p_{1,j} p_{2,i} x_{1,i} x_{2,j} + f_{10} p_{1,i} p_{2,j} x_{1,i} x_{2,j} + \\ f_{22} p_{1,j} p_{2,j} x_{1,i} x_{2,j} + f_{31} p_{1,i} p_{2,i} x_{1,j} x_{2,j} + f_{43} p_{1,j} p_{2,i} x_{1,j} x_{2,j} + f_{34} p_{1,i} p_{2,j} x_{1,j} x_{2,j} + \\ f_{46} p_{1,j} p_{2,j} x_{1,j} x_{2,j} + f_{51} p_{3,i} x_{3,i} + f_{54} p_{3,j} x_{3,i} + f_2 p_{1,i} p_{3,i} x_{1,i} x_{3,i} + f_{14} p_{1,j} p_{3,i} x_{1,i} x_{3,i} + \\ f_5 p_{1,i} p_{3,j} x_{1,i} x_{3,i} + f_{17} p_{1,j} p_{3,j} x_{1,i} x_{3,i} + f_{26} p_{1,i} p_{3,i} x_{1,j} x_{3,i} + f_{38} p_{1,j} p_{3,i} x_{1,j} x_{3,i} + \\ f_{29} p_{1,i} p_{3,j} x_{1,j} x_{3,i} + f_{41} p_{1,j} p_{3,j} x_{1,j} x_{3,i} + f_3 p_{2,i} p_{3,i} x_{2,i} x_{3,i} + f_{15} p_{2,j} p_{3,i} x_{2,i} x_{3,i} + \\ f_6 p_{2,i} p_{3,j} x_{2,i} x_{3,i} + f_{18} p_{2,j} p_{3,j} x_{2,i} x_{3,i} + f_{27} p_{2,i} p_{3,i} x_{2,j} x_{3,i} + f_{39} p_{2,j} p_{3,i} x_{2,j} x_{3,i} + \\ f_{30} p_{2,i} p_{3,j} x_{2,j} x_{3,i} + f_{42} p_{2,j} p_{3,j} x_{2,j} x_{3,i} + f_{57} p_{3,i} x_{3,j} + f_{60} p_{3,j} x_{3,j} + \\ f_8 p_{1,i} p_{3,i} x_{1,i} x_{3,j} + f_{20} p_{1,j} p_{3,i} x_{1,i} x_{3,j} + f_{11} p_{1,i} p_{3,j} x_{1,i} x_{3,j} + f_{23} p_{1,j} p_{3,j} x_{1,i} x_{3,j} + \\ f_{32} p_{1,i} p_{3,i} x_{1,j} x_{3,j} + f_{44} p_{1,j} p_{3,i} x_{1,j} x_{3,j} + f_{35} p_{1,i} p_{3,j} x_{1,j} x_{3,j} + f_{47} p_{1,j} p_{3,j} x_{1,j} x_{3,j} + \\ f_9 p_{2,i} p_{3,i} x_{2,i} x_{3,j} + f_{21} p_{2,j} p_{3,i} x_{2,i} x_{3,j} + f_{12} p_{2,i} p_{3,j} x_{2,i} x_{3,j} + f_{24} p_{2,j} p_{3,j} x_{2,i} x_{3,j} + \\ f_{33} p_{2,i} p_{3,i} x_{2,j} x_{3,j} + f_{45} p_{2,j} p_{3,i} x_{2,j} x_{3,j} + f_{36} p_{2,i} p_{3,j} x_{2,j} x_{3,j} + f_{48} p_{2,j} p_{3,j} x_{2,j} x_{3,j} \left. \right] \right]$$

In[*]:= **lhs** =

Cases [$\mathcal{F}[i, j] \times \mathcal{L} / @ (X_{i,j}[1] X_{i^+,j^+}[-1]) \text{ d} \{ \mathbf{vs}_i, \mathbf{vs}_j, \mathbf{vs}_{i^+}, \mathbf{vs}_{j^+} \}, \text{eSeries}[_ , \mathcal{E}_-] \Rightarrow \mathcal{E}, \infty$]

Out[*]=

$$\left\{ \frac{a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - a_2 b_5 T_1 - c_{51} T_1 - c_{54} T_1 - a_2 b_3 T_1^2 + \dots 154 \dots + f_{47} T_1^2 T_2^2 + f_{48} T_1^2 T_2^2 + f_{49} T_1^2 T_2^2 + f_{50} T_1^2 T_2^2 + f_{51} T_1^2 T_2^2 + f_{58} T_1^2 T_2^2 + f_{59} T_1^2 T_2^2 + f_{60} T_1^2 T_2^2}{(-1+T_1) T_1^2 (-1+T_2) T_2} + \right.$$

$$\left. \frac{\dots 163 \dots + f_{49} T_1^2 T_2^2 - f_{55} T_1^2 T_2^2}{(-1+T_1) T_1 (-1+T_2) (-1+T_1 T_2)} p_{1, \dots} \pi_{1,i} + \frac{\dots 72 \dots}{T_1 T_2^2} + \frac{(-f_{33} + f_{33} T_1 T_2 + f_{36} T_1 T_2) \dots 4 \dots}{T_1 T_2^2} + \frac{\dots 1 \dots}{T_1 T_2^2} \right\}$$

Full expression not available (original memory size: 1 MB)

In[*]:= **eqn** = **CF** [**lhs** - **err**]

Out[*]=

$$\frac{-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 + c_{51} T_1 + c_{54} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + \dots 163 \dots}{(-1+T_1) T_1^2 (-1+T_2) T_2} + \frac{(\dots 163 \dots + f_{49} T_1^2 T_2^2 - f_{55} T_1^2 T_2^2) p_{1, \dots} \pi_{1,i} + \dots 1 \dots}{(-1+T_1) T_1 (-1+T_2) (-1+T_1 T_2)} + \frac{\dots 1 \dots}{\dots 1 \dots} +$$

$$\frac{\dots 76 \dots}{T_1 T_2^2} + \frac{(\dots 1 \dots) \dots 4 \dots}{T_1 T_2^2} + \frac{(f_{33} - f_{33} T_2 - f_{45} T_2 - f_{33} T_1 T_2 - f_{36} T_1 T_2 + f_{33} T_1 T_2^2 + f_{36} T_1 T_2^2 + f_{45} T_1 T_2^2 + f_{48} T_1 T_2^2) p_{2, \dots} p_{3, \dots} \pi_{2,j} \pi_{3,j}}{T_1 T_2^2}$$

Full expression not available (original memory size: 1 MB)

In[*]:= **cvs** = **Union**@**Cases** [**eqn**, **p**__ | **π**__, **∞**]

Out[*]=

{**p**_{1,2+i}, **p**_{1,2+j}, **p**_{2,2+i}, **p**_{2,2+j}, **p**_{3,2+i}, **p**_{3,2+j}, **π**_{1,i}, **π**_{1,j}, **π**_{2,i}, **π**_{2,j}, **π**_{3,i}, **π**_{3,j}}

In[*]:= eqns = CoefficientRules[eqn, cvs] /. (_ -> c_) :-> (c == 0)

Out[*]=

$$\left\{ \begin{aligned} & f_1 - f_7 - f_{25} + f_{31} + \frac{f_{25}}{T_1} - \frac{f_{31}}{T_1} + \frac{f_7}{T_2} - \frac{f_{31}}{T_2} + \frac{f_{31}}{T_1 T_2} = 0, \frac{f_7}{T_2} - \frac{f_{31}}{T_2} + \frac{f_{31}}{T_1 T_2} = 0, \dots 1 \dots = 0, \dots 55 \dots, \\ & \dots 1 \dots, \dots 1 \dots = 0, -\frac{2 a_4 b_3}{(1-T_1)(1-T_2)} + \frac{2 c_{49}}{(1-T_1)(1-T_2)} + \frac{2 c_{50}}{(1-T_1)(1-T_2)} + \frac{c_{52}}{(1-T_1)(1-T_2)} + \frac{c_{53}}{(1-T_1)(1-T_2)} - \frac{c_{54}}{(1-T_1)(1-T_2)} + \\ & \frac{f_1}{(1-T_1)(1-T_2)} + \dots 159 \dots + \frac{f_{49} T_1 T_2}{(1-T_1)(1-T_2)} + \frac{f_{50} T_1 T_2}{(1-T_1)(1-T_2)} + \frac{f_{51} T_1 T_2}{(1-T_1)(1-T_2)} + \frac{f_{58} T_1 T_2}{(1-T_1)(1-T_2)} + \frac{f_{59} T_1 T_2}{(1-T_1)(1-T_2)} + \frac{f_{60} T_1 T_2}{(1-T_1)(1-T_2)} = 0 \end{aligned} \right\}$$

Full expression not available (original memory size: 2.5 MB)

In[*]:= vars = Union@Cases[r42[-1, i, j], f_, ∞]

Out[*]=

- {f₁, f₂, f₃, f₄, f₅, f₆, f₇, f₈, f₉, f₁₀, f₁₁, f₁₂, f₁₃, f₁₄, f₁₅, f₁₆, f₁₇, f₁₈, f₁₉, f₂₀, f₂₁, f₂₂, f₂₃, f₂₄, f₂₅, f₂₆, f₂₇, f₂₈, f₂₉, f₃₀, f₃₁, f₃₂, f₃₃, f₃₄, f₃₅, f₃₆, f₃₇, f₃₈, f₃₉, f₄₀, f₄₁, f₄₂, f₄₃, f₄₄, f₄₅, f₄₆, f₄₇, f₄₈, f₄₉, f₅₀, f₅₁, f₅₂, f₅₃, f₅₄, f₅₅, f₅₆, f₅₇, f₅₈, f₅₉, f₆₀}

In[*]:= {sol} = Solve[eqns, vars]

Out[*]=

$$\left\{ \begin{aligned} & f_1 \rightarrow 0, f_2 \rightarrow 0, f_3 \rightarrow 0, \\ & f_4 \rightarrow -\left(\frac{(-a_2 b_3 + a_4 b_3 + a_2 b_5 - a_4 b_5 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 - c_{13} T_2 + a_4 b_3 T_1 T_2 - c_{13} T_1 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2)}{((-1 + T_1) T_2 (-1 + T_1 T_2))} \right), \\ & f_5 \rightarrow -\left(\frac{(-a_2 b_3 + 2 a_2 b_3 T_1 - a_4 b_3 T_1 - a_2 b_5 T_1 + a_4 b_5 T_1 + c_{14} T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - a_4 b_3 T_2 + 3 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - 2 a_4 b_5 T_1 T_2 - c_{14} T_1 T_2 - 2 a_4 b_3 T_1^2 T_2 - c_{14} T_1^2 T_2 + a_4 b_5 T_1 T_2^2 + c_{14} T_1^2 T_2^2)}{((-1 + T_1) T_1^2 (-1 + T_2) T_2)} \right), \\ & f_6 \rightarrow -\left(\frac{(-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - a_2 b_5 T_1 + a_4 b_5 T_1 + c_{15} T_1 - c_{15} T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - 3 a_4 b_5 T_1 T_2 - a_4 b_3 T_1^2 T_2 - c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2)}{((-1 + T_1) T_1^2 (-1 + T_2) T_2)} \right), f_7 \rightarrow 0, f_8 \rightarrow 0, f_9 \rightarrow 0, \\ & f_{10} \rightarrow -\left(\frac{(-a_2 b_3 + a_4 b_3 + a_2 b_5 - a_4 b_5 - c_4 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 + c_4 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 - c_{13} T_2 + a_4 b_3 T_1 T_2 + c_4 T_1 T_2 - c_{13} T_1 T_2 - c_4 T_1^2 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2)}{((-1 + T_1) (-1 + T_2) (-1 + T_1 T_2))} \right), f_{11} \rightarrow -\frac{a_2 b_5 - a_4 b_5 + c_5 - c_{14} - c_5 T_1 + a_4 b_5 T_2 + c_{14} T_1 T_2}{(-1 + T_1) (-1 + T_1 T_2)}, \\ & f_{12} \rightarrow -\frac{a_2 b_3 + c_6 T_1 - c_{15} T_1 + a_4 b_3 T_2 - a_4 b_3 T_1 T_2 - c_6 T_1 T_2 + c_{15} T_1^2 T_2}{T_1 (-1 + T_2) (-1 + T_1 T_2)}, \\ & f_{13} \rightarrow -\left(\frac{(a_2 b_3 - a_2 b_5 - a_2 b_3 T_1 + c_4 T_1 - c_4 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - 2 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 - c_4 T_1^2 T_2 + c_4 T_1^3 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2)}{(T_1^2 (-1 + T_2) (-1 + T_1 T_2))} \right), \\ & f_{14} \rightarrow -\left(\frac{(-a_2 b_3 + a_2 b_5 + 2 a_2 b_3 T_1 - a_4 b_3 T_1 + c_5 T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - c_5 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 3 a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 - c_5 T_1 T_2 - 2 a_4 b_3 T_1^2 T_2 + c_5 T_1^2 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2)}{(T_1^2 (-1 + T_2) (-1 + T_1 T_2))} \right), \\ & f_{15} \rightarrow -\left(\frac{(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - a_2 b_5 T_1 + a_4 b_5 T_1 + c_6 T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - c_6 T_1^2 - a_2 b_5 T_2 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - 3 a_4 b_5 T_1 T_2 - c_6 T_1 T_2 - a_4 b_3 T_1^2 T_2 + c_6 T_1^2 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2)}{((-1 + T_1) T_1 T_2 (-1 + T_1 T_2))} \right), \\ & f_{16} \rightarrow -\frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \left(-a_2 b_3 + a_2 b_3 T_1 - a_2 b_5 T_1 + a_4 b_5 T_1 - c_4 T_1 - c_{13} T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 + c_4 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - 2 a_4 b_5 T_1 T_2 + c_{13} T_1 T_2 - \right) \end{aligned} \right.$$

$$\begin{aligned}
 & 2 a_4 b_3 T_1^2 T_2 + c_4 T_1^2 T_2 + c_{13} T_1^2 T_2 - c_4 T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2 - c_{13} T_1^2 T_2^2), \\
 f_{17} \rightarrow & -\frac{1}{T_1^3 (-1 + T_2) T_2} (a_2 b_3 - 3 a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_5 T_1 - c_{14} T_1 + \\
 & 2 a_2 b_3 T_1^2 - 2 a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 - 4 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 2 a_4 b_5 T_1 T_2 + \\
 & c_5 T_1 T_2 + c_{14} T_1 T_2 + 4 a_4 b_3 T_1^2 T_2 - c_5 T_1^2 T_2 + c_{14} T_1^2 T_2 - a_4 b_5 T_1 T_2^2 - c_{14} T_1^2 T_2^2), \\
 f_{18} \rightarrow & -\frac{1}{(-1 + T_1) T_1^2 T_2^2} (a_2 b_3 - a_2 b_5 - a_2 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 - c_{15} T_1 + c_6 T_1^2 + \\
 & c_{15} T_1^2 + a_4 b_3 T_2 + 2 a_2 b_5 T_2 - a_4 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - 2 a_2 b_5 T_1 T_2 + 4 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + \\
 & a_4 b_3 T_1^2 T_2 - c_6 T_1^2 T_2 + c_{15} T_1^2 T_2 - c_{15} T_1^3 T_2 + 2 a_4 b_5 T_2^2 - 4 a_4 b_5 T_1 T_2^2), f_{19} \rightarrow 0, f_{20} \rightarrow 0, \\
 f_{21} \rightarrow & 0, f_{22} \rightarrow -\left((a_2 b_3 - a_2 b_5 + a_4 b_5 - c_{13} - a_2 b_3 T_1 + a_4 b_3 T_1 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_5 T_2 + \right. \\
 & \left. c_{13} T_2 - 2 a_4 b_3 T_1 T_2 + c_{13} T_1 T_2 + a_4 b_5 T_2^2 - c_{13} T_1 T_2^2) / (T_1 (-1 + T_2) (-1 + T_1 T_2)) \right), \\
 f_{23} \rightarrow & -\left((-a_2 b_3 + a_2 b_5 - a_4 b_5 - c_{14} + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 + \right. \\
 & \left. c_{14} T_2 + 2 a_4 b_3 T_1 T_2 + c_{14} T_1 T_2 - a_4 b_5 T_2^2 - c_{14} T_1 T_2^2) / (T_1 (-1 + T_2) (-1 + T_1 T_2)) \right), \\
 f_{24} \rightarrow & -\left((a_2 b_3 - a_2 b_3 T_1 - c_{15} T_1 + c_{15} T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + \right. \\
 & \left. a_4 b_3 T_1^2 T_2 + c_{15} T_1^2 T_2 - c_{15} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) / ((-1 + T_1) T_1 T_2 (-1 + T_1 T_2)) \right), \\
 f_{25} \rightarrow & 0, f_{26} \rightarrow 0, f_{27} \rightarrow 0, f_{28} \rightarrow 0, f_{29} \rightarrow -\frac{-a_2 b_3 - a_4 b_3 T_2}{T_1 (-1 + T_2) T_2}, \\
 f_{30} \rightarrow & -\frac{a_4 b_5}{(-1 + T_1) T_1}, \\
 f_{31} \rightarrow & 0, \\
 f_{32} \rightarrow & 0, \\
 f_{33} \rightarrow & 0, \\
 f_{34} \rightarrow & 0, \\
 f_{35} \rightarrow & 0, \\
 f_{36} \rightarrow & 0, \\
 f_{37} \rightarrow & -\left((a_2 b_3 - a_2 b_5 - a_2 b_3 T_1 + c_4 T_1 - c_{13} T_1 - c_4 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - \right. \\
 & \left. 2 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_{13} T_1 T_2 + a_4 b_3 T_1^2 T_2 - c_4 T_1^2 T_2 + c_{13} T_1^2 T_2 + c_4 T_1^3 T_2 + \right. \\
 & \left. a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2 - c_{13} T_1^2 T_2^2) / ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) \right), \\
 f_{38} \rightarrow & -\left((-a_2 b_3 + a_2 b_5 + 2 a_2 b_3 T_1 - a_4 b_3 T_1 + c_5 T_1 - c_{14} T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - c_5 T_1^2 - \right. \\
 & \left. a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 3 a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 - c_5 T_1 T_2 + c_{14} T_1 T_2 - 2 a_4 b_3 T_1^2 T_2 + \right. \\
 & \left. c_5 T_1^2 T_2 + c_{14} T_1^2 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2 - c_{14} T_1^2 T_2^2) / ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) \right), \\
 f_{39} \rightarrow & -\left((a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - a_2 b_5 T_1 + a_4 b_5 T_1 + c_6 T_1 - c_{15} T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - c_6 T_1^2 + \right. \\
 & \left. c_{15} T_1^2 - a_2 b_5 T_2 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - 3 a_4 b_5 T_1 T_2 - c_6 T_1 T_2 - a_4 b_3 T_1^2 T_2 + c_6 T_1^2 \right. \\
 & \left. T_2 + c_{15} T_1^2 T_2 - c_{15} T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2) / ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) \right), \\
 f_{40} \rightarrow & -\left((-a_2 b_3 + a_2 b_3 T_1 - c_4 T_1 + c_4 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2 - \right. \\
 & \left. a_4 b_3 T_1^2 T_2 + c_4 T_1^2 T_2 - c_4 T_1^3 T_2 - a_4 b_5 T_2^2 + 2 a_4 b_5 T_1 T_2^2) / ((-1 + T_1) T_1 T_2 (-1 + T_1 T_2)) \right), \\
 f_{41} \rightarrow & -\left((a_2 b_3 - 3 a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + 2 a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 - 4 a_4 b_3 T_1 T_2 + \right. \\
 & \left. a_4 b_5 T_1 T_2 + c_5 T_1 T_2 + 3 a_4 b_3 T_1^2 T_2 - c_5 T_1^2 T_2 - a_4 b_5 T_1 T_2^2) / ((-1 + T_1) T_1^2 (-1 + T_2) T_2) \right), \\
 f_{42} \rightarrow & -\left((-a_2 b_5 - a_2 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 + a_2 b_3 T_1^2 + c_6 T_1^2 + a_2 b_5 T_2 - \right. \\
 & \left. a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 4 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + a_4 b_3 T_1^2 T_2 - \right. \\
 & \left. c_6 T_1^2 T_2 + a_4 b_5 T_2^2 - 3 a_4 b_5 T_1 T_2^2) / ((-1 + T_1) T_1^2 (-1 + T_2) T_2) \right), \\
 f_{43} \rightarrow & 0, f_{44} \rightarrow 0, f_{45} \rightarrow 0, f_{46} \rightarrow 0, f_{47} \rightarrow 0, f_{48} \rightarrow 0, f_{49} \rightarrow -c_{49}, \\
 f_{50} \rightarrow & -c_{50},
 \end{aligned}$$

$$\begin{aligned}
 f_{51} &\rightarrow -c_{51}, \\
 f_{52} &\rightarrow -\frac{c_{49} + c_{52} - c_{49} T_1^2}{T_1^2}, \\
 f_{53} &\rightarrow -\frac{c_{50} + c_{53} - c_{50} T_2^2}{T_2^2}, \\
 f_{54} &\rightarrow -\frac{1}{T_1^3 T_2^2} \left(-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + c_{51} T_1 + c_{54} T_1 - a_4 b_5 T_2 - \right. \\
 &\quad \left. a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 - a_4 b_5 T_1 T_2^2 - c_{51} T_1^3 T_2^2 \right), \\
 f_{55} &\rightarrow 0, f_{56} \rightarrow 0, f_{57} \rightarrow 0, f_{58} \rightarrow -\frac{-c_{49} - c_{52}}{T_1}, f_{59} \rightarrow -\frac{-c_{50} - c_{53}}{T_2}, \\
 f_{60} &\rightarrow -\frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \left(-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + c_{51} T_1 + \right. \\
 &\quad \left. c_{54} T_1 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 - c_{51} T_1^2 T_2 - c_{54} T_1^2 T_2 \right) \} \}
 \end{aligned}$$

In[*]:= Length[sol]

Out[*]=
60

In[*]:= sol /. (v_ -> val_) :-> (v = CF[val]);

In[*]:= CF[r42[-1, i, j]]

Out[*]=

$$\begin{aligned}
 &-c_{49} p_{1,i} x_{1,i} + \frac{(-c_{49} - c_{52} + c_{49} T_1^2) p_{1,j} x_{1,i}}{T_1^2} + \\
 &\frac{(c_{49} + c_{52}) p_{1,j} x_{1,j}}{T_1} - c_{50} p_{2,i} x_{2,i} + \frac{(-c_{50} - c_{53} + c_{50} T_2^2) p_{2,j} x_{2,i}}{T_2^2} - \\
 &\left((a_2 b_3 - a_2 b_5 - a_2 b_3 T_1 + c_4 T_1 - c_4 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - 2 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 - \right. \\
 &\quad \left. T_2 - c_4 T_1^2 T_2 + c_4 T_1^3 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2) p_{1,j} p_{2,i} x_{1,i} x_{2,i} \right) / (T_1^2 (-1 + T_2) (-1 + T_1 T_2)) - \\
 &\left((-a_2 b_3 + a_4 b_3 + a_2 b_5 - a_4 b_5 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 - c_{13} T_2 + \right. \\
 &\quad \left. a_4 b_3 T_1 T_2 - c_{13} T_1 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2) p_{1,i} p_{2,j} x_{1,i} x_{2,i} \right) / ((-1 + T_1) T_2 (-1 + T_1 T_2)) + \\
 &\frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \left(a_2 b_3 - a_2 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 + c_4 T_1 + c_{13} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 - \right. \\
 &\quad c_4 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 2 a_4 b_5 T_1 T_2 - c_{13} T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 - \\
 &\quad \left. c_4 T_1^2 T_2 - c_{13} T_1^2 T_2 + c_4 T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2 + c_{13} T_1^2 T_2^2 \right) p_{1,j} p_{2,j} x_{1,i} x_{2,i} + \\
 &\left((-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - c_4 T_1 + c_{13} T_1 + c_4 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 - \right. \\
 &\quad \left. a_4 b_5 T_1 T_2 - c_{13} T_1 T_2 - a_4 b_3 T_1^2 T_2 + c_4 T_1^2 T_2 - c_{13} T_1^2 T_2 - c_4 T_1^3 T_2 - a_4 b_5 T_2^2 + a_4 b_5 T_1 T_2^2 + c_{13} T_1^2 T_2^2) \right) \\
 &\quad p_{1,j} p_{2,i} x_{1,j} x_{2,i} / ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) + \\
 &\left((a_2 b_3 - a_2 b_3 T_1 + c_4 T_1 - c_4 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + \right. \\
 &\quad \left. a_4 b_3 T_1^2 T_2 - c_4 T_1^2 T_2 + c_4 T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{1,j} p_{2,j} x_{1,j} x_{2,i} \right) / \\
 &\left((-1 + T_1) T_1 T_2 (-1 + T_1 T_2) \right) + \frac{(c_{50} + c_{53}) p_{2,j} x_{2,j}}{T_2} - \\
 &\left((-a_2 b_3 + a_4 b_3 + a_2 b_5 - a_4 b_5 - c_4 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 + c_4 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + \right.
 \end{aligned}$$

$$\begin{aligned}
 & (2 a_4 b_5 T_2 - c_{13} T_2 + a_4 b_3 T_1 T_2 + c_4 T_1 T_2 - c_{13} T_1 T_2 - c_4 T_1^2 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2) \\
 & p_{1,i} p_{2,j} x_{1,i} x_{2,j}) / ((-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)) + \\
 & ((-a_2 b_3 + a_2 b_5 - a_4 b_5 + c_{13} + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 - c_{13} T_2 + \\
 & 2 a_4 b_3 T_1 T_2 - c_{13} T_1 T_2 - a_4 b_5 T_2^2 + c_{13} T_1 T_2^2) p_{1,j} p_{2,j} x_{1,i} x_{2,j}) / (T_1 (-1 + T_2) (-1 + T_1 T_2)) - \\
 & c_{51} p_{3,i} x_{3,i} + \frac{1}{T_1^3 T_2^2} (a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - c_{51} T_1 - c_{54} T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 + \\
 & a_2 b_5 T_1 T_2 - a_4 b_5 T_1 T_2 - a_4 b_3 T_1^2 T_2 + a_4 b_5 T_1 T_2^2 + c_{51} T_1^3 T_2^2) p_{3,j} x_{3,i} + \\
 & ((a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - \\
 & a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_5 T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 - c_5 T_1^2 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2) \\
 & p_{1,j} p_{3,i} x_{1,i} x_{3,i}) / (T_1^2 (-1 + T_2) (-1 + T_1 T_2)) + \\
 & ((a_2 b_3 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_{14} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_4 b_3 T_2 - 3 a_4 b_3 T_1 T_2 - \\
 & a_2 b_5 T_1 T_2 + 2 a_4 b_5 T_1 T_2 + c_{14} T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 + c_{14} T_1^2 T_2 - a_4 b_5 T_1 T_2^2 - c_{14} T_1^2 T_2^2) \\
 & p_{1,i} p_{3,j} x_{1,i} x_{3,i}) / ((-1 + T_1) T_1^2 (-1 + T_2) T_2) - \frac{1}{T_1^3 (-1 + T_2) T_2} \\
 & (a_2 b_3 - 3 a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_5 T_1 - c_{14} T_1 + 2 a_2 b_3 T_1^2 - 2 a_4 b_3 T_1^2 + \\
 & c_5 T_1^2 + a_4 b_3 T_2 - 4 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 2 a_4 b_5 T_1 T_2 + c_5 T_1 T_2 + c_{14} T_1 T_2 + \\
 & 4 a_4 b_3 T_1^2 T_2 - c_5 T_1^2 T_2 + c_{14} T_1^2 T_2 - a_4 b_5 T_1 T_2^2 - c_{14} T_1^2 T_2^2) p_{1,j} p_{3,j} x_{1,i} x_{3,i} + \\
 & ((a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + c_{14} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 + \\
 & a_2 b_5 T_2 - a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + c_5 T_1 T_2 - c_{14} T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 - \\
 & c_5 T_1^2 T_2 - c_{14} T_1^2 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2 + c_{14} T_1^2 T_2^2) p_{1,j} p_{3,i} x_{1,j} x_{3,i}) / \\
 & ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) + \frac{b_3 (a_2 + a_4 T_2) p_{1,i} p_{3,j} x_{1,j} x_{3,i}}{T_1 (-1 + T_2) T_2} - \\
 & ((a_2 b_3 - 3 a_2 b_3 T_1 + a_4 b_3 T_1 - c_5 T_1 + 2 a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_5 T_1^2 + a_4 b_3 T_2 - 4 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + \\
 & c_5 T_1 T_2 + 3 a_4 b_3 T_1^2 T_2 - c_5 T_1^2 T_2 - a_4 b_5 T_1 T_2^2) p_{1,j} p_{3,j} x_{1,j} x_{3,i}) / ((-1 + T_1) T_1^2 (-1 + T_2) T_2) + \\
 & ((-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_6 T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - \\
 & a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + a_4 b_3 T_1^2 T_2 - c_6 T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) \\
 & p_{2,j} p_{3,i} x_{2,i} x_{3,i}) / ((-1 + T_1) T_1 T_2 (-1 + T_1 T_2)) + \\
 & ((a_2 b_3 - a_2 b_5 - a_2 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_{15} T_1 + c_{15} T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - \\
 & 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 + c_{15} T_1^2 T_2 - c_{15} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) \\
 & p_{2,i} p_{3,j} x_{2,i} x_{3,i}) / ((-1 + T_1) T_1^2 (-1 + T_2) T_2) + \frac{1}{(-1 + T_1) T_1^2 T_2^2} \\
 & (-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1 - a_2 b_5 T_1 + a_4 b_5 T_1 + c_6 T_1 + c_{15} T_1 - c_6 T_1^2 - c_{15} T_1^2 - a_4 b_3 T_2 - \\
 & 2 a_2 b_5 T_2 + a_4 b_5 T_2 + 2 a_4 b_3 T_1 T_2 + 2 a_2 b_5 T_1 T_2 - 4 a_4 b_5 T_1 T_2 - c_6 T_1 T_2 - a_4 b_3 T_1^2 T_2 + \\
 & c_6 T_1^2 T_2 - c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 - 2 a_4 b_5 T_2^2 + 4 a_4 b_5 T_1 T_2^2) p_{2,j} p_{3,j} x_{2,i} x_{3,i} + \\
 & ((-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 + c_{15} T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + c_6 T_1^2 - \\
 & c_{15} T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + \\
 & a_4 b_3 T_1^2 T_2 - c_6 T_1^2 T_2 - c_{15} T_1^2 T_2 + c_{15} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{2,j} p_{3,i} x_{2,j} x_{3,i}) / \\
 & ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) - \frac{a_4 b_5 p_{2,i} p_{3,j} x_{2,j} x_{3,i}}{(-1 + T_1) T_1} - \\
 & ((-a_2 b_5 - a_2 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 - c_6 T_1 + a_2 b_3 T_1^2 + c_6 T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - \\
 & a_2 b_5 T_1 T_2 + 4 a_4 b_5 T_1 T_2 + c_6 T_1 T_2 + a_4 b_3 T_1^2 T_2 - c_6 T_1^2 T_2 + a_4 b_5 T_2^2 - 3 a_4 b_5 T_1 T_2^2)
 \end{aligned}$$

$$\begin{aligned} & p_{2,j} p_{3,j} x_{2,j} x_{3,i} \Big/ \left((-1 + T_1) T_1^2 (-1 + T_2) T_2 \right) + \frac{1}{T_1^2 T_2 (-1 + T_1 T_2)} \\ & \left(a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - c_{51} T_1 - c_{54} T_1 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_1 T_2 + c_{51} T_1^2 T_2 + c_{54} T_1^2 T_2 \right) \\ & p_{3,j} x_{3,j} - \\ & \frac{\left(a_2 b_5 - a_4 b_5 + c_5 - c_{14} - c_5 T_1 + a_4 b_5 T_2 + c_{14} T_1 T_2 \right) p_{1,i} p_{3,j} x_{1,i} x_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} - \\ & \left(-a_2 b_3 + a_2 b_5 - a_4 b_5 - c_{14} + a_2 b_3 T_1 - a_4 b_3 T_1 - a_4 b_3 T_2 - a_2 b_5 T_2 + 2 a_4 b_5 T_2 + c_{14} T_2 + 2 a_4 b_3 T_1 T_2 + \right. \\ & \quad \left. c_{14} T_1 T_2 - a_4 b_5 T_2^2 - c_{14} T_1 T_2^2 \right) p_{1,j} p_{3,j} x_{1,i} x_{3,j} \Big/ \left(T_1 (-1 + T_2) (-1 + T_1 T_2) \right) + \\ & \left(-a_2 b_3 - c_6 T_1 + c_{15} T_1 - a_4 b_3 T_2 + a_4 b_3 T_1 T_2 + c_6 T_1 T_2 - c_{15} T_1^2 T_2 \right) p_{2,i} p_{3,j} x_{2,i} x_{3,j} \Big/ \\ & \left(T_1 (-1 + T_2) (-1 + T_1 T_2) \right) - \\ & \left(a_2 b_3 - a_2 b_3 T_1 - c_{15} T_1 + c_{15} T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - 2 a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 + \right. \\ & \quad \left. c_{15} T_1^2 T_2 - c_{15} T_1^3 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2 \right) p_{2,j} p_{3,j} x_{2,i} x_{3,j} \Big/ \left((-1 + T_1) T_1 T_2 (-1 + T_1 T_2) \right) \end{aligned}$$

In[*]:= CF[r₄₂[-1, i, j] / .

$$\{a_4 \rightarrow \theta, b_3 \rightarrow \theta, b_5 \rightarrow (T_1 - 1) (T_2 - 1) (T_3 - 1) a_2^{-1}, c_{4|5|6|13|14|15|49|50|51|52|53|54} \rightarrow \theta\}$$

Out[*]=

$$\begin{aligned} & - \frac{(-1 + T_1) (-1 + T_2) p_{1,j} p_{2,i} x_{1,i} x_{2,i}}{T_1^2} + \frac{(-1 + T_2)^2 p_{1,i} p_{2,j} x_{1,i} x_{2,i}}{T_2} - \\ & \frac{(-1 + T_1) (-1 + T_2) (-T_1 - T_2 + T_1 T_2) p_{1,j} p_{2,j} x_{1,i} x_{2,i}}{T_1^2 T_2} - \frac{(-1 + T_2) p_{1,j} p_{2,i} x_{1,j} x_{2,i}}{T_1} - \\ & \frac{(-1 + T_1) (-1 + T_2) p_{1,j} p_{2,j} x_{1,j} x_{2,i}}{T_1} + (-1 + T_2) p_{1,i} p_{2,j} x_{1,i} x_{2,j} - \\ & \frac{(-1 + T_1) (-1 + T_2) p_{1,j} p_{2,j} x_{1,i} x_{2,j}}{T_1} + \frac{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) (1 + T_1 T_2) p_{3,j} x_{3,i}}{T_1^3 T_2^2} + \\ & \frac{(-1 + T_1) (-1 + T_2) p_{1,j} p_{3,i} x_{1,i} x_{3,i}}{T_1^2} - \frac{(-1 + T_2) (-1 + T_1 T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,i}}{T_1 T_2} + \\ & \frac{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,i}}{T_1^2 T_2} + \frac{(-1 + T_2) p_{1,j} p_{3,i} x_{1,j} x_{3,i}}{T_1} - \\ & \frac{(-1 + T_1) (-1 + T_2)^2 p_{2,j} p_{3,i} x_{2,i} x_{3,i}}{T_1 T_2} - \frac{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) p_{2,i} p_{3,j} x_{2,i} x_{3,i}}{T_1^2 T_2} + \\ & \frac{(-1 + T_1) (-1 + T_2) (-1 + 2 T_2) (-1 + T_1 T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,i}}{T_1^2 T_2^2} - \\ & \frac{(-1 + T_1) (-1 + T_2) p_{2,j} p_{3,i} x_{2,j} x_{3,i}}{T_1} + \frac{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) p_{2,j} p_{3,j} x_{2,j} x_{3,i}}{T_1^2 T_2} + \\ & \frac{(-1 + T_1) (-1 + T_2) p_{3,j} x_{3,j}}{T_1^2 T_2} + (1 - T_2) p_{1,i} p_{3,j} x_{1,i} x_{3,j} + \\ & \frac{(-1 + T_1) (-1 + T_2) p_{1,j} p_{3,j} x_{1,i} x_{3,j}}{T_1} + \frac{(-1 + T_1) (-1 + T_2) p_{2,j} p_{3,j} x_{2,i} x_{3,j}}{T_1} \end{aligned}$$

In[*]:= $\mathcal{L}[X_{i,j}[s_-]] := T_3^5 \mathbb{E} [q[s, i, j] + r_0[s, i, j] + e r_1[s, i, j] - e r_{42}[s, i, j] + 0[e]^2];$
 $\mathcal{L}[X_{i,j}[1]] // \text{Short}$
 $\mathcal{L}[X_{i,j}[-1]] // \text{Short}$

Out[*]//Short=

$$T_1 T_2 \mathbb{E} \left[\text{eSeries} \left[\left(-p_{1,i} + p_{1,1+i} \right) x_{1,i} + \left(-1 + T_1 \right) \left(p_{1,1+i} - p_{1,1+j} \right) x_{1,i} + \right. \right. \\ \left. \left. \ll 9 \gg + \left(-1 + T_1 T_2 \right) \left(p_{3,1+i} - p_{3,1+j} \right) x_{3,i} + \left(-p_{3,j} + p_{3,1+j} \right) x_{3,j}, \ll 52 \gg + \frac{\ll 1 \gg}{\ll 1 \gg} \right] \right]$$

Out[*]//Short=

$$\frac{\mathbb{E} \left[\text{eSeries} \left[\left(-p_{1,i} + p_{1,1+i} \right) x_{1,i} + \ll 11 \gg + \left(-p_{3,j} + p_{3,1+j} \right) x_{3,j}, \ll 1 \gg \right] \right]}{T_1 T_2}$$

In[*]:= $\text{Echo} / @ \left(\text{CF} @ \text{CoefficientList} \left[\text{Cases} \left[\int \mathcal{F}[i, j] \times \mathcal{L} / @ \left(X_{i,j}[1] X_{i^+,j^+}[-1] \right) d\{vs_i, vs_j, vs_{i^+}, vs_{j^+}\}, \right. \right. \right. \\ \left. \left. \left. \text{eSeries}[_ , \mathcal{E}_-] \Rightarrow \mathcal{E}, \infty \right] \left[[1] / . \{a_{i_-} \Rightarrow \lambda a_i, b_{i_-} \Rightarrow \lambda b_i\}, \lambda \right] \right);$

» 0

»
$$\frac{1}{(-1 + T_1) T_1 (-1 + T_2) T_2 (-1 + T_1 T_2)}$$

$$\left(a_2 c_{51} + a_2 c_{54} - a_2 c_{51} T_1 - a_2 c_{54} T_1 - a_2 c_4 T_2 - 3 a_2 c_5 T_2 + 2 a_2 c_{13} T_2 + a_2 c_{14} T_2 - a_2 c_{51} T_2 + a_2 c_{52} T_2 - \right. \\ a_2 c_{54} T_2 + 2 a_2 c_4 T_1 T_2 - 2 a_2 c_4 T_1 T_2 + 2 a_2 c_5 T_1 T_2 - 4 a_2 c_6 T_1 T_2 + 3 a_4 c_6 T_1 T_2 - 2 a_2 c_{13} T_1 T_2 + \\ a_4 c_{13} T_1 T_2 + 3 a_2 c_{14} T_1 T_2 + 4 a_2 c_{15} T_1 T_2 - a_4 c_{15} T_1 T_2 - a_2 c_{49} T_1 T_2 - a_2 c_{50} T_1 T_2 - a_2 c_{52} T_1 T_2 - \\ a_4 c_{53} T_1 T_2 - a_2 c_4 T_1^2 T_2 + 2 a_4 c_4 T_1^2 T_2 + a_2 c_5 T_1^2 T_2 + 4 a_2 c_6 T_1^2 T_2 - 3 a_4 c_6 T_1^2 T_2 - 4 a_2 c_{15} T_1^2 T_2 + \\ a_4 c_{15} T_1^2 T_2 + a_2 c_{49} T_1^2 T_2 + a_2 c_{50} T_1^2 T_2 + a_2 c_{51} T_1^2 T_2 + a_4 c_{53} T_1^2 T_2 + a_2 c_{54} T_1^2 T_2 - a_4 c_4 T_2^2 + 3 a_2 c_5 T_2^2 - \\ 3 a_4 c_5 T_2^2 - 2 a_2 c_{13} T_2^2 + 2 a_4 c_{13} T_2^2 - a_2 c_{14} T_2^2 + a_4 c_{14} T_2^2 - a_2 c_{52} T_2^2 + a_4 c_{52} T_2^2 + a_2 c_4 T_1 T_2^2 + \\ 4 a_4 c_4 T_1 T_2^2 - 2 a_2 c_5 T_1 T_2^2 + 6 a_4 c_5 T_1 T_2^2 + 4 a_2 c_6 T_1 T_2^2 - 6 a_4 c_6 T_1 T_2^2 - 4 a_4 c_{13} T_1 T_2^2 - 4 a_2 c_{14} T_1 T_2^2 - \\ a_4 c_{14} T_1 T_2^2 + a_4 c_{15} T_1 T_2^2 + a_2 c_{49} T_1 T_2^2 + a_2 c_{50} T_1 T_2^2 + a_2 c_{51} T_1 T_2^2 - a_4 c_{52} T_1 T_2^2 + a_4 c_{53} T_1 T_2^2 + \\ a_2 c_{54} T_1 T_2^2 - 2 a_2 c_4 T_1^2 T_2^2 - a_4 c_4 T_1^2 T_2^2 - a_2 c_5 T_1^2 T_2^2 - 3 a_4 c_5 T_1^2 T_2^2 - 4 a_2 c_6 T_1^2 T_2^2 + 6 a_4 c_6 T_1^2 T_2^2 + \\ 2 a_2 c_{13} T_1^2 T_2^2 - a_4 c_{13} T_1^2 T_2^2 - 3 a_2 c_{14} T_1^2 T_2^2 - 4 a_2 c_{15} T_1^2 T_2^2 - a_2 c_{51} T_1^2 T_2^2 + a_2 c_{52} T_1^2 T_2^2 - a_2 c_{54} T_1^2 T_2^2 + \\ a_2 c_4 T_1^3 T_2^2 - 2 a_4 c_4 T_1^3 T_2^2 + 4 a_2 c_{15} T_1^3 T_2^2 - a_4 c_{15} T_1^3 T_2^2 - a_2 c_{49} T_1^3 T_2^2 - a_2 c_{50} T_1^3 T_2^2 - a_4 c_{53} T_1^3 T_2^2 + \\ 3 a_4 c_5 T_2^3 - 2 a_4 c_{13} T_2^3 - a_4 c_{14} T_2^3 - a_4 c_{52} T_2^3 + a_4 c_4 T_1 T_2^3 - 6 a_4 c_5 T_1 T_2^3 + 3 a_4 c_6 T_1 T_2^3 + 2 a_2 c_{13} T_1 T_2^3 + \\ a_4 c_{13} T_1 T_2^3 + a_2 c_{14} T_1 T_2^3 + a_2 c_{52} T_1 T_2^3 - 4 a_4 c_4 T_1^2 T_2^3 + 3 a_4 c_5 T_1^2 T_2^3 - 3 a_4 c_6 T_1^2 T_2^3 - 2 a_2 c_{13} T_1^2 T_2^3 + \\ 4 a_4 c_{13} T_1^2 T_2^3 + 3 a_2 c_{14} T_1^2 T_2^3 + a_4 c_{14} T_1^2 T_2^3 - a_4 c_{15} T_1^2 T_2^3 - a_2 c_{49} T_1^2 T_2^3 - a_2 c_{50} T_1^2 T_2^3 - a_2 c_{52} T_1^2 T_2^3 + \\ a_4 c_{52} T_1^2 T_2^3 - a_4 c_{53} T_1^2 T_2^3 + 3 a_4 c_4 T_1^3 T_2^3 + a_4 c_{15} T_1^3 T_2^3 + a_2 c_{49} T_1^3 T_2^3 + a_2 c_{50} T_1^3 T_2^3 + a_4 c_{53} T_1^3 T_2^3 + \\ 2 a_4 c_{13} T_1 T_2^4 + a_4 c_{14} T_1 T_2^4 + a_4 c_{52} T_1 T_2^4 - 3 a_4 c_{13} T_1^2 T_2^4 - a_4 c_{14} T_1^2 T_2^4 - a_4 c_{52} T_1^2 T_2^4 \Big) p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i} - \\ \left(a_4 c_4 + a_2 c_5 - a_2 c_{14} - a_4 c_4 T_1 - a_2 c_5 T_1 - a_4 c_4 T_1 T_2 + a_2 c_{14} T_1 T_2 + a_4 c_4 T_1^2 T_2 \right) p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i} -$$

$$\frac{1}{(-1 + T_1) (-1 + T_1 T_2)}$$

$$\frac{1}{-1 + T_1 T_2}$$

$$\left(a_4 c_4 + a_2 c_5 - a_2 c_{13} + a_4 c_{13} - a_4 c_4 T_1 - a_2 c_5 T_1 - \right. \\ \left. a_4 c_{13} T_2 - a_4 c_4 T_1 T_2 + a_2 c_{13} T_1 T_2 - a_4 c_{13} T_1 T_2 + a_4 c_4 T_1^2 T_2 + a_4 c_{13} T_1 T_2^2 \right) \\ p_{1,2+j} p_{3,2+j} \pi_{1,i}^2 \pi_{2,i} + \frac{1}{(-1 + T_1) T_1^2 (-1 + T_2) T_2 (-1 + T_1 T_2)}$$

$$\left(a_2 + a_4 T_2 \right) \\ \left(c_{51} + c_{54} - c_{51} T_1 - c_{54} T_1 + c_{49} T_2 - c_{51} T_2 + c_{52} T_2 - c_{54} T_2 + 3 c_4 T_1 T_2 - 4 c_6 T_1 T_2 - 3 c_{13} T_1 T_2 + 4 c_{15} T_1 T_2 - \right. \\ c_{49} T_1 T_2 - c_{50} T_1 T_2 - c_{52} T_1 T_2 - 3 c_4 T_1^2 T_2 + 4 c_6 T_1^2 T_2 - 4 c_{15} T_1^2 T_2 + c_{50} T_1^2 T_2 + c_{51} T_1^2 T_2 + c_{54} T_1^2 T_2 - \\ c_{49} T_2^2 - c_{52} T_2^2 + 4 c_6 T_1 T_2^2 + 3 c_{13} T_1 T_2^2 + c_{50} T_1 T_2^2 + c_{51} T_1 T_2^2 + c_{54} T_1 T_2^2 - 3 c_4 T_1^2 T_2^2 - 4 c_6 T_1^2 T_2^2 +$$

$$\begin{aligned}
 & \frac{3 c_{13} T_1^2 T_2^2 - 4 c_{15} T_1^2 T_2^2 + c_{49} T_1^2 T_2^2 - c_{51} T_1^2 T_2^2 + c_{52} T_1^2 T_2^2 - c_{54} T_1^2 T_2^2 + 3 c_4 T_1^3 T_2^2 + 4 c_{15} T_1^3 T_2^2 - c_{50} T_1^3 T_2^2 + c_{49} T_1 T_2^3 + c_{52} T_1 T_2^3 - 3 c_{13} T_1^2 T_2^3 - c_{49} T_1^2 T_2^3 - c_{50} T_1^2 T_2^3 - c_{52} T_1^2 T_2^3 + c_{50} T_1^3 T_2^3}{(a_2 + a_4 T_2) (-c_5 + c_{14} + c_5 T_1 - c_{14} T_1 T_2) p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{1,j} \pi_{2,i}} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(-a_2 c_5 + a_2 c_{13} + a_2 c_4 T_1 + 2 a_2 c_5 T_1 - 2 a_2 c_{13} T_1 + a_4 c_{13} T_1 - a_2 c_4 T_1^2 - a_2 c_5 T_1^2 + a_2 c_5 T_2 - a_4 c_5 T_2 - a_2 c_{13} T_2 + a_4 c_{13} T_2 - 2 a_2 c_5 T_1 T_2 + 2 a_4 c_5 T_1 T_2 + a_2 c_{13} T_1 T_2 - 3 a_4 c_{13} T_1 T_2 - a_2 c_4 T_1^2 T_2 + a_2 c_5 T_1^2 T_2 - a_4 c_5 T_1^2 T_2 + 2 a_2 c_{13} T_1^2 T_2 - a_4 c_{13} T_1^2 T_2 + a_2 c_4 T_1^3 T_2 + a_4 c_5 T_2^2 - a_4 c_{13} T_2^2 - 2 a_4 c_5 T_1 T_2^2 + a_2 c_{13} T_1 T_2^2 + a_4 c_{13} T_1 T_2^2 + a_4 c_5 T_1^2 T_2^2 - 2 a_2 c_{13} T_1^2 T_2^2 + 3 a_4 c_{13} T_1^2 T_2^2 + a_4 c_{13} T_1 T_2^3 - 2 a_4 c_{13} T_1^2 T_2^3) p_{1,2+j} p_{3,2+j} \pi_{1,i} \pi_{1,j} \pi_{2,i} + (a_2 + a_4 T_2) (-c_4 + c_{13} + c_4 T_1 - c_{13} T_2) p_{1,2+j} p_{3,2+j} \pi_{1,j}^2 \pi_{2,i}}{(-1 + T_1) T_1 (-1 + T_2)} \\
 & \frac{1}{T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(a_2 c_{13} - a_2 c_6 T_1 + a_2 c_{15} T_1 - a_2 c_{13} T_2 + a_4 c_{13} T_2 + a_2 c_6 T_1 T_2 - a_2 c_{13} T_1 T_2 - a_2 c_{15} T_1^2 T_2 - a_4 c_{13} T_2^2 + a_2 c_{13} T_1 T_2^2 - a_4 c_{13} T_1 T_2^2 + a_4 c_{13} T_1 T_2^3) p_{2,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i} - \frac{1}{T_1 (-1 + T_1 T_2)}}{(-a_2 c_4 - a_2 c_{13} + a_2 c_6 T_1 - a_4 c_4 T_2 + a_2 c_{13} T_2 - a_4 c_{13} T_2 + a_2 c_4 T_1 T_2 + a_4 c_4 T_1 T_2 - a_2 c_6 T_1 T_2 + a_2 c_{13} T_1 T_2 + a_4 c_{13} T_2^2 + a_4 c_4 T_1 T_2^2 - a_2 c_{13} T_1 T_2^2 + a_4 c_{13} T_1 T_2^2 - a_4 c_4 T_1^2 T_2^2 - a_4 c_{13} T_1 T_2^3) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,i} + ((a_2 + a_4 T_2) (c_4 - c_6 - c_{13} + c_{15} - c_4 T_1 + c_6 T_1 - c_{15} T_1 + c_6 T_2 + c_{13} T_2 - c_4 T_1 T_2 - c_6 T_1 T_2 + c_{13} T_1 T_2 - c_{15} T_1 T_2 + c_4 T_1^2 T_2 + c_{15} T_1^2 T_2 - c_{13} T_1 T_2^2) p_{2,2+i} p_{3,2+j} \pi_{1,j} \pi_{2,i}) / ((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)) - (-1 + T_2) (a_2 + a_4 T_2) (-c_6 - c_{13} + c_6 T_1 + c_{13} T_1 T_2) p_{2,2+j} p_{3,2+j} \pi_{1,j} \pi_{2,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) T_2 (-1 + T_1 T_2)} \\
 & \frac{a_4 (-c_{51} - c_{54} - c_{50} T_1 + c_{51} T_1 - c_{53} T_1 + c_{54} T_1 + c_{50} T_1^2 + c_{53} T_1^2 + c_{51} T_2 + c_{54} T_2 + 3 c_4 T_1 T_2 + 4 c_5 T_1 T_2 - 3 c_{13} T_1 T_2 - 4 c_{14} T_1 T_2 + c_{49} T_1 T_2 + c_{50} T_1 T_2 + c_{53} T_1 T_2 - 3 c_4 T_1^2 T_2 - 4 c_5 T_1^2 T_2 - c_{49} T_1^2 T_2 - c_{51} T_1^2 T_2 - c_{54} T_1^2 T_2 - c_{50} T_1^3 T_2 - c_{53} T_1^3 T_2 - 4 c_5 T_1 T_2^2 + 3 c_{13} T_1 T_2^2 + 4 c_{14} T_1 T_2^2 - c_{49} T_1 T_2^2 - c_{51} T_1 T_2^2 - c_{54} T_1 T_2^2 - 3 c_4 T_1^2 T_2^2 + 4 c_5 T_1^2 T_2^2 + 3 c_{13} T_1^2 T_2^2 + 4 c_{14} T_1^2 T_2^2 - c_{50} T_1^2 T_2^2 + c_{51} T_1^2 T_2^2 - c_{53} T_1^2 T_2^2 + c_{54} T_1^2 T_2^2 + 3 c_4 T_1^3 T_2^2 + c_{49} T_1^3 T_2^2 + c_{50} T_1^3 T_2^2 + c_{53} T_1^3 T_2^2 - 3 c_{13} T_1^2 T_2^3 - 4 c_{14} T_1^2 T_2^3 + c_{49} T_1^2 T_2^3 - c_{49} T_1^3 T_2^3) p_{3,2+j} \pi_{1,i} \pi_{2,j} + (a_4 (c_4 + c_5 - c_{13} - c_{14} - c_4 T_1 - c_5 T_1 - c_5 T_2 + c_{13} T_2 + c_{14} T_2 - c_4 T_1 T_2 + c_5 T_1 T_2 + c_{13} T_1 T_2 + c_{14} T_1 T_2 + c_4 T_1^2 T_2 - c_{13} T_1 T_2^2 - c_{14} T_1 T_2^2) p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,j}) / ((-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)) + \frac{a_4 (-1 + T_1) (-c_4 - c_5 + c_5 T_2 + c_4 T_1 T_2) p_{1,2+j} p_{3,2+j} \pi_{1,i}^2 \pi_{2,j}}{(-1 + T_2) (-1 + T_1 T_2)} + a_4 (-c_6 + c_{15} + c_6 T_2 - c_{15} T_1 T_2) p_{2,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{2,j}}{(-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(a_2 c_4 - a_2 c_4 T_1 + a_4 c_4 T_1 - a_4 c_6 T_1 - a_2 c_{13} T_1 - a_4 c_4 T_1^2 + a_4 c_6 T_1^2 + a_4 c_4 T_2 - a_2 c_4 T_1 T_2 - 3 a_4 c_4 T_1 T_2 + 2 a_4 c_6 T_1 T_2 + a_2 c_{13} T_1 T_2 + a_2 c_4 T_1^2 T_2 + a_4 c_4 T_1^2 T_2 - 2 a_4 c_6 T_1^2 T_2 + a_2 c_{13} T_1^2 T_2 + a_4 c_4 T_1^3 T_2 - a_4 c_4 T_1 T_2^2 - a_4 c_6 T_1 T_2^2 + 3 a_4 c_4 T_1^2 T_2^2 + a_4 c_6 T_1^2 T_2^2 - a_2 c_{13} T_1^2 T_2^2 - 2 a_4 c_4 T_1^3 T_2^2) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{2,j} + a_4 (-c_4 + c_{13} + c_4 T_1 - c_{13} T_2) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,j}^2 - (-a_2 c_{14} + a_4 c_{15} T_1 - a_4 c_{14} T_2) p_{3,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{3,i}}{(-1 + T_1) (-1 + T_2) T_1}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{T_1} \\
 & \left(a_2 c_5 + a_2 c_{14} + a_2 c_6 T_1 - a_4 c_6 T_1 - a_4 c_{15} T_1 + a_4 c_5 T_2 + a_4 c_{14} T_2 - \right. \\
 & \quad \left. a_4 c_5 T_1 T_2 + a_4 c_6 T_1 T_2 - a_2 c_{14} T_1 T_2 + a_4 c_{15} T_1^2 T_2 - a_4 c_{14} T_1 T_2^2 \right) p_{3,2+j}^2 \pi_{1,i} \pi_{2,i} \pi_{3,i} + \\
 & \frac{(a_2 + a_4 T_2) (-c_5 + c_{14} + c_5 T_1 - c_{14} T_1 T_2) p_{3,2+i} p_{3,2+j} \pi_{1,j} \pi_{2,i} \pi_{3,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} + \\
 & \frac{(a_2 + a_4 T_2) (-c_6 + c_{14} + c_6 T_1 - c_{14} T_1 T_2) p_{3,2+j}^2 \pi_{1,j} \pi_{2,i} \pi_{3,i}}{(-1 + T_1) T_1} - \\
 & \frac{a_4 (-c_6 + c_{15} + c_6 T_2 - c_{15} T_1 T_2) p_{3,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,j} \pi_{3,i}}{(-1 + T_2) (-1 + T_1 T_2)} - \\
 & \frac{a_4 (-c_5 + c_{15} + c_5 T_2 - c_{15} T_1 T_2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,j} \pi_{3,i}}{-1 + T_2} + \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \left(a_2 c_5 - a_2 c_5 T_1 + a_2 c_6 T_1 - a_4 c_6 T_1 - a_2 c_{14} T_1 - a_2 c_{15} T_1 - a_2 c_6 T_1^2 + a_4 c_6 T_1^2 + a_2 c_{15} T_1^2 - \right. \\
 & \quad a_2 c_5 T_2 + a_4 c_5 T_2 + a_2 c_5 T_1 T_2 - 2 a_4 c_5 T_1 T_2 - a_2 c_6 T_1 T_2 + 2 a_4 c_6 T_1 T_2 + a_2 c_{14} T_1 T_2 + \\
 & \quad \left. a_4 c_5 T_1^2 T_2 + a_2 c_6 T_1^2 T_2 - 2 a_4 c_6 T_1^2 T_2 + a_2 c_{14} T_1^2 T_2 + a_2 c_{15} T_1^2 T_2 - a_2 c_{15} T_1^3 T_2 - a_4 c_5 T_2^2 + \right. \\
 & \quad \left. 2 a_4 c_5 T_1 T_2^2 - a_4 c_6 T_1 T_2^2 - a_4 c_5 T_1^2 T_2^2 + a_4 c_6 T_1^2 T_2^2 - a_2 c_{14} T_1^2 T_2^2 \right) p_{3,2+j}^2 \pi_{1,i} \pi_{2,i} \pi_{3,j} - \\
 & \frac{(a_2 + a_4 T_2) (-c_6 + c_{15} + c_6 T_2 - c_{15} T_1 T_2) p_{3,2+j}^2 \pi_{1,j} \pi_{2,i} \pi_{3,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} + \\
 & \frac{a_4 (-c_5 + c_{14} + c_5 T_1 - c_{14} T_1 T_2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,j} \pi_{3,j}}{(-1 + T_1) (-1 + T_1 T_2)} \\
 & \left(-a_2 a_4 c_4 + a_2^2 c_{13} - a_2 a_4 c_{13} - a_4^2 c_4 T_2 + 2 a_2 a_4 c_{13} T_2 - a_4^2 c_{13} T_2 + a_4^2 c_4 T_1 T_2 + a_4^2 c_{13} T_2^2 \right) p_{3,2+j}^2 \pi_{1,i}^2 \pi_{2,i}^2 - \\
 & \frac{1}{(-1 + T_1) T_1^2 (-1 + T_2)} (a_2 + a_4 T_2) (-a_2 c_{13} - a_2 c_4 T_1 + 2 a_2 c_{13} T_1 - a_4 c_{13} T_1 + a_2 c_4 T_1^2 + a_2 c_{13} T_2 - \\
 & \quad a_4 c_{13} T_2 - 2 a_2 c_{13} T_1 T_2 + 3 a_4 c_{13} T_1 T_2 + a_4 c_{13} T_2^2 - 2 a_4 c_{13} T_1 T_2^2) p_{3,2+j}^2 \pi_{1,i} \pi_{1,j} \pi_{2,i}^2 + \\
 & \frac{(a_2 + a_4 T_2)^2 (-c_4 + c_{13} + c_4 T_1 - c_{13} T_2) p_{3,2+j}^2 \pi_{1,j}^2 \pi_{2,i}^2}{(-1 + T_1) T_1^2 (-1 + T_2)} - \frac{1}{(-1 + T_1) T_1 (-1 + T_2)} \\
 & a_4 (-a_2 c_4 + a_2 c_4 T_1 - a_4 c_4 T_1 + a_2 c_{13} T_1 + a_4 c_4 T_1^2 - a_4 c_4 T_2 + 3 a_4 c_4 T_1 T_2 - a_2 c_{13} T_1 T_2 - 2 a_4 c_4 T_1^2 T_2) \\
 & p_{3,2+j}^2 \pi_{1,i}^2 \pi_{2,i} \pi_{2,j} - \frac{a_4^2 (-c_4 + c_{13} + c_4 T_1 - c_{13} T_2) p_{3,2+j}^2 \pi_{1,i}^2 \pi_{2,j}^2}{(-1 + T_1) (-1 + T_2)} \\
 & \frac{1}{(-1 + T_1) T_1^2 (-1 + T_2) T_2 (-1 + T_1 T_2)} \\
 & \left(a_2^2 b_5 + a_2^2 b_3 T_1 - a_2 a_4 b_3 T_1 - a_2^2 b_5 T_1 - a_2^2 b_3 T_1^2 + a_2 a_4 b_3 T_1^2 - a_2^2 b_5 T_2 + a_2 a_4 b_5 T_2 + 5 a_2^2 b_3 T_1 T_2 + \right. \\
 & \quad a_2 a_4 b_3 T_1 T_2 + 2 a_2^2 b_5 T_1 T_2 - 3 a_2 a_4 b_5 T_1 T_2 - 7 a_2^2 b_3 T_1^2 T_2 + a_2 a_4 b_3 T_1^2 T_2 + 3 a_2^2 b_5 T_1^2 T_2 - \\
 & \quad 2 a_2 a_4 b_5 T_1^2 T_2 + 2 a_2^2 b_3 T_1^3 T_2 - 2 a_2 a_4 b_3 T_1^3 T_2 - a_2 a_4 b_5 T_2^2 + 8 a_2 a_4 b_3 T_1 T_2^2 - a_4^2 b_3 T_1 T_2^2 - \\
 & \quad a_2^2 b_5 T_1 T_2^2 + 7 a_2 a_4 b_5 T_1 T_2^2 - a_4^2 b_5 T_1 T_2^2 - 18 a_2 a_4 b_3 T_1^2 T_2^2 + 4 a_4^2 b_3 T_1^2 T_2^2 - 3 a_2^2 b_5 T_1^2 T_2^2 + \\
 & \quad 2 a_2 a_4 b_5 T_1^2 T_2^2 + 3 a_4^2 b_5 T_1^2 T_2^2 + 10 a_2 a_4 b_3 T_1^3 T_2^2 - 3 a_4^2 b_3 T_1^3 T_2^2 + 3 a_4^2 b_3 T_1 T_2^3 - 4 a_2 a_4 b_5 T_1 T_2^3 + \\
 & \quad \left. 4 a_4^2 b_5 T_1 T_2^3 - 8 a_4^2 b_3 T_1 T_2^3 - 8 a_4^2 b_5 T_1 T_2^3 + 5 a_4^2 b_3 T_1^2 T_2^3 - 3 a_4^2 b_5 T_1 T_2^4 + 5 a_4^2 b_5 T_1^2 T_2^4 \right) \\
 & p_{3,2+j} \pi_{1,i} \pi_{2,i} - \frac{a_2 b_5 (a_2 - a_4 + a_4 T_2) p_{1,2+i} p_{3,2+j} \pi_{1,i}^2 \pi_{2,i}}{(-1 + T_1) (-1 + T_1 T_2)} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(a_2 - a_4 + a_4 T_2) (-a_2 b_3 + a_2 b_3 T_1 - a_4 b_3 T_2 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_2^2) p_{1,2+j} p_{3,2+j} \pi_{1,i}^2 \pi_{2,i}}{(-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1^3 (-1 + T_2) T_2 (-1 + T_1 T_2)} \\
 & \frac{(a_2 + a_4 T_2) (a_2 b_5 + a_2 b_3 T_1 - a_4 b_3 T_1 - a_2 b_5 T_1 - a_2 b_3 T_1^2 + a_4 b_3 T_1^2 - a_2 b_5 T_2 + a_4 b_5 T_2 + a_2 b_3 T_1 T_2 + 2 a_4 b_3 T_1 T_2 + 3 a_2 b_5 T_1 T_2 - 2 a_4 b_5 T_1 T_2 - 2 a_2 b_3 T_1^2 T_2 - a_4 b_3 T_1^2 T_2 + a_2 b_5 T_1^2 T_2 + a_2 b_3 T_1^3 T_2 - a_4 b_3 T_1^3 T_2 - a_4 b_5 T_2^2 + a_4 b_3 T_1 T_2^2 - 2 a_2 b_5 T_1 T_2^2 + 4 a_4 b_5 T_1 T_2^2 - 4 a_4 b_3 T_1^2 T_2^2 - a_2 b_5 T_1^2 T_2^2 - a_4 b_5 T_1^2 T_2^2 + 3 a_4 b_3 T_1^3 T_2^2 - 2 a_4 b_5 T_1 T_2^3 + a_4 b_5 T_1^2 T_2^3) p_{3,2+j} \pi_{1,j} \pi_{2,i} + b_5 (a_2 + a_4 T_2) (a_2 - a_4 + a_4 T_2) p_{1,2+i} p_{3,2+j} \pi_{1,i} \pi_{1,j} \pi_{2,i}}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(a_2^2 b_5 - a_2 a_4 b_5 - a_2^2 b_3 T_1 + a_2 a_4 b_3 T_1 + a_2^2 b_3 T_1^2 - a_2 a_4 b_3 T_1^2 + a_2 a_4 b_3 T_2 - a_2^2 b_5 T_2 + 2 a_2 a_4 b_5 T_2 - a_4^2 b_5 T_2 - 3 a_2 a_4 b_3 T_1 T_2 + a_4^2 b_3 T_1 T_2 + 2 a_2 a_4 b_3 T_1^2 T_2 - a_4^2 b_3 T_1^2 T_2 + a_4^2 b_3 T_2^2 - a_2 a_4 b_5 T_2^2 + a_4^2 b_5 T_2^2 - 3 a_4^2 b_3 T_1 T_2^2 + a_4^2 b_5 T_1 T_2^2 + 2 a_4^2 b_3 T_1^2 T_2^2 - a_4^2 b_5 T_1 T_2^3) p_{1,2+j} p_{3,2+j} \pi_{1,i} \pi_{1,j} \pi_{2,i} - (a_2 + a_4 T_2) (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{1,2+j} p_{3,2+j} \pi_{1,j}^2 \pi_{2,i}}{(-1 + T_1) T_1^2 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{a_2 b_3 (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{2,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i}^2}{T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(a_2 + a_4 T_2 - a_4 T_1 T_2) (-a_2 b_3 + a_2 b_3 T_1 - a_4 b_3 T_2 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_2^2) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,i}^2}{(-1 + T_1) T_1 (-1 + T_1 T_2)} \\
 & \frac{b_5 (a_2 + a_4 T_2) (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{2,2+i} p_{3,2+j} \pi_{1,j} \pi_{2,i}^2}{(-1 + T_1) T_1^2 (-1 + T_1 T_2)} \\
 & \frac{b_5 (-1 + T_2) (a_2 + a_4 T_2) (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{2,2+j} p_{3,2+j} \pi_{1,j} \pi_{2,i}^2}{(-1 + T_1) T_1^2 (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1^2 (-1 + T_2) T_2 (-1 + T_1 T_2)} \\
 & \frac{a_4 (-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - 2 a_4 b_3 T_1 T_2 + 2 a_4 b_5 T_1 T_2 - 2 a_2 b_3 T_1^2 T_2 + 4 a_4 b_3 T_1^2 T_2 - 2 a_2 b_5 T_1^2 T_2 + a_4 b_5 T_1^2 T_2 + 2 a_2 b_3 T_1^3 T_2 - 2 a_4 b_3 T_1^3 T_2 + a_4 b_5 T_2^2 - a_2 b_5 T_1 T_2^2 - a_4 b_5 T_1 T_2^2 - a_4 b_3 T_1^2 T_2^2 + 2 a_2 b_5 T_1^2 T_2^2 - 4 a_4 b_5 T_1^2 T_2^2 + a_4 b_3 T_1^3 T_2^2 - a_4 b_5 T_1 T_2^3 + 3 a_4 b_5 T_1^2 T_2^3) p_{3,2+j} \pi_{1,i} \pi_{2,j} - a_4 b_3 (a_2 - a_4 + a_4 T_2) p_{1,2+i} p_{3,2+j} \pi_{1,i}^2 \pi_{2,j} - a_4 b_3 (-1 + T_1) (a_2 - a_4 + a_4 T_2) p_{1,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,j}}{(-1 + T_2) (-1 + T_1 T_2) (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{a_4 b_3 (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{2,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{2,j}}{T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(a_2 a_4 b_3 - a_2^2 b_3 T_1 + a_2^2 b_5 T_1 - a_2 a_4 b_5 T_1 + a_2^2 b_3 T_1^2 - a_2 a_4 b_3 T_1^2 + a_2^2 b_3 T_2 - a_2 a_4 b_5 T_2 - 2 a_2 a_4 b_3 T_1 T_2 - a_4^2 b_3 T_1 T_2 - a_2^2 b_5 T_1 T_2 + 3 a_2 a_4 b_5 T_1 T_2 - a_4^2 b_5 T_1 T_2 + 2 a_2 a_4 b_3 T_1^2 T_2 + a_2 a_4 b_5 T_2^2 - a_4^2 b_5 T_2^2 - a_4^2 b_3 T_1 T_2^2 - 2 a_2 a_4 b_5 T_1 T_2^2 + 3 a_4^2 b_3 T_1^2 T_2^2 + a_4^2 b_5 T_2^3 - 2 a_4^2 b_5 T_1 T_2^3) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{2,j} - a_4 (a_2 - a_4 + a_4 T_2) (-b_3 + b_5 + b_3 T_1 - b_5 T_2) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,j}^2}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2) (-1 + T_1) T_1 (-1 + T_2)} \\
 & \frac{(a_2^2 b_3 - 2 a_2^2 b_3 T_1 + a_2^2 b_3 T_1^2 + 2 a_2 a_4 b_3 T_2 - 4 a_2 a_4 b_3 T_1 T_2 + a_4^2 b_5 T_1 T_2 + 2 a_2 a_4 b_3 T_1^2 T_2 + 2 a_2 a_4 b_5 T_2^2 - a_4^2 b_5 T_2^2 - a_4^2 b_3 T_1 T_2^2 - 2 a_2 a_4 b_5 T_1 T_2^2 + 3 a_4^2 b_3 T_1^2 T_2^2 + a_4^2 b_5 T_2^3) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{2,j}}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2)} \\
 & \frac{(a_2^2 b_3 - 2 a_2^2 b_3 T_1 + a_2^2 b_3 T_1^2 + 2 a_2 a_4 b_3 T_2 - 4 a_2 a_4 b_3 T_1 T_2 + a_4^2 b_5 T_1 T_2 + 2 a_2 a_4 b_3 T_1^2 T_2 + 2 a_2 a_4 b_5 T_2^2 - a_4^2 b_5 T_2^2 - a_4^2 b_3 T_1 T_2^2 - 2 a_2 a_4 b_5 T_1 T_2^2 + 3 a_4^2 b_3 T_1^2 T_2^2 + a_4^2 b_5 T_2^3) p_{2,2+j} p_{3,2+j} \pi_{1,i} \pi_{2,i} \pi_{2,j}}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{a_4^2 b_3 T_2^2 - 2 a_4^2 b_3 T_1 T_2^2 - 2 a_4^2 b_5 T_1 T_2^2 + a_4^2 b_3 T_1^2 T_2^2 + a_4^2 b_5 T_1 T_2^3}{(a_2 + a_4 T_2) (a_2 b_3 - a_2 b_5 - 2 a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - 3 a_4 b_3 T_1 T_2 + a_4 b_5 T_1 T_2 + 2 a_4 b_3 T_1^2 T_2 + a_4 b_5 T_2^2 - a_4 b_5 T_1 T_2^2)} p_{3,2+j}^2 \pi_{1,i} \pi_{2,i} \pi_{3,i} - \\
 & \frac{p_{3,2+i} p_{3,2+j} \pi_{1,j} \pi_{2,i} \pi_{3,i}}{((-1 + T_1) T_1^2 (-1 + T_2) (-1 + T_1 T_2))} + \\
 & \frac{(a_2 + a_4 T_2) (-a_2 b_3 + a_2 b_5 + a_2 b_3 T_1^2 - a_4 b_3 T_2 - a_2 b_5 T_2 + a_4 b_5 T_2 + a_4 b_3 T_1 T_2 - a_4 b_5 T_2^2) p_{3,2+j}^2 \pi_{1,j} \pi_{2,i} \pi_{3,i}}{(-1 + T_1) T_1^2 (-1 + T_2)} + \\
 & \frac{(a_4 (-a_2 b_5 - a_2 b_3 T_1 + a_4 b_3 T_1 + a_2 b_5 T_1 - a_4 b_5 T_1 + a_2 b_3 T_1^2 - a_4 b_3 T_1^2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 - a_2 b_5 T_1 T_2 + 3 a_4 b_5 T_1 T_2 + a_4 b_3 T_1^2 T_2 + a_4 b_5 T_2^2 - 2 a_4 b_5 T_1 T_2^2) p_{3,2+i} p_{3,2+j} \pi_{1,i} \pi_{2,j} \pi_{3,i})}{((-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2))} - \\
 & \frac{a_4 (-a_4 b_3 - a_2 b_5 + a_4 b_5 + a_4 b_3 T_1 + a_2 b_5 T_2 - a_4 b_5 T_2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,j} \pi_{3,i}}{(-1 + T_1) (-1 + T_2)} + \\
 & \frac{a_4 (a_2 + a_4 T_2) (-b_3 - b_5 + b_3 T_1 + b_5 T_2) p_{3,2+j}^2 \pi_{1,j} \pi_{2,j} \pi_{3,i}}{(-1 + T_1) T_1 (-1 + T_2)} + \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{(2 a_2^2 b_3 - 3 a_2^2 b_3 T_1 + a_2 a_4 b_3 T_1 + a_2^2 b_5 T_1 - a_2 a_4 b_5 T_1 + a_2^2 b_3 T_1^2 - a_2 a_4 b_3 T_1^2 + 3 a_2 a_4 b_3 T_2 + a_2 a_4 b_5 T_2 - 7 a_2 a_4 b_3 T_1 T_2 + a_4^2 b_3 T_1 T_2 - a_2^2 b_5 T_1 T_2 + a_2 a_4 b_5 T_1 T_2 + a_4^2 b_5 T_1 T_2 + 4 a_2 a_4 b_3 T_1^2 T_2 - a_4^2 b_3 T_1^2 T_2 + a_4^2 b_3 T_2^2 - a_2 a_4 b_5 T_2^2 + a_4^2 b_5 T_2^2 - 3 a_4^2 b_3 T_1 T_2^2 - 3 a_4^2 b_5 T_1 T_2^2 + 2 a_4^2 b_3 T_1^2 T_2^2 - a_4^2 b_5 T_2^3 + 2 a_4^2 b_5 T_1 T_2^3) p_{3,2+j}^2 \pi_{1,i} \pi_{2,i} \pi_{3,j} + b_3 (a_2 + a_4 T_2) (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{3,2+j}^2 \pi_{1,j} \pi_{2,i} \pi_{3,j} - a_4 b_5 (a_2 - a_4 + a_4 T_2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,j} \pi_{3,j}}{T_1^2 (-1 + T_2) (-1 + T_1 T_2) (-1 + T_1) (-1 + T_1 T_2)} \\
 & \frac{a_4 (a_2 + a_4 T_2) (a_2 b_3 + a_4 b_3 T_2 + a_2 b_5 T_2 - a_4 b_5 T_2 - a_4 b_3 T_1 T_2 + a_4 b_5 T_2^2) p_{3,2+j}^2 \pi_{1,i} \pi_{2,i}^2}{T_1 (-1 + T_1 T_2)} + \\
 & \frac{(a_2 + a_4 T_2) (-a_2^2 b_3 + a_2^2 b_5 - a_2 a_4 b_5 + a_2^2 b_3 T_1 - a_2 a_4 b_3 T_2 - a_2^2 b_5 T_2 + 2 a_2 a_4 b_5 T_2 - a_4^2 b_5 T_2 + 2 a_2 a_4 b_3 T_1 T_2 - a_2 a_4 b_3 T_1^2 T_2 - a_2 a_4 b_5 T_2^2 + a_4^2 b_5 T_2^2 + a_4^2 b_5 T_1 T_2^2 - a_4^2 b_5 T_1 T_2^3) p_{3,2+j}^2 \pi_{1,i} \pi_{1,j} \pi_{2,i}^2}{((-1 + T_1) T_1^2 (-1 + T_2) (-1 + T_1 T_2))} - \\
 & \frac{(a_2 + a_4 T_2)^2 (-b_3 + b_5 + b_3 T_1 - b_5 T_2) (a_2 + a_4 T_2 - a_4 T_1 T_2) p_{3,2+j}^2 \pi_{1,j}^2 \pi_{2,i}^2}{(-1 + T_1) T_1^3 (-1 + T_2) (-1 + T_1 T_2)} + \\
 & \frac{1}{(-1 + T_1) T_1 (-1 + T_2) (-1 + T_1 T_2)} \\
 & \frac{a_4 (a_2 a_4 b_3 - a_2^2 b_3 T_1 + a_2^2 b_5 T_1 - a_2 a_4 b_5 T_1 + a_2^2 b_3 T_1^2 - a_2 a_4 b_3 T_1^2 + a_4^2 b_3 T_2 - 2 a_2 a_4 b_3 T_1 T_2 - a_4^2 b_3 T_1 T_2 - a_2^2 b_5 T_1 T_2 + 2 a_2 a_4 b_5 T_1 T_2 + 2 a_2 a_4 b_3 T_1^2 T_2 - a_4^2 b_3 T_1 T_2^2 - a_2 a_4 b_5 T_1 T_2^2 + a_4^2 b_3 T_1^2 T_2^2) p_{3,2+j}^2 \pi_{1,i}^2 \pi_{2,i} \pi_{2,j} + a_4^2 (a_2 - a_4 + a_4 T_2) (-b_3 + b_5 + b_3 T_1 - b_5 T_2) p_{3,2+j}^2 \pi_{1,i}^2 \pi_{2,j}^2}{(-1 + T_1) (-1 + T_2) (-1 + T_1 T_2)}
 \end{aligned}$$