Pensieve header: Finding the A2 $d=1$ invariant using undetermined coefficients.

Initialization

In[•]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\HigherRank"];
Once[<< KnotTheory`; << Rot.m];
<< FormalGaussianIntegration.m;
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.

Loading Rot.m from http://drorbn.net/AP/Projects/HigherRank to compute rotation numbers.
```

In[•]:=
```
Features[Knot[8, 17]]
```

Out[•]=

$$\text{Features}[18,\ C_6[-1]\,C_{14}[-1]\,X_{1,7}[1]\,X_{3,9}[-1]\,X_{5,13}[-1]\,X_{8,16}[1]\,X_{10,4}[-1]\,X_{12,18}[1]\,X_{15,2}[-1]\,X_{17,11}[1]]$$

Producing generic perturbations

In[•]:=
```
mons0 = MonomialList[
    p3 x1 x2 /.
     {(v : p | x)_α_ :> v_α,i + v_α,j}
   ] /. c_Integer * mon_ :> mon;
mons1 = MonomialList[
    1 + p1 x1 + p2 x2 + p3 x3 + p1 p1 x1 x1 +
      p2 p2 x2 x2 + p1 p2 x1 x2 + p1 p3 x1 x3 + p2 p3 x2 x3 + p3 p3 x3 x3 + p1 p2 x3 /.
     {(v : p | x)_α_ :> v_α,i + v_α,j}
   ] /. c_Integer * mon_ :> mon;
k = 0;
r1[1, i_, j_] := Evaluate[Sum[c_++k mon, {mon, mons0}] + ϵ Sum[c_++k mon, {mon, mons1}]];
r1[-1, i_, j_] := Evaluate[Sum[c_++k mon, {mon, mons0}] + ϵ Sum[c_++k mon, {mon, mons1}]];
r1[-1, 4, 7]
```

Out[•]=

$$
\begin{aligned}
&c_{105}\,p_{3,4}\,x_{1,4}\,x_{2,4} + c_{109}\,p_{3,7}\,x_{1,4}\,x_{2,4} + c_{107}\,p_{3,4}\,x_{1,7}\,x_{2,4} + c_{111}\,p_{3,7}\,x_{1,7}\,x_{2,4} +\\
&c_{106}\,p_{3,4}\,x_{1,4}\,x_{2,7} + c_{110}\,p_{3,7}\,x_{1,4}\,x_{2,7} + c_{108}\,p_{3,4}\,x_{1,7}\,x_{2,7} + c_{112}\,p_{3,7}\,x_{1,7}\,x_{2,7} +\\
&\epsilon\,\big(c_{208} + c_{127}\,p_{1,4}\,x_{1,4} + c_{152}\,p_{1,7}\,x_{1,4} + c_{113}\,p_{1,4}^2\,x_{1,4}^2 + c_{116}\,p_{1,4}\,p_{1,7}\,x_{1,4}^2 +\\
&\quad c_{141}\,p_{1,7}^2\,x_{1,4}^2 + c_{136}\,p_{1,4}\,x_{1,7} + c_{161}\,p_{1,7}\,x_{1,7} + c_{114}\,p_{1,4}^2\,x_{1,4}\,x_{1,7} + c_{117}\,p_{1,4}\,p_{1,7}\,x_{1,4}\,x_{1,7} +\\
&\quad c_{142}\,p_{1,7}^2\,x_{1,4}\,x_{1,7} + c_{115}\,p_{1,4}^2\,x_{1,7}^2 + c_{118}\,p_{1,4}\,p_{1,7}\,x_{1,7}^2 + c_{143}\,p_{1,7}^2\,x_{1,7}^2 + c_{176}\,p_{2,4}\,x_{2,4} +\\
&\quad c_{189}\,p_{2,7}\,x_{2,4} + c_{119}\,p_{1,4}\,p_{2,4}\,x_{1,4}\,x_{2,4} + c_{144}\,p_{1,7}\,p_{2,4}\,x_{1,4}\,x_{2,4} + c_{121}\,p_{1,4}\,p_{2,7}\,x_{1,4}\,x_{2,4} +\\
&\quad c_{146}\,p_{1,7}\,p_{2,7}\,x_{1,4}\,x_{2,4} + c_{128}\,p_{1,4}\,p_{2,4}\,x_{1,7}\,x_{2,4} + c_{153}\,p_{1,7}\,p_{2,4}\,x_{1,7}\,x_{2,4} + c_{130}\,p_{1,4}\,p_{2,7}\,x_{1,7}\,x_{2,4} +\\
&\quad c_{155}\,p_{1,7}\,p_{2,7}\,x_{1,7}\,x_{2,4} + c_{166}\,p_{2,4}^2\,x_{2,4}^2 + c_{169}\,p_{2,4}\,p_{2,7}\,x_{2,4}^2 + c_{182}\,p_{2,7}^2\,x_{2,4}^2 + c_{181}\,p_{2,4}\,x_{2,7} +\\
&\quad c_{194}\,p_{2,7}\,x_{2,7} + c_{120}\,p_{1,4}\,p_{2,4}\,x_{1,4}\,x_{2,7} + c_{145}\,p_{1,7}\,p_{2,4}\,x_{1,4}\,x_{2,7} + c_{122}\,p_{1,4}\,p_{2,7}\,x_{1,4}\,x_{2,7} +\\
&\quad c_{147}\,p_{1,7}\,p_{2,7}\,x_{1,4}\,x_{2,7} + c_{129}\,p_{1,4}\,p_{2,4}\,x_{1,7}\,x_{2,7} + c_{154}\,p_{1,7}\,p_{2,4}\,x_{1,7}\,x_{2,7} + c_{131}\,p_{1,4}\,p_{2,7}\,x_{1,7}\,x_{2,7} +\\
&\quad c_{156}\,p_{1,7}\,p_{2,7}\,x_{1,7}\,x_{2,7} + c_{167}\,p_{2,4}^2\,x_{2,4}\,x_{2,7} + c_{170}\,p_{2,4}\,p_{2,7}\,x_{2,4}\,x_{2,7} + c_{183}\,p_{2,7}^2\,x_{2,4}\,x_{2,7} +\\
&\quad c_{168}\,p_{2,4}^2\,x_{2,7}^2 + c_{171}\,p_{2,4}\,p_{2,7}\,x_{2,7}^2 + c_{184}\,p_{2,7}^2\,x_{2,7}^2 + c_{137}\,p_{1,4}\,p_{2,4}\,x_{3,4} + c_{162}\,p_{1,7}\,p_{2,4}\,x_{3,4} +\\
&\quad c_{139}\,p_{1,4}\,p_{2,7}\,x_{3,4} + c_{164}\,p_{1,7}\,p_{2,7}\,x_{3,4} + c_{201}\,p_{3,4}\,x_{3,4} + c_{202}\,p_{3,7}\,x_{3,4} + c_{123}\,p_{1,4}\,p_{3,4}\,x_{1,4}\,x_{3,4} +\\
&\quad c_{148}\,p_{1,7}\,p_{3,4}\,x_{1,4}\,x_{3,4} + c_{124}\,p_{1,4}\,p_{3,7}\,x_{1,4}\,x_{3,4} + c_{149}\,p_{1,7}\,p_{3,7}\,x_{1,4}\,x_{3,4} + c_{132}\,p_{1,4}\,p_{3,4}\,x_{1,7}\,x_{3,4} +\\
&\quad c_{157}\,p_{1,7}\,p_{3,4}\,x_{1,7}\,x_{3,4} + c_{133}\,p_{1,4}\,p_{3,7}\,x_{1,7}\,x_{3,4} + c_{158}\,p_{1,7}\,p_{3,7}\,x_{1,7}\,x_{3,4} + c_{172}\,p_{2,4}\,p_{3,4}\,x_{2,4}\,x_{3,4} +\\
&\quad c_{185}\,p_{2,7}\,p_{3,4}\,x_{2,4}\,x_{3,4} + c_{173}\,p_{2,4}\,p_{3,7}\,x_{2,4}\,x_{3,4} + c_{186}\,p_{2,7}\,p_{3,7}\,x_{2,4}\,x_{3,4} + c_{177}\,p_{2,4}\,p_{3,4}\,x_{2,7}\,x_{3,4} +\\
&\quad c_{190}\,p_{2,7}\,p_{3,4}\,x_{2,7}\,x_{3,4} + c_{178}\,p_{2,4}\,p_{3,7}\,x_{2,7}\,x_{3,4} + c_{191}\,p_{2,7}\,p_{3,7}\,x_{2,7}\,x_{3,4} + c_{195}\,p_{3,4}^2\,x_{3,4}^2 +\\
&\quad c_{196}\,p_{3,4}\,p_{3,7}\,x_{3,4}^2 + c_{197}\,p_{3,7}^2\,x_{3,4}^2 + c_{138}\,p_{1,4}\,p_{2,4}\,x_{3,7} + c_{163}\,p_{1,7}\,p_{2,4}\,x_{3,7} +\\
&\quad c_{140}\,p_{1,4}\,p_{2,7}\,x_{3,7} + c_{165}\,p_{1,7}\,p_{2,7}\,x_{3,7} + c_{206}\,p_{3,4}\,x_{3,7} + c_{207}\,p_{3,7}\,x_{3,7} + c_{125}\,p_{1,4}\,p_{3,4}\,x_{1,4}\,x_{3,7} +\\
&\quad c_{150}\,p_{1,7}\,p_{3,4}\,x_{1,4}\,x_{3,7} + c_{126}\,p_{1,4}\,p_{3,7}\,x_{1,4}\,x_{3,7} + c_{151}\,p_{1,7}\,p_{3,7}\,x_{1,4}\,x_{3,7} + c_{134}\,p_{1,4}\,p_{3,4}\,x_{1,7}\,x_{3,7} +\\
&\quad c_{159}\,p_{1,7}\,p_{3,4}\,x_{1,7}\,x_{3,7} + c_{135}\,p_{1,4}\,p_{3,7}\,x_{1,7}\,x_{3,7} + c_{160}\,p_{1,7}\,p_{3,7}\,x_{1,7}\,x_{3,7} + c_{174}\,p_{2,4}\,p_{3,4}\,x_{2,4}\,x_{3,7} +\\
&\quad c_{187}\,p_{2,7}\,p_{3,4}\,x_{2,4}\,x_{3,7} + c_{175}\,p_{2,4}\,p_{3,7}\,x_{2,4}\,x_{3,7} + c_{188}\,p_{2,7}\,p_{3,7}\,x_{2,4}\,x_{3,7} + c_{179}\,p_{2,4}\,p_{3,4}\,x_{2,7}\,x_{3,7} +\\
&\quad c_{192}\,p_{2,7}\,p_{3,4}\,x_{2,7}\,x_{3,7} + c_{180}\,p_{2,4}\,p_{3,7}\,x_{2,7}\,x_{3,7} + c_{193}\,p_{2,7}\,p_{3,7}\,x_{2,7}\,x_{3,7} + c_{198}\,p_{3,4}^2\,x_{3,4}\,x_{3,7} +\\
&\quad c_{199}\,p_{3,4}\,p_{3,7}\,x_{3,4}\,x_{3,7} + c_{200}\,p_{3,7}^2\,x_{3,4}\,x_{3,7} + c_{203}\,p_{3,4}^2\,x_{3,7}^2 + c_{204}\,p_{3,4}\,p_{3,7}\,x_{3,7}^2 + c_{205}\,p_{3,7}^2\,x_{3,7}^2\big)
\end{aligned}
$$

The A2 Integrand

Adopted from pensieve://Talks//Oaxaca-2210/Rho.nb.

```
In[•]:= T3 = T1 T2;
        S = {x__, p__};
        q[s_, i_, j_] :=
          Sum[xα,i (pα,i - pα,i+1) + xα,j (pα,j - pα,j+1) + xα,i ((1 - Tα^s) pα,i+1 + (Tα^s - 1) pα,j+1), {α, 3}];
        γ1[φ_, k_] := ϵ φ (3 / 2 - x1,k p1,k - x2,k p2,k - x3,k p3,k);
        ℒ[Xi_,j_[s_]] := T3^s 𝔼[-q[s, i, j] + r1[s, i, j] + O[ϵ]^1];
        ℒ[Ck_[φ_]] :=
          T3^φ 𝔼[-x1,k (p1,k - p1,k+1) - x2,k (p2,k - p2,k+1) - x3,k (p3,k - p3,k+1) + γ1[φ, k] + O[ϵ]^1];
        ℒ[K_] := (2 π)^-Features[K][[1]] CF[ℒ /@ Features[K][[2]]];
        vsi_ := Sequence[p1,i, x1,i, p2,i, x2,i, p3,i, x3,i];
        vs[K_] := Union @@ Table[{vsi}, {i, Features[K][[1]]}]
```

```
In[•]:= Features[Knot[3, 1]]
```

```
Out[•]=
```

$$\text{Features}[7, C_4[-1]\, X_{2,6}[-1]\, X_{5,1}[-1]\, X_{7,3}[-1]]$$

```
In[•]:= Short[ℒ[Knot[3, 1]], 10]
```

```
Out[•]//Short=
```

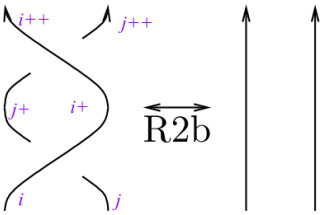
$$\frac{1}{128\,\pi^7\,T_1^4\,T_2^4}\,\mathbb{E}\Big[\epsilon\text{Series}\Big[-p_{1,1}\,x_{1,1}+p_{1,2}\,x_{1,1}-p_{1,2}\,x_{1,2}+\frac{p_{1,3}\,x_{1,2}}{T_1}+\frac{(-1+T_1)\,p_{1,7}\,x_{1,2}}{T_1}-p_{1,3}\,x_{1,3}+p_{1,4}\,x_{1,3}-$$
$$p_{1,4}\,x_{1,4}+p_{1,5}\,x_{1,4}+\frac{(-1+T_1)\,p_{1,2}\,x_{1,5}}{T_1}-p_{1,5}\,x_{1,5}+\frac{p_{1,6}\,x_{1,5}}{T_1}-p_{1,6}\,x_{1,6}+p_{1,7}\,x_{1,6}+$$
$$\frac{(-1+T_1)\,p_{1,4}\,x_{1,7}}{T_1}-p_{1,7}\,x_{1,7}+\frac{p_{1,8}\,x_{1,7}}{T_1}-p_{2,1}\,x_{2,1}+p_{2,2}\,x_{2,1}+c_{112}\,p_{3,1}\,x_{1,1}\,x_{2,1}+\text{<<41>>}+$$
$$c_{107}\,p_{3,7}\,x_{1,3}\,x_{2,7}+c_{109}\,p_{3,3}\,x_{1,7}\,x_{2,7}+c_{105}\,p_{3,7}\,x_{1,7}\,x_{2,7}-p_{3,1}\,x_{3,1}+p_{3,2}\,x_{3,1}-p_{3,2}\,x_{3,2}+\frac{p_{3,3}\,x_{3,2}}{T_1\,T_2}+$$
$$\frac{(-1+T_1\,T_2)\,p_{3,7}\,x_{3,2}}{T_1\,T_2}-p_{3,3}\,x_{3,3}+p_{3,4}\,x_{3,3}-p_{3,4}\,x_{3,4}+p_{3,5}\,x_{3,4}+\frac{(-1+T_1\,T_2)\,p_{3,2}\,x_{3,5}}{T_1\,T_2}-$$
$$p_{3,5}\,x_{3,5}+\frac{p_{3,6}\,x_{3,5}}{T_1\,T_2}-p_{3,6}\,x_{3,6}+p_{3,7}\,x_{3,6}+\frac{(-1+T_1\,T_2)\,p_{3,4}\,x_{3,7}}{T_1\,T_2}-p_{3,7}\,x_{3,7}+\frac{p_{3,8}\,x_{3,7}}{T_1\,T_2},\ \text{<<1>>}\Big]\Big]$$

```
In[•]:= vs[Knot[3, 1]]
```

```
Out[•]=
```

$$\{p_{1,1}, p_{1,2}, p_{1,3}, p_{1,4}, p_{1,5}, p_{1,6}, p_{1,7}, p_{2,1}, p_{2,2}, p_{2,3}, p_{2,4}, p_{2,5}, p_{2,6},$$
$$p_{2,7}, p_{3,1}, p_{3,2}, p_{3,3}, p_{3,4}, p_{3,5}, p_{3,6}, p_{3,7}, x_{1,1}, x_{1,2}, x_{1,3}, x_{1,4}, x_{1,5}, x_{1,6}, x_{1,7},$$
$$x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4}, x_{2,5}, x_{2,6}, x_{2,7}, x_{3,1}, x_{3,2}, x_{3,3}, x_{3,4}, x_{3,5}, x_{3,6}, x_{3,7}\}$$

Invariance Under Reidemeister 2b



Territory: $(x \mid p)_{(i\mid j)^+}$, $x_{i\mid j}$.

In[•]:=
```
{lhs} = Cases[
  ∫𝔼[Sum[π_{α,i} p_{α,i} + π_{α,j} p_{α,j}, {α, 3}]] × ℒ /@ (X_{i,j}[1] X_{i+1,j+1}[-1]) 𝕕{vs_i, vs_j, vs_{i+1}, vs_{j+1}},
  ϵSeries[ℰ_] :> ℰ, ∞]
```

Out[•]=

$$\Big\{p_{1,2+i}\,\pi_{1,i} + p_{1,2+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} +$$
$$\frac{1}{T_1\,T_2}\,(c_{108} + c_{106}\,T_1 - c_{108}\,T_1 + c_{107}\,T_2 - c_{108}\,T_2 + c_1\,T_1\,T_2 + c_{105}\,T_1\,T_2 - c_{106}\,T_1\,T_2 - c_{107}\,T_1\,T_2 + c_{108}\,T_1\,T_2)$$
$$p_{3,2+i}\,\pi_{1,i}\,\pi_{2,i} + \frac{1}{T_1\,T_2}$$
$$\big(-c_{108} - c_{106}\,T_1 + c_{108}\,T_1 - c_{107}\,T_2 + c_{108}\,T_2 + c_5\,T_1\,T_2 - c_{105}\,T_1\,T_2 + c_{106}\,T_1\,T_2 + c_{107}\,T_1\,T_2 + c_{112}\,T_1\,T_2 +$$
$$c_{106}\,T_1^2\,T_2 - c_{108}\,T_1^2\,T_2 + c_{110}\,T_1^2\,T_2 - c_{112}\,T_1^2\,T_2 + c_{107}\,T_1\,T_2^2 - c_{108}\,T_1\,T_2^2 + c_{111}\,T_1\,T_2^2 - c_{112}\,T_1\,T_2^2 +$$
$$c_{105}\,T_1^2\,T_2^2 - c_{106}\,T_1^2\,T_2^2 - c_{107}\,T_1^2\,T_2^2 + c_{108}\,T_1^2\,T_2^2 + c_{109}\,T_1^2\,T_2^2 - c_{110}\,T_1^2\,T_2^2 - c_{111}\,T_1^2\,T_2^2 + c_{112}\,T_1^2\,T_2^2\big)$$
$$p_{3,2+j}\,\pi_{1,i}\,\pi_{2,i} + \frac{(c_{108} + c_{107}\,T_2 - c_{108}\,T_2 + c_3\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,i}}{T_1\,T_2} +$$
$$\frac{1}{T_1\,T_2}\,\big(-c_{108} - c_{107}\,T_2 + c_{108}\,T_2 + c_7\,T_1\,T_2 + c_{108}\,T_1\,T_2 + c_{112}\,T_1\,T_2 + c_{107}\,T_1\,T_2^2 -$$
$$c_{108}\,T_1\,T_2^2 + c_{111}\,T_1\,T_2^2 - c_{112}\,T_1\,T_2^2\big)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,i} +$$
$$p_{2,2+j}\,\pi_{2,j} + \frac{(c_{108} + c_{106}\,T_1 - c_{108}\,T_1 + c_2\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,j}}{T_1\,T_2} +$$
$$\frac{1}{T_1\,T_2}\,\big(-c_{108} - c_{106}\,T_1 + c_{108}\,T_1 + c_6\,T_1\,T_2 + c_{108}\,T_1\,T_2 + c_{112}\,T_1\,T_2 +$$
$$c_{106}\,T_1^2\,T_2 - c_{108}\,T_1^2\,T_2 + c_{110}\,T_1^2\,T_2 - c_{112}\,T_1^2\,T_2\big)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,j} +$$
$$\frac{(c_{108} + c_4\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2} + \frac{(-c_{108} + c_8\,T_1\,T_2 + c_{108}\,T_1\,T_2 + c_{112}\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2} +$$
$$p_{3,2+i}\,\pi_{3,i} + p_{3,2+j}\,\pi_{3,j}\Big\}$$

In[•]:=
```
{rhs} = Cases[∫𝔼[Sum[π_{α,i} p_{α,i} + π_{α,j} p_{α,j}, {α, 3}]] ×
  ℒ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[0]) 𝕕{vs_i, vs_j, vs_{i+1}, vs_{j+1}}, ϵSeries[ℰ_] :> ℰ, ∞]
```

Out[•]=

$$\{p_{1,2+i}\,\pi_{1,i} + p_{1,2+j}\,\pi_{1,j} + p_{2,2+i}\,\pi_{2,i} + p_{2,2+j}\,\pi_{2,j} + p_{3,2+i}\,\pi_{3,i} + p_{3,2+j}\,\pi_{3,j}\}$$

In[•]:=
```
eqn = CF[lhs - rhs]
```

Out[•]=

$$\frac{1}{T_1\,T_2}\left(c_{108}+c_{106}\,T_1-c_{108}\,T_1+c_{107}\,T_2-c_{108}\,T_2+c_1\,T_1\,T_2+c_{105}\,T_1\,T_2-c_{106}\,T_1\,T_2-c_{107}\,T_1\,T_2+c_{108}\,T_1\,T_2\right)$$
$$p_{3,2+i}\,\pi_{1,i}\,\pi_{2,i}+$$
$$\frac{1}{T_1\,T_2}\left(-c_{108}-c_{106}\,T_1+c_{108}\,T_1-c_{107}\,T_2+c_{108}\,T_2+c_5\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2+c_{112}\,T_1\,T_2+\right.$$
$$c_{106}\,T_1^2\,T_2-c_{108}\,T_1^2\,T_2+c_{110}\,T_1^2\,T_2-c_{112}\,T_1^2\,T_2+c_{107}\,T_1\,T_2^2-c_{108}\,T_1\,T_2^2+c_{111}\,T_1\,T_2^2-c_{112}\,T_1\,T_2^2+$$
$$\left.c_{105}\,T_1^2\,T_2^2-c_{106}\,T_1^2\,T_2^2-c_{107}\,T_1^2\,T_2^2+c_{108}\,T_1^2\,T_2^2+c_{109}\,T_1^2\,T_2^2-c_{110}\,T_1^2\,T_2^2-c_{111}\,T_1^2\,T_2^2+c_{112}\,T_1^2\,T_2^2\right)$$
$$p_{3,2+j}\,\pi_{1,i}\,\pi_{2,i}+\frac{\left(c_{108}+c_{107}\,T_2-c_{108}\,T_2+c_3\,T_1\,T_2\right)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,i}}{T_1\,T_2}+$$
$$\frac{1}{T_1\,T_2}\left(-c_{108}-c_{107}\,T_2+c_{108}\,T_2+c_7\,T_1\,T_2+c_{108}\,T_1\,T_2+c_{112}\,T_1\,T_2+\right.$$
$$\left.c_{107}\,T_1\,T_2^2-c_{108}\,T_1\,T_2^2+c_{111}\,T_1\,T_2^2-c_{112}\,T_1\,T_2^2\right)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,i}+$$
$$\frac{\left(c_{108}+c_{106}\,T_1-c_{108}\,T_1+c_2\,T_1\,T_2\right)\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,j}}{T_1\,T_2}+\frac{1}{T_1\,T_2}\left(-c_{108}-c_{106}\,T_1+c_{108}\,T_1+c_6\,T_1\,T_2+\right.$$
$$\left.c_{108}\,T_1\,T_2+c_{112}\,T_1\,T_2+c_{106}\,T_1^2\,T_2-c_{108}\,T_1^2\,T_2+c_{110}\,T_1^2\,T_2-c_{112}\,T_1^2\,T_2\right)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,j}+$$
$$\frac{\left(c_{108}+c_4\,T_1\,T_2\right)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2}+\frac{\left(-c_{108}+c_8\,T_1\,T_2+c_{108}\,T_1\,T_2+c_{112}\,T_1\,T_2\right)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,j}}{T_1\,T_2}$$

In[•]:=
```
cvs = Union@Cases[eqn, p__ | π__, ∞]
```

Out[•]=

$$\{p_{3,2+i},\ p_{3,2+j},\ \pi_{1,i},\ \pi_{1,j},\ \pi_{2,i},\ \pi_{2,j}\}$$

In[•]:=
```
eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0)
```

Out[•]=

$$\left\{c_1+c_{105}-c_{106}-c_{107}+c_{108}+\frac{c_{107}}{T_1}-\frac{c_{108}}{T_1}+\frac{c_{106}}{T_2}-\frac{c_{108}}{T_2}+\frac{c_{108}}{T_1\,T_2}==0,\right.$$
$$c_2+\frac{c_{106}}{T_2}-\frac{c_{108}}{T_2}+\frac{c_{108}}{T_1\,T_2}==0,\ c_3+\frac{c_{107}}{T_1}-\frac{c_{108}}{T_1}+\frac{c_{108}}{T_1\,T_2}==0,\ c_4+\frac{c_{108}}{T_1\,T_2}==0,$$
$$c_5-c_{105}+c_{106}+c_{107}+c_{112}-\frac{c_{107}}{T_1}+\frac{c_{108}}{T_1}+c_{106}\,T_1-c_{108}\,T_1+c_{110}\,T_1-$$
$$c_{112}\,T_1-\frac{c_{106}}{T_2}+\frac{c_{108}}{T_2}-\frac{c_{108}}{T_1\,T_2}+c_{107}\,T_2-c_{108}\,T_2+c_{111}\,T_2-c_{112}\,T_2+c_{105}\,T_1\,T_2-$$
$$c_{106}\,T_1\,T_2-c_{107}\,T_1\,T_2+c_{108}\,T_1\,T_2+c_{109}\,T_1\,T_2-c_{110}\,T_1\,T_2-c_{111}\,T_1\,T_2+c_{112}\,T_1\,T_2==0,$$
$$c_6+c_{108}+c_{112}+c_{106}\,T_1-c_{108}\,T_1+c_{110}\,T_1-c_{112}\,T_1-\frac{c_{106}}{T_2}+\frac{c_{108}}{T_2}-\frac{c_{108}}{T_1\,T_2}==0,$$
$$\left.c_7+c_{108}+c_{112}-\frac{c_{107}}{T_1}+\frac{c_{108}}{T_1}-\frac{c_{108}}{T_1\,T_2}+c_{107}\,T_2-c_{108}\,T_2+c_{111}\,T_2-c_{112}\,T_2==0,\ c_8+c_{108}+c_{112}-\frac{c_{108}}{T_1\,T_2}==0\right\}$$

In[•]:=
```
vars = Union@Cases[eqn, c_, ∞]
```

Out[•]=

$$\{c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_6,\ c_7,\ c_8,\ c_{105},\ c_{106},\ c_{107},\ c_{108},\ c_{109},\ c_{110},\ c_{111},\ c_{112}\}$$

In[•]:=
```
{sol} = Solve[eqns, vars]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\left\{\left\{c_2 \to -\frac{c_1}{-1+T_2}-\frac{c_{105}}{-1+T_2}-\frac{c_{107}\,(1-T_1)}{T_1\,(-1+T_2)},\ c_3 \to -\frac{c_1}{-1+T_1}-\frac{c_{105}}{-1+T_1}-\frac{c_{106}\,(1-T_2)}{(-1+T_1)\,T_2},\right.\right.$$
$$c_4 \to \frac{c_1}{(-1+T_1)\,(-1+T_2)}+\frac{c_{105}}{(-1+T_1)\,(-1+T_2)}-\frac{c_{107}}{T_1\,(-1+T_2)}-\frac{c_{106}}{(-1+T_1)\,T_2},$$
$$c_6 \to -c_{112}\,(1-T_1)-c_{110}\,T_1-\frac{c_1\,(-1+T_1\,T_2)}{-1+T_2}-\frac{c_{105}\,(-1+T_1\,T_2)}{-1+T_2}-\frac{c_{107}\left(-1+T_1+T_1\,T_2-T_1^2\,T_2\right)}{T_1\,(-1+T_2)},$$
$$c_7 \to -c_{112}\,(1-T_2)-c_{111}\,T_2-\frac{c_1\,(-1+T_1\,T_2)}{-1+T_1}-\frac{c_{105}\,(-1+T_1\,T_2)}{-1+T_1}-\frac{c_{106}\left(-1+T_2+T_1\,T_2-T_1\,T_2^2\right)}{(-1+T_1)\,T_2},$$
$$c_8 \to -c_{112}-\frac{c_1\,(1-T_1\,T_2)}{(-1+T_1)\,(-1+T_2)}-\frac{c_{105}\,(1-T_1\,T_2)}{(-1+T_1)\,(-1+T_2)}-\frac{c_{107}\,(-1+T_1\,T_2)}{T_1\,(-1+T_2)}-\frac{c_{106}\,(-1+T_1\,T_2)}{(-1+T_1)\,T_2},$$
$$c_{108} \to \frac{c_{106}\,T_1}{-1+T_1}+\frac{c_{107}\,T_2}{-1+T_2}-\frac{c_1\,T_1\,T_2}{(-1+T_1)\,(-1+T_2)}-\frac{c_{105}\,T_1\,T_2}{(-1+T_1)\,(-1+T_2)},$$
$$\left.\left.c_{109} \to -\frac{c_{111}\,(1-T_1)}{T_1}-\frac{c_5}{T_1\,T_2}-\frac{c_{110}\,(1-T_2)}{T_2}-\frac{c_1\,(1-T_1\,T_2)}{T_1\,T_2}-\frac{c_{112}\,(1-T_1-T_2+T_1\,T_2)}{T_1\,T_2}\right\}\right\}$$

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val])
```

Out[•]=

$$\left\{-\frac{c_{107}+c_1\,T_1+c_{105}\,T_1-c_{107}\,T_1}{T_1\,(-1+T_2)},\ -\frac{c_{106}+c_1\,T_2+c_{105}\,T_2-c_{106}\,T_2}{(-1+T_1)\,T_2},\right.$$
$$\frac{c_{106}\,T_1+c_{107}\,T_2+c_1\,T_1\,T_2+c_{105}\,T_1\,T_2-c_{106}\,T_1\,T_2-c_{107}\,T_1\,T_2}{(-1+T_1)\,T_1\,(-1+T_2)\,T_2},$$
$$\frac{1}{T_1\,(-1+T_2)}\left(c_{107}+c_1\,T_1+c_{105}\,T_1-c_{107}\,T_1+c_{112}\,T_1+c_{110}\,T_1^2-c_{112}\,T_1^2-\right.$$
$$\left.c_{107}\,T_1\,T_2-c_{112}\,T_1\,T_2-c_1\,T_1^2\,T_2-c_{105}\,T_1^2\,T_2+c_{107}\,T_1^2\,T_2-c_{110}\,T_1^2\,T_2+c_{112}\,T_1^2\,T_2\right),$$
$$\frac{1}{(-1+T_1)\,T_2}\left(c_{106}+c_1\,T_2+c_{105}\,T_2-c_{106}\,T_2+c_{112}\,T_2-c_{106}\,T_1\,T_2-c_{112}\,T_1\,T_2+\right.$$
$$\left.c_{111}\,T_2^2-c_{112}\,T_2^2-c_1\,T_1\,T_2^2-c_{105}\,T_1\,T_2^2+c_{106}\,T_1\,T_2^2-c_{111}\,T_1\,T_2^2+c_{112}\,T_1\,T_2^2\right),$$
$$-\left(\left(c_{106}\,T_1+c_{107}\,T_2+c_1\,T_1\,T_2+c_{105}\,T_1\,T_2-c_{106}\,T_1\,T_2-c_{107}\,T_1\,T_2+c_{112}\,T_1\,T_2-c_{106}\,T_1^2\,T_2-\right.\right.$$
$$\left.\left.c_{112}\,T_1^2\,T_2-c_{107}\,T_1\,T_2^2-c_{112}\,T_1\,T_2^2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2+c_{112}\,T_1^2\,T_2^2\right)\Big/\right.$$
$$\left.\left((-1+T_1)\,T_1\,(-1+T_2)\,T_2\right)\right),\ \frac{-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2}{(-1+T_1)\,(-1+T_2)},$$
$$\left.\frac{1}{T_1\,T_2}\left(-c_1-c_5-c_{112}-c_{110}\,T_1+c_{112}\,T_1-c_{111}\,T_2+c_{112}\,T_2+c_1\,T_1\,T_2+c_{110}\,T_1\,T_2+c_{111}\,T_1\,T_2-c_{112}\,T_1\,T_2\right)\right\}$$

Invariance Under Reidemeister 3b

In[•]:=
```
Clear[k]
```

In[•]:=
```
{lhs} =
 Cases[∫ 𝔼[Sum[π_{α,i} p_{α,i} + π_{α,j} p_{α,j} + π_{α,k} p_{α,k}, {α, 3}]] × ℒ /@ (X_{i,j}[1] X_{i+1,k}[1] X_{j+1,k+1}[1])
   𝕕{vs_i, vs_j, vs_k, vs_{i+1}, vs_{j+1}, vs_{k+1}}, ϵSeries[ℰ_] :> ℰ, ∞]
```

Out[•]=

$$\Big\{T_1^2\, p_{1,2+i}\, \pi_{1,i} - (-1+T_1)\, T_1\, p_{1,2+j}\, \pi_{1,i} + (1-T_1)\, p_{1,2+k}\, \pi_{1,i} + T_1\, p_{1,2+j}\, \pi_{1,j} +$$
$$(1-T_1)\, p_{1,2+k}\, \pi_{1,j} + p_{1,2+k}\, \pi_{1,k} + T_2^2\, p_{2,2+i}\, \pi_{2,i} - (-1+T_2)\, T_2\, p_{2,2+j}\, \pi_{2,i} +$$
$$(1-T_2)\, p_{2,2+k}\, \pi_{2,i} + 2\, c_1\, T_1^2\, T_2^2\, p_{3,2+i}\, \pi_{1,i}\, \pi_{2,i} + \Big(c_{106}\, T_1^2 + 2\, c_1\, T_1\, T_2 + c_5\, T_1\, T_2 + c_{105}\, T_1^2\, T_2 -$$
$$c_{106}\, T_1^2\, T_2 + c_{107}\, T_2^2 + c_{105}\, T_1\, T_2^2 - c_{107}\, T_1\, T_2^2 - c_1\, T_1^2\, T_2^2 - c_{105}\, T_1^2\, T_2^2\Big)\, p_{3,2+j}\, \pi_{1,i}\, \pi_{2,i} +$$
$$\Big(2\, c_1 + 2\, c_5 - c_1\, T_1 - c_5\, T_1 - c_{112}\, T_1 + c_{112}\, T_1^2 - c_1\, T_2 - c_5\, T_2 - c_{112}\, T_2 + c_5\, T_1\, T_2 - c_{110}\, T_1\, T_2 -$$
$$c_{111}\, T_1\, T_2 + 3\, c_{112}\, T_1\, T_2 + c_1\, T_1^2\, T_2 + c_{110}\, T_1^2\, T_2 + c_{111}\, T_1^2\, T_2 - 2\, c_{112}\, T_1^2\, T_2 + c_{112}\, T_2^2 + c_1\, T_1\, T_2^2 +$$
$$c_{110}\, T_1\, T_2^2 + c_{111}\, T_1\, T_2^2 - 2\, c_{112}\, T_1\, T_2^2 - 2\, c_1\, T_1^2\, T_2^2 - c_{110}\, T_1^2\, T_2^2 - c_{111}\, T_1^2\, T_2^2 + c_{112}\, T_1^2\, T_2^2\Big)$$
$$p_{3,2+k}\, \pi_{1,i}\, \pi_{2,i} + \frac{T_1^2\, T_2\, (-c_{106} - c_1\, T_2 - c_{105}\, T_2 + c_{106}\, T_2)\, p_{3,2+i}\, \pi_{1,j}\, \pi_{2,i}}{-1+T_1} +$$
$$T_2\, (c_1\, T_1 - c_{112}\, T_1 + c_{107}\, T_2 + c_{105}\, T_1\, T_2 - c_{107}\, T_1\, T_2 - c_{111}\, T_1\, T_2 + c_{112}\, T_1\, T_2)\, p_{3,2+j}\, \pi_{1,j}\, \pi_{2,i} +$$
$$\frac{1}{-1+T_1}\Big(-c_1 - c_5 + c_{112} + c_1\, T_1 + c_5\, T_1 - c_{106}\, T_1 - c_{112}\, T_1 + c_1\, T_2 + c_5\, T_2 + c_{111}\, T_2 - c_1\, T_1\, T_2 -$$
$$c_5\, T_1\, T_2 - c_{105}\, T_1\, T_2 + c_{106}\, T_1\, T_2 + c_{110}\, T_1\, T_2 - c_{111}\, T_1\, T_2 - 2\, c_{112}\, T_1\, T_2 - c_1\, T_1^2\, T_2 + c_{106}\, T_1^2\, T_2 -$$
$$c_{110}\, T_1^2\, T_2 + 2\, c_{112}\, T_1^2\, T_2 - c_{112}\, T_2^2 - c_1\, T_1\, T_2^2 - c_{110}\, T_1\, T_2^2 - c_{111}\, T_1\, T_2^2 + 3\, c_{112}\, T_1\, T_2^2 +$$
$$2\, c_1\, T_1^2\, T_2^2 + c_{105}\, T_1^2\, T_2^2 - c_{106}\, T_1^2\, T_2^2 + c_{110}\, T_1^2\, T_2^2 + c_{111}\, T_1^2\, T_2^2 - 2\, c_{112}\, T_1^2\, T_2^2\Big)\, p_{3,2+k}\, \pi_{1,j}\, \pi_{2,i} +$$
$$\frac{T_1\, T_2\, (-c_{106} - c_1\, T_2 - c_{105}\, T_2 + c_{106}\, T_2)\, p_{3,2+i}\, \pi_{1,k}\, \pi_{2,i}}{-1+T_1} +$$
$$\frac{\Big(-c_{106}\, T_1 - c_1\, T_1\, T_2 - c_{105}\, T_1\, T_2 + c_{106}\, T_1\, T_2 - c_{107}\, T_2^2 + c_{107}\, T_1\, T_2^2\Big)\, p_{3,2+j}\, \pi_{1,k}\, \pi_{2,i}}{-1+T_1} +$$
$$\Big(-c_{112} - c_{111}\, T_2 + c_{112}\, T_2^2\Big)\, p_{3,2+k}\, \pi_{1,k}\, \pi_{2,i} +$$
$$T_2\, p_{2,2+j}\, \pi_{2,j} + (1-T_2)\, p_{2,2+k}\, \pi_{2,j} +$$
$$\frac{T_1\, (-c_{107} - c_1\, T_1 - c_{105}\, T_1 + c_{107}\, T_1)\, T_2^2\, p_{3,2+i}\, \pi_{1,i}\, \pi_{2,j}}{-1+T_2} -$$
$$T_1\, (-c_{106}\, T_1 - c_1\, T_2 + c_{112}\, T_2 - c_{105}\, T_1\, T_2 + c_{106}\, T_1\, T_2 + c_{110}\, T_1\, T_2 - c_{112}\, T_1\, T_2)\, p_{3,2+j}\, \pi_{1,i}\, \pi_{2,j} +$$
$$\frac{1}{-1+T_2}\Big(-c_1 - c_5 + c_{112} + c_1\, T_1 + c_5\, T_1 + c_{110}\, T_1 - c_{112}\, T_1^2 + c_1\, T_2 + c_5\, T_2 - c_{107}\, T_2 - c_{112}\, T_2 - c_1\, T_1\, T_2 -$$
$$c_5\, T_1\, T_2 - c_{105}\, T_1\, T_2 + c_{107}\, T_1\, T_2 - c_{110}\, T_1\, T_2 + c_{111}\, T_1\, T_2 - 2\, c_{112}\, T_1\, T_2 - c_1\, T_1^2\, T_2 - c_{110}\, T_1^2\, T_2 -$$
$$c_{111}\, T_1^2\, T_2 + 3\, c_{112}\, T_1^2\, T_2 - c_1\, T_1\, T_2^2 + c_{107}\, T_1\, T_2^2 - c_{111}\, T_1\, T_2^2 + 2\, c_{112}\, T_1\, T_2^2 + 2\, c_1\, T_1^2\, T_2^2 + c_{105}\, T_1^2\, T_2^2 -$$
$$c_{107}\, T_1^2\, T_2^2 + c_{110}\, T_1^2\, T_2^2 + c_{111}\, T_1^2\, T_2^2 - 2\, c_{112}\, T_1^2\, T_2^2\Big)\, p_{3,2+k}\, \pi_{1,i}\, \pi_{2,j} - \frac{1}{(-1+T_1)\,(-1+T_2)}$$
$$T_1\, T_2\, (-c_{106}\, T_1 - c_{107}\, T_2 - c_1\, T_1\, T_2 - c_{105}\, T_1\, T_2 + c_{106}\, T_1\, T_2 + c_{107}\, T_1\, T_2)\, p_{3,2+i}\, \pi_{1,j}\, \pi_{2,j} +$$
$$(c_1 - c_{112})\, T_1\, T_2\, p_{3,2+j}\, \pi_{1,j}\, \pi_{2,j} - \frac{1}{(-1+T_1)\,(-1+T_2)}$$
$$\Big(-c_1 - c_5 + c_{112} + c_1\, T_1 + c_5\, T_1 - c_{106}\, T_1 - c_{112}\, T_1 + c_1\, T_2 + c_5\, T_2 - c_{107}\, T_2 - c_{112}\, T_2 - c_1\, T_1\, T_2 - c_5\, T_1\, T_2 -$$
$$c_{105}\, T_1\, T_2 + c_{106}\, T_1\, T_2 + c_{107}\, T_1\, T_2 - c_1\, T_1^2\, T_2 + c_{106}\, T_1^2\, T_2 + c_{112}\, T_1^2\, T_2 - c_1\, T_1\, T_2^2 + c_{107}\, T_1\, T_2^2 +$$
$$c_{112}\, T_1\, T_2^2 + 2\, c_1\, T_1^2\, T_2^2 + c_{105}\, T_1^2\, T_2^2 - c_{106}\, T_1^2\, T_2^2 - c_{107}\, T_1^2\, T_2^2 - c_{112}\, T_1^2\, T_2^2\Big)\, p_{3,2+k}\, \pi_{1,j}\, \pi_{2,j} +$$

$$\frac{T_1\,(-c_{106}-c_1\,T_2-c_{105}\,T_2+c_{106}\,T_2)\,p_{3,2+j}\,\pi_{1,k}\,\pi_{2,j}}{-1+T_1}+(-c_{112}-c_{111}\,T_2+c_{112}\,T_2)\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,j}+$$
$$p_{2,2+k}\,\pi_{2,k}+\frac{T_1\,(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,T_2\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,k}}{-1+T_2}+$$
$$\frac{\left(-c_{106}\,T_1^2-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{107}\,T_1\,T_2+c_{106}\,T_1^2\,T_2\right)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,k}}{-1+T_2}+$$
$$\left(-c_{112}-c_{110}\,T_1+c_{112}\,T_1^2\right)\,p_{3,2+k}\,\pi_{1,i}\,\pi_{2,k}+$$
$$\frac{(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,T_2\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,k}}{-1+T_2}+$$
$$(-c_{112}-c_{110}\,T_1+c_{112}\,T_1)\,p_{3,2+k}\,\pi_{1,j}\,\pi_{2,k}-$$
$$\frac{(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,k}\,\pi_{2,k}}{(-1+T_1)\,(-1+T_2)}-$$
$$\frac{(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,k}\,\pi_{2,k}}{(-1+T_1)\,(-1+T_2)}-$$
$$2\,c_{112}\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,k}+T_1^2\,T_2^2\,p_{3,2+i}\,\pi_{3,i}-T_1\,T_2\,(-1+T_1\,T_2)\,p_{3,2+j}\,\pi_{3,i}+$$
$$(1-T_1\,T_2)\,p_{3,2+k}\,\pi_{3,i}+T_1\,T_2\,p_{3,2+j}\,\pi_{3,j}+(1-T_1\,T_2)\,p_{3,2+k}\,\pi_{3,j}+p_{3,2+k}\,\pi_{3,k}\Big\}$$

In[•]:=
```
{rhs} =
 Cases[∫𝔼[Sum[π_α,i p_α,i + π_α,j p_α,j + π_α,k p_α,k, {α, 3}]] × ℒ /@ (X_j,k[1] X_i,k+1[1] X_i+1,j+1[1])
   𝕕{vs_i, vs_j, vs_k, vs_i+1, vs_j+1, vs_k+1}, ϵSeries[ℰ_] :↦ ℰ, ∞]
```

Out[•]=

$$\Big\{T_1^2\,p_{1,2+i}\,\pi_{1,i}-(-1+T_1)\,T_1\,p_{1,2+j}\,\pi_{1,i}+(1-T_1)\,p_{1,2+k}\,\pi_{1,i}+T_1\,p_{1,2+j}\,\pi_{1,j}+(1-T_1)\,p_{1,2+k}\,\pi_{1,j}+$$
$$p_{1,2+k}\,\pi_{1,k}+T_2^2\,p_{2,2+i}\,\pi_{2,i}-(-1+T_2)\,T_2\,p_{2,2+j}\,\pi_{2,i}+(1-T_2)\,p_{2,2+k}\,\pi_{2,i}+2\,c_1\,T_1^2\,T_2^2\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,i}-$$
$$T_1\,T_2\,(-2\,c_1-c_5+2\,c_1\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,i}+(c_1+c_5-c_1\,T_1\,T_2)\,p_{3,2+k}\,\pi_{1,i}\,\pi_{2,i}-$$
$$\frac{(-2+T_1)\,T_1^2\,T_2\,(-c_{106}-c_1\,T_2-c_{105}\,T_2+c_{106}\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,i}}{-1+T_1}+T_1$$
$$\left(c_{106}+c_1\,T_2+c_{105}\,T_2-c_{106}\,T_2-c_{112}\,T_2-c_{106}\,T_1\,T_2-c_{111}\,T_2^2+c_{112}\,T_2^2-c_1\,T_1\,T_2^2-c_{105}\,T_1\,T_2^2+c_{106}\,T_1\,T_2^2\right)$$
$$p_{3,2+j}\,\pi_{1,j}\,\pi_{2,i}-(-1+T_1)\,(-c_{112}-c_{111}\,T_2+c_{112}\,T_2)\,p_{3,2+k}\,\pi_{1,j}\,\pi_{2,i}+$$
$$\frac{T_1^2\,T_2\,(-c_{106}-c_1\,T_2-c_{105}\,T_2+c_{106}\,T_2)\,p_{3,2+i}\,\pi_{1,k}\,\pi_{2,i}}{-1+T_1}-$$
$$\frac{T_1\,(-c_{106}-c_1\,T_2-c_{105}\,T_2+c_{106}\,T_2)\,(-1+T_1\,T_2)\,p_{3,2+j}\,\pi_{1,k}\,\pi_{2,i}}{-1+T_1}+$$
$$(-c_{112}-c_{111}\,T_2+c_{112}\,T_2)\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,i}+T_2\,p_{2,2+j}\,\pi_{2,j}+(1-T_2)\,p_{2,2+k}\,\pi_{2,j}-$$
$$\frac{T_1\,(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,(-2+T_2)\,T_2^2\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,j}}{-1+T_2}+T_2$$
$$\left(c_{107}+c_1\,T_1+c_{105}\,T_1-c_{107}\,T_1-c_{112}\,T_1-c_{110}\,T_1^2+c_{112}\,T_1^2-c_{107}\,T_1\,T_2-c_1\,T_1^2\,T_2-c_{105}\,T_1^2\,T_2+c_{107}\,T_1^2\,T_2\right)$$
$$p_{3,2+j}\,\pi_{1,i}\,\pi_{2,j}-(-c_{112}-c_{110}\,T_1+c_{112}\,T_1)\,(-1+T_2)\,p_{3,2+k}\,\pi_{1,i}\,\pi_{2,j}-$$
$$(T_1\,T_2\,(2-T_1-T_2+T_1\,T_2)\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,j}$$
$$\pi_{2,j})/((-1+T_1)\,(-1+T_2))+\left(c_{106}\,T_1+c_{107}\,T_2+2\,c_1\,T_1\,T_2+c_{105}\,T_1\,T_2-c_{106}\,T_1\,T_2-c_{107}\,T_1\,T_2-\right.$$
$$\left.c_{112}\,T_1\,T_2-c_{106}\,T_1^2\,T_2-c_{107}\,T_1\,T_2^2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2\right)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,j}+$$

$$
\begin{aligned}
&(c_1+c_5-c_{112}+c_{112}\,T_1+c_{112}\,T_2-c_1\,T_1\,T_2-c_{112}\,T_1\,T_2)\;p_{3,2+k}\,\pi_{1,j}\,\pi_{2,j}+\\
&\frac{1}{-1+T_1}\,T_1\,T_2\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\;p_{3,2+i}\,\pi_{1,k}\,\pi_{2,j}-\\
&\frac{1}{-1+T_1}\,\big(2\,c_{106}\,T_1+c_{107}\,T_2+2\,c_1\,T_1\,T_2+2\,c_{105}\,T_1\,T_2-2\,c_{106}\,T_1\,T_2-c_{107}\,T_1\,T_2-\\
&\qquad c_{106}\,T_1^2\,T_2-c_{107}\,T_1\,T_2^2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2\big)\;p_{3,2+j}\,\pi_{1,k}\,\pi_{2,j}+\\
&(-2\,c_{112}-c_{111}\,T_2+2\,c_{112}\,T_2)\;p_{3,2+k}\,\pi_{1,k}\,\pi_{2,j}+p_{2,2+k}\,\pi_{2,k}+\\
&\frac{T_1\,(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,T_2^2\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,k}}{-1+T_2}-\\
&\frac{(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,T_2\,(-1+T_1\,T_2)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,k}}{-1+T_2}+\\
&(-c_{112}-c_{110}\,T_1+c_{112}\,T_1)\;p_{3,2+k}\,\pi_{1,i}\,\pi_{2,k}+\\
&\frac{1}{-1+T_2}\,T_1\,T_2\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\;p_{3,2+i}\,\pi_{1,j}\,\pi_{2,k}-\\
&\frac{1}{-1+T_2}\,\big(c_{106}\,T_1+2\,c_{107}\,T_2+2\,c_1\,T_1\,T_2+2\,c_{105}\,T_1\,T_2-c_{106}\,T_1\,T_2-2\,c_{107}\,T_1\,T_2-\\
&\qquad c_{106}\,T_1^2\,T_2-c_{107}\,T_1\,T_2^2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2\big)\;p_{3,2+j}\,\pi_{1,j}\,\pi_{2,k}+\\
&(-2\,c_{112}-c_{110}\,T_1+2\,c_{112}\,T_1)\;p_{3,2+k}\,\pi_{1,j}\,\pi_{2,k}-\frac{1}{(-1+T_1)\,(-1+T_2)}\\
&\quad T_1\,T_2\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\;p_{3,2+i}\,\pi_{1,k}\,\pi_{2,k}+\\
&\frac{1}{(-1+T_1)\,(-1+T_2)}\,(-2+T_1\,T_2)\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\\
&\quad p_{3,2+j}\,\pi_{1,k}\,\pi_{2,k}-2\,c_{112}\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,k}+T_1^2\,T_2^2\,p_{3,2+i}\,\pi_{3,i}-T_1\,T_2\,(-1+T_1\,T_2)\,p_{3,2+j}\,\pi_{3,i}+\\
&(1-T_1\,T_2)\,p_{3,2+k}\,\pi_{3,i}+T_1\,T_2\,p_{3,2+j}\,\pi_{3,j}+(1-T_1\,T_2)\,p_{3,2+k}\,\pi_{3,j}+p_{3,2+k}\,\pi_{3,k}\Big\}
\end{aligned}
$$

In[•]:=
```
eqn = CF[lhs - rhs]
```

Out[•]=

$$
\begin{aligned}
&\big(c_{106}\,T_1^2+c_{105}\,T_1^2\,T_2-c_{106}\,T_1^2\,T_2+c_{107}\,T_2^2+c_{105}\,T_1\,T_2^2-c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2\big)\;p_{3,2+j}\,\pi_{1,i}\,\pi_{2,i}+\\
&\big(c_1+c_5-c_1\,T_1-c_5\,T_1-c_{112}\,T_1+c_{112}\,T_1^2-c_1\,T_2-c_5\,T_2-c_{112}\,T_2+c_1\,T_1\,T_2+c_5\,T_1\,T_2-c_{110}\,T_1\,T_2-\\
&\qquad c_{111}\,T_1\,T_2+3\,c_{112}\,T_1\,T_2+c_1\,T_1^2\,T_2+c_{110}\,T_1^2\,T_2+c_{111}\,T_1^2\,T_2-2\,c_{112}\,T_1^2\,T_2+c_{112}\,T_2^2+c_1\,T_1\,T_2^2+\\
&\qquad c_{110}\,T_1\,T_2^2+c_{111}\,T_1\,T_2^2-2\,c_{112}\,T_1\,T_2^2-2\,c_1\,T_1^2\,T_2^2-c_{110}\,T_1^2\,T_2^2-c_{111}\,T_1^2\,T_2^2+c_{112}\,T_1^2\,T_2^2\big)\;p_{3,2+k}\,\pi_{1,i}\,\pi_{2,i}+\\
&T_1^2\,T_2\,(-c_{106}-c_1\,T_2-c_{105}\,T_2+c_{106}\,T_2)\;p_{3,2+i}\,\pi_{1,j}\,\pi_{2,i}+\big(-c_{106}\,T_1-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+\\
&\qquad c_{106}\,T_1^2\,T_2+c_{107}\,T_2^2+c_{105}\,T_1\,T_2^2-c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2+c_{105}\,T_1^2\,T_2^2-c_{106}\,T_1^2\,T_2^2\big)\;p_{3,2+j}\,\pi_{1,j}\,\pi_{2,i}-\\
&\frac{1}{-1+T_1}\,\big(c_1+c_5-c_1\,T_1-c_5\,T_1+c_{106}\,T_1-c_{112}\,T_1+c_{112}\,T_1^2-c_1\,T_2-c_5\,T_2-c_{112}\,T_2+c_1\,T_1\,T_2+\\
&\qquad c_5\,T_1\,T_2+c_{105}\,T_1\,T_2-c_{106}\,T_1\,T_2-c_{110}\,T_1\,T_2-c_{111}\,T_1\,T_2+4\,c_{112}\,T_1\,T_2+c_1\,T_1^2\,T_2-c_{106}\,T_1^2\,T_2+\\
&\qquad c_{110}\,T_1^2\,T_2+c_{111}\,T_1^2\,T_2-3\,c_{112}\,T_1^2\,T_2+c_{112}\,T_2^2+c_1\,T_1\,T_2^2+c_{110}\,T_1\,T_2^2+c_{111}\,T_1\,T_2^2-3\,c_{112}\,T_1\,T_2^2-\\
&\qquad 2\,c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2-c_{110}\,T_1^2\,T_2^2-c_{111}\,T_1^2\,T_2^2+2\,c_{112}\,T_1^2\,T_2^2\big)\;p_{3,2+k}\,\pi_{1,j}\,\pi_{2,i}-\\
&T_1\,T_2\,(-c_{106}-c_1\,T_2-c_{105}\,T_2+c_{106}\,T_2)\;p_{3,2+i}\,\pi_{1,k}\,\pi_{2,i}+\\
&\frac{T_2\,\big(-c_{106}\,T_1^2-c_{107}\,T_2+c_{107}\,T_1\,T_2-c_1\,T_1^2\,T_2-c_{105}\,T_1^2\,T_2+c_{106}\,T_1^2\,T_2\big)\;p_{3,2+j}\,\pi_{1,k}\,\pi_{2,i}}{-1+T_1}+\\
&c_{112}\,(-1+T_2)\,T_2\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,i}+
\end{aligned}
$$

$$T_1\,(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,T_2^2\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,j}+$$
$$\left(c_{106}\,T_1^2-c_{107}\,T_2-c_{105}\,T_1\,T_2+c_{107}\,T_1\,T_2+c_{105}\,T_1^2\,T_2-\right.$$
$$\left.c_{106}\,T_1^2\,T_2+c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2+c_{105}\,T_1^2\,T_2^2-c_{107}\,T_1^2\,T_2^2\right)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,j}-$$
$$\frac{1}{-1+T_2}\left(c_1+c_5-c_1\,T_1-c_5\,T_1-c_{112}\,T_1+c_{112}\,T_1^2-c_1\,T_2-c_5\,T_2+c_{107}\,T_2-c_{112}\,T_2+c_1\,T_1\,T_2+\right.$$
$$c_5\,T_1\,T_2+c_{105}\,T_1\,T_2-c_{107}\,T_1\,T_2-c_{110}\,T_1\,T_2-c_{111}\,T_1\,T_2+4\,c_{112}\,T_1\,T_2+c_1\,T_1^2\,T_2+c_{110}\,T_1^2\,T_2+$$
$$c_{111}\,T_1^2\,T_2-3\,c_{112}\,T_1^2\,T_2+c_{112}\,T_2^2+c_1\,T_1\,T_2^2-c_{107}\,T_1\,T_2^2+c_{110}\,T_1\,T_2^2+c_{111}\,T_1\,T_2^2-3\,c_{112}\,T_1\,T_2^2-$$
$$\left.2\,c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2-c_{110}\,T_1^2\,T_2^2-c_{111}\,T_1^2\,T_2^2+2\,c_{112}\,T_1^2\,T_2^2\right)\,p_{3,2+k}\,\pi_{1,i}\,\pi_{2,j}+$$
$$T_1\,T_2\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,j}-$$
$$(-1+T_1\,T_2)\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,j}-$$
$$\frac{1}{(-1+T_1)\,(-1+T_2)}\left(-c_{106}\,T_1+c_{112}\,T_1-c_{112}\,T_1^2-c_{107}\,T_2+c_{112}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+\right.$$
$$c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2-4\,c_{112}\,T_1\,T_2+c_{106}\,T_1^2\,T_2+3\,c_{112}\,T_1^2\,T_2-c_{112}\,T_2^2+c_{107}\,T_1\,T_2^2+$$
$$\left.3\,c_{112}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2+c_{105}\,T_1^2\,T_2^2-c_{106}\,T_1^2\,T_2^2-c_{107}\,T_1^2\,T_2^2-2\,c_{112}\,T_1^2\,T_2^2\right)\,p_{3,2+k}\,\pi_{1,j}\,\pi_{2,j}-$$
$$\frac{1}{-1+T_1}\,T_1\,T_2\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,k}\,\pi_{2,j}+$$
$$\frac{1}{-1+T_1}\,(-1+T_1\,T_2)\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,k}\,\pi_{2,j}-$$
$$c_{112}\,(-1+T_2)\,p_{3,2+k}\,\pi_{1,k}\,\pi_{2,j}-$$
$$T_1\,(-c_{107}-c_1\,T_1-c_{105}\,T_1+c_{107}\,T_1)\,T_2\,p_{3,2+i}\,\pi_{1,i}\,\pi_{2,k}+$$
$$\frac{T_1\left(-c_{106}\,T_1+c_{106}\,T_1\,T_2-c_{107}\,T_2^2-c_1\,T_1\,T_2^2-c_{105}\,T_1\,T_2^2+c_{107}\,T_1\,T_2^2\right)\,p_{3,2+j}\,\pi_{1,i}\,\pi_{2,k}}{-1+T_2}+$$
$$c_{112}\,(-1+T_1)\,T_1\,p_{3,2+k}\,\pi_{1,i}\,\pi_{2,k}-$$
$$\frac{1}{-1+T_2}\,T_1\,T_2\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,j}\,\pi_{2,k}+$$
$$\frac{1}{-1+T_2}\,(-1+T_1\,T_2)\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,j}\,\pi_{2,k}-$$
$$c_{112}\,(-1+T_1)\,p_{3,2+k}\,\pi_{1,j}\,\pi_{2,k}+\frac{1}{(-1+T_1)\,(-1+T_2)}\,(-1+T_1\,T_2)$$
$$(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+i}\,\pi_{1,k}\,\pi_{2,k}-\frac{1}{(-1+T_1)\,(-1+T_2)}$$
$$(-1+T_1\,T_2)\,(-c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2)\,p_{3,2+j}\,\pi_{1,k}\,\pi_{2,k}$$

In[•]:=
```
cvs = Union@Cases[eqn, p__ | π__, ∞]
```

Out[•]=
$$\{p_{3,2+i},\,p_{3,2+j},\,p_{3,2+k},\,\pi_{1,i},\,\pi_{1,j},\,\pi_{1,k},\,\pi_{2,i},\,\pi_{2,j},\,\pi_{2,k}\}$$

In[•]:=
```
eqns = CoefficientRules[eqn, cvs] /. (_ → c_) :> (c == 0)
```

Out[•]=
$$\{-c_{107}\,T_1\,T_2^2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2==0,$$
$$c_{107}\,T_1\,T_2+c_1\,T_1^2\,T_2+c_{105}\,T_1^2\,T_2-c_{107}\,T_1^2\,T_2==0,\;-c_{106}\,T_1^2\,T_2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2==0,$$
$$-c_{106}\,T_1^2\,T_2-c_{107}\,T_1\,T_2^2-c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2+c_{106}\,T_1^2\,T_2^2+c_{107}\,T_1^2\,T_2^2==0,$$

$$-\frac{c_{106}\,T_1^2\,T_2}{1-T_2}-\frac{c_{107}\,T_1\,T_2^2}{1-T_2}-\frac{c_1\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{105}\,T_1^2\,T_2^2}{1-T_2}+\frac{c_{106}\,T_1^2\,T_2^2}{1-T_2}+\frac{c_{107}\,T_1^2\,T_2^2}{1-T_2}==0,$$

$$c_{106}\,T_1\,T_2+c_1\,T_1\,T_2^2+c_{105}\,T_1\,T_2^2-c_{106}\,T_1\,T_2^2==0,$$

$$-\frac{c_{106}\,T_1^2\,T_2}{1-T_1}-\frac{c_{107}\,T_1\,T_2^2}{1-T_1}-\frac{c_1\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{105}\,T_1^2\,T_2^2}{1-T_1}+\frac{c_{106}\,T_1^2\,T_2^2}{1-T_1}+\frac{c_{107}\,T_1^2\,T_2^2}{1-T_1}==0,$$

$$\frac{c_{106}\,T_1}{(1-T_1)\,(1-T_2)}+\frac{c_{107}\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_1\,T_1\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{105}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{106}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{107}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{106}\,T_1^2\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{107}\,T_1\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{c_1\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{c_{105}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{c_{106}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{c_{107}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}==0,$$

$$c_{106}\,T_1^2+c_{105}\,T_1^2\,T_2-c_{106}\,T_1^2\,T_2+c_{107}\,T_2^2+c_{105}\,T_1\,T_2^2-c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2-c_{105}\,T_1^2\,T_2^2==0,$$

$$c_{106}\,T_1^2-c_{107}\,T_2-c_{105}\,T_1\,T_2+c_{107}\,T_1\,T_2+c_{105}\,T_1^2\,T_2-c_{106}\,T_1^2\,T_2+c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2+c_{105}\,T_1^2\,T_2^2-c_{107}\,T_1^2\,T_2^2==0,\ \frac{c_{106}\,T_1^2}{1-T_2}-\frac{c_{106}\,T_1^2\,T_2}{1-T_2}+\frac{c_{107}\,T_1\,T_2^2}{1-T_2}+\frac{c_1\,T_1^2\,T_2^2}{1-T_2}+\frac{c_{105}\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{107}\,T_1^2\,T_2^2}{1-T_2}==0,$$

$$-c_{106}\,T_1-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{106}\,T_1^2\,T_2+c_{107}\,T_2^2+c_{105}\,T_1\,T_2^2-c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2+c_{105}\,T_1^2\,T_2^2-c_{106}\,T_1^2\,T_2^2==0,\ -c_{106}\,T_1-c_{107}\,T_2-c_1\,T_1\,T_2-c_{105}\,T_1\,T_2+c_{106}\,T_1\,T_2+c_{107}\,T_1\,T_2+c_{106}\,T_1^2\,T_2+c_{107}\,T_1\,T_2^2+c_1\,T_1^2\,T_2^2+c_{105}\,T_1^2\,T_2^2-c_{106}\,T_1^2\,T_2^2-c_{107}\,T_1^2\,T_2^2==0,$$

$$-\frac{c_{106}\,T_1}{1-T_2}-\frac{c_{107}\,T_2}{1-T_2}-\frac{c_1\,T_1\,T_2}{1-T_2}-\frac{c_{105}\,T_1\,T_2}{1-T_2}+\frac{c_{106}\,T_1\,T_2}{1-T_2}+\frac{c_{107}\,T_1\,T_2}{1-T_2}+\frac{c_{106}\,T_1^2\,T_2}{1-T_2}+\frac{c_{107}\,T_1\,T_2^2}{1-T_2}+\frac{c_1\,T_1^2\,T_2^2}{1-T_2}+\frac{c_{105}\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{106}\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{107}\,T_1^2\,T_2^2}{1-T_2}==0,$$

$$\frac{c_{106}\,T_1^2\,T_2}{1-T_1}+\frac{c_{107}\,T_2^2}{1-T_1}-\frac{c_{107}\,T_1\,T_2^2}{1-T_1}+\frac{c_1\,T_1^2\,T_2^2}{1-T_1}+\frac{c_{105}\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{106}\,T_1^2\,T_2^2}{1-T_1}==0,$$

$$-\frac{c_{106}\,T_1}{1-T_1}-\frac{c_{107}\,T_2}{1-T_1}-\frac{c_1\,T_1\,T_2}{1-T_1}-\frac{c_{105}\,T_1\,T_2}{1-T_1}+\frac{c_{106}\,T_1\,T_2}{1-T_1}+\frac{c_{107}\,T_1\,T_2}{1-T_1}+\frac{c_{106}\,T_1^2\,T_2}{1-T_1}+\frac{c_{107}\,T_1\,T_2^2}{1-T_1}+\frac{c_1\,T_1^2\,T_2^2}{1-T_1}+\frac{c_{105}\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{106}\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{107}\,T_1^2\,T_2^2}{1-T_1}==0,$$

$$-\frac{c_{106}\,T_1}{(1-T_1)\,(1-T_2)}-\frac{c_{107}\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_1\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{105}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{106}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{107}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{106}\,T_1^2\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{107}\,T_1\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{c_1\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{c_{105}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{c_{106}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{c_{107}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}==0,$$

$$c_1+c_5-c_1\,T_1-c_5\,T_1-c_{112}\,T_1+c_{112}\,T_1^2-c_1\,T_2-c_5\,T_2-c_{112}\,T_2+c_1\,T_1\,T_2+c_5\,T_1\,T_2-c_{110}\,T_1\,T_2-c_{111}\,T_1\,T_2+3\,c_{112}\,T_1\,T_2+c_1\,T_1^2\,T_2+c_{110}\,T_1^2\,T_2+c_{111}\,T_1^2\,T_2-2\,c_{112}\,T_1^2\,T_2+c_{112}\,T_2^2+c_1\,T_1\,T_2^2+c_{110}\,T_1\,T_2^2+c_{111}\,T_1\,T_2^2-2\,c_{112}\,T_1\,T_2^2-2\,c_1\,T_1^2\,T_2^2-c_{110}\,T_1^2\,T_2^2-c_{111}\,T_1^2\,T_2^2+c_{112}\,T_1^2\,T_2^2==0,$$

$$\frac{c_1}{1-T_2}+\frac{c_5}{1-T_2}-\frac{c_1\,T_1}{1-T_2}-\frac{c_5\,T_1}{1-T_2}-\frac{c_{112}\,T_1}{1-T_2}+\frac{c_{112}\,T_1^2}{1-T_2}-\frac{c_1\,T_2}{1-T_2}-\frac{c_5\,T_2}{1-T_2}+\frac{c_{107}\,T_2}{1-T_2}-\frac{c_{112}\,T_2}{1-T_2}+\frac{c_1\,T_1\,T_2}{1-T_2}+$$

$$\frac{c_5\,T_1\,T_2}{1-T_2}+\frac{c_{105}\,T_1\,T_2}{1-T_2}-\frac{c_{107}\,T_1\,T_2}{1-T_2}-\frac{c_{110}\,T_1\,T_2}{1-T_2}-\frac{c_{111}\,T_1\,T_2}{1-T_2}+\frac{4\,c_{112}\,T_1\,T_2}{1-T_2}+\frac{c_1\,T_1^2\,T_2}{1-T_2}+\frac{c_{110}\,T_1^2\,T_2}{1-T_2}+$$
$$\frac{c_{111}\,T_1^2\,T_2}{1-T_2}-\frac{3\,c_{112}\,T_1^2\,T_2}{1-T_2}+\frac{c_{112}\,T_2^2}{1-T_2}+\frac{c_1\,T_1\,T_2^2}{1-T_2}-\frac{c_{107}\,T_1\,T_2^2}{1-T_2}+\frac{c_{110}\,T_1\,T_2^2}{1-T_2}+\frac{c_{111}\,T_1\,T_2^2}{1-T_2}-\frac{3\,c_{112}\,T_1\,T_2^2}{1-T_2}-$$
$$\frac{2\,c_1\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{105}\,T_1^2\,T_2^2}{1-T_2}+\frac{c_{107}\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{110}\,T_1^2\,T_2^2}{1-T_2}-\frac{c_{111}\,T_1^2\,T_2^2}{1-T_2}+\frac{2\,c_{112}\,T_1^2\,T_2^2}{1-T_2}==0,\ -c_{112}\,T_1+c_{112}\,T_1^2==0,$$
$$\frac{c_1}{1-T_1}+\frac{c_5}{1-T_1}-\frac{c_1\,T_1}{1-T_1}-\frac{c_5\,T_1}{1-T_1}+\frac{c_{106}\,T_1}{1-T_1}-\frac{c_{112}\,T_1}{1-T_1}+\frac{c_{112}\,T_1^2}{1-T_1}-\frac{c_1\,T_2}{1-T_1}-\frac{c_5\,T_2}{1-T_1}-\frac{c_{112}\,T_2}{1-T_1}+$$
$$\frac{c_1\,T_1\,T_2}{1-T_1}+\frac{c_5\,T_1\,T_2}{1-T_1}+\frac{c_{105}\,T_1\,T_2}{1-T_1}-\frac{c_{106}\,T_1\,T_2}{1-T_1}-\frac{c_{110}\,T_1\,T_2}{1-T_1}-\frac{c_{111}\,T_1\,T_2}{1-T_1}+\frac{4\,c_{112}\,T_1\,T_2}{1-T_1}+\frac{c_1\,T_1^2\,T_2}{1-T_1}-$$
$$\frac{c_{106}\,T_1^2\,T_2}{1-T_1}+\frac{c_{110}\,T_1^2\,T_2}{1-T_1}+\frac{c_{111}\,T_1^2\,T_2}{1-T_1}-\frac{3\,c_{112}\,T_1^2\,T_2}{1-T_1}+\frac{c_{112}\,T_2^2}{1-T_1}+\frac{c_1\,T_1\,T_2^2}{1-T_1}+\frac{c_{110}\,T_1\,T_2^2}{1-T_1}+\frac{c_{111}\,T_1\,T_2^2}{1-T_1}-$$
$$\frac{3\,c_{112}\,T_1\,T_2^2}{1-T_1}-\frac{2\,c_1\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{105}\,T_1^2\,T_2^2}{1-T_1}+\frac{c_{106}\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{110}\,T_1^2\,T_2^2}{1-T_1}-\frac{c_{111}\,T_1^2\,T_2^2}{1-T_1}+\frac{2\,c_{112}\,T_1^2\,T_2^2}{1-T_1}==0,$$
$$\frac{c_{106}\,T_1}{(1-T_1)\,(1-T_2)}-\frac{c_{112}\,T_1}{(1-T_1)\,(1-T_2)}+\frac{c_{112}\,T_1^2}{(1-T_1)\,(1-T_2)}+\frac{c_{107}\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{112}\,T_2}{(1-T_1)\,(1-T_2)}+$$
$$\frac{c_1\,T_1\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{105}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{106}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-\frac{c_{107}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}+\frac{4\,c_{112}\,T_1\,T_2}{(1-T_1)\,(1-T_2)}-$$
$$\frac{c_{106}\,T_1^2\,T_2}{(1-T_1)\,(1-T_2)}-\frac{3\,c_{112}\,T_1^2\,T_2}{(1-T_1)\,(1-T_2)}+\frac{c_{112}\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{c_{107}\,T_1\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{3\,c_{112}\,T_1\,T_2^2}{(1-T_1)\,(1-T_2)}-$$
$$\frac{c_1\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}-\frac{c_{105}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{c_{106}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{c_{107}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}+\frac{2\,c_{112}\,T_1^2\,T_2^2}{(1-T_1)\,(1-T_2)}==0,$$
$$c_{112}-c_{112}\,T_1==0,\ -c_{112}\,T_2+c_{112}\,T_2^2==0,\ c_{112}-c_{112}\,T_2==0\Big\}$$

In[•]:=
```
vars = Union@Cases[eqn, c_, ∞]
```

Out[•]=

$$\{c_1,\ c_5,\ c_{105},\ c_{106},\ c_{107},\ c_{110},\ c_{111},\ c_{112}\}$$

In[•]:=
```
{sol} = Solve[eqns, vars]
```

Solve: Equations may not give solutions for all "solve" variables.

Out[•]=

$$\left\{\left\{c_1\to 0,\ c_{105}\to 0,\ c_{106}\to 0,\ c_{107}\to 0,\ c_{111}\to -c_{110}+\frac{c_5}{T_1\,T_2},\ c_{112}\to 0\right\}\right\}$$

In[•]:=
```
sol /. (v_ → val_) :> (v = CF[val])
```

Out[•]=

$$\left\{0,\ 0,\ 0,\ 0,\ -\frac{-c_5+c_{110}\,T_1\,T_2}{T_1\,T_2},\ 0\right\}$$

In[•]:=
```
vars = Union@Cases[eqn, c_, ∞]
```

Out[•]=

$$\{c_5,\ c_{110}\}$$

```
In[•]:= Factor@Cases[ℒ[X_{i,j}[1]], ϵSeries[ℰ_] :> Coefficient[ℰ, c_5], ∞]
```

Out[•]=

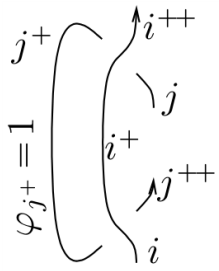
$$\left\{\frac{p_{3,j}\,(T_1\,x_{1,i}-x_{1,j})\,x_{2,i}}{T_1}\right\}$$

```
In[•]:= Factor@Cases[ℒ[X_{i,j}[1]], ϵSeries[ℰ_] :> Coefficient[ℰ, c_110], ∞]
```

Out[•]=

$$\{-p_{3,j}\,(-T_2\,x_{1,j}\,x_{2,i}+T_1\,x_{1,i}\,x_{2,j})\}$$

Invariance Under R2c




```
In[•]:= lhs = ∫ 𝔼[π_i p_i + π_j p_j] × ℒ /@ (X_{i+1,j}[1] X_{i,j+2}[-1] C_{j+1}[1])
            𝕕{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{j+2}, p_{j+2}}
        rhs = ∫ 𝔼[π_i p_i + π_j p_j] × ℒ /@ (C_i[0] C_{i+1}[0] C_j[0] C_{j+1}[1] C_{j+2}[0])
            𝕕{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{j+2}, p_{j+2}};
        lhs == rhs
```

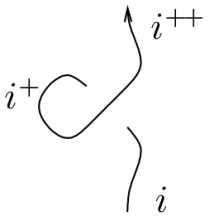
Out[•]=

$$-32\,i\,\pi^5\,\sqrt{T}\,\mathbb{E}\left[\epsilon\text{Series}\left[p_{2+i}\,\pi_i+p_{3+j}\,\pi_j,\ -\frac{1}{2}-p_{3+j}\,\pi_j,\ \frac{1}{2}\,p_{3+j}\,\pi_j\right]\right]$$

Out[•]=

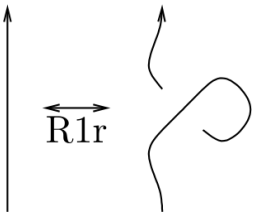
True

Invariance Under R1l



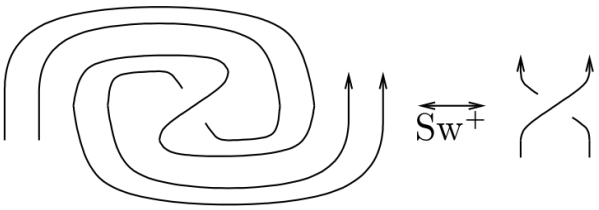
```
In[•]:= lhs = ∫ 𝔼[πi pi] × ℒ /@ (Xi+2,i[1] Ci+1[1]) 𝕕{xi, pi, xi+1, pi+1, xi+2, pi+2}
         rhs = ∫ 𝔼[πi pi] × ℒ /@ (Ci[0] Ci+1[0] Ci+2[0]) 𝕕{xi, pi, xi+1, pi+1, xi+2, pi+2};
         lhs == rhs
Out[•]=
         -8 i π^3 𝔼[∈Series[p3+i πi, 0, 0]]
Out[•]=
         True
```

Invariance Under R1r




```
In[•]:= lhs = ∫ 𝔼[πi pi] × ℒ /@ (Xi,i+2[1] Ci+1[-1]) 𝕕{xi, pi, xi+1, pi+1, xi+2, pi+2}
         rhs = ∫ 𝔼[πi pi] × ℒ /@ (Ci[0] Ci+1[0] Ci+2[0]) 𝕕{xi, pi, xi+1, pi+1, xi+2, pi+2};
         lhs == rhs
Out[•]=
         -8 i π^3 𝔼[∈Series[p3+i πi, 0, 0]]
Out[•]=
         True
```

Invariance Under Sw



In[•]:=
```
lhs = ∫𝔼[π_i p_i + π_j p_j] × ℒ /@ (X_{i+1,j+1}[1] C_i[-1] C_j[-1] C_{i+2}[1] C_{j+2}[1])
    𝕕{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{i+2}, p_{i+2}, x_{j+2}, p_{j+2}}
rhs = ∫𝔼[π_i p_i + π_j p_j] × ℒ /@ (X_{i+1,j+1}[1] C_i[0] C_j[0] C_{i+2}[0] C_{j+2}[0])
    𝕕{x_i, x_j, p_i, p_j, x_{i+1}, x_{j+1}, p_{i+1}, p_{j+1}, x_{i+2}, p_{i+2}, x_{j+2}, p_{j+2}};
lhs == rhs
```

Out[•]=

$$64\,\pi^6\,\sqrt{T}\;\mathbb{E}\Big[\in\text{Series}\Big[T\,p_{3+i}\,\pi_i+p_{3+j}\,(\pi_i-T\,\pi_i+\pi_j),\\ -\frac{1}{2}+\frac{1}{2}\,T\,p_{3+i}\,p_{3+j}\,\pi_i\,(-\pi_i+T\,\pi_i-2\,\pi_j)-\frac{1}{2}\,T\,p_{3+j}^2\,\pi_i\,(-\pi_i+T\,\pi_i-2\,\pi_j)+p_{3+j}\,(T\,\pi_i-\pi_j),\\ \frac{1}{4}\,T\,p_{3+j}^2\,\pi_i\,(-3\,\pi_i+5\,T\,\pi_i-10\,\pi_j)-\frac{1}{4}\,T\,p_{3+i}\,p_{3+j}\,\pi_i\,(-\pi_i+3\,T\,\pi_i-6\,\pi_j)-\\ \frac{1}{6}\,T^2\,p_{3+i}^2\,p_{3+j}\,\pi_i^2\,(-\pi_i+T\,\pi_i-3\,\pi_j)+\frac{1}{2}\,p_{3+j}\,(-T\,\pi_i+\pi_j)+\\ \frac{1}{6}\,T\,p_{3+i}\,p_{3+j}^2\,\pi_i\,\left(\pi_i^2-5\,T\,\pi_i^2+4\,T^2\,\pi_i^2+3\,\pi_i\,\pi_j-12\,T\,\pi_i\,\pi_j+3\,\pi_j^2\right)-\\ \frac{1}{6}\,T\,p_{3+j}^3\,\pi_i\,\left(\pi_i^2-4\,T\,\pi_i^2+3\,T^2\,\pi_i^2+3\,\pi_i\,\pi_j-9\,T\,\pi_i\,\pi_j+3\,\pi_j^2\right)\Big]\Big]$$

Out[•]=
```
True
```