Pensieve header: Exponentiation in ybax algebras.

Startup

In[∘]:=
```
Date[]
SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\FullDoPeGDO"];
Once[<< KnotTheory`];
Once[Get@"../Profile/Profile.m"];
BeginProfile[];
$k = 1;
<< Engine.m
<< Objects.m
<< KT.m
HL[ℰ_] := Style[ℰ, Background → If[TrueQ@ℰ, ■, ■]];
```

Out[∘]= {2021, 8, 13, 10, 52, 53.4911172}

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
This is Profile.m of http://www.drorbn.net/AcademicPensieve/Projects/Profile/.
This version: April 2020. Original version: July 1994.
```

Exponentials

Task. Define $\text{Exp}_m[U\_(\mathbb{U}_{\{\_\_\_\}\to\{i\_\}})]$ to compute $e^{\mathbb{O}(U)}$ to order $\epsilon^{\text{Length}@\{U\}-1}$ using the $m_{i,i\to i}$ multiplication, where U is an ϵ-dependent near-docile element, giving the answer in $\mathbb{E}$-form.
Example: $\text{Exp}_{\text{dm},1}[\mathbb{U}_{\{\}\to\{2\}}[b_2\, a_2 + y_2\, x_2, 0]]$ is the exponential of the arrow on strand 2, computed to degree 1.

In[∘]:= `m = cm; U = 𝕌{}→{i}[ai bi + xi yi, xi + yi, xi^2 yi]`

Out[∘]= $\mathbb{U}_{\{\}\to\{i\}}[a_i\, b_i + x_i\, y_i,\ x_i + y_i,\ x_i^2\, y_i]$

In[∘]:= `k = Length@{U} - 1`

Out[∘]= 0

In[∘]:=
```
Fa = 𝔼{}→{i}[fa[λ] ai];
Fa
Fa /. {λ → μ, i → j}
```

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}[\text{fa}[\lambda]\, a_i]$

Out[∘]= $\mathbb{E}_{\{\}\to\{j\}}[\text{fa}[\mu]\, a_j]$

In[∘]:= `Fa (Fa /. {λ → μ, i → j}) // mi,j→i`

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}[(\text{fa}[\lambda] + \text{fa}[\mu])\, a_i]$

In[∘]:= $\text{la} = \left(\partial_\mu \text{First}\left[\text{Fa}\ (\text{Fa}\ /.\ \{\lambda \to \mu,\ i \to j\})\ //\ m_{i,j\to i}\right]\right)\ /.\ \mu \to 0\ /.\ \{\text{fa}[0] \to 0,\ \text{fa}'[0] \to \partial_{a_i} U[[1]]\}$

Out[∘]= $a_i\, b_i$

In[∘]:= $\text{ra} = (\partial_\mu \text{First}[\text{Fa}\ /.\ \lambda \to \lambda + \mu])\ /.\ \mu \to 0$

Out[∘]= $a_i\, \text{fa}'[\lambda]$

In[∘]:= $\text{Sa} = \text{DSolve}[\text{la} == \text{ra} \land \text{fa}[0] == 0,\ \text{fa},\ \lambda][[1, 1]]$

Out[∘]= $\text{fa} \to \text{Function}[\{\lambda\},\ \lambda\, b_i]$

In[∘]:= $\text{F0} = \mathbb{E}_{\{\}\to\{i\}}[\text{f}[\lambda] + \text{fa}[\lambda]\, a_i + \text{fx}[\lambda]\, x_i + \text{fy}[\lambda]\, y_i + \text{fxy}[\lambda]\, x_i\, y_i]\ /.\ \text{Sa};$
F0
$\text{F0}\ /.\ \{\lambda \to \mu,\ i \to j\}$

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}[\text{f}[\lambda] + \lambda\, a_i\, b_i + \text{fx}[\lambda]\, x_i + \text{fy}[\lambda]\, y_i + \text{fxy}[\lambda]\, x_i\, y_i]$

Out[∘]= $\mathbb{E}_{\{\}\to\{j\}}[\text{f}[\mu] + \mu\, a_j\, b_j + \text{fx}[\mu]\, x_j + \text{fy}[\mu]\, y_j + \text{fxy}[\mu]\, x_j\, y_j]$

In[∘]:= $\text{F0}\ (\text{F0}\ /.\ \{\lambda \to \mu,\ i \to j\})\ //\ m_{i,j\to i}$

Out[∘]= $\mathbb{E}_{\{\}\to\{i\}}\Big[\text{f}[\lambda] + \text{f}[\mu] + \text{fx}[\lambda] \times \text{fy}[\mu]\, b_i + (\lambda + \mu)\, a_i\, b_i +$
$\left(\text{fx}[\mu] + \text{fx}[\lambda] \times \text{fxy}[\mu]\, b_i + \text{fx}[\lambda]\, B_i^{\mu/\hbar}\right) x_i + \left(\text{fy}[\lambda] + \text{fxy}[\lambda] \times \text{fy}[\mu]\, b_i + \text{fy}[\mu]\, B_i^{\lambda/\hbar}\right) y_i +$
$\left(\text{fxy}[\lambda] \times \text{fxy}[\mu]\, b_i + \text{fxy}[\mu]\, B_i^{\lambda/\hbar} + \text{fxy}[\lambda]\, B_i^{\mu/\hbar}\right) x_i\, y_i\Big]$

In[∘]:= $\text{l0} = \left(\partial_\mu \text{First}\left[\text{F0}\ (\text{F0}\ /.\ \{\lambda \to \mu,\ i \to j\})\ //\ m_{i,j\to i}\right]\right)\ /.\ \mu \to 0\ /.\ \{(\text{f} \mid \text{fx} \mid \text{fy} \mid \text{fxy})[0] \to 0\}\ /.$
$\text{Thread}\Big[\{\text{f}'[0],\ \text{fx}'[0],\ \text{fy}'[0],\ \text{fxy}'[0]\} \to$
$\left(\left\{U[[1]],\ \partial_{x_i} U[[1]],\ \partial_{y_i} U[[1]],\ \partial_{x_i,y_i} U[[1]]\right\}\ /.\ (a \mid x \mid y)_i \to 0\right)\Big]$

Out[∘]= $a_i\, b_i + \left(\frac{\text{fx}[\lambda] \times \text{Log}[B_i]}{\hbar} + \text{fx}[\lambda]\, b_i\right) x_i + \left(\frac{\text{fxy}[\lambda] \times \text{Log}[B_i]}{\hbar} + \text{fxy}[\lambda]\, b_i + B_i^{\lambda/\hbar}\right) x_i\, y_i$

In[∘]:= $\text{r0} = (\partial_\mu \text{First}[\text{F0}\ /.\ \lambda \to \lambda + \mu])\ /.\ \mu \to 0$

Out[∘]= $a_i\, b_i + \text{f}'[\lambda] + x_i\, \text{fx}'[\lambda] + x_i\, y_i\, \text{fxy}'[\lambda] + y_i\, \text{fy}'[\lambda]$

In[∘]:= $\text{UnDot}[\mathcal{E}\_,\ vs\_] := \text{Thread}\left[\left\{\text{Times}\ @@\ vs^{\#[[1]]},\ \#[[2]]\right\}\ \&\ /@\ \text{CoefficientRules}[\mathcal{E},\ vs]\right];$
$\text{UnDot}[U[[1]],\ \{a_i,\ x_i,\ y_i\}]$

Out[∘]= $\{\{a_i,\ x_i\, y_i\},\ \{b_i,\ 1\}\}$

In[∘]:= $\text{UnDot}[\text{l0} - \text{r0},\ \{a_i,\ x_i,\ y_i\}]$

Out[∘]= $\Big\{\{x_i\, y_i,\ x_i,\ y_i,\ 1\},\ \Big\{\frac{\text{fxy}[\lambda] \times \text{Log}[B_i]}{\hbar} + \text{fxy}[\lambda]\, b_i + B_i^{\lambda/\hbar} - \text{fxy}'[\lambda],$
$\frac{\text{fx}[\lambda] \times \text{Log}[B_i]}{\hbar} + \text{fx}[\lambda]\, b_i - \text{fx}'[\lambda],\ -\text{fy}'[\lambda],\ -\text{f}'[\lambda]\Big\}\Big\}$

In[•]:=
```
{S0} = DSolve[(# == 0) & /@ UnDot[l0 - r0, {a_i, x_i, y_i}][[2]] ⋃
   {f[0] == 0, fx[0] == 0, fy[0] == 0, fxy[0] == 0}, {f, fx, fy, fxy}, λ]
```

Out[•]=
$$\left\{\left\{f\to \text{Function}[\{\lambda\},0],\ fx\to \text{Function}[\{\lambda\},0],\ fxy\to \text{Function}\left[\{\lambda\},\frac{\left(-1+e^{\lambda b_i}\right)B_i^{\lambda/\hbar}}{b_i}\right],\ fy\to \text{Function}[\{\lambda\},0]\right\}\right\}$$

In[•]:=
```
(F0 /. {λ → μ, i → j}) /. S0
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{j\}}\left[\mu\, a_j\, b_j+\frac{\left(-1+e^{\mu b_i}\right)B_i^{\mu/\hbar}\, x_j\, y_j}{b_i}\right]$$

In[•]:=
```
κ = 1;
F1 = Append[F0 /. S0, Total@Flatten@
    Table[f_{m,p,q}[λ] a_i^m x_i^p y_i^q, {m, 0, 2κ + 2, 2}, {p, 0, 2κ + 2 - 2m}, {q, 0, 2κ + 2 - 2m - p}]]
```

Out[•]=
$$\mathbb{E}_{\{\}\to\{i\}}\left[\lambda\, a_i\, b_i+\frac{\left(-1+e^{\lambda b_i}\right)B_i^{\lambda/\hbar}\, x_i\, y_i}{b_i},\right.$$
$$f_{0,0,0}[\lambda]+y_i f_{0,0,1}[\lambda]+y_i^2 f_{0,0,2}[\lambda]+y_i^3 f_{0,0,3}[\lambda]+y_i^4 f_{0,0,4}[\lambda]+x_i f_{0,1,0}[\lambda]+ x_i y_i f_{0,1,1}[\lambda]+x_i y_i^2 f_{0,1,2}[\lambda]+x_i y_i^3 f_{0,1,3}[\lambda]+x_i^2 f_{0,2,0}[\lambda]+x_i^2 y_i f_{0,2,1}[\lambda]+ x_i^2 y_i^2 f_{0,2,2}[\lambda]+x_i^3 f_{0,3,0}[\lambda]+x_i^3 y_i f_{0,3,1}[\lambda]+x_i^4 f_{0,4,0}[\lambda]+a_i^2 f_{2,0,0}[\lambda]\Big]$$

In[•]:=
```
{mons, fs} = UnDot[Last@F1, {a_i, x_i, y_i}]
```

Out[•]=
$$\{\{a_i^2, x_i^4, x_i^3 y_i, x_i^3, x_i^2 y_i^2, x_i^2 y_i, x_i^2, x_i y_i^3, x_i y_i^2, x_i y_i, x_i, y_i^4, y_i^3, y_i^2, y_i, 1\},$$
$$\{f_{2,0,0}[\lambda], f_{0,4,0}[\lambda], f_{0,3,1}[\lambda], f_{0,3,0}[\lambda], f_{0,2,2}[\lambda], f_{0,2,1}[\lambda], f_{0,2,0}[\lambda], f_{0,1,3}[\lambda], f_{0,1,2}[\lambda], f_{0,1,1}[\lambda], f_{0,1,0}[\lambda], f_{0,0,4}[\lambda], f_{0,0,3}[\lambda], f_{0,0,2}[\lambda], f_{0,0,1}[\lambda], f_{0,0,0}[\lambda]\}\}$$

In[•]:=
```
Alternatives @@ (fs /. λ → 0)
```

Out[•]=
$$f_{2,0,0}[0]\mid f_{0,4,0}[0]\mid f_{0,3,1}[0]\mid f_{0,3,0}[0]\mid f_{0,2,2}[0]\mid f_{0,2,1}[0]\mid f_{0,2,0}[0]\mid f_{0,1,3}[0]\mid f_{0,1,2}[0]\mid f_{0,1,1}[0]\mid f_{0,1,0}[0]\mid f_{0,0,4}[0]\mid f_{0,0,3}[0]\mid f_{0,0,2}[0]\mid f_{0,0,1}[0]\mid f_{0,0,0}[0]$$

In[•]:=
```
mis = Flatten@Table[MI[m, p, q], {m, 0, 2κ + 2, 2}, {p, 0, 2κ + 2 - 2m}, {q, 0, 2κ + 2 - 2m - p}]
```

Out[•]=
```
{MI[0, 0, 0], MI[0, 0, 1], MI[0, 0, 2], MI[0, 0, 3], MI[0, 0, 4],
 MI[0, 1, 0], MI[0, 1, 1], MI[0, 1, 2], MI[0, 1, 3], MI[0, 2, 0],
 MI[0, 2, 1], MI[0, 2, 2], MI[0, 3, 0], MI[0, 3, 1], MI[0, 4, 0], MI[2, 0, 0]}
```

In[•]:=
```
MI /: Coefficient[ℰ_, MI[m_, p_, q_]] :=
  Coefficient[Coefficient[Coefficient[ℰ, a_i, m], x_i, p], y_i, q]
```

In[•]:=
```
axy /: axy^MI[m_,p_,q_] := a_i^m x_i^p y_i^q
```

In[•]:=
```
Table[f_{Sequence@@mi}'[0] → Coefficient[U[[κ + 1]], mi], {mi, mis}]
```

Out[•]=
$$\{f_{0,0,0}'[0]\to 0, f_{0,0,1}'[0]\to 1, f_{0,0,2}'[0]\to 0, f_{0,0,3}'[0]\to 0, f_{0,0,4}'[0]\to 0, f_{0,1,0}'[0]\to 1, f_{0,1,1}'[0]\to 0, f_{0,1,2}'[0]\to 0, f_{0,1,3}'[0]\to 0, f_{0,2,0}'[0]\to 0, f_{0,2,1}'[0]\to 0, f_{0,2,2}'[0]\to 0, f_{0,3,0}'[0]\to 0, f_{0,3,1}'[0]\to 0, f_{0,4,0}'[0]\to 0, f_{2,0,0}'[0]\to 0\}$$

In[◦]:= **l1 =**
(∂_μ Last[F1 (F1 /. {$\lambda \to \mu$, i → j}) // $m_{i,j\to i}$]) /. $\mu \to 0$ /. Alternatives @@ (fs /. $\lambda \to 0$) → 0 /.
Table[$f_{\text{Sequence@@mi}}$'[0] → Coefficient[U⟦$\varkappa$ + 1⟧, mi], {mi, mis}]

Out[◦]=
$$e^{\lambda b_i} B_i^{\lambda/\hbar} y_i + x_i \left(1 + \frac{\text{Log}[B_i]\, f_{0,1,0}[\lambda]}{\hbar} + b_i f_{0,1,0}[\lambda]\right) + x_i y_i^2 \left(\frac{\text{Log}[B_i]\, f_{0,1,2}[\lambda]}{\hbar} + b_i f_{0,1,2}[\lambda]\right) + x_i y_i^3 \left(\frac{\text{Log}[B_i]\, f_{0,1,3}[\lambda]}{\hbar} + b_i f_{0,1,3}[\lambda]\right) + x_i^2 \left(\frac{2\,\text{Log}[B_i]\, f_{0,2,0}[\lambda]}{\hbar} + 2 b_i f_{0,2,0}[\lambda]\right) + x_i^2 y_i \left(\frac{2\,\text{Log}[B_i]\, f_{0,2,1}[\lambda]}{\hbar} + 2 b_i f_{0,2,1}[\lambda]\right) + \frac{x_i^2 y_i^2 \left(-2 b_i B_i^{\frac{2\lambda}{\hbar}} + 2 e^{\lambda b_i} b_i B_i^{\frac{2\lambda}{\hbar}} - 2 \lambda b_i^2 B_i^{\frac{2\lambda}{\hbar}} + \frac{4\,\text{Log}[B_i]\, b_i^3 f_{0,2,2}[\lambda]}{\hbar} + 4 b_i^4 f_{0,2,2}[\lambda]\right)}{2 b_i^3} + x_i^3 \left(\frac{3\,\text{Log}[B_i]\, f_{0,3,0}[\lambda]}{\hbar} + 3 b_i f_{0,3,0}[\lambda]\right) + x_i^3 y_i \left(\frac{3\,\text{Log}[B_i]\, f_{0,3,1}[\lambda]}{\hbar} + 3 b_i f_{0,3,1}[\lambda]\right) + x_i^4 \left(\frac{4\,\text{Log}[B_i]\, f_{0,4,0}[\lambda]}{\hbar} + 4 b_i f_{0,4,0}[\lambda]\right) - \frac{a_i B_i^{\lambda/\hbar} x_i y_i \left(\lambda b_i^2 + 2 b_i^2 f_{2,0,0}[\lambda]\right)}{b_i^2} + \frac{x_i y_i \left(-B_i^{\lambda/\hbar} + e^{\lambda b_i} B_i^{\lambda/\hbar} + \lambda b_i B_i^{\lambda/\hbar} + \frac{\text{Log}[B_i]\, b_i f_{0,1,1}[\lambda]}{\hbar} + b_i^2 f_{0,1,1}[\lambda] + b_i B_i^{\lambda/\hbar} f_{2,0,0}[\lambda]\right)}{b_i}$$

In[◦]:= **r1 = (∂_μ Last[F1 /. $\lambda \to \lambda + \mu$]) /. $\mu \to 0$**

Out[◦]=
$$f_{0,0,0}'[\lambda] + y_i f_{0,0,1}'[\lambda] + y_i^2 f_{0,0,2}'[\lambda] + y_i^3 f_{0,0,3}'[\lambda] + y_i^4 f_{0,0,4}'[\lambda] + x_i f_{0,1,0}'[\lambda] + x_i y_i f_{0,1,1}'[\lambda] + x_i y_i^2 f_{0,1,2}'[\lambda] + x_i y_i^3 f_{0,1,3}'[\lambda] + x_i^2 f_{0,2,0}'[\lambda] + x_i^2 y_i f_{0,2,1}'[\lambda] + x_i^2 y_i^2 f_{0,2,2}'[\lambda] + x_i^3 f_{0,3,0}'[\lambda] + x_i^3 y_i f_{0,3,1}'[\lambda] + x_i^4 f_{0,4,0}'[\lambda] + a_i^2 f_{2,0,0}'[\lambda]$$

In[∘]:= **l1 - r1 // CF**

Out[∘]= $-a_i B_i^{\lambda/\hbar} x_i y_i (\lambda + 2 f_{2,0,0}[\lambda]) - f_{0,0,0}'[\lambda] + y_i \left(e^{\lambda b_i} B_i^{\lambda/\hbar} - f_{0,0,1}'[\lambda]\right) - y_i^2 f_{0,0,2}'[\lambda] - y_i^3 f_{0,0,3}'[\lambda] - y_i^4 f_{0,0,4}'[\lambda] + \frac{x_i (\hbar + \text{Log}[B_i] f_{0,1,0}[\lambda] + \hbar b_i f_{0,1,0}[\lambda] - \hbar f_{0,1,0}'[\lambda])}{\hbar} + \frac{1}{\hbar b_i} x_i y_i \left(-\hbar B_i^{\lambda/\hbar} + e^{\lambda b_i} \hbar B_i^{\lambda/\hbar} + \lambda \hbar b_i B_i^{\lambda/\hbar} + \text{Log}[B_i] b_i f_{0,1,1}[\lambda] + \hbar b_i^2 f_{0,1,1}[\lambda] + \hbar b_i B_i^{\lambda/\hbar} f_{2,0,0}[\lambda] - \hbar b_i f_{0,1,1}'[\lambda]\right) + \frac{x_i y_i^2 (\text{Log}[B_i] f_{0,1,2}[\lambda] + \hbar b_i f_{0,1,2}[\lambda] - \hbar f_{0,1,2}'[\lambda])}{\hbar} + \frac{x_i y_i^3 (\text{Log}[B_i] f_{0,1,3}[\lambda] + \hbar b_i f_{0,1,3}[\lambda] - \hbar f_{0,1,3}'[\lambda])}{\hbar} + \frac{x_i^2 (2\,\text{Log}[B_i] f_{0,2,0}[\lambda] + 2 \hbar b_i f_{0,2,0}[\lambda] - \hbar f_{0,2,0}'[\lambda])}{\hbar} + \frac{x_i^2 y_i (2\,\text{Log}[B_i] f_{0,2,1}[\lambda] + 2 \hbar b_i f_{0,2,1}[\lambda] - \hbar f_{0,2,1}'[\lambda])}{\hbar} - \frac{x_i^2 y_i^2 \left(\hbar B_i^{\frac{2\lambda}{\hbar}} - e^{\lambda b_i} \hbar B_i^{\frac{2\lambda}{\hbar}} + \lambda \hbar b_i B_i^{\frac{2\lambda}{\hbar}} - 2\,\text{Log}[B_i] b_i^2 f_{0,2,2}[\lambda] - 2 \hbar b_i^3 f_{0,2,2}[\lambda] + \hbar b_i^2 f_{0,2,2}'[\lambda]\right)}{\hbar b_i^2} + \frac{x_i^3 (3\,\text{Log}[B_i] f_{0,3,0}[\lambda] + 3 \hbar b_i f_{0,3,0}[\lambda] - \hbar f_{0,3,0}'[\lambda])}{\hbar} + \frac{x_i^3 y_i (3\,\text{Log}[B_i] f_{0,3,1}[\lambda] + 3 \hbar b_i f_{0,3,1}[\lambda] - \hbar f_{0,3,1}'[\lambda])}{\hbar} + \frac{x_i^4 (4\,\text{Log}[B_i] f_{0,4,0}[\lambda] + 4 \hbar b_i f_{0,4,0}[\lambda] - \hbar f_{0,4,0}'[\lambda])}{\hbar} - a_i^2 f_{2,0,0}'[\lambda]$

In[∘]:= **{S1} = DSolve[Table[Coefficient[l1 - r1, mi] == 0 ∧ f_{Sequence@@mi}[0] == 0, {mi, mis}], Table[f_{Sequence@@mi}, {mi, mis}], λ]**

Out[∘]= $\left\{\left\{f_{0,0,0} \to \text{Function}[\{\lambda\}, 0],\ f_{0,0,1} \to \text{Function}\left[\{\lambda\}, \frac{\left(-1 + e^{\lambda\left(\frac{\text{Log}[B_i]}{\hbar} + b_i\right)}\right)\hbar}{\text{Log}[B_i] + \hbar b_i}\right],\ f_{0,0,2} \to \text{Function}[\{\lambda\}, 0],\ f_{0,0,3} \to \text{Function}[\{\lambda\}, 0],\ f_{0,0,4} \to \text{Function}[\{\lambda\}, 0],\ f_{0,1,0} \to \text{Function}\left[\{\lambda\}, \frac{e^{\lambda b_i - \frac{\lambda(\text{Log}[B_i] + \hbar b_i)}{\hbar}}\left(-1 + e^{\frac{\lambda(\text{Log}[B_i] + \hbar b_i)}{\hbar}}\right)\hbar B_i^{\lambda/\hbar}}{\text{Log}[B_i] + \hbar b_i}\right],\ f_{0,1,1} \to \text{Function}\left[\{\lambda\}, \frac{\left(-1 + e^{\lambda b_i}\right)\lambda B_i^{\lambda/\hbar}}{b_i}\right],\ f_{2,0,0} \to \text{Function}[\{\lambda\}, 0],\ f_{0,1,2} \to \text{Function}[\{\lambda\}, 0],\ f_{0,1,3} \to \text{Function}[\{\lambda\}, 0],\ f_{0,2,0} \to \text{Function}[\{\lambda\}, 0],\ f_{0,2,1} \to \text{Function}[\{\lambda\}, 0],\ f_{0,2,2} \to \text{Function}\left[\{\lambda\}, \frac{\left(3 - 4 e^{\lambda b_i} + e^{2\lambda b_i} + 2\lambda b_i\right) B_i^{\frac{2\lambda}{\hbar}}}{4 b_i^3}\right],\ f_{0,3,0} \to \text{Function}[\{\lambda\}, 0],\ f_{0,3,1} \to \text{Function}[\{\lambda\}, 0],\ f_{0,4,0} \to \text{Function}[\{\lambda\}, 0]\right\}\right\}$

In[∘]:=
```
κ = 2;
F2 = Append[l2U[F1 /. S1], Total@Flatten@
    Table[f_{m,p,q}[λ] a_i^m x_i^p y_i^q, {m, 0, 2 κ + 2, 2}, {p, 0, 2 κ + 2 - 2 m}, {q, 0, 2 κ + 2 - 2 m - p}]]
```

Out[∘]=
$$\mathbb{E}_{\{\}\to\{i\}}\Big[\lambda a_i b_i + \frac{B_i^{\lambda/\hbar}\left(-1+B_i^{-\frac{\lambda}{\hbar}}\right) x_i y_i}{b_i},\ \frac{\lambda B_i^{\lambda/\hbar}\left(-1+B_i^{-\frac{\lambda}{\hbar}}\right) x_i y_i}{b_i} + \frac{B_i^{\frac{2\lambda}{\hbar}}\left(3+2\lambda b_i + B_i^{-\frac{2\lambda}{\hbar}} - 4 B_i^{-\frac{\lambda}{\hbar}}\right) x_i^2 y_i^2}{4 b_i^3},$$
$$f_{0,0,0}[\lambda] + y_i f_{0,0,1}[\lambda] + y_i^2 f_{0,0,2}[\lambda] + y_i^3 f_{0,0,3}[\lambda] + y_i^4 f_{0,0,4}[\lambda] + y_i^5 f_{0,0,5}[\lambda] + y_i^6 f_{0,0,6}[\lambda] +$$
$$x_i f_{0,1,0}[\lambda] + x_i y_i f_{0,1,1}[\lambda] + x_i y_i^2 f_{0,1,2}[\lambda] + x_i y_i^3 f_{0,1,3}[\lambda] + x_i y_i^4 f_{0,1,4}[\lambda] +$$
$$x_i y_i^5 f_{0,1,5}[\lambda] + x_i^2 f_{0,2,0}[\lambda] + x_i^2 y_i f_{0,2,1}[\lambda] + x_i^2 y_i^2 f_{0,2,2}[\lambda] + x_i^2 y_i^3 f_{0,2,3}[\lambda] +$$
$$x_i^2 y_i^4 f_{0,2,4}[\lambda] + x_i^3 f_{0,3,0}[\lambda] + x_i^3 y_i f_{0,3,1}[\lambda] + x_i^3 y_i^2 f_{0,3,2}[\lambda] + x_i^3 y_i^3 f_{0,3,3}[\lambda] + x_i^4 f_{0,4,0}[\lambda] +$$
$$x_i^4 y_i f_{0,4,1}[\lambda] + x_i^4 y_i^2 f_{0,4,2}[\lambda] + x_i^5 f_{0,5,0}[\lambda] + x_i^5 y_i f_{0,5,1}[\lambda] + x_i^6 f_{0,6,0}[\lambda] + a_i^2 f_{2,0,0}[\lambda] +$$
$$a_i^2 y_i f_{2,0,1}[\lambda] + a_i^2 y_i^2 f_{2,0,2}[\lambda] + a_i^2 x_i f_{2,1,0}[\lambda] + a_i^2 x_i y_i f_{2,1,1}[\lambda] + a_i^2 x_i^2 f_{2,2,0}[\lambda]\Big]$$

In[∘]:=
```
mis = Flatten@Table[MI[m, p, q], {m, 0, 2 κ + 2, 2}, {p, 0, 2 κ + 2 - 2 m}, {q, 0, 2 κ + 2 - 2 m - p}]
```

Out[∘]=
```
{MI[0, 0, 0], MI[0, 0, 1], MI[0, 0, 2], MI[0, 0, 3], MI[0, 0, 4], MI[0, 0, 5], MI[0, 0, 6],
 MI[0, 1, 0], MI[0, 1, 1], MI[0, 1, 2], MI[0, 1, 3], MI[0, 1, 4], MI[0, 1, 5], MI[0, 2, 0],
 MI[0, 2, 1], MI[0, 2, 2], MI[0, 2, 3], MI[0, 2, 4], MI[0, 3, 0], MI[0, 3, 1], MI[0, 3, 2],
 MI[0, 3, 3], MI[0, 4, 0], MI[0, 4, 1], MI[0, 4, 2], MI[0, 5, 0], MI[0, 5, 1], MI[0, 6, 0],
 MI[2, 0, 0], MI[2, 0, 1], MI[2, 0, 2], MI[2, 1, 0], MI[2, 1, 1], MI[2, 2, 0]}
```

In[∘]:=
```
l2 = U2l[
   (∂_μ Last[F2 (F2 /. {λ → μ, i → j}) // m_{i,j→i}]) /. μ → 0 /. Alternatives @@ (fs /. λ → 0) → 0 /.
    Table[f_{Sequence@@mi}'[0] → Coefficient[U〚κ + 1〛, mi], {mi, mis}]]
```

Out[∘]=
$$-\frac{\left(-1+e^{-\lambda b_i}\right) x_i y_i}{b_i} - \frac{e^{-\lambda b_i}\lambda \operatorname{Log}\left[e^{-\hbar b_i}\right] x_i y_i}{\hbar b_i} +$$
$$\frac{a_i\left(-\frac{\operatorname{Log}\left[e^{-\hbar b_i}\right]}{\hbar} + \frac{e^{-\lambda b_i}\operatorname{Log}\left[e^{-\hbar b_i}\right]}{\hbar} - b_i + e^{-\lambda b_i} b_i + \frac{e^{-\lambda b_i}\lambda \operatorname{Log}\left[e^{-\hbar b_i}\right] b_i}{\hbar}\right) x_i y_i}{b_i^2} +$$
$$\frac{\left(\frac{2\operatorname{Log}\left[e^{-\hbar b_i}\right]}{\hbar} + \frac{8 e^{-2\lambda b_i}\operatorname{Log}\left[e^{-\hbar b_i}\right]}{\hbar} - \frac{8 e^{-\lambda b_i}\operatorname{Log}\left[e^{-\hbar b_i}\right]}{\hbar} + 2 b_i + 4 e^{-2\lambda b_i} b_i - 4 e^{-\lambda b_i} b_i + \frac{4 e^{-2\lambda b_i}\lambda \operatorname{Log}\left[e^{-\hbar b_i}\right] b_i}{\hbar}\right) x_i^2 y_i^2}{4 b_i^3}$$

In[∘]:=
```
r2 = (∂_μ Last[F2 /. λ → λ + μ]) /. μ → 0
```

Out[∘]=
$$f_{0,0,0}'[\lambda] + y_i f_{0,0,1}'[\lambda] + y_i^2 f_{0,0,2}'[\lambda] + y_i^3 f_{0,0,3}'[\lambda] + y_i^4 f_{0,0,4}'[\lambda] + y_i^5 f_{0,0,5}'[\lambda] + y_i^6 f_{0,0,6}'[\lambda] +$$
$$x_i f_{0,1,0}'[\lambda] + x_i y_i f_{0,1,1}'[\lambda] + x_i y_i^2 f_{0,1,2}'[\lambda] + x_i y_i^3 f_{0,1,3}'[\lambda] + x_i y_i^4 f_{0,1,4}'[\lambda] +$$
$$x_i y_i^5 f_{0,1,5}'[\lambda] + x_i^2 f_{0,2,0}'[\lambda] + x_i^2 y_i f_{0,2,1}'[\lambda] + x_i^2 y_i^2 f_{0,2,2}'[\lambda] + x_i^2 y_i^3 f_{0,2,3}'[\lambda] +$$
$$x_i^2 y_i^4 f_{0,2,4}'[\lambda] + x_i^3 f_{0,3,0}'[\lambda] + x_i^3 y_i f_{0,3,1}'[\lambda] + x_i^3 y_i^2 f_{0,3,2}'[\lambda] + x_i^3 y_i^3 f_{0,3,3}'[\lambda] + x_i^4 f_{0,4,0}'[\lambda] +$$
$$x_i^4 y_i f_{0,4,1}'[\lambda] + x_i^4 y_i^2 f_{0,4,2}'[\lambda] + x_i^5 f_{0,5,0}'[\lambda] + x_i^5 y_i f_{0,5,1}'[\lambda] + x_i^6 f_{0,6,0}'[\lambda] + a_i^2 f_{2,0,0}'[\lambda] +$$
$$a_i^2 y_i f_{2,0,1}'[\lambda] + a_i^2 y_i^2 f_{2,0,2}'[\lambda] + a_i^2 x_i f_{2,1,0}'[\lambda] + a_i^2 x_i y_i f_{2,1,1}'[\lambda] + a_i^2 x_i^2 f_{2,2,0}'[\lambda]$$

In[∘]:=
```
{S2} = DSolve[Table[Coefficient[l2 - r2, mi] == 0 ∧ f_Sequence@@mi[0] == 0, {mi, mis}],
  Table[f_Sequence@@mi, {mi, mis}], λ]
```

Out[∘]=

$$\Big\{\Big\{f_{0,0,0}\to \text{Function}[\{\lambda\},0],\ f_{0,0,1}\to \text{Function}[\{\lambda\},0],\ f_{0,0,2}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,0,3}\to \text{Function}[\{\lambda\},0],\ f_{0,0,4}\to \text{Function}[\{\lambda\},0],\ f_{0,0,5}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,0,6}\to \text{Function}[\{\lambda\},0],\ f_{0,1,0}\to \text{Function}[\{\lambda\},0],\ f_{0,1,1}\to \text{Function}\Big[\{\lambda\},$$
$$\frac{e^{-\lambda b_i}\left(\text{Log}\left[e^{-\hbar b_i}\right]-e^{\lambda b_i}\,\text{Log}\left[e^{-\hbar b_i}\right]+\hbar b_i-e^{\lambda b_i}\hbar b_i+\lambda\,\text{Log}\left[e^{-\hbar b_i}\right]b_i+e^{\lambda b_i}\lambda\hbar b_i^2\right)}{\hbar b_i^3}\Big],$$
$$f_{0,1,2}\to \text{Function}[\{\lambda\},0],\ f_{0,1,3}\to \text{Function}[\{\lambda\},0],\ f_{0,1,4}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,1,5}\to \text{Function}[\{\lambda\},0],\ f_{0,2,0}\to \text{Function}[\{\lambda\},0],\ f_{0,2,1}\to \text{Function}[\{\lambda\},0],\ f_{0,2,2}\to$$
$$\text{Function}\Big[\{\lambda\},\frac{1}{4\hbar b_i^4}e^{-2\lambda b_i}\Big(-5\,\text{Log}\left[e^{-\hbar b_i}\right]+8e^{\lambda b_i}\,\text{Log}\left[e^{-\hbar b_i}\right]-3e^{2\lambda b_i}\,\text{Log}\left[e^{-\hbar b_i}\right]-2\hbar b_i+$$
$$4e^{\lambda b_i}\hbar b_i-2e^{2\lambda b_i}\hbar b_i-2\lambda\,\text{Log}\left[e^{-\hbar b_i}\right]b_i+2e^{2\lambda b_i}\lambda\,\text{Log}\left[e^{-\hbar b_i}\right]b_i+2e^{2\lambda b_i}\lambda\hbar b_i^2\Big)\Big],$$
$$f_{0,2,3}\to \text{Function}[\{\lambda\},0],\ f_{0,2,4}\to \text{Function}[\{\lambda\},0],\ f_{0,3,0}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,3,1}\to \text{Function}[\{\lambda\},0],\ f_{0,3,2}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,3,3}\to \text{Function}[\{\lambda\},0],\ f_{0,4,0}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,4,1}\to \text{Function}[\{\lambda\},0],\ f_{0,4,2}\to \text{Function}[\{\lambda\},0],\ f_{0,5,0}\to \text{Function}[\{\lambda\},0],$$
$$f_{0,5,1}\to \text{Function}[\{\lambda\},0],\ f_{0,6,0}\to \text{Function}[\{\lambda\},0],\ f_{2,0,0}\to \text{Function}[\{\lambda\},0],$$
$$f_{2,0,1}\to \text{Function}[\{\lambda\},0],\ f_{2,0,2}\to \text{Function}[\{\lambda\},0],\ f_{2,1,0}\to \text{Function}[\{\lambda\},0],$$
$$f_{2,1,1}\to \text{Function}[\{\lambda\},0],\ f_{2,2,0}\to \text{Function}[\{\lambda\},0]\Big\}\Big\}$$

In[∘]:=
```
l2U[F2 /. S2 /. λ → 1]
```

Out[∘]=

$$\mathbb{E}_{\{\}\to\{i\}}\Big[a_i b_i+\frac{B_i^{\frac{1}{\hbar}}\left(-1+B_i^{-1/\hbar}\right)x_i y_i}{b_i},\ \frac{B_i^{\frac{1}{\hbar}}\left(-1+B_i^{-1/\hbar}\right)x_i y_i}{b_i}+\frac{B_i^{2/\hbar}\left(3+2b_i+B_i^{-2/\hbar}-4B_i^{-1/\hbar}\right)x_i^2 y_i^2}{4b_i^3},$$
$$\frac{B_i^{\frac{1}{\hbar}}\left(\text{Log}[B_i]+\hbar b_i+\text{Log}[B_i]\,b_i-\text{Log}[B_i]\,B_i^{-1/\hbar}-\hbar b_i B_i^{-1/\hbar}+\hbar b_i^2 B_i^{-1/\hbar}\right)x_i y_i}{\hbar b_i^3}+$$
$$\frac{1}{4\hbar b_i^4}B_i^{2/\hbar}\Big(-5\,\text{Log}[B_i]-2\hbar b_i-2\,\text{Log}[B_i]\,b_i-3\,\text{Log}[B_i]\,B_i^{-2/\hbar}-2\hbar b_i B_i^{-2/\hbar}+$$
$$2\,\text{Log}[B_i]\,b_i B_i^{-2/\hbar}+2\hbar b_i^2 B_i^{-2/\hbar}+8\,\text{Log}[B_i]\,B_i^{-1/\hbar}+4\hbar b_i B_i^{-1/\hbar}\Big)x_i^2 y_i^2\Big]$$

Previous attempt

In[∘]:=
```
mons[i_] := Flatten@
   Table[ϵ^κ a_i^m x_i^p y_i^q, {κ, 0, k}, {m, 0, 2 κ + 2, 2}, {p, 0, 2 κ + 2 - 2 m}, {q, 0, 2 κ + 2 - 2 m - p}];
mons[
 1]
```

Out[∘]= $\{1, y_1, y_1^2, x_1, x_1 y_1, x_1^2\}$

In[∘]:=
```
fs[λ_] := Flatten@
   Table[f_{κ,m,p,q}[λ], {κ, 0, k}, {m, 0, 2κ + 2, 2}, {p, 0, 2κ + 2 - 2m}, {q, 0, 2κ + 2 - 2m - p}];
fs[
 μ]
```

Out[∘]= $\{f_{0,0,0,0}[\mu], f_{0,0,0,1}[\mu], f_{0,0,0,2}[\mu], f_{0,0,1,0}[\mu], f_{0,0,1,1}[\mu], f_{0,0,2,0}[\mu]\}$

In[∘]:=
```
F[λ_, i_] := Λ2𝔼_{{}→{i}}[fs[λ].mons[i]];
F[λ, i]
F[μ, j]
```

Out[∘]= $\mathbb{E}_{\{\}\to\{2\}}\left[f_{0,0,0,0}[\lambda] + y_2 f_{0,0,0,1}[\lambda] + y_2^2 f_{0,0,0,2}[\lambda] + x_2 f_{0,0,1,0}[\lambda] + x_2 y_2 f_{0,0,1,1}[\lambda] + x_2^2 f_{0,0,2,0}[\lambda], 0\right]$

Out[∘]= $\mathbb{E}_{\{\}\to\{j\}}\left[f_{0,0,0,0}[\mu] + y_j f_{0,0,0,1}[\mu] + y_j^2 f_{0,0,0,2}[\mu] + x_j f_{0,0,1,0}[\mu] + x_j y_j f_{0,0,1,1}[\mu] + x_j^2 f_{0,0,2,0}[\mu], 0\right]$

In[∘]:=
```
F[λ, i] × F[μ, j] // m_{i,j→i}
```

Out[∘]=

··· 1 ···

large output | show less | show more | show all | set size limit...

In[∘]:=
```
l1 = (∂_μ List @@ (F[λ, i] × F[μ, j] // m_{i,j→i})) /. μ → 0 /. f___[0] → 0
```

Out[∘]=
$$\left\{-\frac{f_{0,0,0,0}[\lambda]\left(-4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)}{\hbar^2} - \right.$$

$$\frac{y_2 f_{0,0,0,1}[\lambda]\left(-4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)}{\hbar^2}$$

$$- \frac{y_2^2 f_{0,0,0,2}[\lambda]\left(-4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)}{\hbar^2}$$

$$- \frac{x_2 f_{0,0,1,0}[\lambda]\left(-4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)}{\hbar^2}$$

$$- \frac{1}{\hbar^2} x_2 y_2 f_{0,0,1,1}[\lambda] \left(-4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right) -$$

$$\frac{x_2^2 f_{0,0,2,0}[\lambda]\left(-4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)}{\hbar^2}$$

$$+ \frac{1}{\hbar^2} y_2 \left(\hbar^2 f_{0,0,0,1}'[0] + \hbar f_{0,0,1,1}[\lambda] f_{0,0,0,1}'[0] - \hbar B_2 f_{0,0,1,1}[\lambda] f_{0,0,0,1}'[0] + \right.$$
$$2 \hbar f_{0,0,1,0}[\lambda] f_{0,0,0,2}'[0] - 2 \hbar B_2 f_{0,0,1,0}[\lambda] f_{0,0,0,2}'[0] +$$
$$2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] - 4 B_2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] +$$
$$2 B_2^2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] - 4 f_{0,0,0,1}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] +$$
$$\left. 8 B_2 f_{0,0,0,1}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,0,1}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right) +$$

$$\frac{1}{\hbar^2} y_2^2 \left(\hbar^2 f_{0,0,0,2}'[0] + 2\hbar f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] - 2\hbar B_2 f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] + \right.$$
$$f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0] - 2 B_2 f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0] +$$
$$B_2^2 f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0] - 4 f_{0,0,0,2}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] +$$
$$\left. 8 B_2 f_{0,0,0,2}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,0,2}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right) +$$
$$\frac{1}{\hbar^2} \left(\hbar^2 f_{0,0,0,0}'[0] + \hbar f_{0,0,1,0}[\lambda] f_{0,0,0,1}'[0] - \hbar B_2 f_{0,0,1,0}[\lambda] f_{0,0,0,1}'[0] + \right.$$
$$f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0] - 2 B_2 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0] +$$
$$B_2^2 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0] - 4 f_{0,0,0,0}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] +$$
$$8 B_2 f_{0,0,0,0}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2^2 f_{0,0,0,0}[\lambda] f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] +$$
$$\left.\frac{1}{2} \times \left(4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 8 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + 4 B_2^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)\right) +$$
$$\frac{1}{\hbar^2} x_2 \left(2\hbar f_{0,0,2,0}[\lambda] f_{0,0,0,1}'[0] - 2\hbar B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,1}'[0] + \hbar^2 f_{0,0,1,0}'[0] + \right.$$
$$\left.\hbar f_{0,0,1,0}[\lambda] f_{0,0,1,1}'[0] - \hbar B_2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}'[0]\right) + \frac{1}{\hbar}$$
$$x_2 y_2 (4 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] + \hbar f_{0,0,1,1}'[0] +$$
$$f_{0,0,1,1}[\lambda] f_{0,0,1,1}'[0] - B_2 f_{0,0,1,1}[\lambda] f_{0,0,1,1}'[0]) +$$
$$\frac{x_2^2 \left(2\hbar f_{0,0,2,0}[\lambda] f_{0,0,1,1}'[0] - 2\hbar B_2 f_{0,0,2,0}[\lambda] f_{0,0,1,1}'[0] + \hbar^2 f_{0,0,2,0}'[0]\right)}{\hbar^2},$$

$$-\frac{(-1 + 3 B_2)\, y_2 f_{0,0,1,0}[\lambda]\, (\hbar + f_{0,0,1,1}[\lambda] - B_2 f_{0,0,1,1}[\lambda])\, f_{0,0,0,2}'[0]}{\hbar} +$$
$$\frac{2 a_2 B_2 y_2^2 f_{0,0,1,1}[\lambda]\, (\hbar + f_{0,0,1,1}[\lambda] - B_2 f_{0,0,1,1}[\lambda])\, f_{0,0,0,2}'[0]}{\hbar} +$$
$$\frac{1}{\hbar^5} x_2 y_2^3 (\hbar + f_{0,0,1,1}[\lambda] - B_2 f_{0,0,1,1}[\lambda])$$
$$\left(2\hbar^5 f_{0,0,1,1}[\lambda] + \hbar^4 f_{0,0,1,1}[\lambda]^2 - 3\hbar^4 B_2 f_{0,0,1,1}[\lambda]^2\right) f_{0,0,0,2}'[0] +$$
$$\frac{1}{\hbar^2} a_2 B_2 y_2 \left(\hbar^2 f_{0,0,1,1}[\lambda] f_{0,0,0,1}'[0] + 2\hbar^2 f_{0,0,1,0}[\lambda] f_{0,0,0,2}'[0] + 4\hbar f_{0,0,1,0}[\lambda] \right.$$
$$\left. f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] - 4\hbar B_2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0]\right) - \frac{1}{4\hbar^5} (-1 + 3 B_2)\, y_2^2$$
$$\left(4\hbar^5 f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] + 2\hbar^4 f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0] - 2\hbar^4 B_2 f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0]\right) +$$
$$\frac{1}{2\hbar^5} x_2 y_2^2 \left(2\hbar^6 f_{0,0,1,1}[\lambda] f_{0,0,0,1}'[0] + \hbar^5 f_{0,0,1,1}[\lambda]^2 f_{0,0,0,1}'[0] - 3\hbar^5 B_2 f_{0,0,1,1}[\lambda]^2 f_{0,0,0,1}'[0] + \right.$$
$$4\hbar^6 f_{0,0,1,0}[\lambda] f_{0,0,0,2}'[0] + 12\hbar^5 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] -$$
$$20\hbar^5 B_2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda] f_{0,0,0,2}'[0] + 6\hbar^4 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0] -$$
$$\left. 24\hbar^4 B_2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0] + 18\hbar^4 B_2^2 f_{0,0,1,0}[\lambda] f_{0,0,1,1}[\lambda]^2 f_{0,0,0,2}'[0]\right) +$$
$$\frac{(-1 + B_2) \times (-1 + 3 B_2) \times \left(2\hbar^6 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0] + 4\hbar^6 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right)}{4\hbar^7} +$$
$$\frac{1}{\hbar^3} a_2 B_2 \left(\hbar^3 f_{0,0,1,0}[\lambda] f_{0,0,0,1}'[0] + 2\hbar^2 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0] - \right.$$
$$\left. 2\hbar^2 B_2 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0] + 4\hbar^2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0] - 4\hbar^2 B_2 f_{0,0,2,0}[\lambda] f_{0,0,0,2}'[0]\right) -$$

$$\frac{1}{2\hbar^5}(-1+3B_2)\,x_2\,\Big(\hbar^5 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,1}'[0]+2\hbar^5 f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]+2\hbar^4 f_{0,0,1,0}[\lambda]^3 f_{0,0,0,2}'[0]-2\hbar^4 B_2 f_{0,0,1,0}[\lambda]^3 f_{0,0,0,2}'[0]+16\hbar^4 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-16\hbar^4 B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]\Big)+$$

$$\frac{1}{\hbar^6}x_2\,y_2\,\Big(\hbar^7 f_{0,0,1,0}[\lambda]\,f_{0,0,0,1}'[0]+\hbar^6 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,0,1}'[0]-3\hbar^6 B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,0,1}'[0]+3\hbar^6 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0]-5\hbar^6 B_2 f_{0,0,1,0}[\lambda]^2 f_{0,0,0,2}'[0]+3\hbar^5 f_{0,0,1,0}[\lambda]^2 f_{0,0,1,1}[\lambda]\,f_{0,0,0,2}'[0]-12\hbar^5 B_2 f_{0,0,1,0}[\lambda]^2 f_{0,0,1,1}[\lambda]\,f_{0,0,0,2}'[0]+9\hbar^5 B_2^2 f_{0,0,1,0}[\lambda]^2 f_{0,0,1,1}[\lambda]\,f_{0,0,0,2}'[0]+8\hbar^6 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-16\hbar^6 B_2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+8\hbar^5 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-32\hbar^5 B_2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+24\hbar^5 B_2^2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]\Big)+$$

$$\frac{2\,a_2 B_2 x_2^2 f_{0,0,2,0}[\lambda]\,(4 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-4B_2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+\hbar f_{0,0,1,1}'[0])}{\hbar}-$$

$$\frac{2\times(-1+3B_2)\,x_2^4 f_{0,0,2,0}[\lambda]^2\,(4 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-4B_2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+\hbar f_{0,0,1,1}'[0])}{\hbar}$$

$$+\frac{1}{\hbar^2}a_2 B_2 x_2\,\Big(2\hbar^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]+8\hbar f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-8\hbar B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+\hbar^2 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}'[0]\Big)-$$

$$\frac{1}{\hbar^4}(-1+3B_2)\,x_2^3 f_{0,0,2,0}[\lambda]\,\Big(2\hbar^4 f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]+12\hbar^3 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-12\hbar^3 B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+2\hbar^4 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}'[0]\Big)+$$

$$\frac{1}{\hbar^2}a_2 B_2 x_2 y_2\,\Big(4\hbar^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+8\hbar f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-8\hbar B_2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+\hbar^2 f_{0,0,1,1}[\lambda]\,f_{0,0,1,1}'[0]\Big)+$$

$$\frac{1}{\hbar^4}x_2^3 y_2 f_{0,0,2,0}[\lambda]\,\Big(12\hbar^4 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-20\hbar^4 B_2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+12\hbar^3 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-48\hbar^3 B_2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+36\hbar^3 B_2^2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+2\hbar^5 f_{0,0,1,1}'[0]+2\hbar^4 f_{0,0,1,1}[\lambda]\,f_{0,0,1,1}'[0]-6\hbar^4 B_2 f_{0,0,1,1}[\lambda]\,f_{0,0,1,1}'[0]\Big)+$$

$$\frac{1}{2\hbar^5}x_2^2 y_2\,\Big(4\hbar^6 f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]+4\hbar^5 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]-12\hbar^5 B_2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]+24\hbar^5 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-40\hbar^5 B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+24\hbar^4 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-96\hbar^4 B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+72\hbar^4 B_2^2 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+2\hbar^6 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}'[0]+2\hbar^5 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,1,1}'[0]-6\hbar^5 B_2 f_{0,0,1,0}[\lambda]\,f_{0,0,1,1}[\lambda]\,f_{0,0,1,1}'[0]\Big)+$$

$$\frac{1}{4\hbar^5}x_2^2 y_2^2\,\Big(16\hbar^6 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+48\hbar^5 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-80\hbar^5 B_2 f_{0,0,1,1}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+24\hbar^4 f_{0,0,1,1}[\lambda]^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-96\hbar^4 B_2 f_{0,0,1,1}[\lambda]^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+72\hbar^4 B_2^2 f_{0,0,1,1}[\lambda]^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+4\hbar^6 f_{0,0,1,1}[\lambda]\,f_{0,0,1,1}'[0]+2\hbar^5 f_{0,0,1,1}[\lambda]^2 f_{0,0,1,1}'[0]-6\hbar^5 B_2 f_{0,0,1,1}[\lambda]^2 f_{0,0,1,1}'[0]\Big)-$$

$$\frac{1}{4\hbar^5}(-1+3B_2)\,x_2^2\left(8\hbar^5 f_{0,0,1,0}[\lambda]\,f_{0,0,2,0}[\lambda]\,f_{0,0,0,1}'[0]+24\hbar^4 f_{0,0,1,0}[\lambda]^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]-24\hbar^4 B_2 f_{0,0,1,0}[\lambda]^2 f_{0,0,2,0}[\lambda]\,f_{0,0,0,2}'[0]+56\hbar^4 f_{0,0,2,0}[\lambda]^2 f_{0,0,0,2}'[0]-56\hbar^4 B_2 f_{0,0,2,0}[\lambda]^2 f_{0,0,0,2}'[0]+2\hbar^5 f_{0,0,1,0}[\lambda]^2 f_{0,0,1,1}'[0]+4\hbar^5 f_{0,0,2,0}[\lambda]\,f_{0,0,1,1}'[0]\right)\}$$

In[∘]:= r1 = (∂_μ List @@ F[λ + μ, i]) /. μ → 0

Out[∘]= $\{f_1'[\lambda]+a_2 f_2'[\lambda]+x_2 f_3'[\lambda]+y_2 f_4'[\lambda]+x_2 y_2 f_5'[\lambda]\}$

In[∘]:= eqs1 = And @@ ((# == 0) & /@ Flatten@CoefficientList[l1 − r1, {a_i, x_i, y_i}]) /. f_[0] → 0

Out[∘]= $$-f_1'[\lambda]+\frac{\hbar f_1'[0]+f_3[\lambda]f_4'[0]-B_2 f_3[\lambda]f_4'[0]}{\hbar}==0\;\&\&$$
$$\frac{e^{-f_2[\lambda]}\left(\hbar f_4'[0]+e^{f_2[\lambda]}f_5[\lambda]f_4'[0]-e^{f_2[\lambda]}B_2 f_5[\lambda]f_4'[0]\right)}{\hbar}-f_4'[\lambda]==0\;\&\&$$
$$-f_3[\lambda]f_2'[0]-f_3'[\lambda]-\frac{-\hbar f_3'[0]-f_3[\lambda]f_5'[0]+B_2 f_3[\lambda]f_5'[0]}{\hbar}==0\;\&\&$$
$$-f_5[\lambda]f_2'[0]-\frac{e^{-f_2[\lambda]}\left(-\hbar f_5'[0]-e^{f_2[\lambda]}f_5[\lambda]f_5'[0]+e^{f_2[\lambda]}B_2 f_5[\lambda]f_5'[0]\right)}{\hbar}-f_5'[\lambda]==0\;\&\&$$
$$f_2'[0]-f_2'[\lambda]==0$$

In[∘]:= l2 = Take[{U}, 1]

Out[∘]= $\{a_2 b_2+x_2 y_2\}$

In[∘]:= r2 = (∂_μ List @@ F[μ, i]) /. μ → 0

Out[∘]= $\{f_1'[0]+a_2 f_2'[0]+x_2 f_3'[0]+y_2 f_4'[0]+x_2 y_2 f_5'[0]\}$

In[∘]:= eqs2 = And @@ ((# == 0) & /@ Flatten@CoefficientList[l2 − r2, {a_i, x_i, y_i}])

Out[∘]= $-f_1'[0]==0\;\&\&-f_4'[0]==0\;\&\&-f_3'[0]==0\;\&\&\,1-f_5'[0]==0\;\&\&\,b_2-f_2'[0]==0$

In[∘]:= eqs3 = eqs1 /. {f_5'[0] → 1, f_2'[0] → b_2, f_'[0] → 0}

Out[∘]= $$-f_1'[\lambda]==0\;\&\&-f_4'[\lambda]==0\;\&\&-b_2 f_3[\lambda]-\frac{-f_3[\lambda]+B_2 f_3[\lambda]}{\hbar}-f_3'[\lambda]==0\;\&\&$$
$$-b_2 f_5[\lambda]-\frac{e^{-f_2[\lambda]}\left(-\hbar-e^{f_2[\lambda]}f_5[\lambda]+e^{f_2[\lambda]}B_2 f_5[\lambda]\right)}{\hbar}-f_5'[\lambda]==0\;\&\&\,b_2-f_2'[\lambda]==0$$

In[∘]:= DSolve[f_1[0] == 0 ∧ f_2[0] == 0 ∧ f_3[0] == 0 ∧ f_4[0] == 0 ∧ f_5[0] == 0 ∧ eqs3,
{f_1[λ], f_2[λ], f_3[λ], f_4[λ], f_5[λ]}, λ]

Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is $-\mathrm{Log}[e^{c_4}]==0$.

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]= $$\left\{\left\{f_1[\lambda]\to 0,\,f_4[\lambda]\to 0,\,f_3[\lambda]\to 0,\,f_2[\lambda]\to\lambda b_2,\,f_5[\lambda]\to\frac{e^{-\frac{\lambda}{\hbar}-\frac{\lambda(-1+\hbar b_2+B_2)}{\hbar}}\left(-e^{\lambda/\hbar}+e^{\frac{\lambda B_2}{\hbar}}\right)\hbar}{-1+B_2}\right\}\right\}$$

In[ ]:= **DSolve[f_2[0] == 0 ∧ b_2 - f_2'[λ] == 0, {f_2[λ]}, λ]**

Out[ ]= $\{\{f_2[\lambda] \to \lambda\, b_2\}\}$

In[ ]:= **ans = FullSimplify[f_5[λ] /. DSolve[$-b_2 f_5[\lambda] - \frac{e^{-\lambda b_2}\left(-\hbar - e^{\lambda b_2} f_5[\lambda] + e^{\lambda b_2} B_2 f_5[\lambda]\right)}{\hbar} - f_5'[\lambda] == 0 \wedge f_5[0] == 0$, f_5[λ], λ]]**

Out[ ]= $\left\{\frac{e^{-\frac{\lambda(-1+\hbar b_2+B_2)}{\hbar}}\left(-1+e^{\frac{\lambda(-1+B_2)}{\hbar}}\right)\hbar}{-1+B_2}\right\}$

In[ ]:= **FullSimplify[ans /. b_2 → 0]**

Out[ ]= $\left\{\frac{\left(1-e^{\frac{\lambda-\lambda B_2}{\hbar}}\right)\hbar}{-1+B_2}\right\}$