The LCS expansion

June-22-15 11:34 AM

From monoblog:

(150624) "Set function $\varphi\colon G \to H$ is affine" means $\varphi(I^n_G) \subset I^n_H$. Makes a category. Group morphisms and translations are affine. $\varphi_i\colon G_i \to H$ affine $\Rightarrow \varphi_1\varphi_2$ affine, so "sorting" on FG is affine, $Id\colon G \times H \to G \ltimes H$ is affine. If $G \ltimes H$ is almost-direct, $Id\colon G \ltimes H \to G \times H$ is affine, so combing braids is bi-affine.

(150517) $G =: G_1$ a group, $G_{n+1} := (G, G_n)$, $\pi_n\colon G_n \to L_n := G_n/G_{n+1}$. Given affine sections $\varphi_n\colon L_n \to G_n$ let $\zeta\colon G \to \hat{L} := \prod L_n$, the "LCS-expansion using φ." ("group-PBW for φ_*"?), by $\zeta_1 := \pi_1$ and $\zeta_n(g) := \pi_n(\varphi\zeta_{<n}(g)^{-1}g)$ where $\varphi(\lambda_1, \lambda_2, \dots) := \varphi_1(\lambda_1)\varphi_2(\lambda_2)\cdots$. Then $\zeta_{<n}(h) = 0$ iff $h \in G_n$ and $g = \varphi\zeta_{<n}(g)$ in G/G_n. Is ζ_n of type n?

Inductive claim. ζ_n vanishes on I^{n+1}, and is linear on I^n, invariant under the right action of G on I^n?

$$b^m a^n = a^n b^m (a,b)^{nm} \cdot \underbrace{(a,(a,b))^{-}}_{\text{roughly } m\binom{n}{2}} (b,(a,b))^{-}$$

$$b^{\epsilon_2} a^{\epsilon_1} = a^{\epsilon_1} b^{\epsilon_2} (ab)^{\epsilon_1\epsilon_2}$$

<u>Note</u> $(\bar{g}, \bar{h}) = \overline{(g,h)}$ in $\mathbb{Q}G$

Is I^2 generated by G_2?