Dror Bar-Natan: Talks: Northeastern-1602:

Help Needed! I wish I could read, or maybe I wish I could write, a survery paper on Taylor expansions for groups with an abstract and a "master table" as below. Oh wait, I'm actually in the process of writing that paper⋆! But it's going hard because I'm underqualified — I don't know the existing general material well enough, and many of the entries of the "master table" require specialized knowledge that I don't have.

Abstract. First year students learn that the Taylor expansion Z_T carries functions into power series, and that it has some nice algebraic properties (e.g. multiplicativity, $Z_T(fg) = Z_T(f)Z_T(g)$). It is less well known that the same game can be played within arbitrary groups: there is a natural way to say "a Taylor expansion Z for elements of an arbitrary group G", and a natural way to carry the algebraic properties of the Taylor expansion to this more general context. In the case of a general G "Taylor expansions" (expansions with the same good properties as Z_T) may or may not exist, may or may not be unique, may or may not separate group elements, and a further good property which is hidden in the case of Z_T, "quadraticity", may or may not hold.

The purpose of this expository note is to properly define all the notions in the above paragraph, to enumerate some classes of groups whose theory of expansions we either understand or wish to understand, to indicate the relationship between these notions and the notions of "finite type invariants" and "unipotent" and "Mal'cev" completions, and to point out (with references) that our generalization of "expansions" to arbitrary groups is merely the tip of an iceberg, for almost everything we say can be generalized further to "expansions for arbitrary algebraic structures".

⋆See `http://drorbn.net/ExQu`. Also see Suciu-Wang, arXiv:1504.08294.

✔:=Yes, ✘:=No, ∼:=it Depends
?:=Unknown (to the author).
Superscripts: see "table footnotes" below.

Group(s) G	**Faithful** Z?	**Taylor** Z?	**Quadratic?**	**See**
1. Finite / torsion groups	✘[1]	✔[2]	✔[2]	Sec. 2.3
2. Free Abelian groups $\mathbb{Z}^n$	✔	✔	✔	Sec. 1.4.2
3. Free groups FG_n	✔	✔	✔	Sec. 1.4.3
4. LOT and LOF groups	∼	✔	✔	Sec. 4.1
5. Knot and pure tangle groups	∼	✔	✔	Sec. 4.2
6. Link groups	✘[3]	✔	✔	Sec. 4.3
7. 2-Knots groups	∼	✔	✔	
8. Pure braid groups PB_n	✔	✔	✔	Sec. 1.5
9. Hyperplane arrangement groups	?	✔	✔	
10. Reduced free groups RF_n	✔	✔	✘	Sec. 4.4
11. Reduced (homotopy) pure braid groups RPB_n	✔	✔	✘	
12. Pure v-braid groups PvB_n	?	✘	✔	
13. Pure w-braid groups PwB_n	✔	✔	✔	
14. Pure f-braid groups PfB_n				Merkov
15. Annular braids				
16. Elliptic pure braid groups PB_n^1 (braids on the torus)	?	✔	✘	
17. Higher genus pure braid groups $PB_n^{>1}$ (braids on high genus surfaces)	?	?	✘	arXiv:math/0309245?
18. Braid commutators $[PuB_n, PuB_n]$				
19. v-Braid commutators $[PvB_n, PvB_n]$				
20. w-Braid commutators $[PwB_n, PwB_n]$				
21. Hilden braids				
22. Mexican plait braids				Kurpita-Murasugi
23. Cactus groups				
24. Fundamental groups of surfaces		✔	✔	
25. Mapping class groups				
26. Torelli groups				Hain
27. Right-angled Artin groups		✔	✔	
28. General Artin groups				
29. Groups from BEER				arXiv:math/0509661
30. Groups from Brochier				arXiv:1209.0417
31. Poly-free groups				arXiv:math/0603470

Table 1: Some groups and their expansion properties.

Table footnotes. 1. Except $G = \{e\}$. 2. In an empty manner. 3. Except $G = \mathbb{Z}^n$.