EMERGENT VERSION OF DRINFELD'S ASSOCIATOR EQUATIONS

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Abstract. It is sometimes beneficial in and around knot theory to think of some knot strands as hard and unmoving, or *fixed*, and some other strands as *flexible*. Further to that, we consider the *emergent quotient*, in which the flexible strands are made to be nearly transparent to themselves and to each other — we don't quite decry that for flexible strands $\nearrow = \nwarrow$ for that would reduce the flexible strands to homotopy classes in the complement of the fixed strands — yet we do mod out by relations that say that $\nearrow = \nwarrow$ is nearly true, and so in the quotient that remains knot theory is just barely visible, or *emergent*.

We show that within this context the Drinfel'd pentagon equation for associators remains meaningful and that furthermore, solutions to the resulting emergent linearized Drinfel'd pentagon equation still lead to solutions of the linearized Kashiwara-Vergne equations.

Our results are adjacent to the results of [BN1, BDHLS] on the relationship between emergent tangles and the Goldman-Turaev Lie bialgebra and we hope that in time they will play a role in relating several bodies of work, on Drinfel'd associators, Kashiwara-Vergne equations, and on expansions for classical tangles, for w-tangles, and for the Goldman-Turaev Lie bialgebra.

1. Introduction

References

[BN1] Dror Bar-Natan, *Tangles in a Pole Dance Studio: A Reading of Massuyeau, Alekseev, and Naef,* talk given in *Algebra, Topology and the Grothendieck-Teichmüller group*, Les Diablerets, September 2022. Handout and video at http://drorbn.net/ld22.

[BDHLS] Dror Bar-Natan, Zsuzsanna Dancso, Tamara Hogan, Jessica Liu, and Nancy Scherich, *Goldman-Turaev formality from the Kontsevitch integral,* in preparation.

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