

```
In[ ]:= WSimp[ $\mathcal{E}$ ] :=  $\mathcal{E}$  /. W[L___, i_, j_, r___] /; i + j == 0 := WSimp@W[L, r]
```

```
In[ ]:= WSimp[W[1, 2, 3, -3, -2, 4] + W[1, -1, 2, -2]]
```

```
Out[ ]:= W[] + W[1, 4]
```

```
Unprotect[NonCommutativeMultiply];
```

```
0 ** _ = 0;
```

```
_ ** 0 = 0;
```

```
(a_ + b_) ** c_ := a ** c + b ** c;
```

```
a_ ** (b_ + c_) := a ** b + a ** c;
```

```
w1_W ** w2_W := WSimp[Join[w1, w2]];
```

```
a_ ** W[] := a;
```

```
W[] ** a_ := a;
```

```
(a_ * b_) ** c_ /; FreeQ[a, W] := a (b ** c);
```

```
a_ ** (b_ * c_) /; FreeQ[b, W] := b (a ** c)
```

```
In[ ]:= W[1, 2, 3] ** W[-3, -2, 4]
```

```
Out[ ]:= W[1, 4]
```

```
In[ ]:=  $\overline{W[i_ , is_ ]} := \overline{W[i]} ** W[is] + \overline{W[is]}$ ;  
 $\overline{W[i_ ]} /; i < 0 := -\overline{W[-i]} ** W[i]$ 
```

```
In[ ]:=  $\overline{W[1, 2, 3, -4]}$ 
```

```
Out[ ]:=  $\overline{W[1]} ** W[2, 3, -4] + \overline{W[2]} ** W[3, -4] + \overline{W[3]} ** W[-4] - \overline{W[4]} ** W[-4]$ 
```

```
In[ ]:=  $\overline{W[is_ ]} ** \overline{W[i_ ]} := \overline{W[is, i]} - \overline{W[is]}$ 
```

```
In[ ]:=  $\overline{W[1, 2, 3]} ** \overline{W[2]}$ 
```

```
Out[ ]:=  $-\overline{W[1]} ** W[2, 3] + \overline{W[1]} ** W[2, 3, 2] - \overline{W[2]} ** W[3] + \overline{W[2]} ** W[3, 2] + \overline{W[3]} ** W[2] + \overline{W[2]} - \overline{W[3]}$ 
```

```
In[ ]:=  $\overline{W[2]} ** \overline{W[3]}$ 
```

```
Out[ ]:=  $\overline{W[2]} ** \overline{W[3]}$ 
```

```
In[ ]:=  $\overline{W[2]} ** \overline{W[3]} ** \overline{W[1]}$ 
```

```
Out[ ]:=  $\overline{W[2]} ** \overline{W[3]} ** \overline{W[1]}$ 
```

```
In[ ]:=  $\overline{W[1]} ** \overline{W[2]} ** \overline{W[3]}$ 
```

```
Out[ ]:=  $-\overline{W[1]} ** \overline{W[2]} + \overline{W[2]} ** \overline{W[3]} + \overline{W[1]} ** \overline{W[2]} ** \overline{W[3]}$ 
```

```
In[ ]:=  $\overline{W[1, 2, -3]} ** \overline{W[4]}$ 
```

```
Out[ ]:=  $\overline{W[1]} ** W[2, -3, 4] + \overline{W[2]} ** W[-3, 4] - \overline{W[3]} ** W[-3, 4]$ 
```

```
In[ ]:=  $\overline{W[1, 2]} ** \overline{W[-1, 2]} ** \overline{W[2, -3]}$ 
```

```
Out[ ]:=  $\overline{W[1]} ** \overline{W[2]} - \overline{W[2]} ** \overline{W[2]} - \overline{W[1]} ** \overline{W[2]} ** W[-3] -$   
 $\overline{W[1]} ** \overline{W[2]} ** W[-1, 2] - \overline{W[1]} ** \overline{W[2]} ** W[2, -3] + \overline{W[1]} ** \overline{W[2]} ** W[-1, 2, 2, -3] +$   
 $\overline{W[1]} ** \overline{W[3]} ** W[-3] + \overline{W[2]} ** \overline{W[1]} ** W[-1, 2] - \overline{W[2]} ** \overline{W[1]} ** W[-1, 2, 2, -3] +$   
 $\overline{W[2]} ** \overline{W[2]} ** W[-3] + \overline{W[2]} ** \overline{W[2]} ** W[2, -3] - \overline{W[2]} ** \overline{W[3]} ** W[-3]$ 
```

```

TPReduce[ρ_, ε_] := ε /. {
  (ws ** W[js___, j_]) ⊗ u_ /; j > 0 ⇒ (ws ** W[js]) ⊗ (u /. ρ[σj,0]),
  (ws ** W[js___, j_]) ⊗ u_ /; j < 0 ⇒ (ws ** W[js]) ⊗ (u /. ρ[σ̄-j,0])
}

```