$$0 \to I \xrightarrow{i} \mathbb{Q}[F_n] \xrightarrow{\pi} \mathbb{Q} \to 0$$

$$\left\{ \sum a_i f_i \;\middle|\; a_i \in \mathbb{Q},\ f_i \in F_n \right\} \in \mathbb{Q}[F_n]$$

$$\pi\left(\sum a_i f_i\right) = \sum a_i$$

$$I = \ker(\pi) = \left\{ \sum a_i f_i : \sum a_i = 0 \right\}$$

$$= \boxed{\langle f - 1 \mid f \in F_n \rangle}$$

proof: $\sum a_i f_i = \sum a_i (f-1) + \cancel{\sum a_i}^{\,0}$

$$f = g_n g_{n-1} \cdots g_1 =$$

$$f - 1 = \sum_{k=1}^{n} (g_k - 1) g_{k-1} \cdots g_1 = \sum_{k=1}^{n} \underbrace{g_k (g_{k-1} \cdots g_1)}_{g_k f} - \underbrace{(g_k \cdots g_1)}_{- f}.$$

T 7pm

W 7am

Th 7pm

start w/ rep

$$\rho : \boxed{F_n \rtimes B_n} \xrightarrow{\subseteq B_{n+1}} GL(V)$$

$\rightsquigarrow$ get rep $\rho^+ : B_n \to GL(\overbrace{V \oplus \cdots \oplus V}^{n \text{ copies}})$

Proof: Have V = rep of both F_n and B_n
$\mathbb{C}$-vectorspace
($\mathbb{Q}$-vecspace for Dror)

Can define rep of B_n on

aug ideal $I \otimes_{\mathbb{Q}[F_n]}$ ✓

by $b\,(i \otimes v) = (b.i) \otimes b.v$

I think the semi direct product makes this action ok?

$I = \{ g_i f - f \mid f \in F_n \}$
$g_i 1 - 1 \in I$

gen of F_n

Not sure where F_n comes into ?

I as a $\mathbb{Q}[F_n]$-module is rank n?

$$I = \{g_i f - f \mid f \in F_n\}$$

$$= \bigoplus (g_i - 1) \cdot \mathbb{Q}[F_n]$$

$$B_n \curvearrowright I \otimes V$$

$$\tau : B_{n+1} \to GL(\mathbb{Q}) \quad \xrightarrow{\text{construct}} \quad \text{Burau}$$

$$\sigma_i \mapsto \text{mult by } t$$

Set $V_i = (g_i - 1) \otimes V$

$I \otimes V \cong V_1 \oplus \cdots \oplus V_n$.

w/ each one is iso to V.

Explicitly: $\rho^+(\sigma_i)$ does this:

g_i's generate F_n

$(v_1 \oplus \cdots \oplus v_n) \to$

$\rho(\sigma_i)v_1 \oplus \cdots \oplus \rho(\sigma_i)v_{i-1} \oplus \rho(g_{i+1}\sigma_i)v_{i+1} \oplus$

$(1 - \rho(g_{i+1}^{-1} g_i g_{i+1}))\rho(\sigma_i)v_{i+1} \oplus \rho(\sigma_i)v_i \oplus \rho(\sigma_i)v_{i+2}$

$\oplus \cdots \oplus \rho(\sigma_i)v_n$

another way

B_{n+1} contains free grp F_n

$\Rightarrow$ all braids which become trivial when the first strand is removed.

generated by $g_1, \ldots, g_n$

identify $B_n \leq B_{n+1}$ as generated by $\sigma_2, \ldots, \sigma_n$

Then $F_n \rtimes B_n$ sits inside B_{n+1}

Think of $B_n \subseteq Aut(F_n)$

then $B_n \curvearrowright \underline{V_1 \oplus \cdots \oplus V_n}$ by conjugation

should yield the same rep ↑.

. . . . .

So, a rep $\rho : B_{n+1} \to GL(V)$

gives a rep of $F_n \rtimes B_n \to GL(V)$

gives a rep of $B_n \to GL(V \oplus \cdots \oplus V)$

$\rho : F_n \rtimes vB_n \longrightarrow Gl(V)$

construct

$\rho^+ : vB_n \longrightarrow Gl(\underbrace{V \oplus \cdots \oplus V}_{n})$

$vB_n \curvearrowright I \otimes_{\mathbb{Q}[F_n]} V$

$b.(i \otimes v) = \underline{(b.i)} \otimes \underline{(b.v)}$ ✓