

The nilpotent part  $M$  of the Borel of the " $U_\epsilon[\mathfrak{sl}_3]$ " algebra illustrates a bug in the definition of the non-commutative multiplication  $**$ . *Mathematica* fails to simplify the highlighted expression during the evaluation, resulting in an infinite loop.

## Implementing M

```
$k = 3
```

```
3
```

```
DeclareAlgebra[M, Generators → {X, Y, Z}, Centralis → {}];
```

```
B[YM, XM] = -ε M@Z;
```

```
B[ZM, XM] = -ε M[X, X, Y] ;
```

```
B[ZM, YM] = -ε M[X, Y, Y] ;
```

```
(ε YM) ** M[X, Y, Y]
```

```
-ε2 M[Y, Y, Z] + ε M[X, Y, Y, Y] + 2 ε3 M[X, Y, Y, Y]
```

```
M[Y] ** M[X, Y, Y]
```

```
-ε M[Y, Y, Z] - ε3 M[Y, Y, Z] + M[X, Y, Y, Y] + 2 ε2 M[X, Y, Y, Y]
```

```
(#1 ** M[Y, Y] &) [-ε M[Z]]
```

```
-ε M[Y, Y, Z] - ε3 M[Y, Y, Z] + 2 ε2 M[X, Y, Y, Y]
```

```
(#1 ** M[Y, Y] &) [M[X, Y]]
```

```
M[X, Y, Y, Y]
```

```
ε M[Z] ** M[Y, Y]
```

```
ε (M[Y, Y, Z] + ε2 M[Y, Y, Z] - 2 ε M[X, Y, Y, Y] - 2 ε3 M[X, Y, Y, Y])
```

```
0 * (qq = 7)
```

```
0
```

```
qq
```

```
7
```

```
SetAttributes[MM, HoldRest]; MM[0, _] = 0; MM[a_, x_] := a x;
```

```
MM[0, qq = 8]
```

```
0
```

```
qq
```

```
7
```

```
MM[3, 4]
```

```
12
```

**MM[ $\epsilon$ ,  $\epsilon$ ]**

$\epsilon^2$

**$\epsilon$**

$\epsilon$

**MM[ $\epsilon^2$ , qq = 9]**

9  $\epsilon^2$

**qq**

9